



CTEQ

# SM Measurements and PDF

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Southern Methodist University

In collaboration with

CTEQ-TEA

Jan 28, 2015

Exploring the Physics Frontier with Circular  
Colliders, Aspen

# CTEQ-TEA group

- CTEQ – Tung et al. (TEA)

in memory of Prof. Wu-Ki Tung, who established CTEQ Collaboration in early 90's

- Current members:

Sayipjamal Dulat (Xinjiang Univ.)

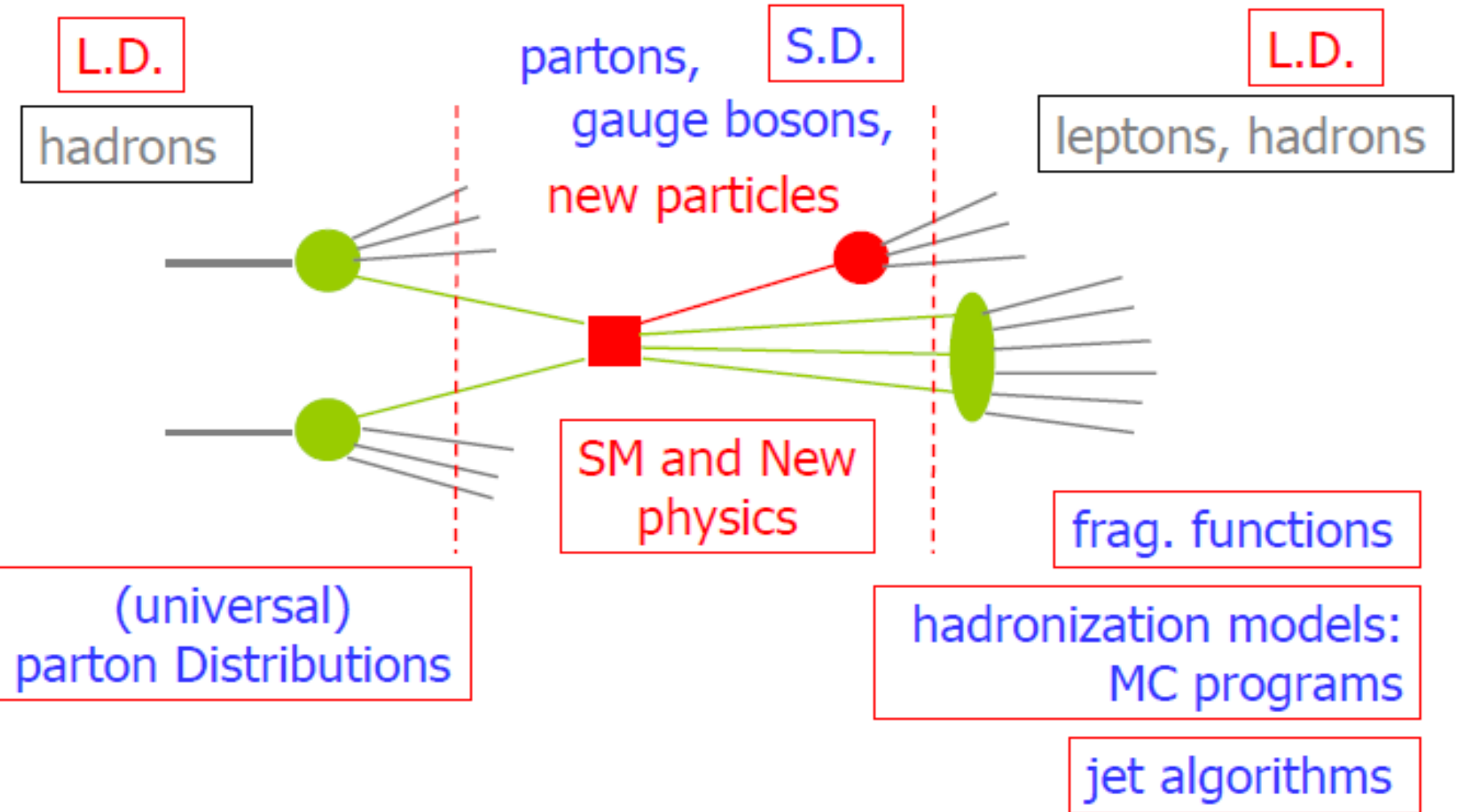
Southern Methodist Univ. -- Pavel Nadolsky, Jun Gao, Marco Guzzi, Tie-Jiun Hou

Michigan State Univ. -- Joey Huston, Jon Pumplin, Dan Stump, Carl Schmidt, C.-P. Yuan

# Parton Distribution Functions

Needed for making theoretical  
calculations to compare with  
experimental data

# Hadron Collider Physics



# Outline

- 1) CT10 NLO/NNLO PDFs review and update  
Gao et al, PRD **89**, 033009 (2014)
- 2) Update of Intrinsic Charm Analysis  
Dulat et al, PRD **89**, 073004 (2014)
- 4) Lagrange Multiplier (LM) Uncertainty Analysis on  $gg \rightarrow H$   
Dulat et al, arXiv:1309.0025[hep-ph]
- 5) Lagrange Multiplier (LM) Uncertainty Analysis on  $gg \rightarrow t \bar{t}$
- 6) PDF Benchmarking
- 7) PDFs for Future Hadron Collider

CT10 NNLO  
PDFs  
and  
Beyond

# CT10 NNLO error PDFs

Available at [http://hep.pa.msu.edu/cteq/public/ct10\\_2012.html](http://hep.pa.msu.edu/cteq/public/ct10_2012.html);  
LHAPDF;

Complements the CT10/CT10W NLO PDF sets (*Lai et al., PRD82, 074024 (2010)*)

- **Includes only “pre-LHC” CT10 data.** Can be used to predict LHC cross sections based on pre-LHC experimental inputs
- Same input parameters, functional forms for input PDFs as in the CT10 NLO PDFs
  - ▶  $\alpha_s(M_Z) = 0.118 \pm 0.002$ ,  $m_c^{pole} = 1.3 \text{ GeV}$ ,  $m_b^{pole} = 4.75 \text{ GeV}$
  - ▶ Simpler assumptions about the PDF flavor composition at  $\mu_0 = m_c^{pole} = 1.3 \text{ GeV}$ , e.g.,  $\bar{u}(x)/\bar{d}(x) \rightarrow 1$  as  $x \rightarrow 0$

# CT10 NNLO PDFs

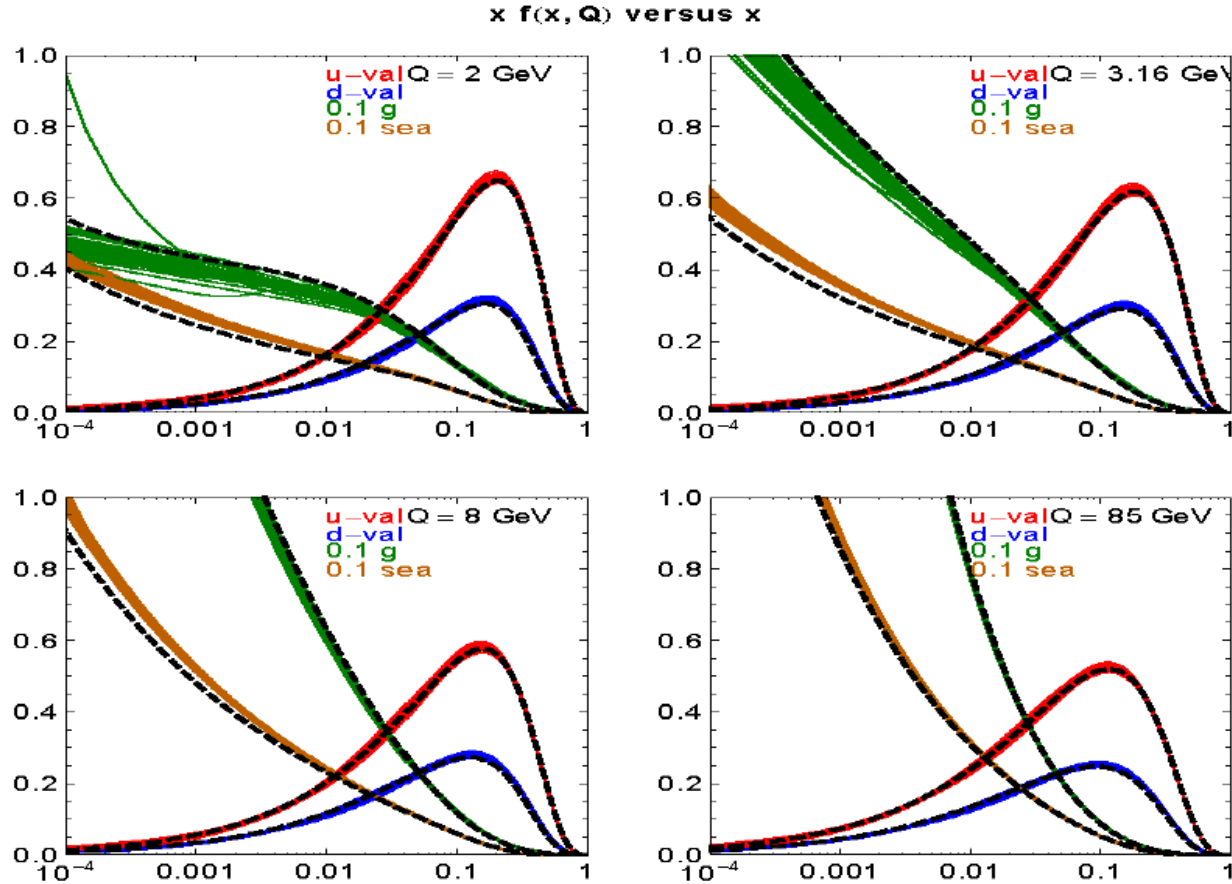


Figure 3: CT10-NNLO parton distribution functions. These figures show the *alternate fits* for the CT10-NNLO analysis. Each graph shows  $x u_{\text{valence}} = x(u - \bar{u})$ ,  $x d_{\text{valence}} = x(d - \bar{d})$ ,  $0.10 x g$  and  $0.10 x \bar{q}_{\text{sea}}$  as functions of  $x$  for a fixed value of  $Q$ . The values of  $Q$  are 2, 3.16, 8, 85 GeV. Sea =  $2(\bar{d} + \bar{u} + \bar{s})$ . The dashed curves are the central NLO fit, CT10.



# DGLAP evolution for PDFs

- Non-perturbative PDFs are determined at the scale of  $Q=1.3$  GeV.
- PDFs at any other scale  $Q$  can be obtained from pQCD, via solving DGLAP evolution equations.
- Due to DGLAP evolution, the PDF error band becomes smaller when the energy scale  $Q$  increases.
- The evolution effect is large in low  $Q$  region, say, from 1.3 GeV to 8 GeV.

## The CT10-NNLO global analysis of QCD

**Parametrization** of PDFs at  $Q = 1.3$  GeV, with 25 parameter values to be chosen; there are from 4 to 6 parameters for each parton type.

**Many data sets**, for short distance interactions.

**Perturbative QCD**, using NNLO approximations wherever available.

Taking account of **experimental errors**, statistical and systematic.

(Not so strong on systematic **theoretical errors**.)

Heavy flavor mass effects are included using the **S-ACOT- $\chi$**  factorization formalism (extended to NNLO).

# Details of the CT10 NNLO computation

- **NNLO hard-scattering contributions in DIS** (in the S-ACOT- $\chi$  mass scheme) and **vector boson production** (NNLO  $K$  factors from FEWZ for  $d\sigma/dy$ ; NNLL/NLO+K from ResBos for  $W$  charge asymmetry)
- NNLO evolution for  $\alpha_s$  and PDFs (HOPPET)
  - ▶ matching coefficients relating the PDFs in  $N_f$  and  $N_{f+1}$  schemes (*Smith, van Neerven, et al.*)
- Pole quark masses or  $\overline{MS}$  quark masses as an input
  - ▶ CT10 NNLO: pole masses  $m_c = 1.3$  GeV,  $m_b = 4.75$  GeV

28 data sets used for  
the CT10-NNLO global  
analysis

CT10-NNLO Table	Ndp	Chi <sup>2</sup>	Nsy	
1/ 159 HERA1X0	579	617.	114	Combined HERA1 NC+CC DIS (2009)
2/ 101 BcdF2pCor	339	392.	5	BCDMS collaboration
3/ 102 BcdF2dCor	251	291.	5	BCDMS collaboration
4/ 103 NmcF2pCor	201	333.	11	NMC collaboration
5/ 104 NmcRatCor	123	151.	5	NMC collaboration
6/ 108 cdhswf2	85	70.5	0	P Berge et al Z Phys C49 187 (1991)
7/ 109 cdhswf3	96	77.9	0	P Berge et al Z Phys C49 187 (1991)
8/ 110 ccfrf2.mi	69	67.8	5	Yang&Bodek model-independent
9/ 111 ccfrf3.md	86	34.8	0	Shaevitz&Seligman model-dependent processed by SK
10/ 201 e605	119	95.7	0	DY Q <sup>3</sup> dSig/dQ dy proton on heavy target
11/ 203 e866f	15	9.7	0	E866 experiment: pd / 2pp
12/ 225 cdfLasy	11	13.4		W production: decay lepton asymmetry CDF Run-1
13/ 140 HN+67F2c	8	9.3	0	H1 neutral current charm
14/ 143 HN+90X0c	10	16.3	8	H1 neutral current charm
15/ 156 ZN+67F2c	18	13.4	0	ZEUS neutral current charm
16/ 157 ZN+80F2c	27	16.7	0	ZEUS neutral current charm
17/ 124 NuTvNuChXN	38	29.6	0	NuTeV Neutrino Dimuon Reduced xSec
18/ 125 NuTvNbChXN	33	28.4	0	NuTeV Neutrino Dimuon Reduced xSec
19/ 126 CcfrNuChXN	40	48.0	0	Ccfr Neutrino Dimuon Reduced xSec
20/ 127 CcfrNbChXN	38	26.4	0	Ccfr Neutrino Dimuon Reduced xSec
21/ 204 e866ppxf	184	234.	0	E866 experiment: DY pp: Q <sup>3</sup> dSig/dQ dx
22/ 260 ZyD02a	28	15.6	6	Z rapidity dist. (D0 TeV II-a)
23/ 261 ZyCDF2	29	46.5	6	Z rapidity dist. (CDF TeV II)
24/ 227 cdfLasy2	11	11.4	0	W production: decay lepton asymmetry CDF Run-2
25/ 231 d02Easy1	12	26.0	0	W production: decay elec asymmetry D0 Run-2 Pt>25
26/ 234 d02Masy1	9	14.8	0	W production: decay muon asymmetry D0 Run-2 Pt>20
27/ 504 cdf2jtCor2	72	101.	24	(run II: cor.err; ptmin & ptmax)
28/ 514 d02jtCor2	110	114.	23	(run II: cor.err; ptmin & ptmax)

## Experimental “Errors” (or, Uncertainties)

An experiment publishes N measurements,

$$\{M_i ; i = 1, 2, 3, \dots, N\}.$$

Each measurement has several parts,

$$M_i = \{D_i ; \sigma_{0i} ; \{\sigma_{1i}, \sigma_{2i}, \sigma_{3i}, \dots\}\}$$

= {central value; SD of statistical error; SDs of correlated systematic errors};

that is,

$$D_i = True_i + \sigma_{0i} r_{0i} + \sum_{k=1}^{N_{sy}} \sigma_{ki} r_k$$

...where  $r_{0i}$  and  $\{r_k\}$  are random variables (gaussian?)

Define 
$$\chi^2 = \sum_i (D_i - \sum_k \sigma_{ki} r_k - T_i)^2 / \sigma_{0i}^2 + \sum_k r_k^2$$

...and minimize with respect to both the normalized systematic shifts  $\{r_k\}$  and the theory parameters.

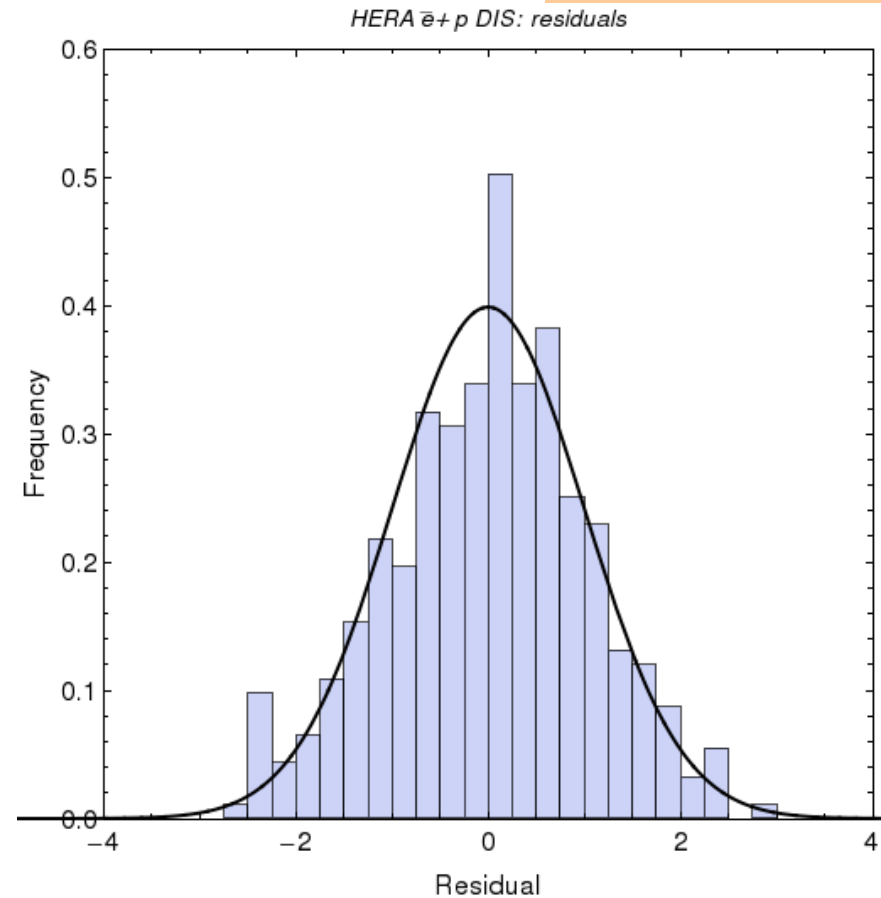
## “Histogram of Residuals”

We define the *residual* by

$$\text{Residual}_i = \frac{sD_i - T_i}{\sigma_{0i}}$$

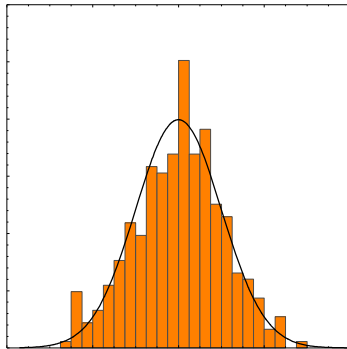
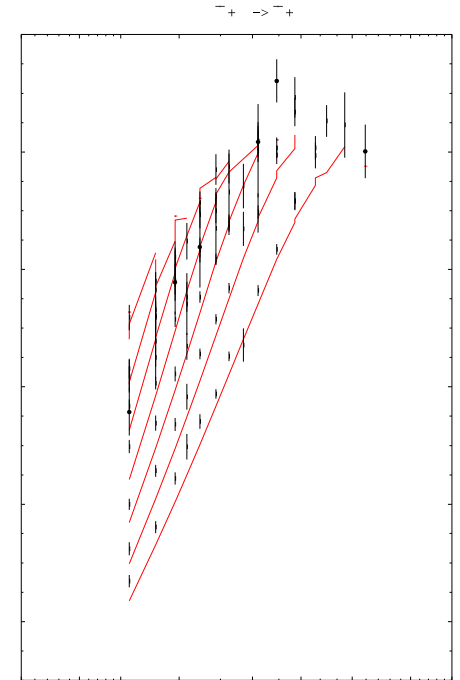
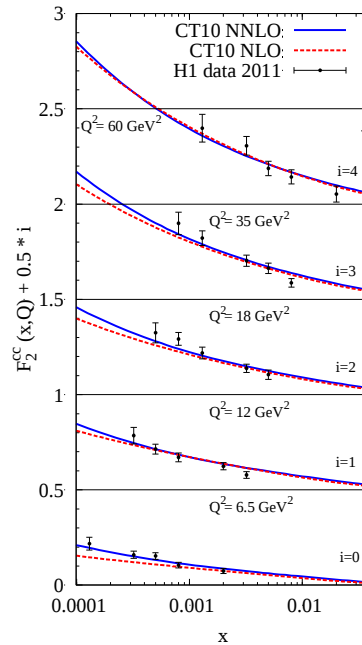
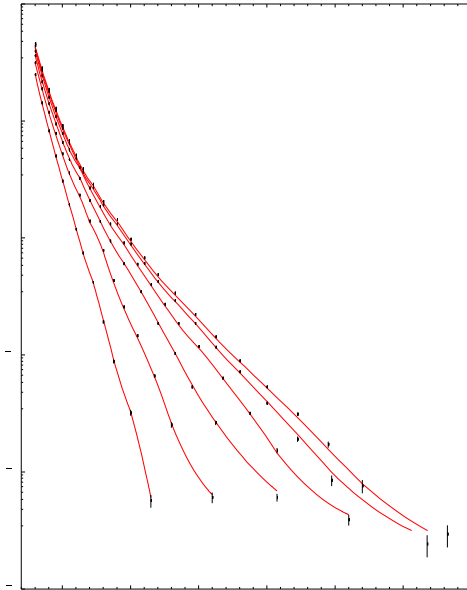
( $i = 1, 2, 3, \dots, NDP$ )

For good agreement between data and theory, the residuals should have a Gaussian distribution with mean = 0 and standard deviation = 1.



Theory = CT10-NNLO, i.e., ***the central fit***;  
 sData = Data **MINUS** the optimized systematic errors;  
 Black curve = ideal Gaussian distribution

# CT10NNLO vs. fitted data



Fits well:  $\chi^2 / N_{pt} = 2950 / 2641 = 1.11$

# CT10 NNLO PDFs

## ● PDF error bands

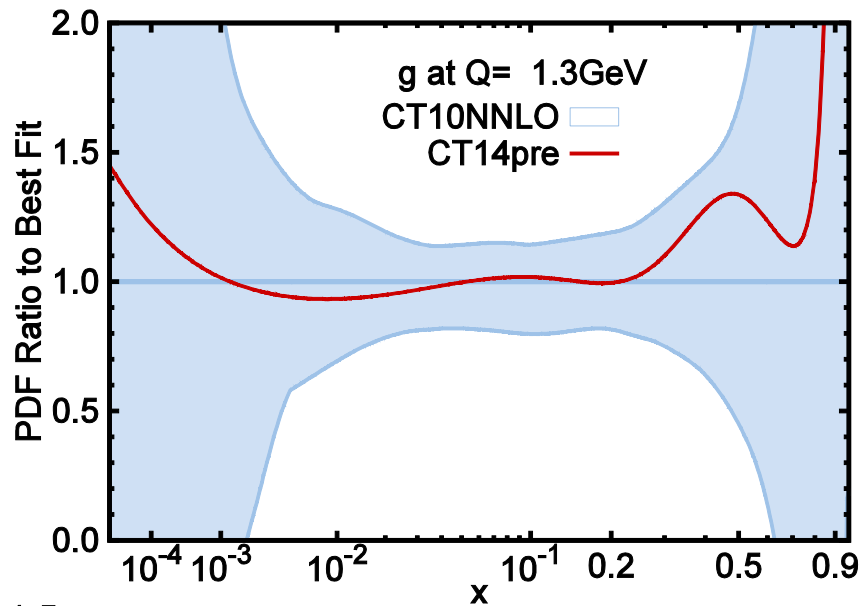
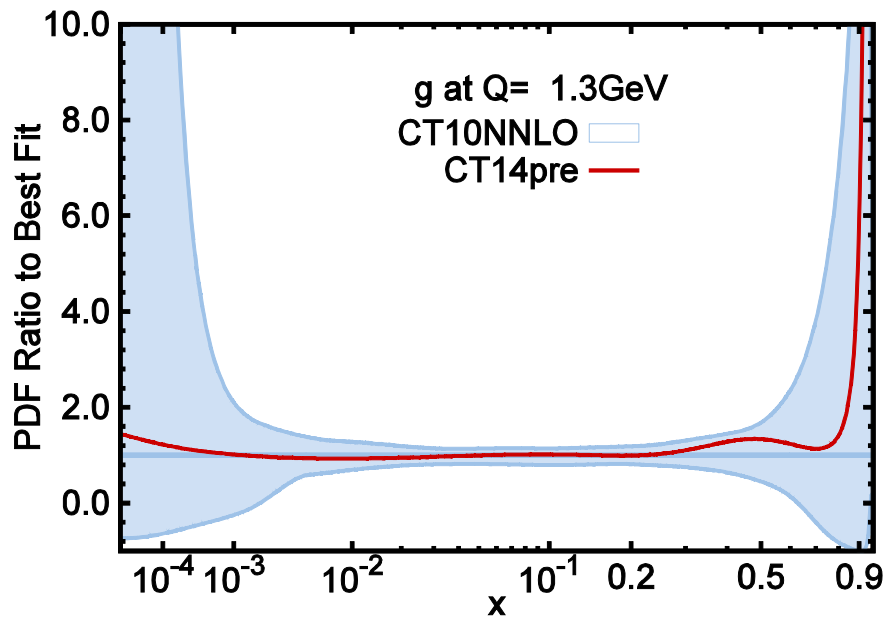
- u and d PDFs are best known
- currently no constraint for  $x$  below  $1E-4$
- large error for  $x$  above 0.3
- larger sea (e.g.,  $\bar{u}$  and  $\bar{d}$ ) quark uncertainties in large  $x$  region
- with non-perturbative parametrization form dependence in small and large  $x$  regions

## ● PDF eigensets

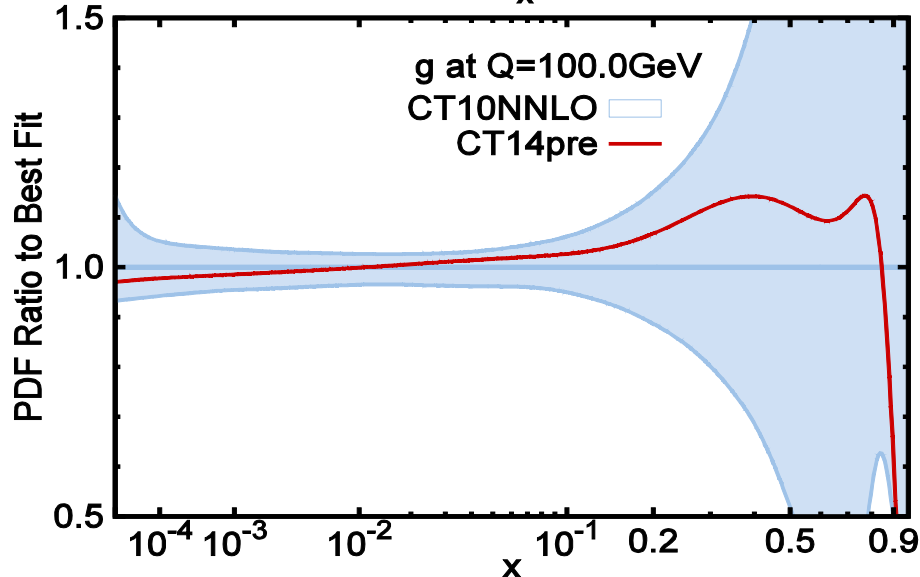
- useful for calculating PDF induced uncertainty
- sensitive to some special (combination of) parton flavor(s).
- (e.g., eigenset 7 is sensitive to  $d/u$  or  $\bar{d}/\bar{u}$ ; hence,  $W$  asymmetry data at Tevatron and LHC.)



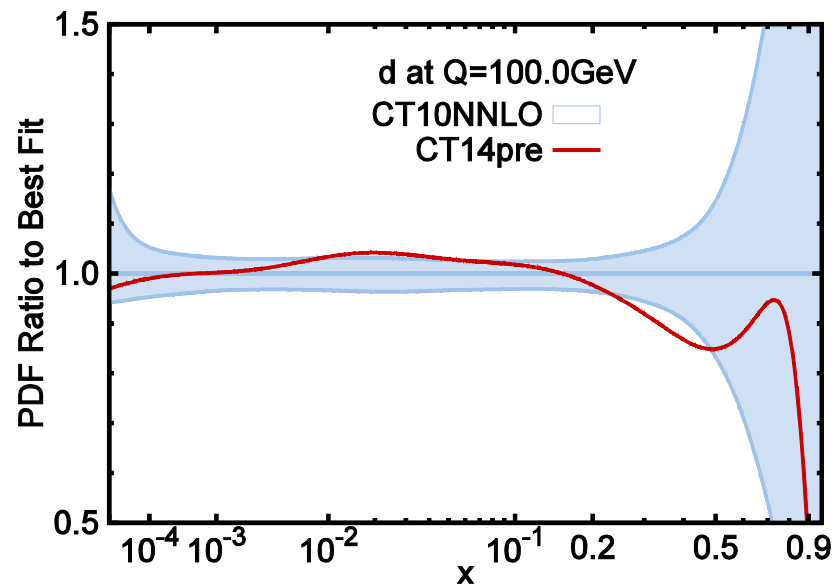
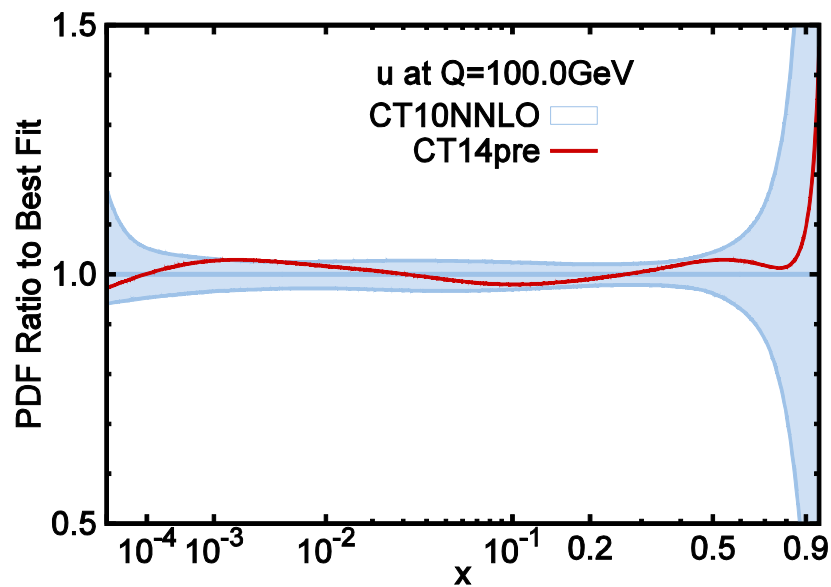
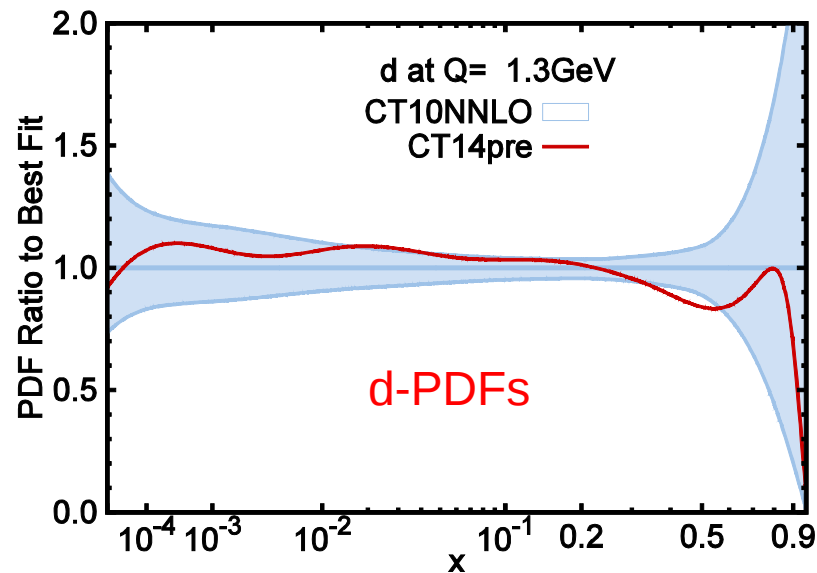
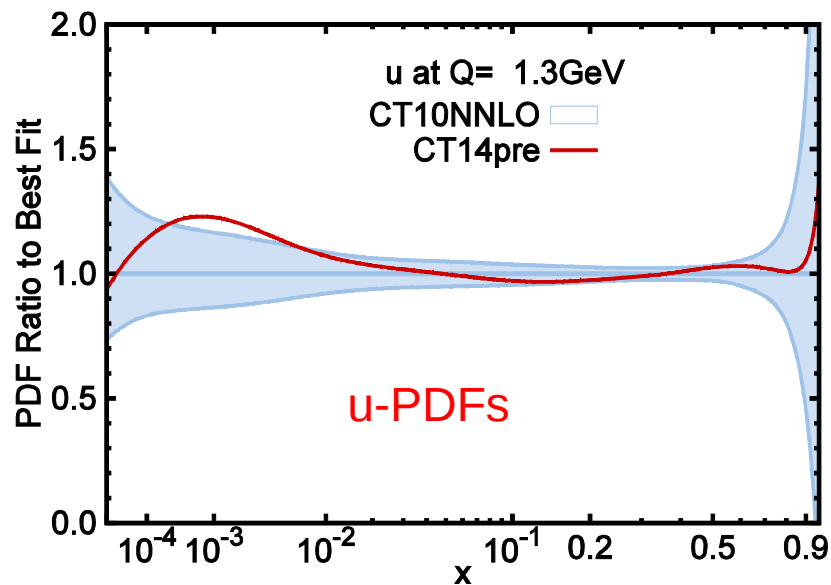
# CT10 and CT14 NNLO PDFs



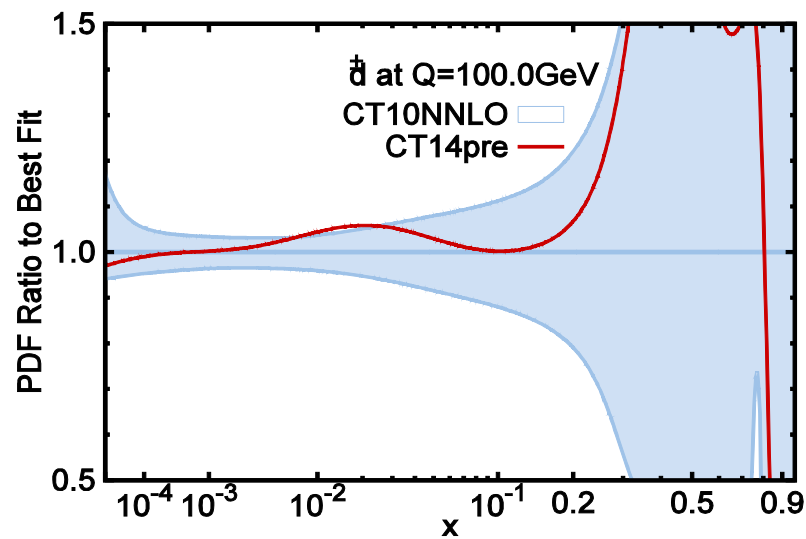
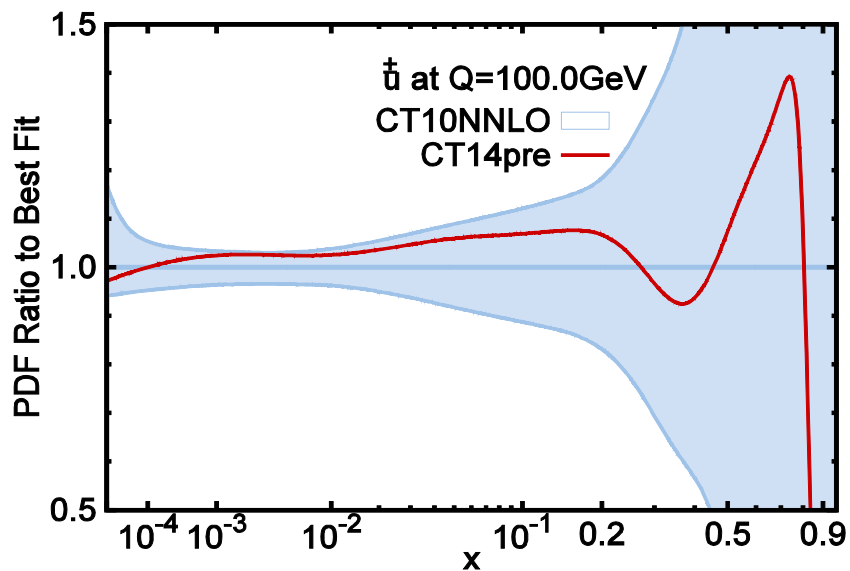
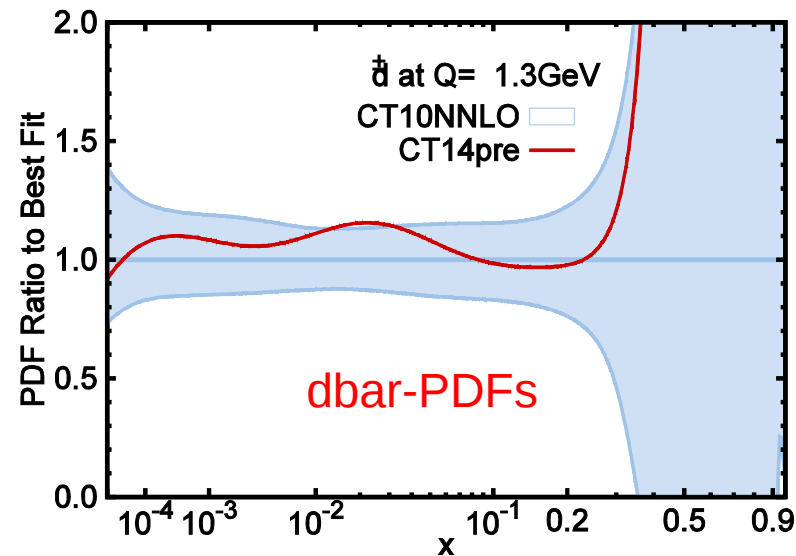
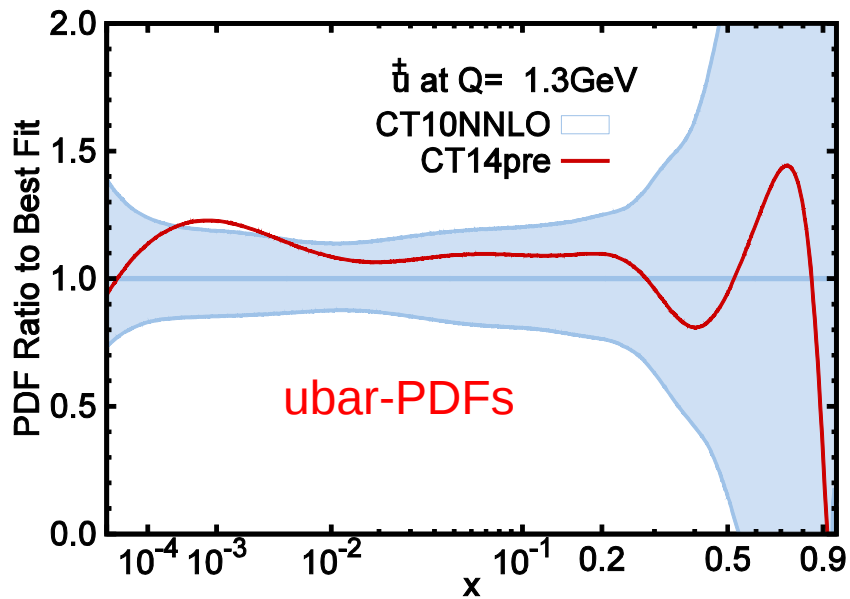
Gluon PDFs



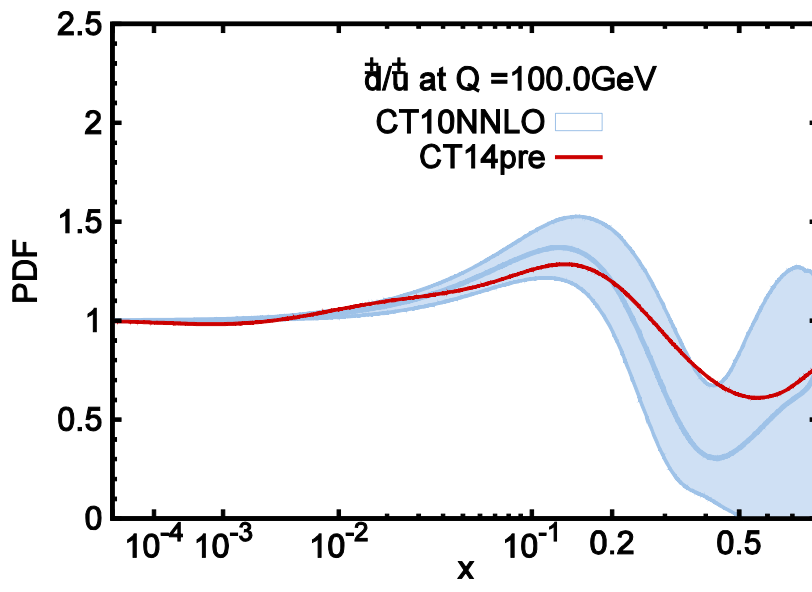
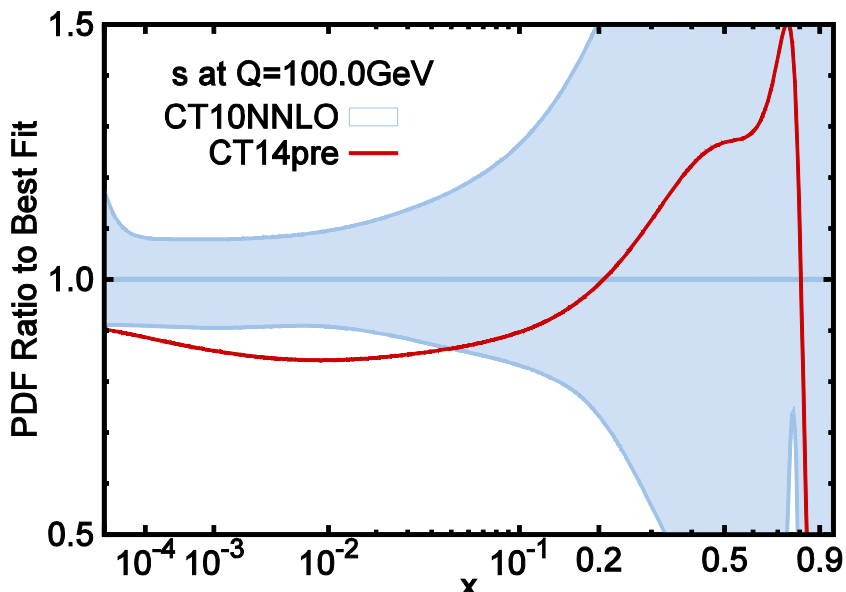
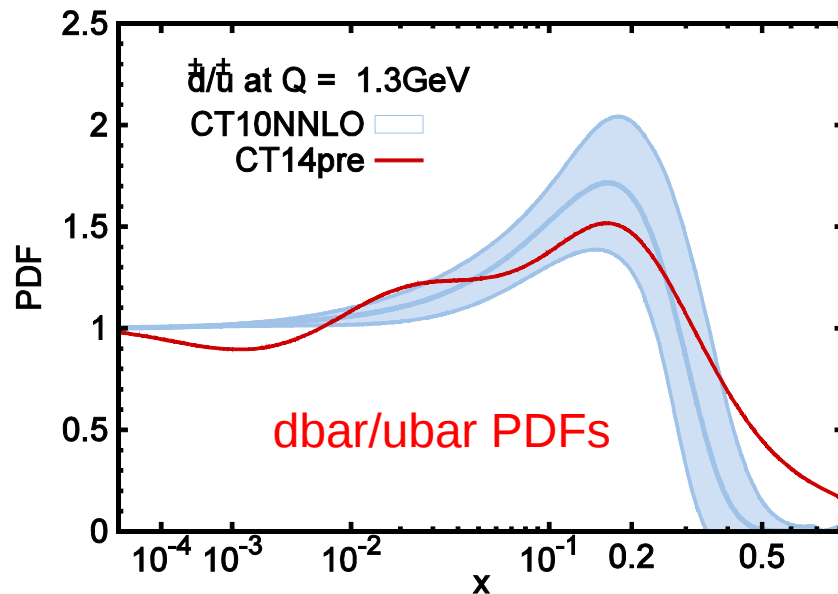
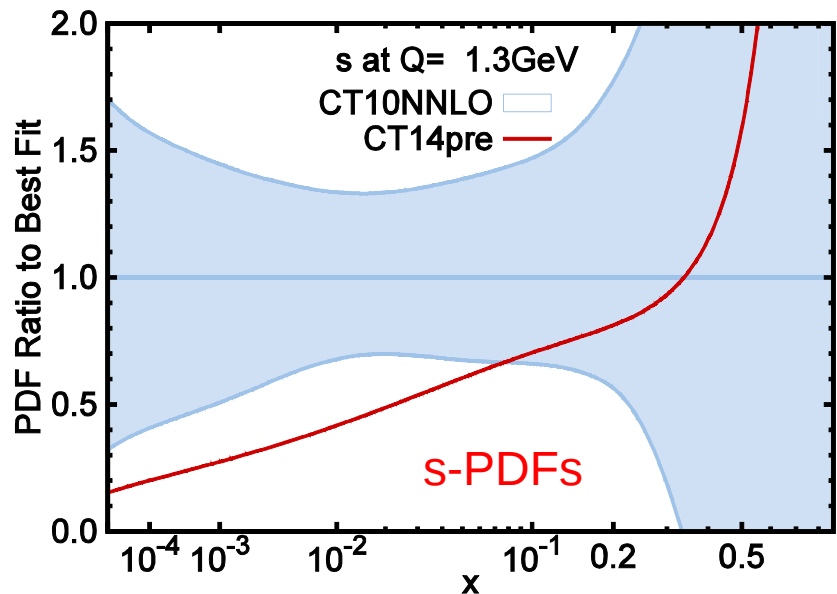
# CT10 and CT14 NNLO PDFs



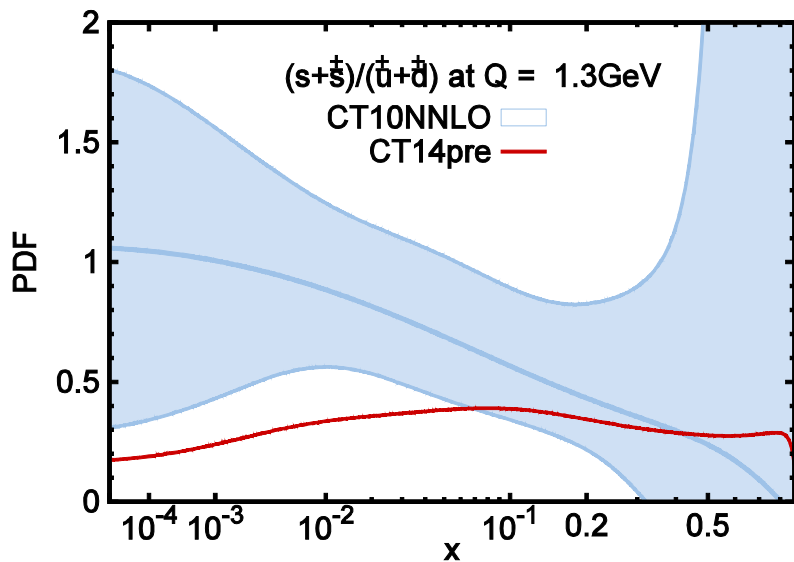
# CT10 and CT14 NNLO PDFs



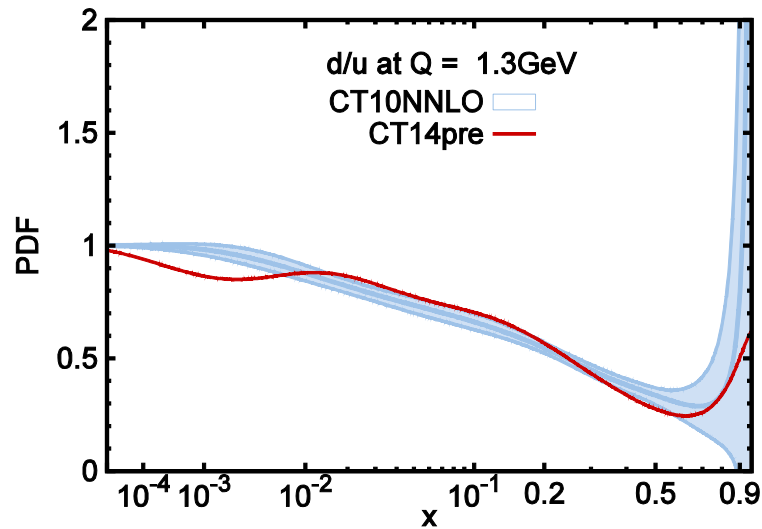
# CT10 and CT14 NNLO PDFs



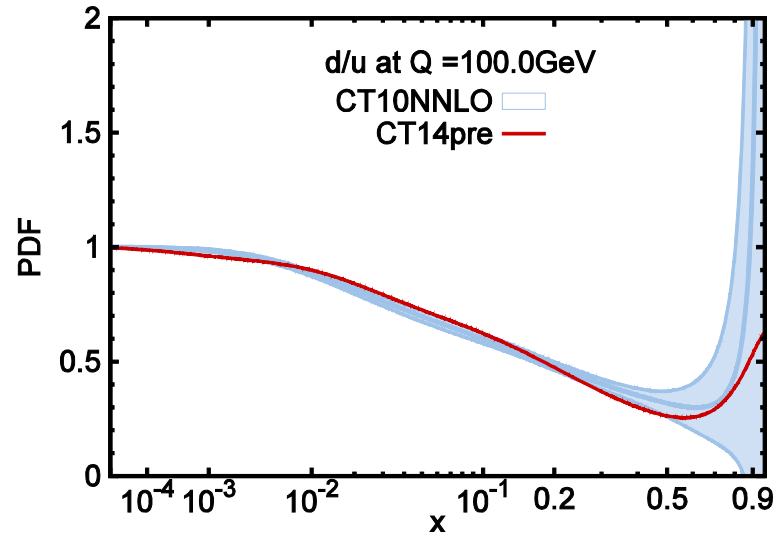
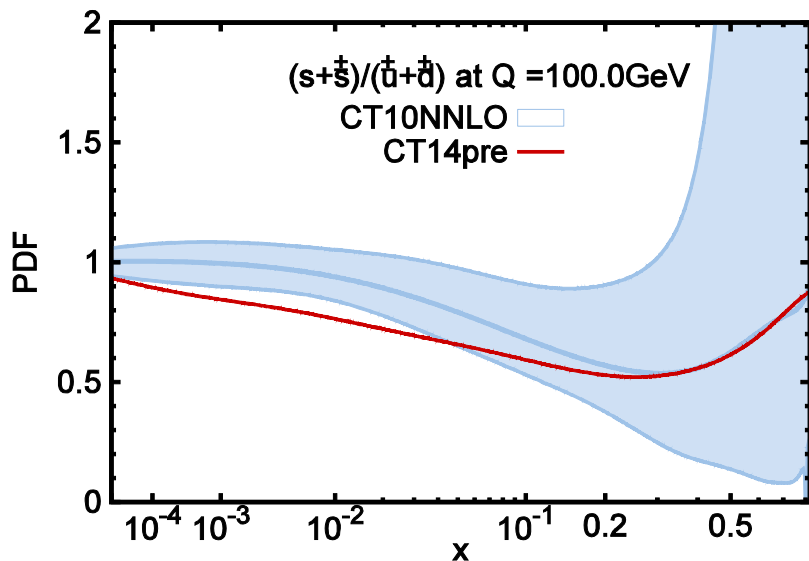
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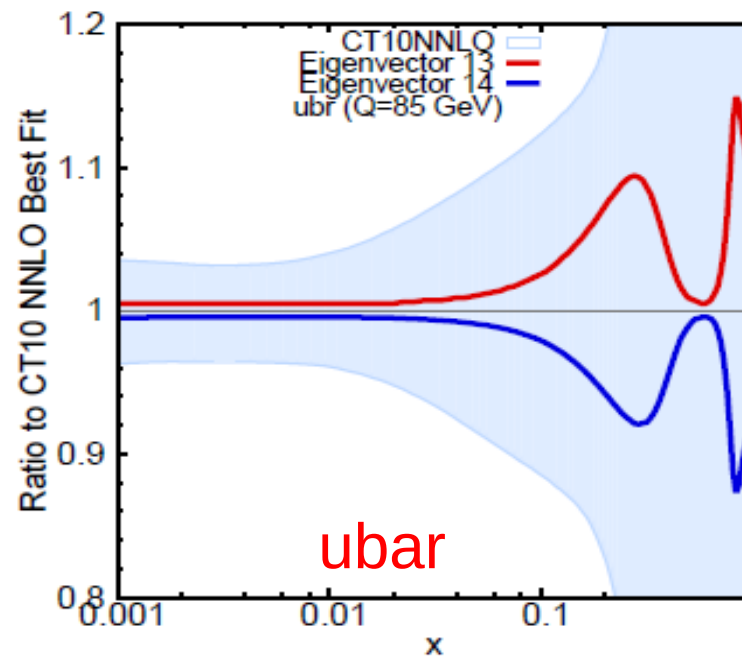
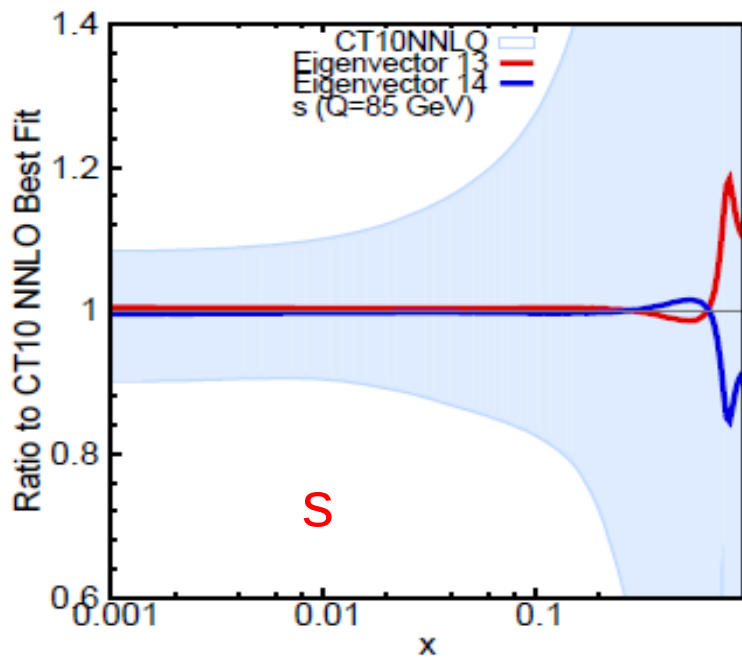
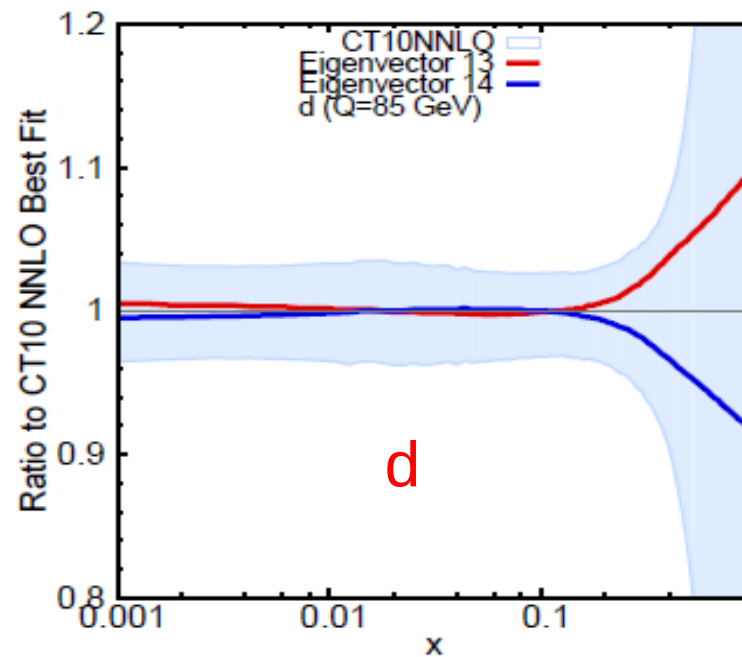
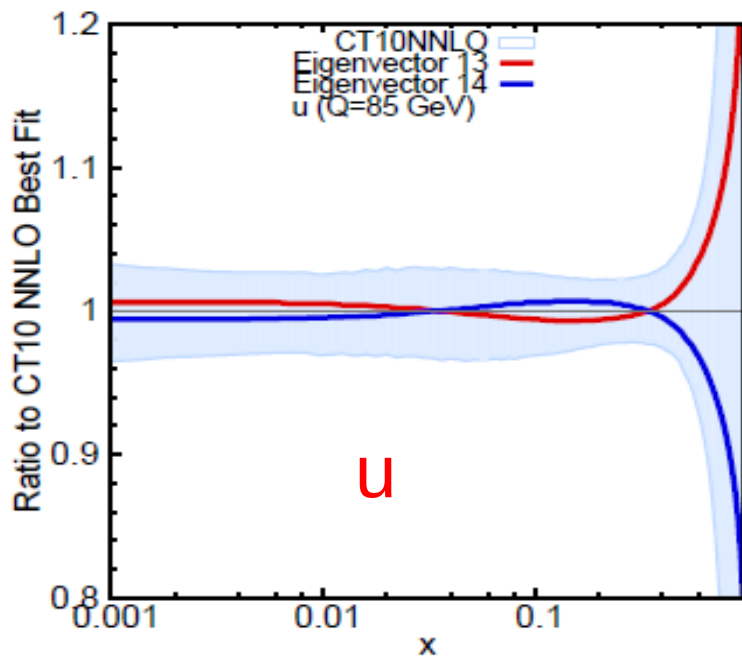


$(s+\bar{s})/(\bar{u}+\bar{d})$  PDFs

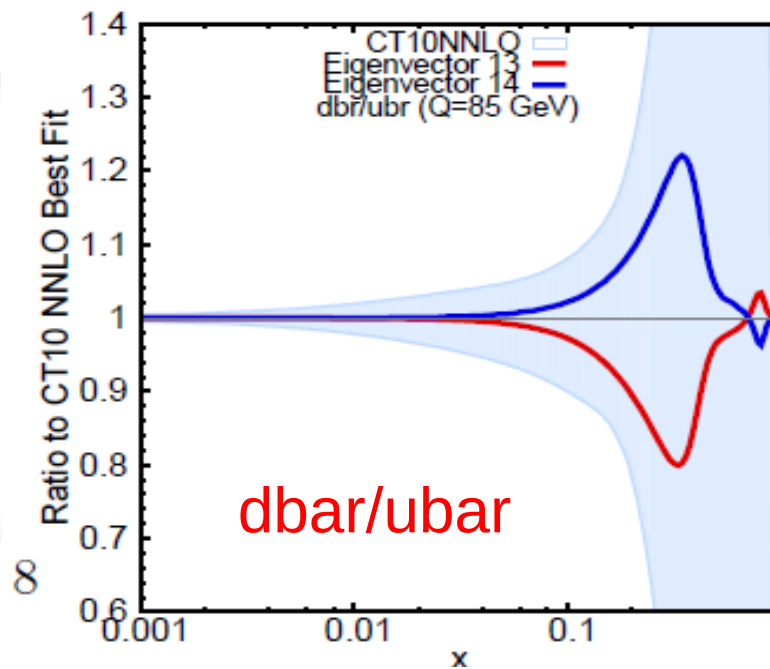
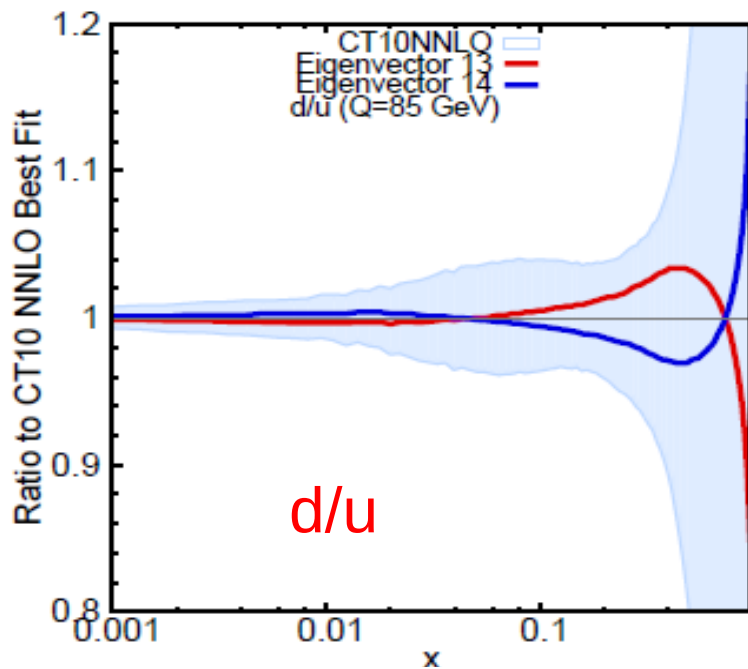
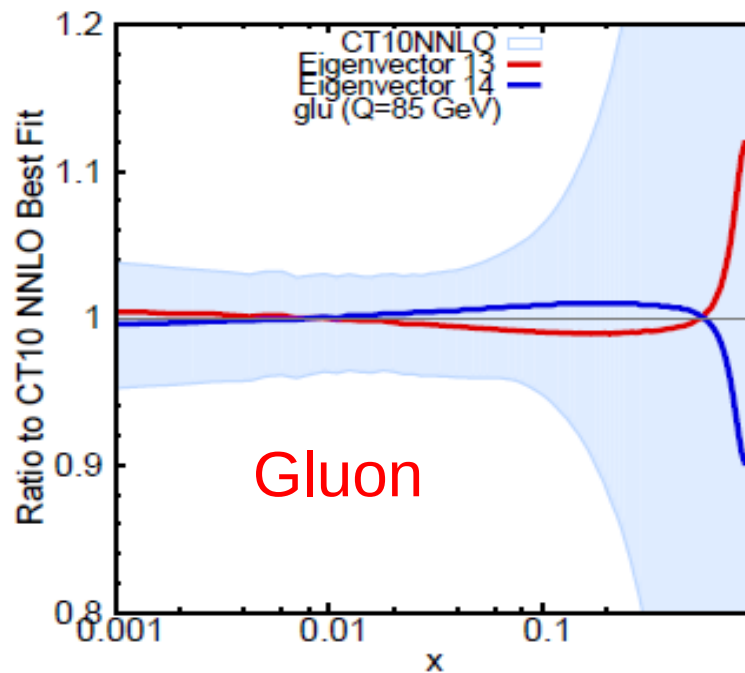
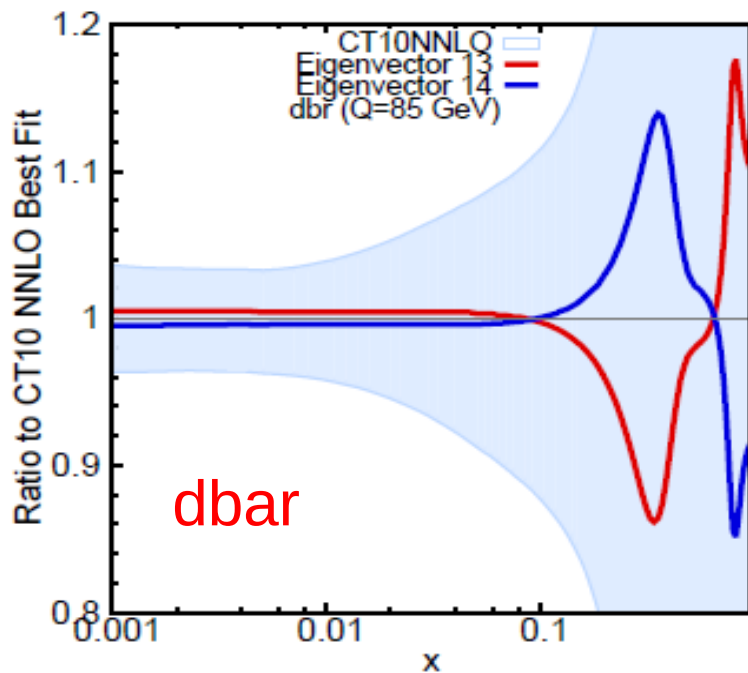


$d/u$  PDFs





7<sup>th</sup> CT10  
Eigenset



7<sup>th</sup> CT10  
Eigenset

# $\alpha_s(m_Z)$

- Right now the Higgs Cross Section Working Group is using a mean value for  $\alpha_s(m_Z)$  of 0.118 with 90% CL error of 0.002 (68%CL error of 0.0012), or an inflation of the world average uncertainties; the  $\alpha_s$  error is added in quadrature with the PDF error
- The world average is dominated by lattice results

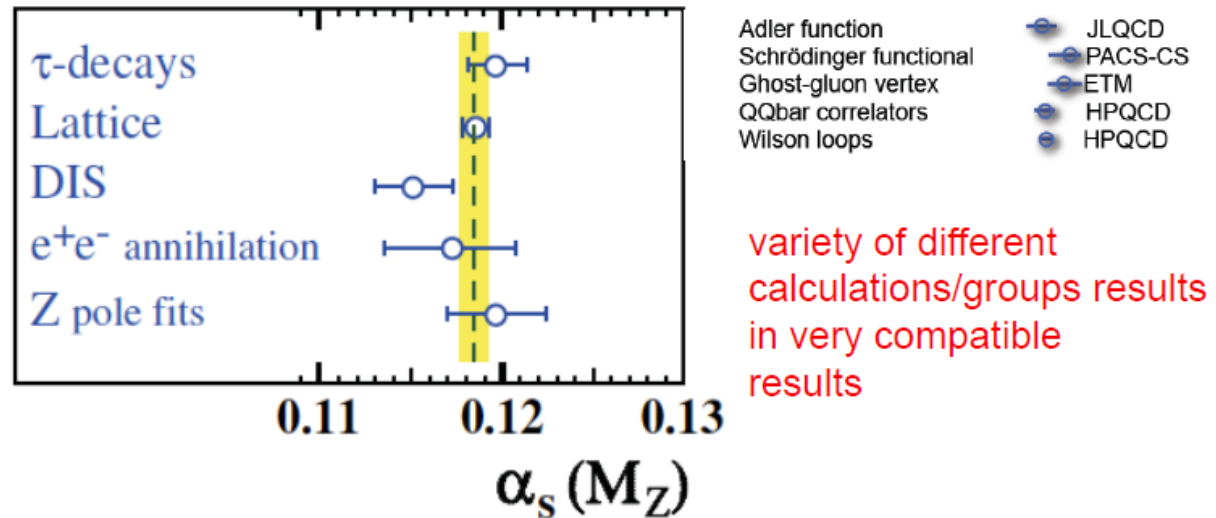


Figure 1-1. Summary of values of  $\alpha_s(M_Z^2)$  obtained for various sub-classes of measurements. The world average value of  $\alpha_s(M_Z^2) = \underline{0.1184 \pm 0.0007}$  is indicated by the dashed line and the shaded band. Figure taken from [1].



# NNLO cross section and PDF induced uncertainty for $gg \rightarrow H$ (using ResBos2 program)

$$\Delta\sigma_{\text{PDF}} = \frac{1}{2} \sqrt{\sum_{i=1}^d \left( \sigma_i^{(+)} - \sigma_i^{(-)} \right)^2}.$$

$$\alpha_s(M_Z) = 0.118$$

arXiv:1205.4311 [hep-ph]

$M=125$  GeV

	CTEQ6.6	CT10 NLO	CT10W NLO	CT10 NNLO	MSTW2008NNLO	NNPDF2.3NNLO
Tevatron	$0.77 \pm 6.9\%$	$0.77 \pm 6.9\%$	$0.76 \pm 7.0\%$	$0.77 \pm 6.9\%$	$0.78 \pm 6.4\%$	$0.80 \pm 4.6\%$
LHC 7 TeV	$12.80 \pm 6.1\%$	$13.33 \pm 6.1\%$	$12.82 \pm 5.1\%$	$12.65 \pm 5.8\%$	$12.69 \pm 4.5\%$	$13.73 \pm 3.0\%$
LHC 8 TeV	$16.31 \pm 5.5\%$	$16.53 \pm 5.5\%$	$16.95 \pm 4.8\%$	$16.63 \pm 5.6\%$	$16.30 \pm 4.5\%$	$16.90 \pm 5.5\%$
LHC 14 TeV	$42.39 \pm 8.5\%$	$42.64 \pm 8.5\%$	$42.91 \pm 7.1\%$	$41.87 \pm 7.7\%$	$43.10 \pm 6.4\%$	$43.28 \pm 5.9\%$

TABLE II: The total cross sections (in pb) for Higgs boson production via  $g + g \rightarrow H + X$  at the Tevatron (1.96 TeV) and LHC (7 TeV, 8 TeV and 14 TeV) by using different PDF sets in ResBos2. The PDF induced uncertainties are estimated at 90% confidence-level, and expressed in the form of percentages.

# Uncertainties of cross sections for $gg \rightarrow H$ (using ResBos2 program)

$$\Delta\sigma_{\alpha_s} = \frac{1}{2} \sqrt{[\sigma_0(A_{-2}) - \sigma_0(A_2)]^2}$$

(at 90% CL, with the range of 0.116 to 0.120)

$\alpha_s(M_Z)$

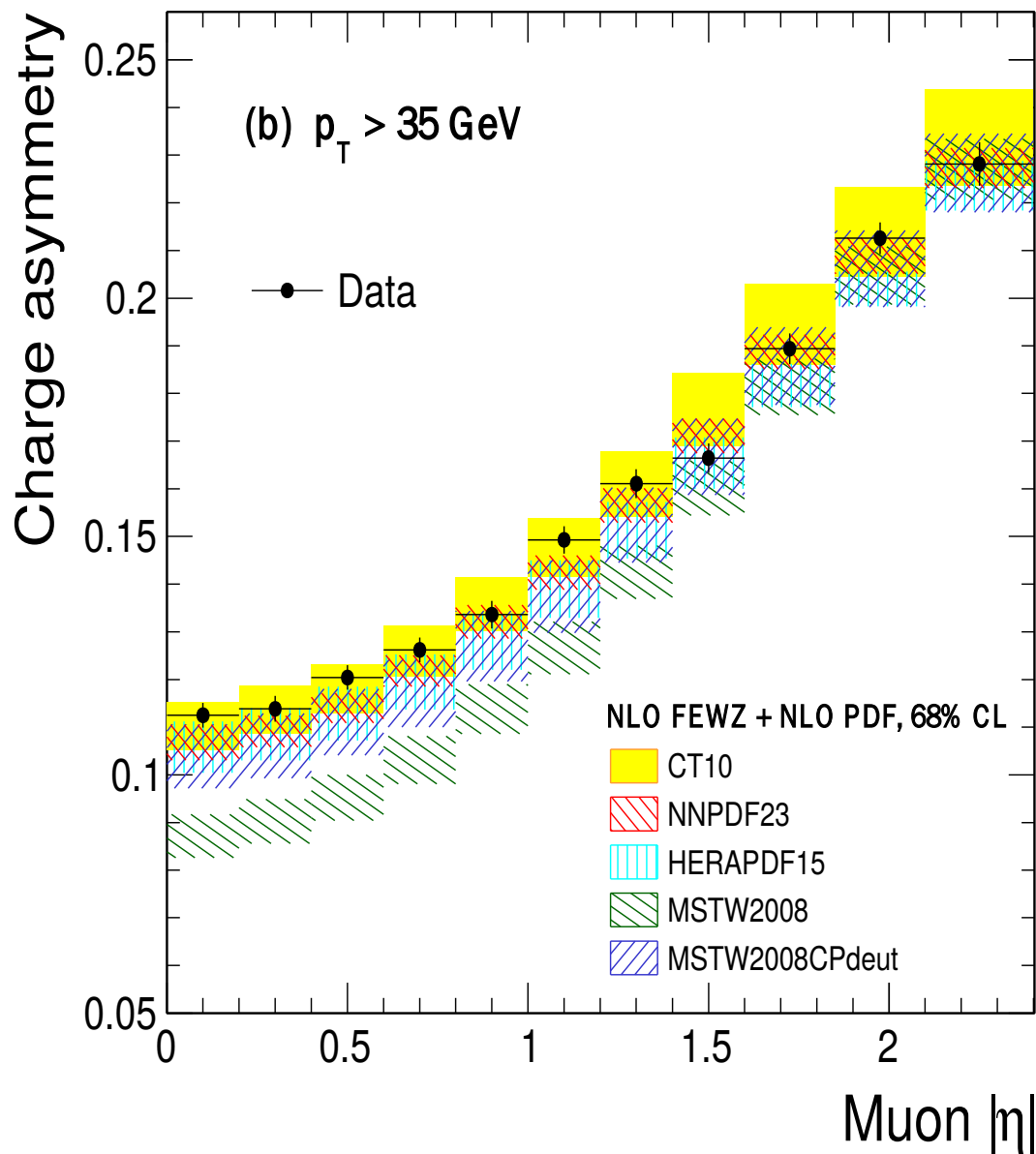
CT10AS PDF sets

$$(\Delta\sigma)^2 = (\Delta\sigma_{\text{PDF}})^2 + (\Delta\sigma_{\alpha_s})^2$$

M=125 GeV

CT10-NNLO	$\sigma_0$	$\Delta\sigma_{\text{PDF}}$	$\Delta\sigma_{\alpha_s}$	$\Delta\sigma = \sqrt{(\Delta\sigma_{\text{PDF}})^2 + (\Delta\sigma_{\alpha_s})^2}$
Tevatron	0.77	$\pm 6.9\%$	$\pm 1.8\%$	$\pm 7.1\%$
LHC 7 TeV	12.65	$\pm 5.8\%$	$\pm 2.5\%$	$\pm 6.3\%$
LHC 8 TeV	16.63	$\pm 5.6\%$	$\pm 3.5\%$	$\pm 6.6\%$
LHC 14 TeV	41.87	$\pm 7.7\%$	$\pm 5.3\%$	$\pm 9.3\%$

CMS,  $L = 4.7 \text{ fb}^{-1}$  at  $\sqrt{s} = 7 \text{ TeV}$



Data is already more precise than current PDF uncertainty.

Will help to determine PDFs in small  $x$  region.

Most useful for determining  $d\bar{b}/u\bar{b}$ .

# Intrinsic Charm and CT10IC

- 1) Update of CTEQ6.5 IC study from 2007 to CT10NNLO
  - includes combined H1 and ZEUS data, HERA inclusive charm

- 2) Recent CT10 global analysis study of charm quark mass:

$$m_c(m_c) = 1.15_{-0.12}^{+0.18} \text{ GeV} \quad \text{Gao et al, Eur.Phys.J. C73 (2013) 2541}$$

Use  $m_c(\text{pole}) = 1.3 \text{ GeV}$  for this study

- some correlation between  $m_c$  and IC

- 3) Two model Intrinsic Charm distributions at  $Q_c = 1.3 \text{ GeV}$

- BHPS valence-like model (Brodsky et al, Phys. Lett. **93B**, 451 (1980))

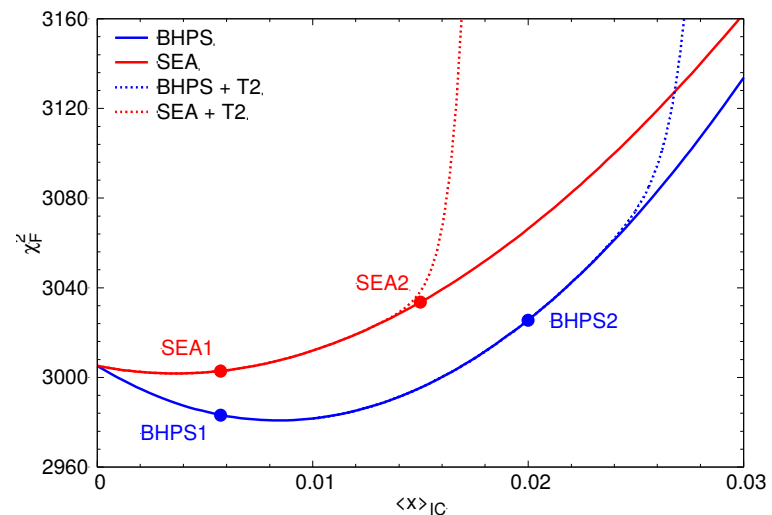
- SEA-like model

$$\langle x \rangle_{\text{IC}} = \int_0^1 x [\alpha(x, Q_c) + \bar{c}(x, Q_c)] dx$$

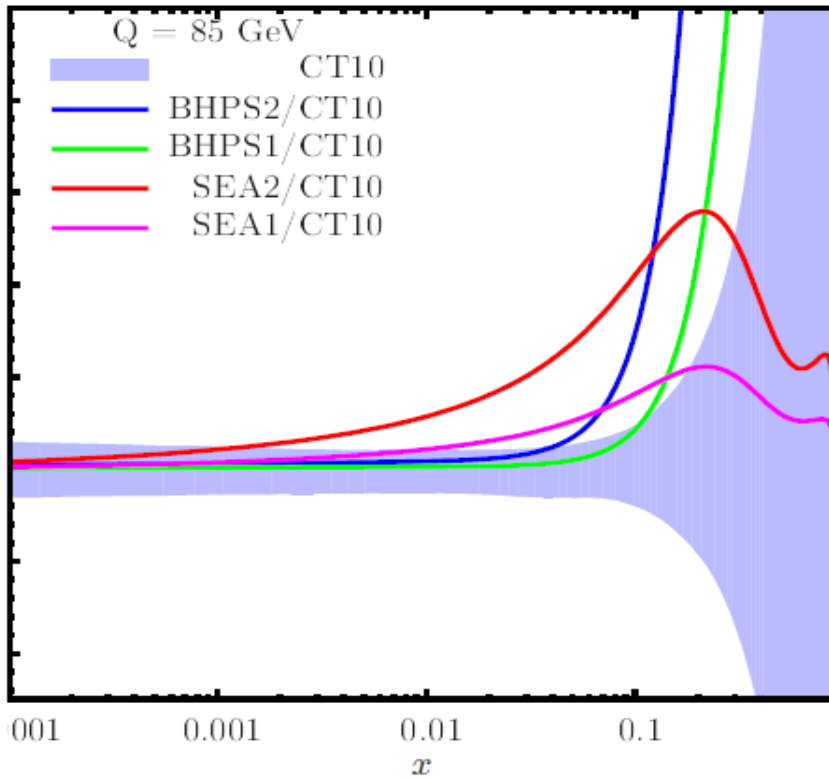
- 4) 90% CL limits:

$$\langle x \rangle_{\text{IC}} \leq 0.025 \quad \text{BHPS}$$

$$\langle x \rangle_{\text{IC}} \leq 0.015 \quad \text{SEA}$$



# Intrinsic Charm at LHC

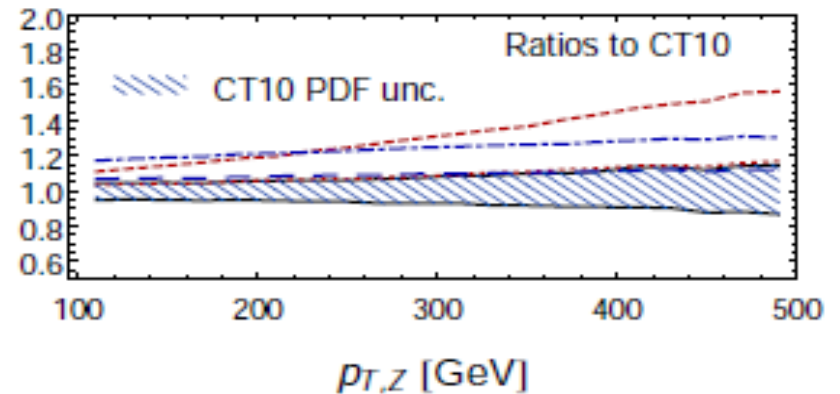
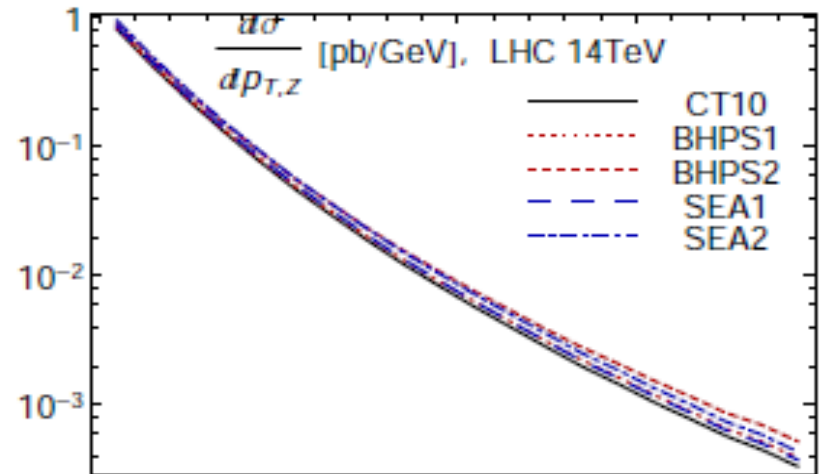


IC vs CT10 charm PDF

SEA1/BHPS1:  $\langle x \rangle_{\text{IC}} = 0.57\%$

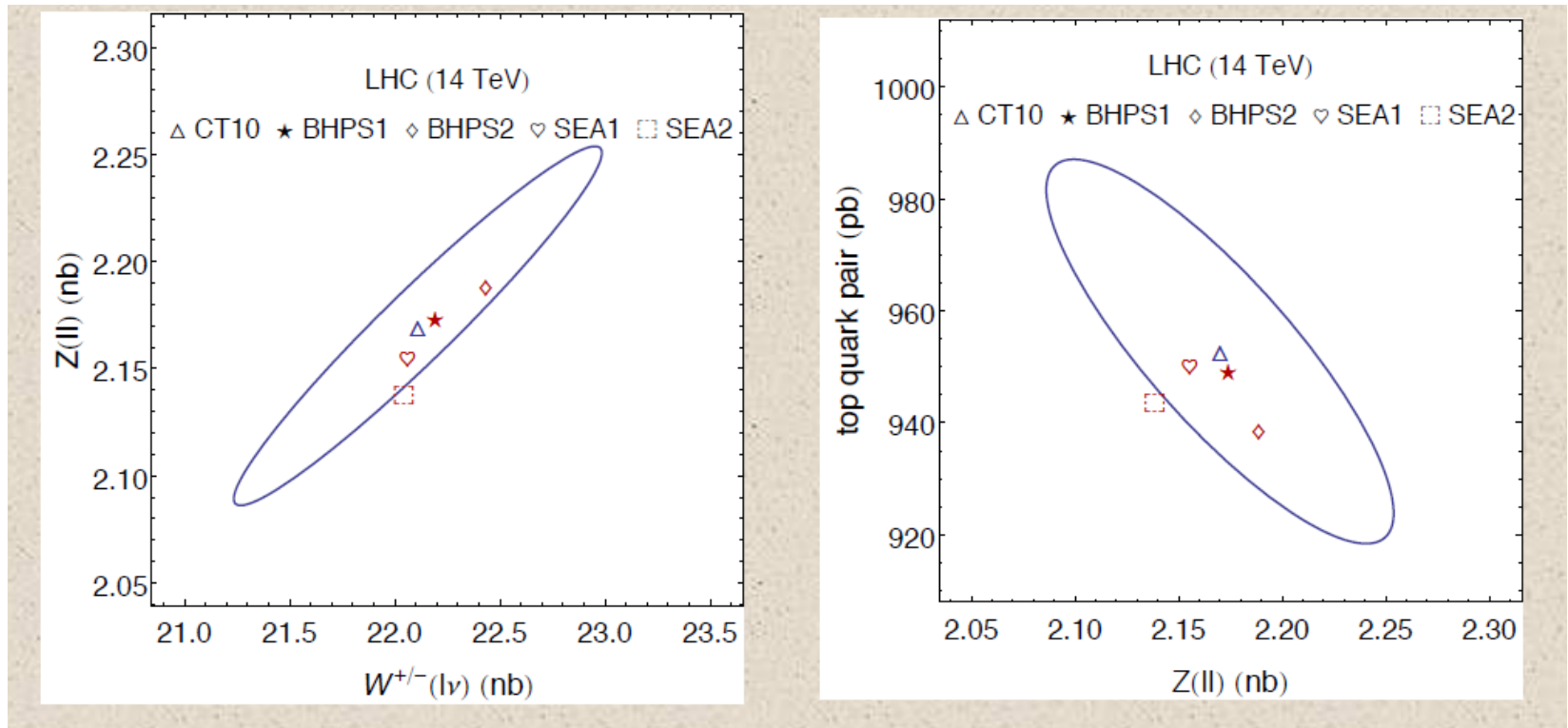
SEA2:  $\langle x \rangle_{\text{IC}} = 1.5\%$

BHPS2:  $\langle x \rangle_{\text{IC}} = 2.0\%$



$pp \rightarrow Zc$  at LHC may further constrain valence-like model

# CT10 IC at LHC



W, Z and top production at LHC

CT10 IC distributions publicly available

# PDF uncertainties in $gg \rightarrow H$

1) Most analyses use Hessian Method (n error PDF sets)

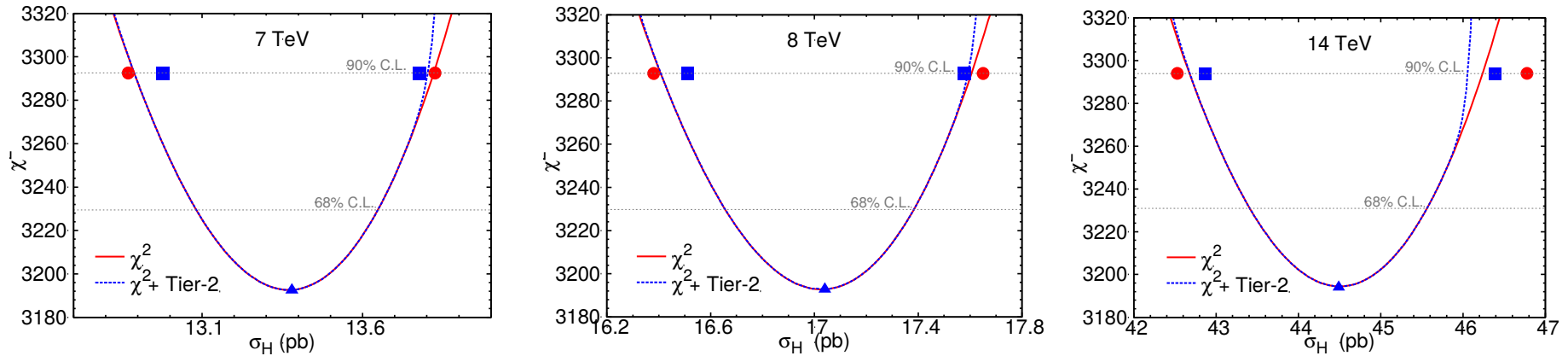
$$(\delta X)^2 = \frac{1}{4} \sum_{k=1}^n \left( X(a_k^+) - X(a_k^-) \right)^2$$

- Error sets can be used by anyone for any observable
- Assume quadratic and linear dependence of  $\chi^2$ ,  $X$  on  $a_k$

2) Lagrange Multiplier (LM) method is more robust

- Find best fit for each constrained value of observable  $X$
- No assumptions on dependence of  $\chi^2$ ,  $X$  on  $a_k$
- Can validate Hessian method
- Can display correlations between PDFs and Observable
- Must calculate separately for each observable

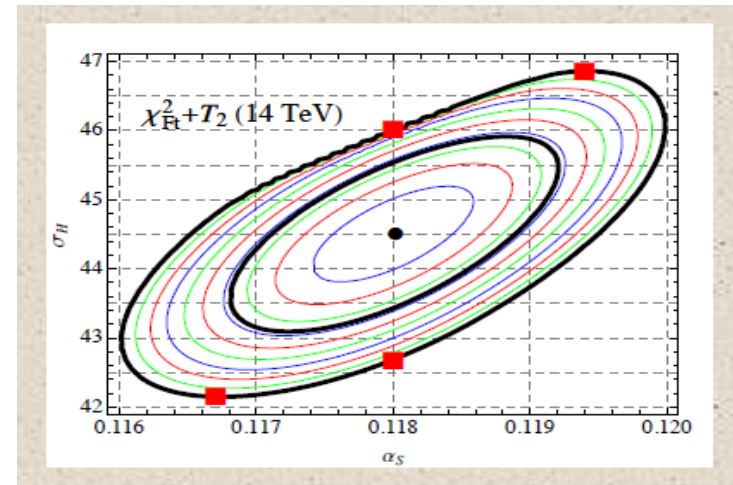
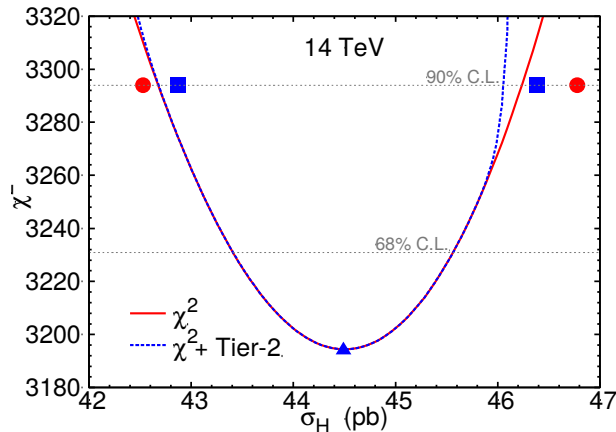
# Uncertainties in $gg \rightarrow H$



- Curves are LM, circles/squares are Hessian
- Red use  $\chi^2$
- Blue add Tier-2 penalty to ensure no specific experiment is too badly fit
- Allowed Tolerance is 100 at 90% CL
  
- Small differences in asymmetries, but in general the two methods agree well for this observable

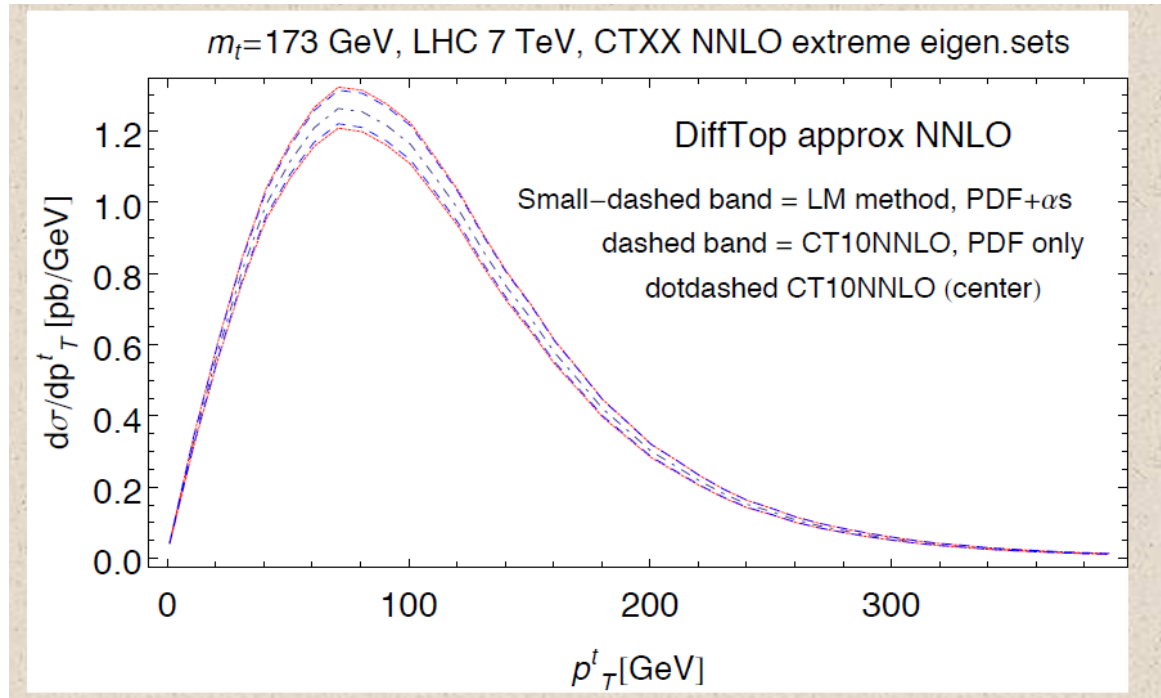


# LM Uncertainty in $gg \rightarrow H$



- Lagrange Multiplier method more robust than Hessian Method
- PDF uncertainty only or PDF+ $\alpha_s$  uncertainty
- But must check for each process
- For Higgs cross section they agree well
  
- PDF sets that give **extreme values of Higgs cross section** at 90% CL are publically available as **CT10H** sets. It contains the combined effect from  $\alpha_s$  and PDF uncertainties.
- Shown on contour plot as **Red squares**
- Useful for efficient  $gg \rightarrow H$  analyses

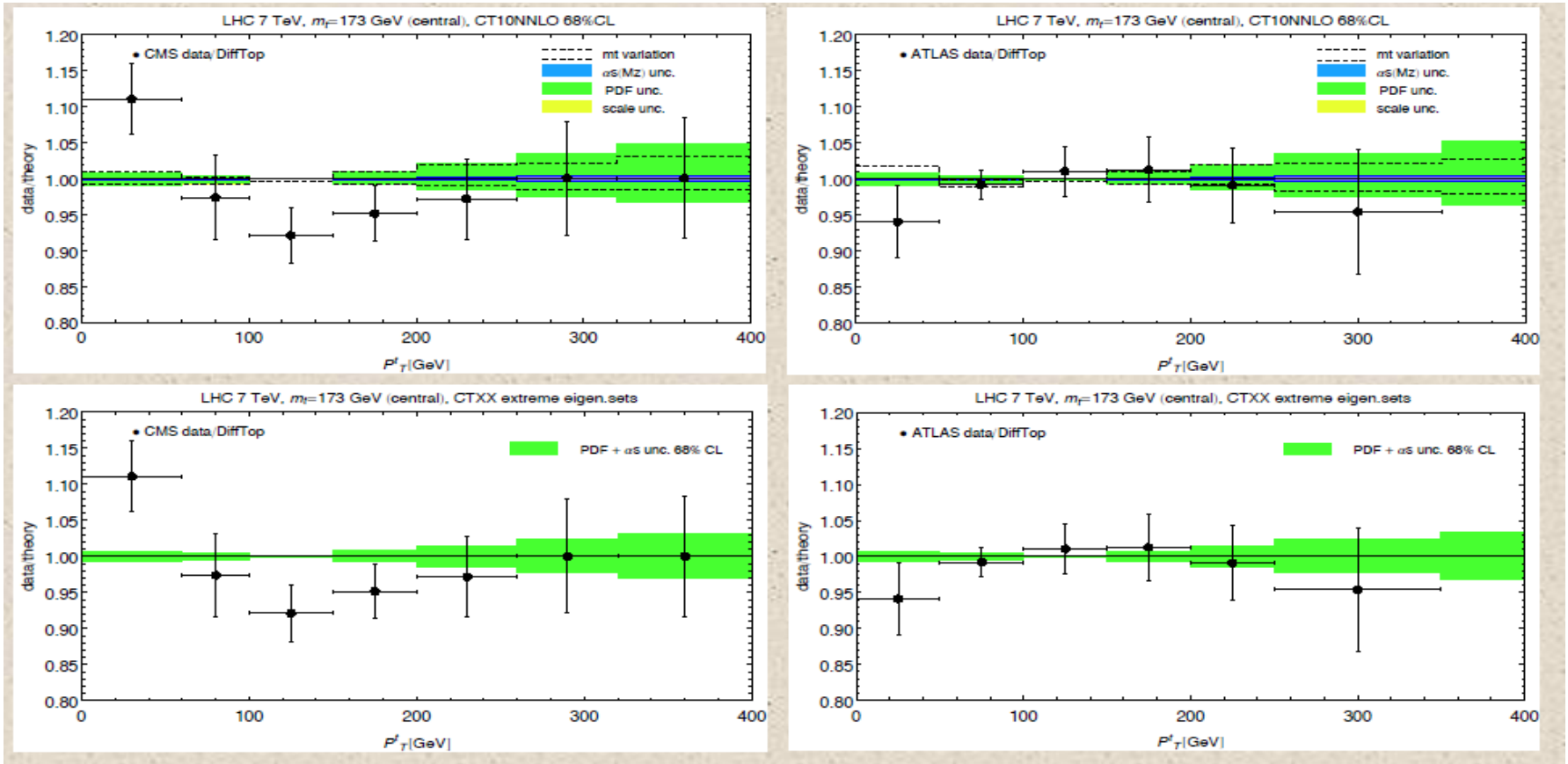
# CT10tt extreme sets



- Pairs of CT10tt extreme sets (PDF, PDF+ $\alpha_s$ ) to be released  
- for focused ttbar analyses

Results provided by DiffTop group (M. Guzzi, K. Lipka, S. Moch)

# CT10tt extreme sets



- Comparison with CMS (left) and ATLAS (right) data
- Hessian top, LM extreme sets bottom
- Extreme sets useful if highly correlated with inclusive  $t\bar{t}$  (note high  $p_T$ )

Results provided by DiffTop group (M. Guzzi, K. Lipka, S. Moch)

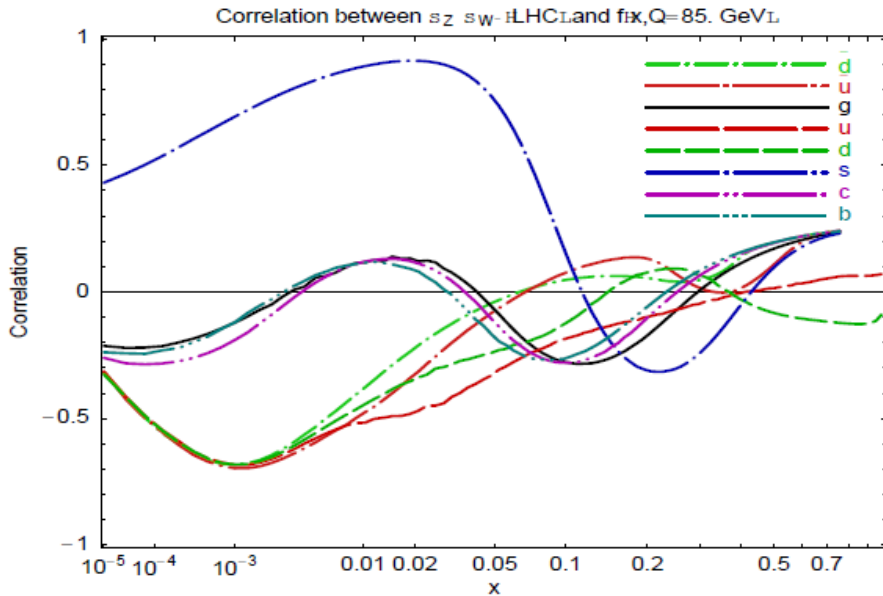
# Recap

- Intrinsic Charm
  - Limits on valence-like and sea-like IC
  - CT10IC PDFs available for further study
  - LHC will probe further
- Lagrange Multiplier Uncertainty analysis
  - Less dependent on assumptions than Hessian analysis
  - Allows study of data correlations with particular observable
  - Test of Hessian results
    - Consistent with Hessian results for both Higgs and  $t\bar{t}$
  - CT10H extreme sets available for focused studies  
(CT10tt extreme sets to come)

# CT10, CT14, and LHC data

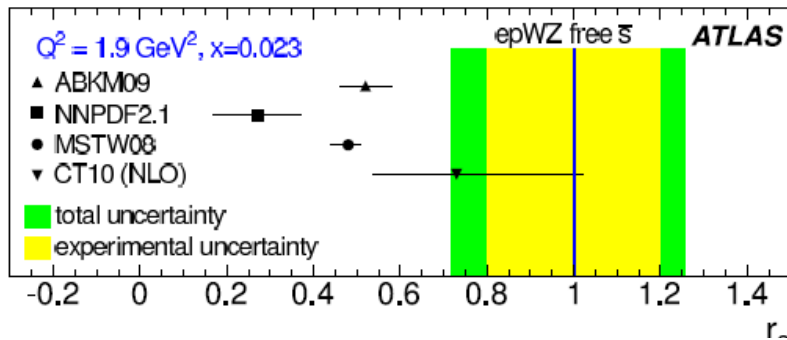
- We have since included early (7 TeV) LHC data: Atlas  $W/Z$  production and asymmetry at 7 TeV, Atlas single jet inclusive, CMS  $W$  asymmetry, HERA  $F_L$  and  $F_2^c$
- New D0 (9.7 1/fb)  $W$ -electron rapidity asymmetry data.
- More flexible parametrization – gluon,  $d/u$  at large  $x$  and both,  $d/u$  and  $d\bar{u}/u\bar{d}$  at small  $x$ , strangeness, and  $s - s\bar{u}$ .
- Improvements modest so far, but expectation from  $t\bar{t}$ ,  $W/Z$ , Higgs, etc.

# Strangeness in CT10 PDFs and LHC W/Z Data



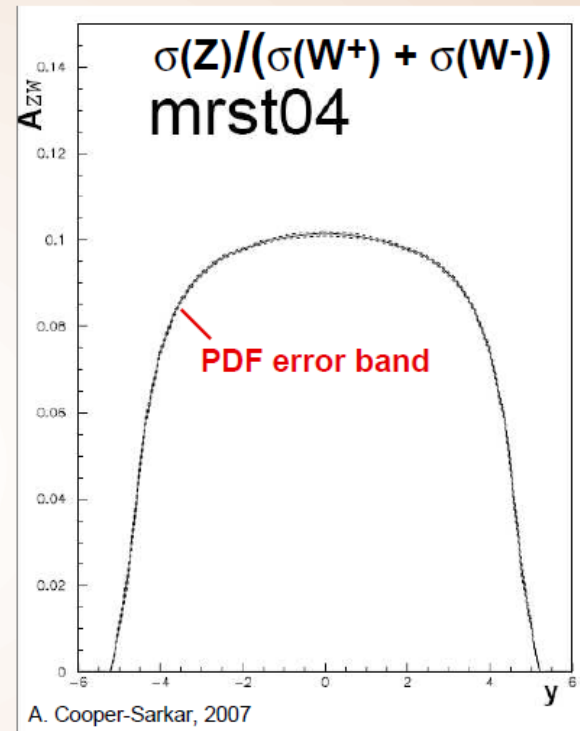
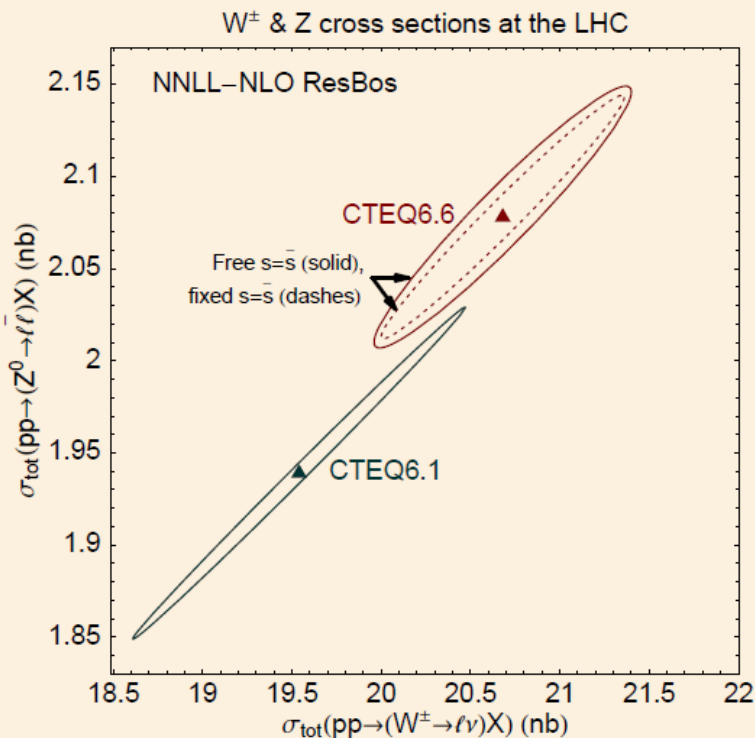
In 2008, our CTEQ6.6 PDF correlation analysis pointed out the sensitivity of ratios  $\sigma_W/\sigma_Z$  at the LHC to the strangeness PDF, with implications to EW precision measurements (Nadolsky, Lai, Cao, Huston, Pumplin, Tung, Yuan, PRD, 78 (2008) 013004).

The ATLAS analysis (arXiv:1203.4051) of  $W$  and  $Z$  production suggests that  $\bar{s}(x, Q)/\bar{d}(x, Q) = 1.00^{+0.25}_{-0.28}$  at  $x = 0.023$  and  $Q^2 = 1.9 \text{ GeV}^2$



# CTEQ6.6 vs. CTEQ6.1

## Correlations and ratio of $W$ and $Z$ cross sections



Radiative contributions, PDF dependence have similar structure in  $W$ ,  $Z$ , and alike cross sections; cancel well in Xsection ratios

# Correlation analysis for collider observables

(J. Pumplin et al., PRD 65, 014013 (2002); P.N. and Z. Sullivan, hep-ph/0110378)

A technique based on the Hessian method

For  $2N$  PDF eigensets and two cross sections  $X$  and  $Y$ :

$$\Delta X = \frac{1}{2} \sqrt{\sum_{i=1}^N \left( X_i^{(+)} - X_i^{(-)} \right)^2}$$

$$\cos \varphi = \frac{1}{4\Delta X \Delta Y} \sum_{i=1}^N \left( X_i^{(+)} - X_i^{(-)} \right) \left( Y_i^{(+)} - Y_i^{(-)} \right)$$

$X_i^{(\pm)}$  are maximal (minimal) values of  $X_i$  tolerated along the  $i$ -th PDF eigenvector direction;  $N = 22$  for the CTEQ6.6 set

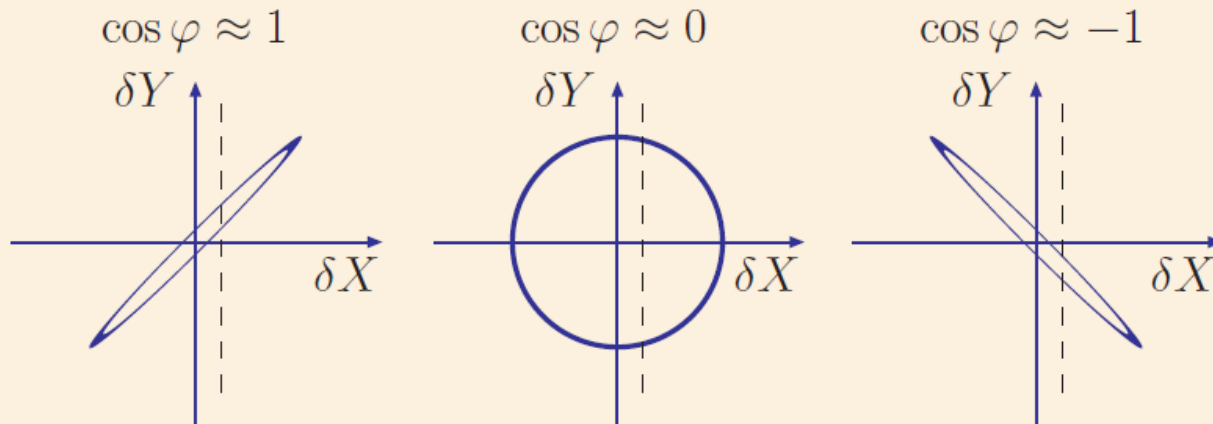


# Correlation angle $\varphi$

Determines the parametric form of the  $X - Y$  correlation ellipse

$$X = X_0 + \Delta X \cos \theta$$

$$Y = Y_0 + \Delta Y \cos(\theta + \varphi)$$



$X_0, Y_0$ : best-fit values

$\Delta X, \Delta Y$ : PDF errors

$\cos \varphi \approx \pm 1$  :

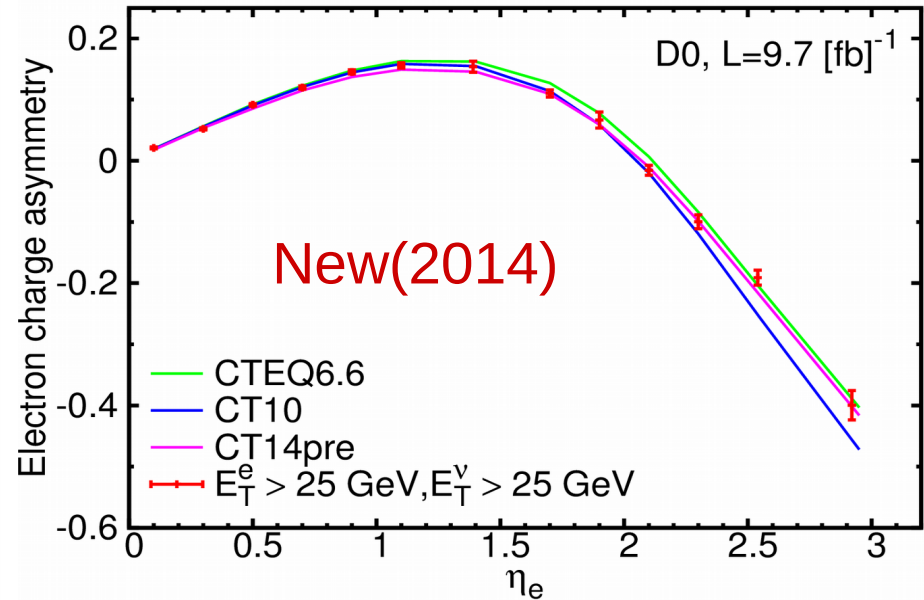
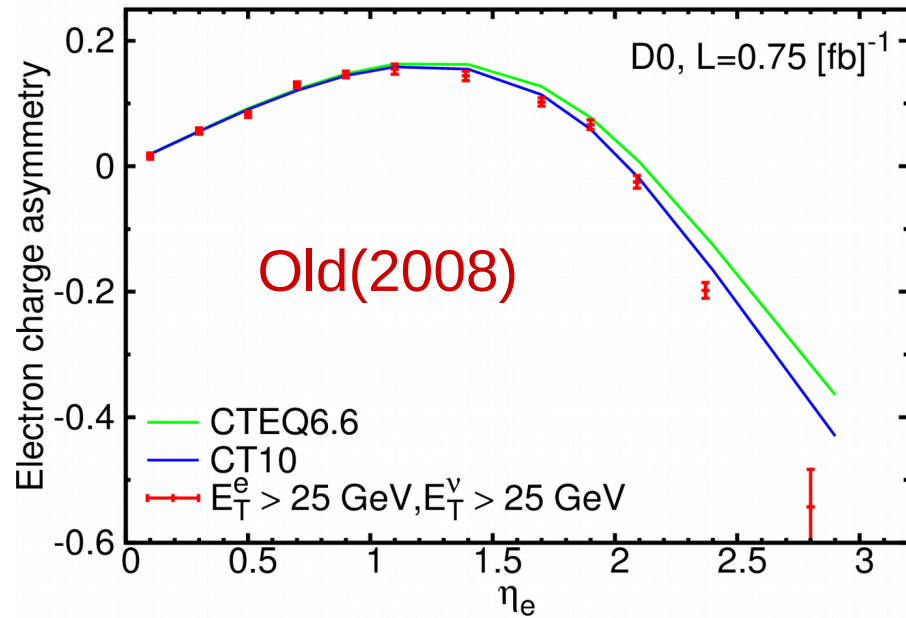
Measurement of  $X$  imposes

tight  
loose

constraints on  $Y$

$\cos \varphi \approx 0$  :

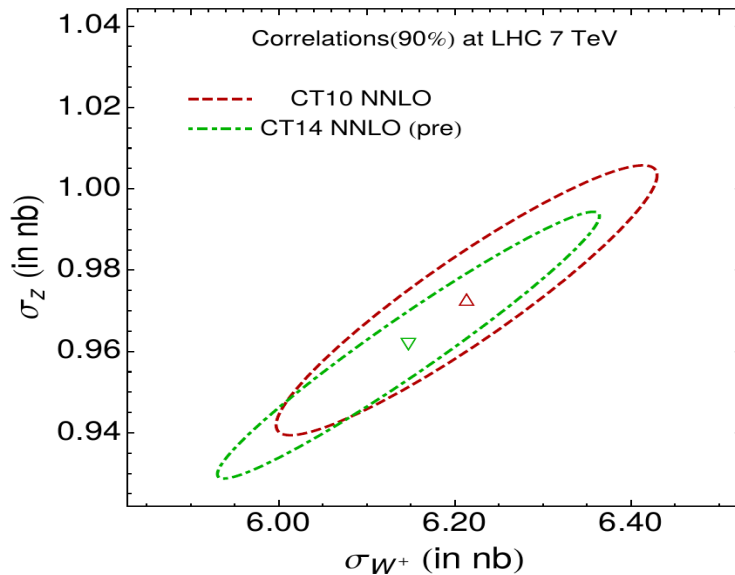
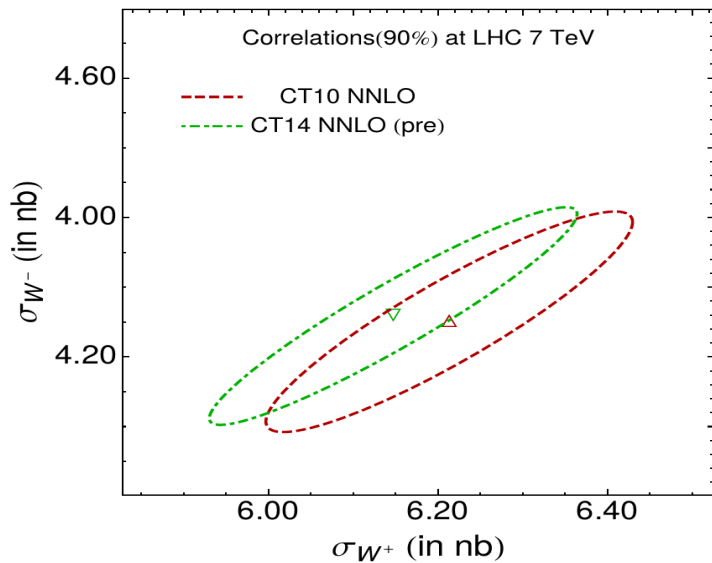
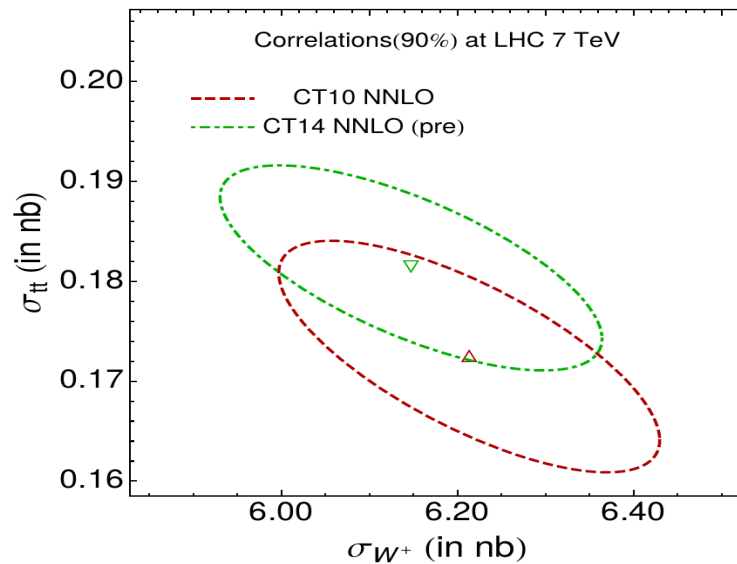
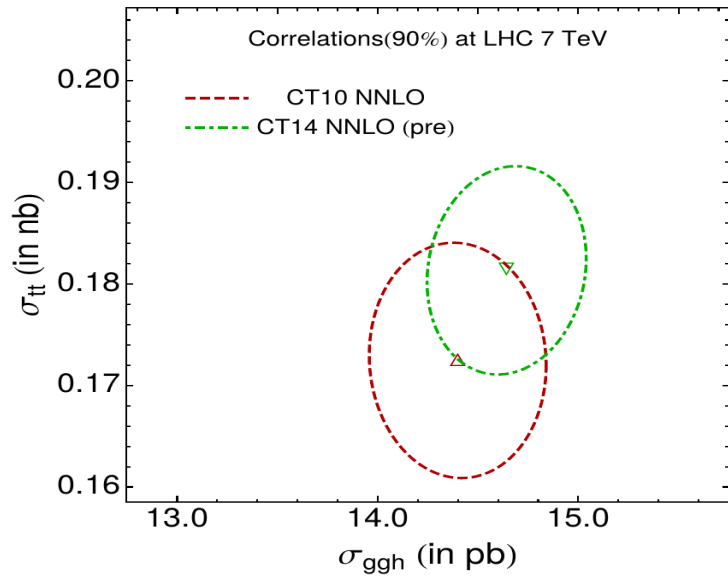
# Compare Old and New D0 W-electron charged asymmetry Data



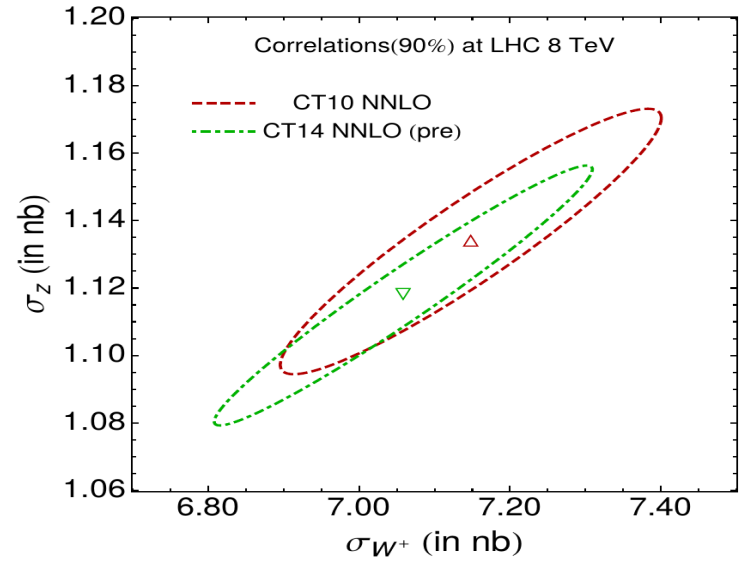
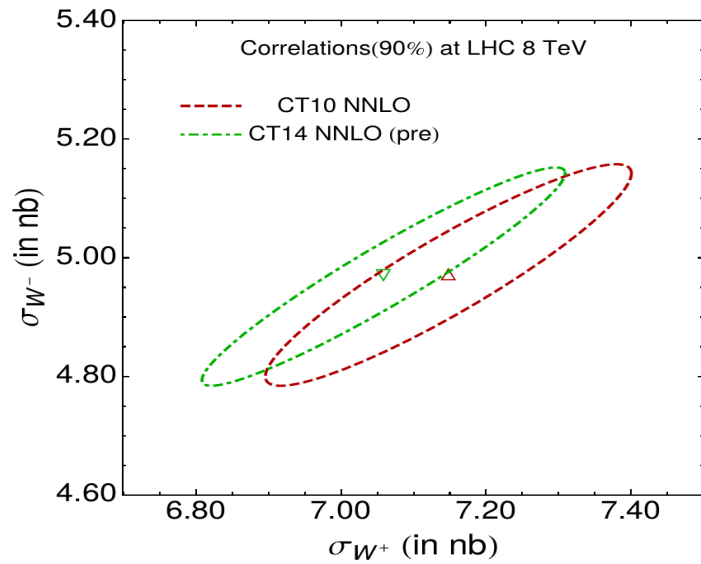
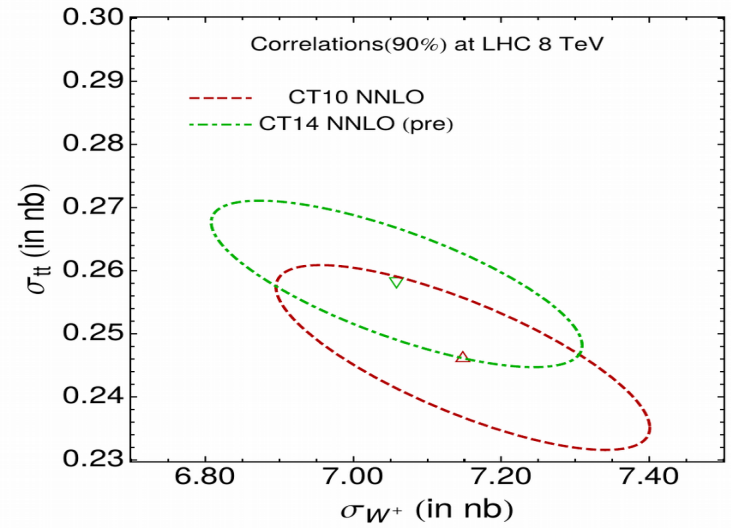
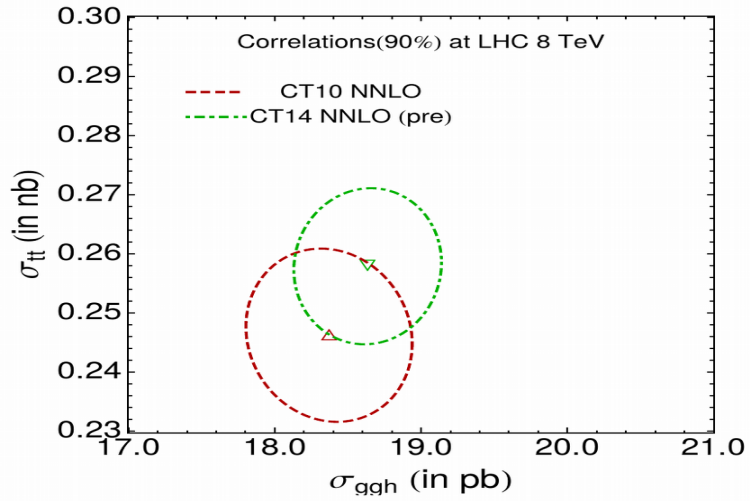
Old D0 data disfavor CTEQ6.6 and requires CT10.

New D0 data disfavor CT10 and requires CT14.

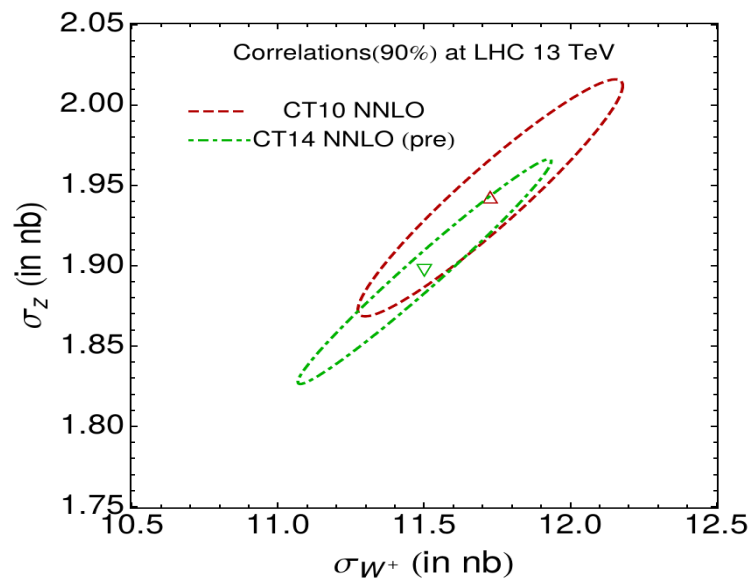
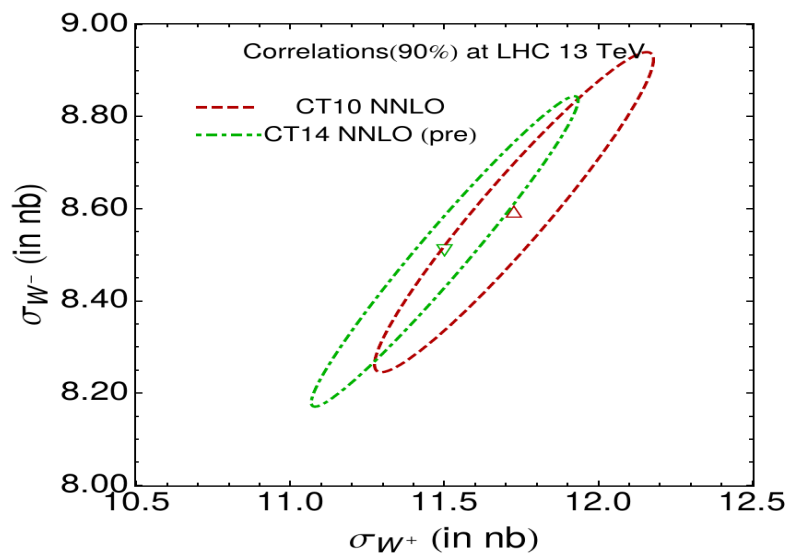
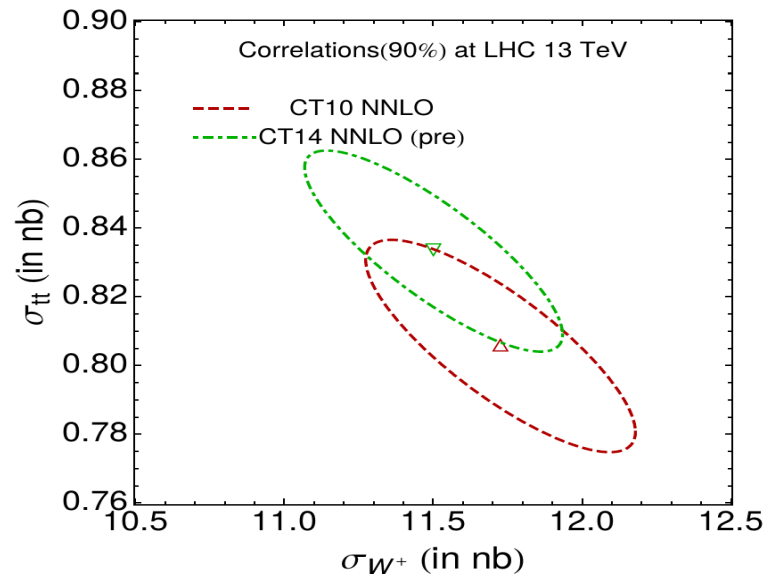
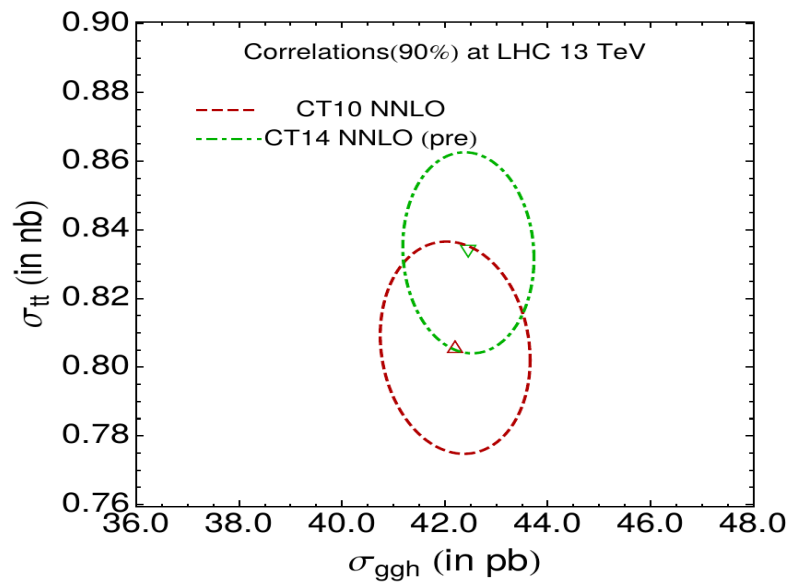
# LHC 7 TeV



# LHC 8 TeV



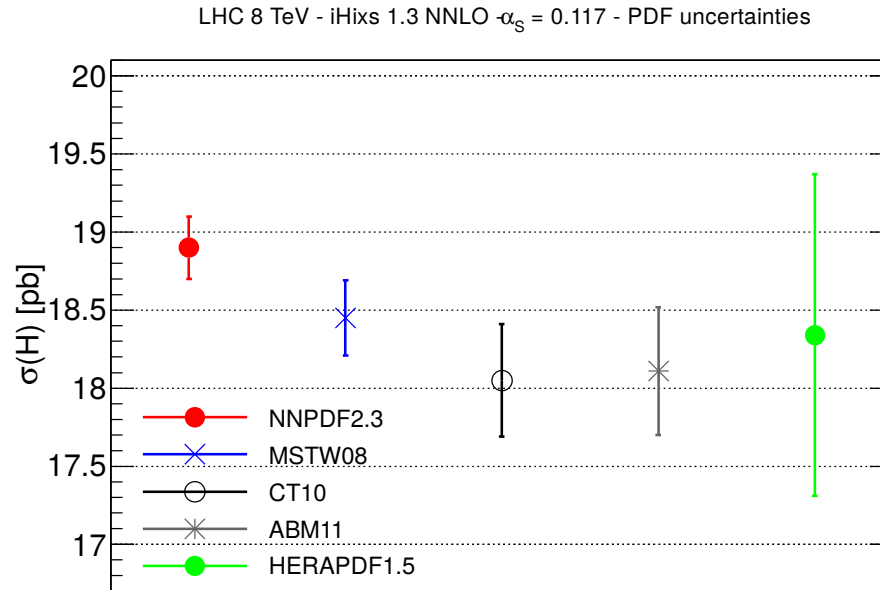
# LHC 13 TeV



PDF Benchmarking:  
compared with other  
PDF global analysis  
groups

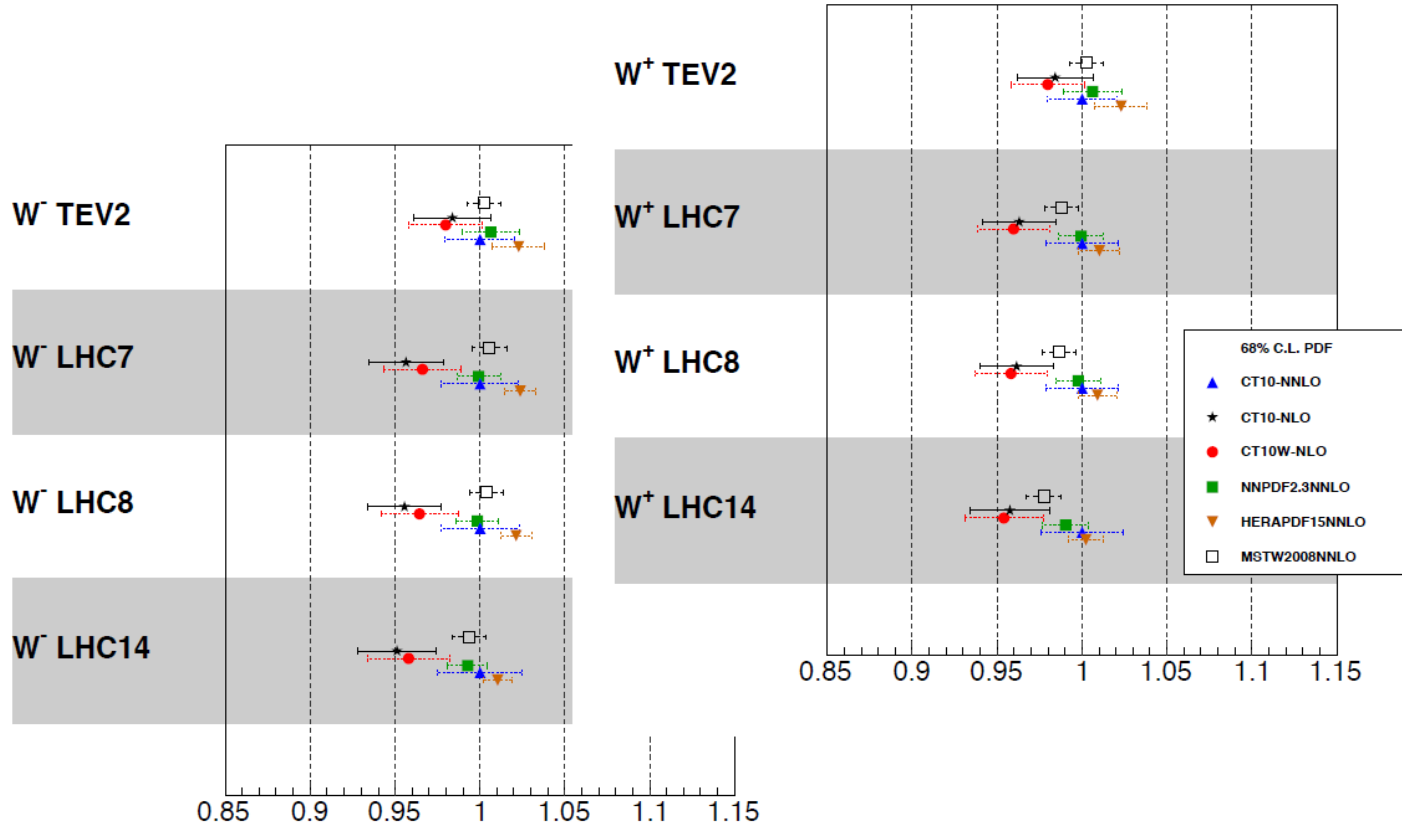
# PDF Benchmarking

gg -> H cross section



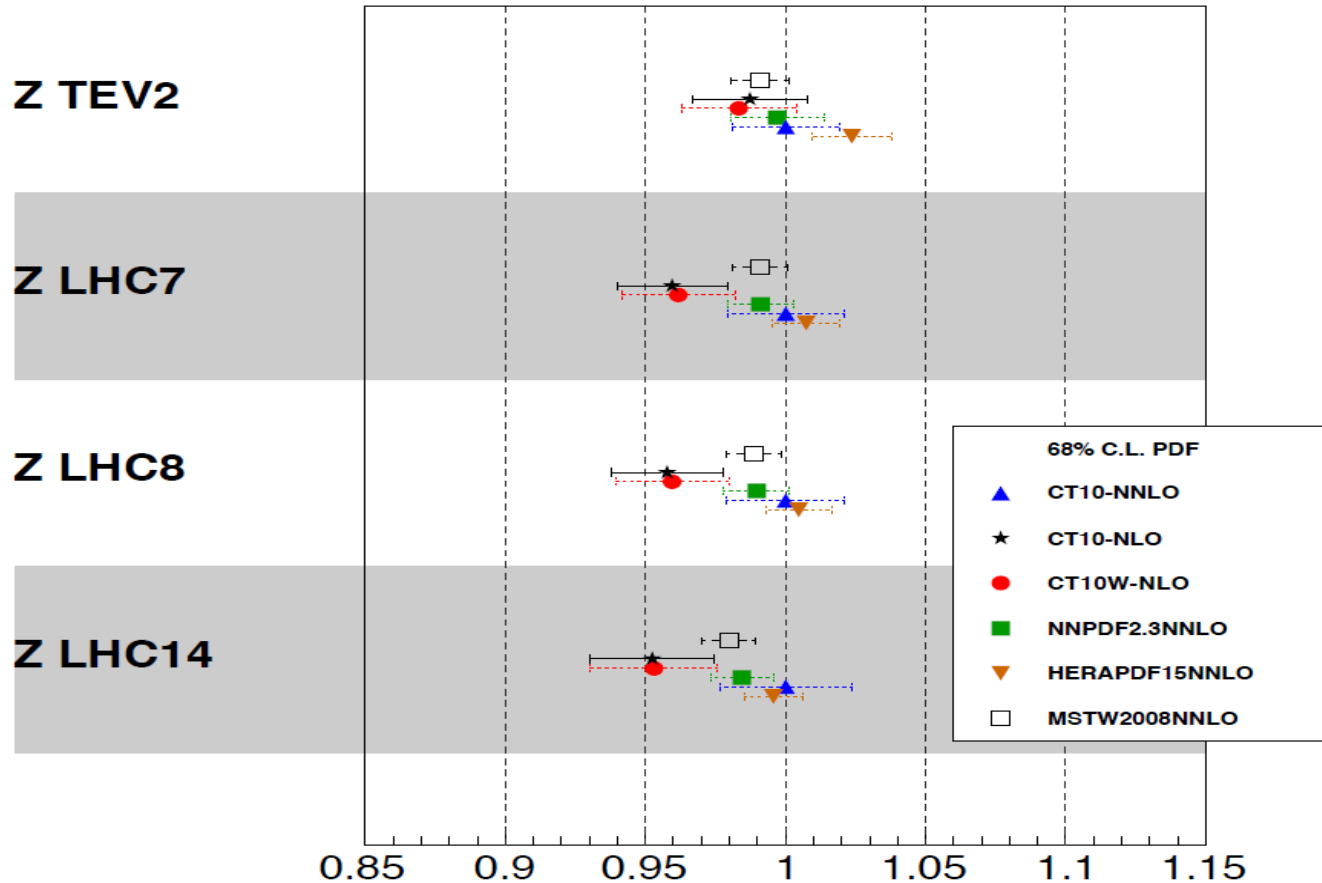
Benchmarking-  
Ongoing study to compare and  
understand differences in PDF  
predictions at LHC

# CT10-NNLO compared with other PDFs for W boson production (NNLO)



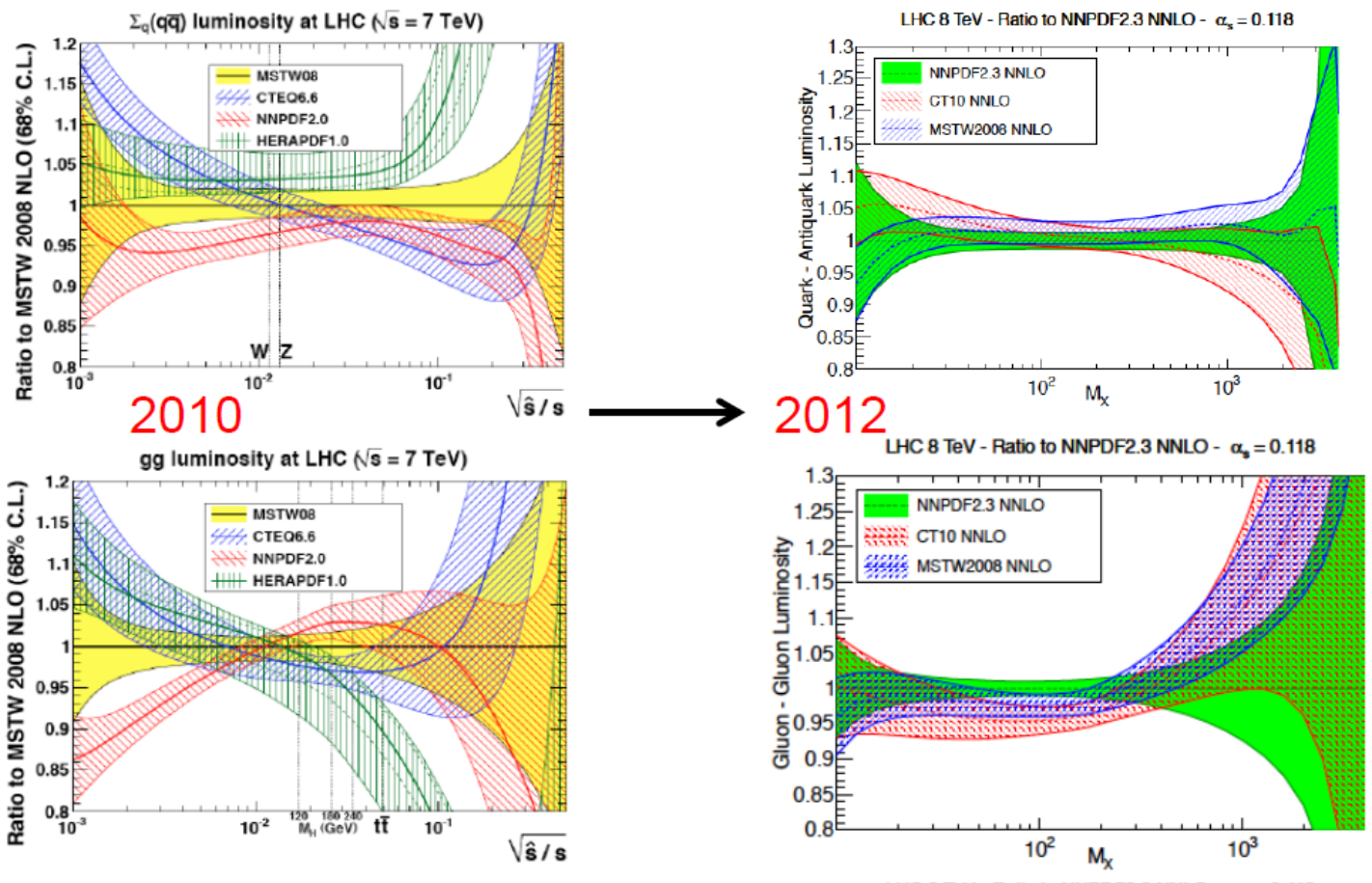


# CT10-NNLO compared with other PDFs for Z boson production (NNLO)



# Uncertainties did improve

- ...with additional data and in going from NLO to NNLO (and other theory improvements)



PDFs  
for  
Future Hadron Colliders

# Some basics about PDFs

- Parton Distribution Function  $f(x, Q)$
- Given a heavy resonance with mass  $Q$  produced at hadron collider with c.m. energy  $\sqrt{S}$
- What's the typical  $x$  value?

$$\langle x \rangle = \frac{Q}{\sqrt{S}} \quad \text{at central rapidity } (y=0)$$

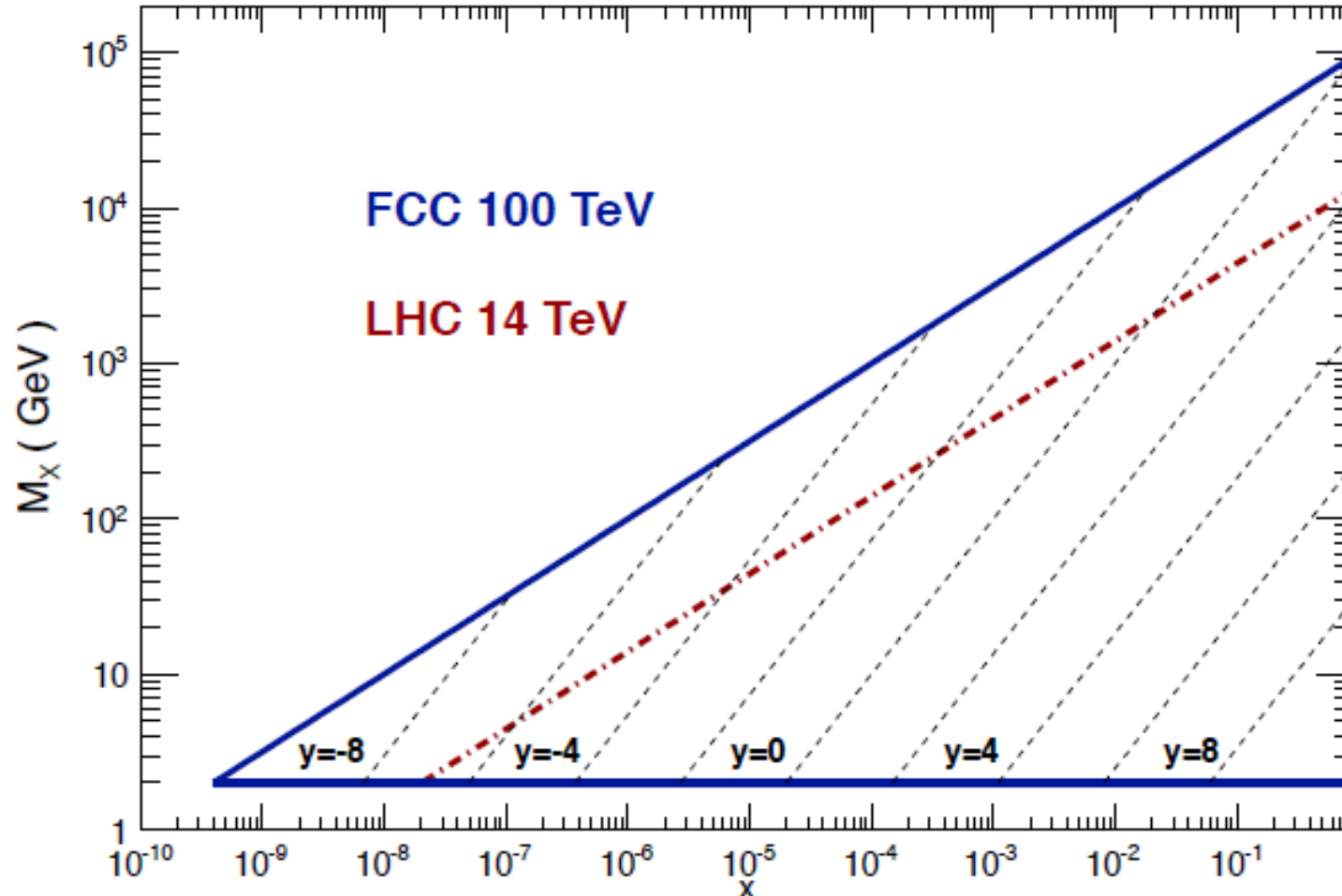
- Generally,  $x_1 = \frac{Q}{\sqrt{S}} e^y$  and  $x_2 = \frac{Q}{\sqrt{S}} e^{-y}$

$$x_1 + x_2 = 2 \frac{Q}{\sqrt{S}} \cosh(y) \quad \longrightarrow \quad y_{\max} : x_1 + x_2 = 1$$

# Kinematics of a 100 TeV SppC

Kinematics of a 100 TeV FCC

Plot by J. Rojo, Dec 2013

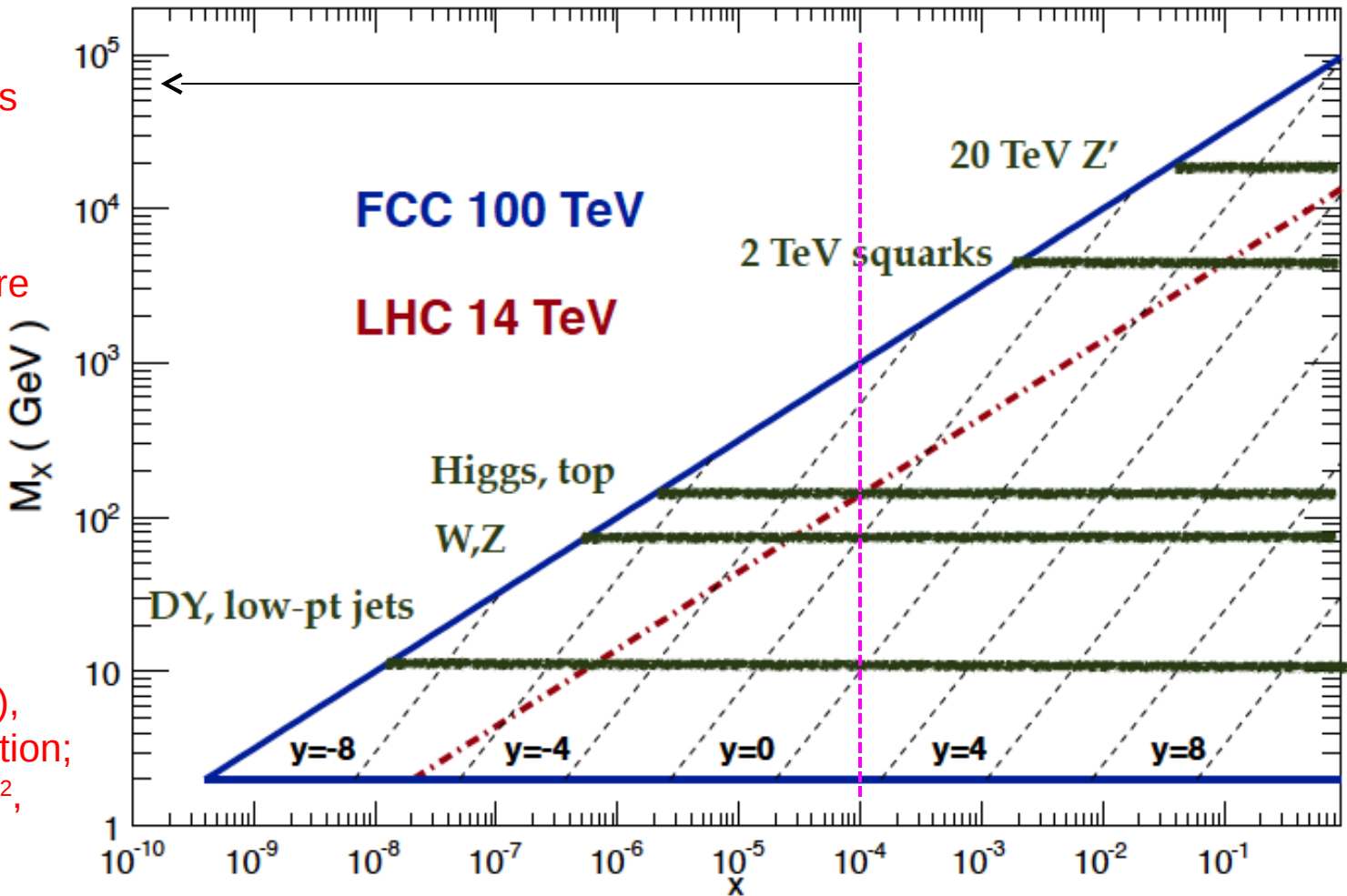


- J. Rojo: kickoff meeting for FCC at CERN, Feb. 2014

# On to a 100 TeV SppC

Kinematics of a 100 TeV FCC

Plot by J. Rojo, Dec 2013



will access smaller  $x$ , larger  $Q^2$

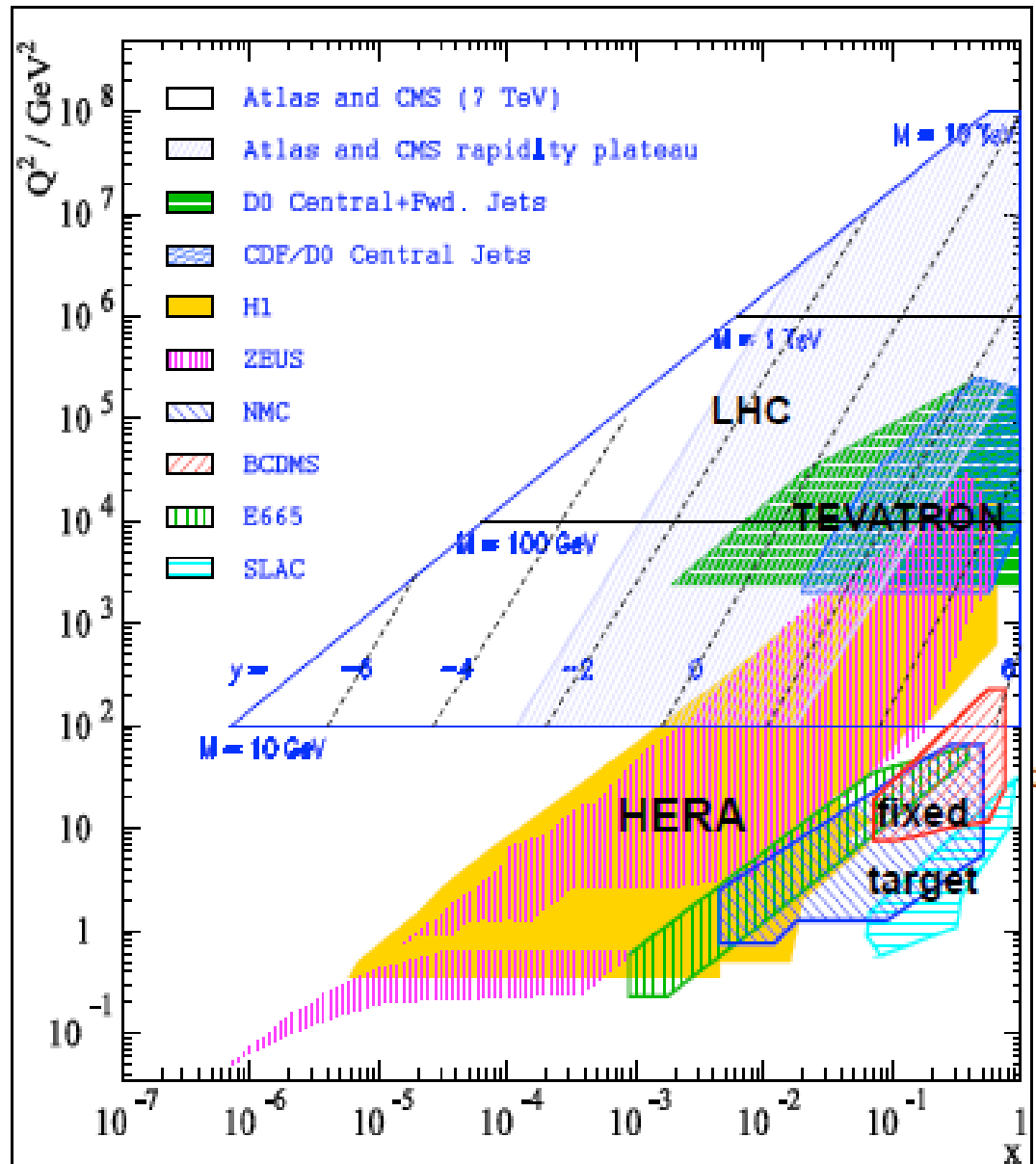
currently have no constraints on PDFs for  $x$  values below  $1E-4$

we don't know where at low  $x$ , BFKL effects start to become important

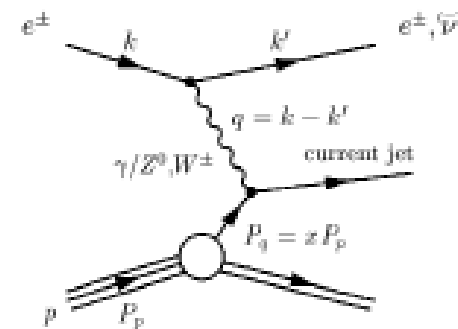
poor constraints (still) as well for high  $x$  PDFs

at high masses ( $Q^2$ ), rely on DLAP evolution; we know at large  $Q^2$ , EW effects also become important

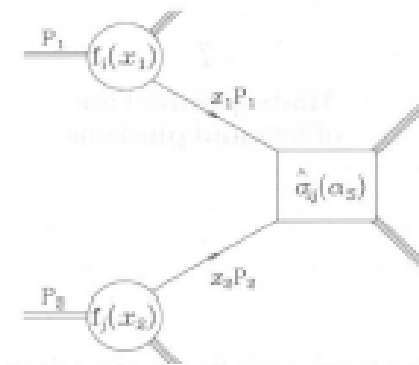
# Experimental access to the proton structure



HERA: low and medium  $x$



LHC: important constraints on  $g(x)$ ,  
flavour separation

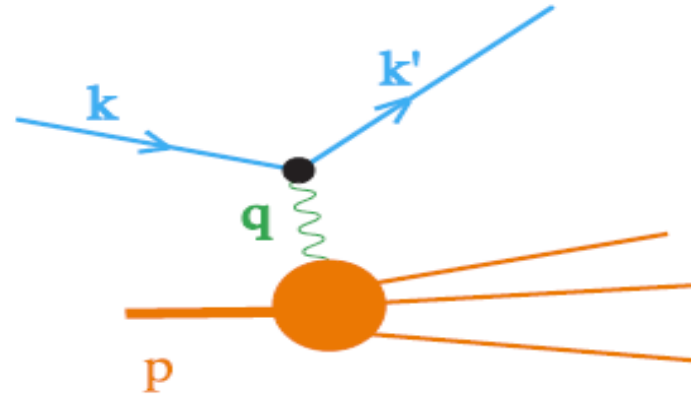


Fixed Target: high  $x$ , nuclear PDFs

# Kinematics of deep inelastic lepton scattering

$$\ell(k) + h(p) \rightarrow \ell'(k') + X.$$

$$\begin{aligned}q^\mu &= k^\mu - k'^\mu \\Q^2 &= -q^2 \\x &= \frac{Q^2}{2p \cdot q} \text{ or } A \frac{Q^2}{2p \cdot q} \\y &= \frac{p \cdot q}{p \cdot k}.\end{aligned}$$



“Deep Inelastic”  $\Rightarrow Q^2 \rightarrow \infty$ ,  $x$  fixed.

Then also  $W^2 = (p + q)^2 = m_h^2 + \frac{1-x}{x}Q^2 \rightarrow \infty$ .

Included here:  $\gamma$ ,  $W$  and  $Z$  exchanges.

In CT analysis, we imposed cuts:

$Q > 2 \text{ GeV}$  and  $W^2 > 12 \text{ GeV}^2$

e.g.,  $Q=2 \text{ GeV}$ ,  $x < 0.25$

Namely, large- $x$  data are not included in the analysis, to avoid large non-perturbative, higher twist, contributions.



# PDF luminosities

$$\sigma = \int dx_1 dx_2 g(x_1, M) g(x_2, M) \hat{\sigma}(M)$$

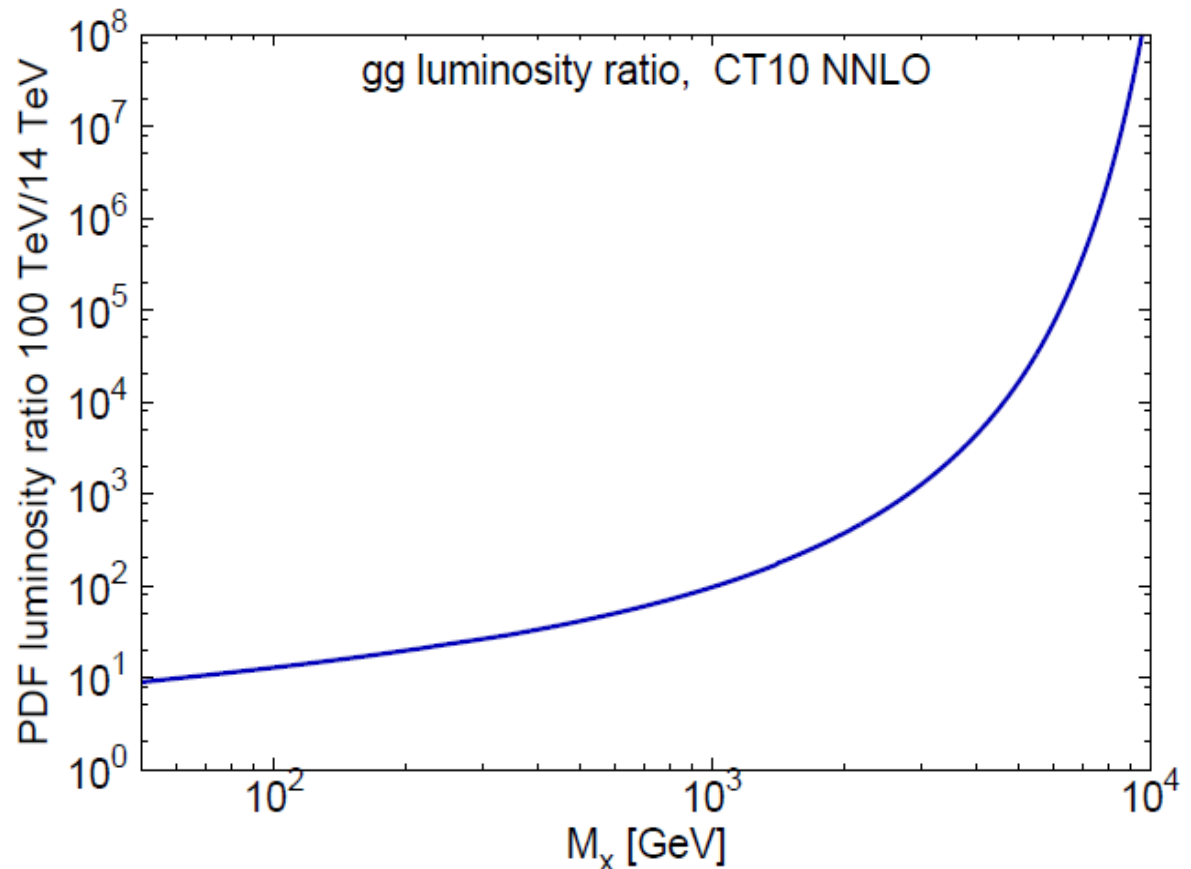
$$= \int d\tau dy g(x_1, M) g(x_2, M) \hat{\sigma}(M)$$

$$\equiv \int dM^2 \frac{dL}{dM^2} \hat{\sigma}(M)$$

PDF Luminosity

$$\tau = x_1 x_2$$

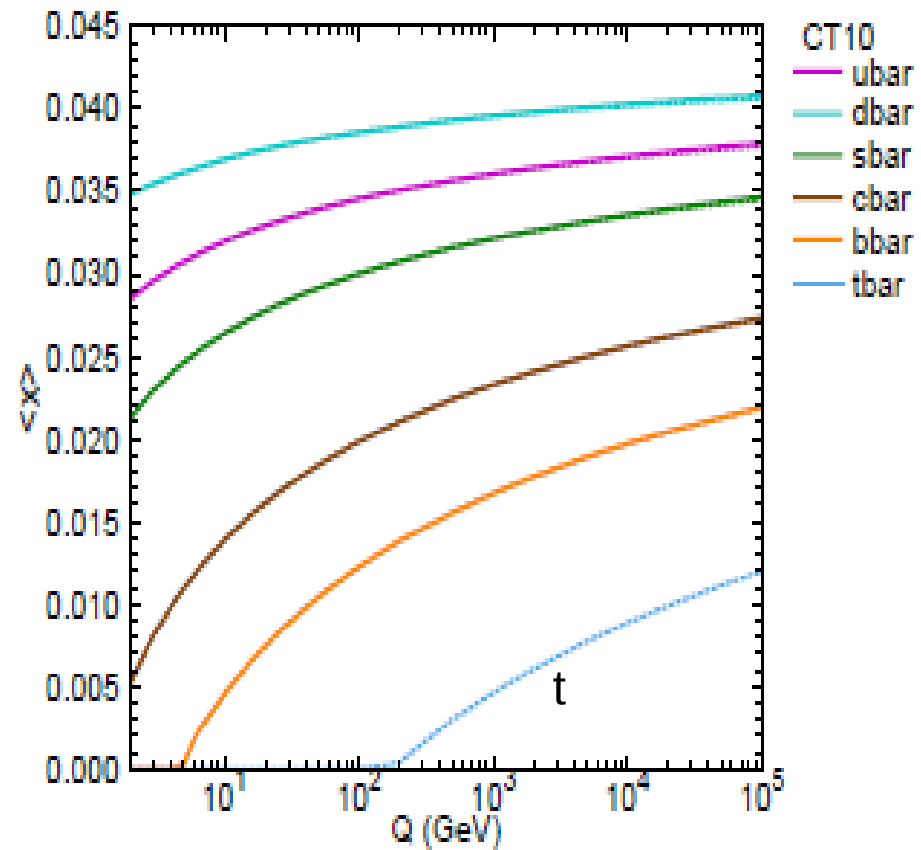
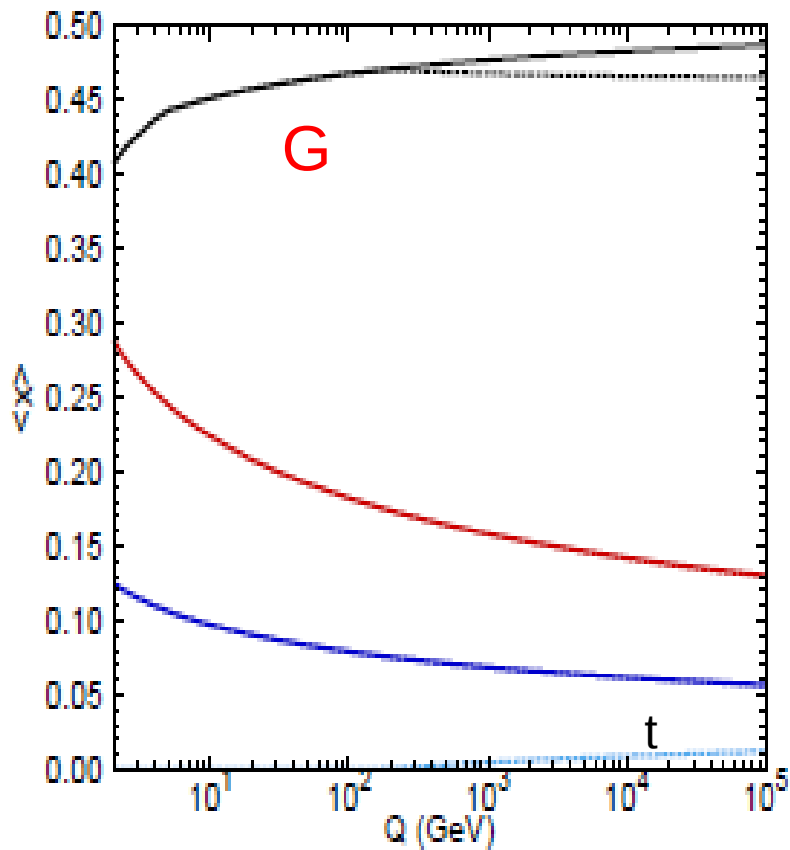
$$y = \frac{1}{2} \ln \left( \frac{x_1}{x_2} \right)$$



# Top quark as a parton

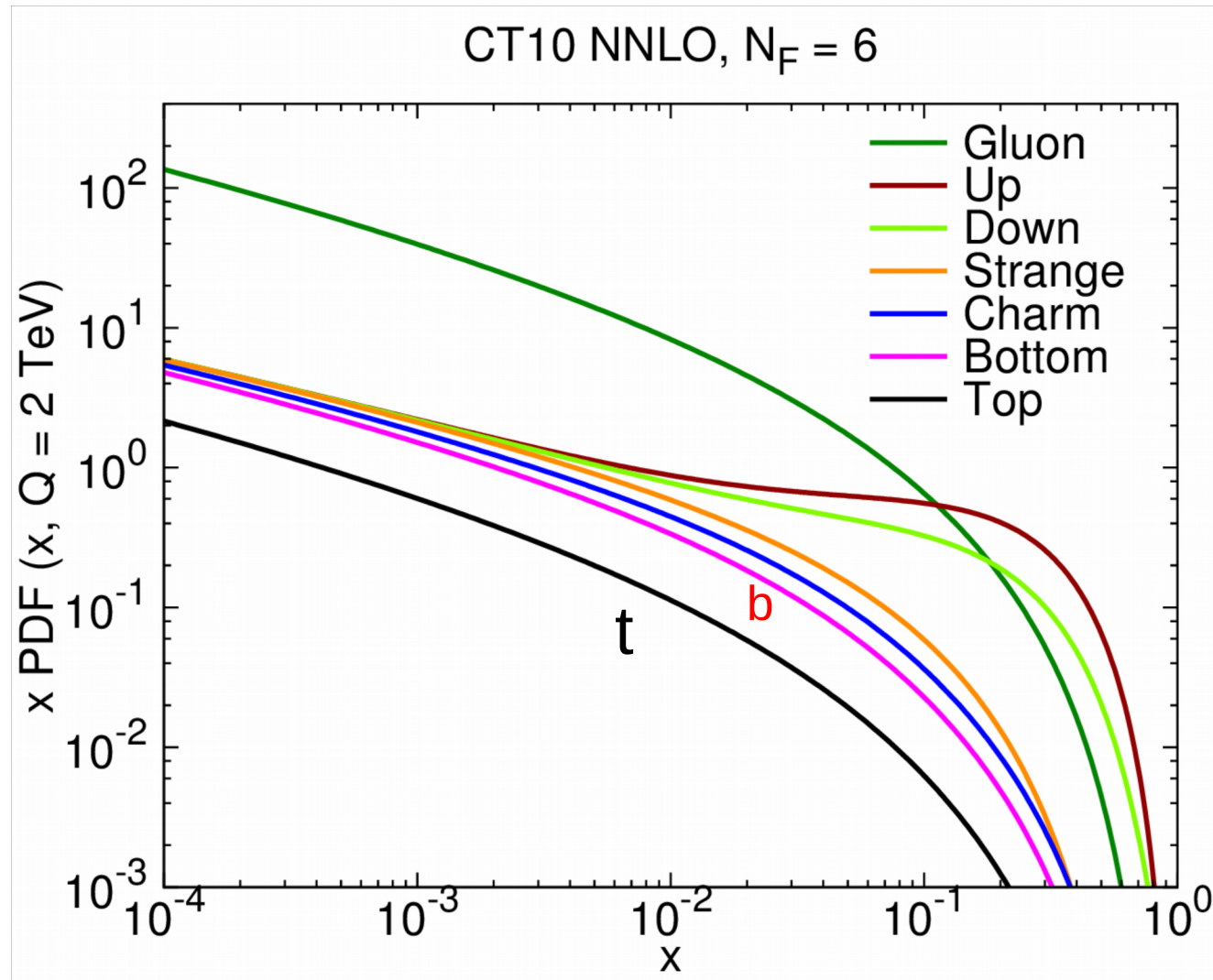
- For a 100 TeV SppC, top mass (172 GeV) can be ignored; top quark, just like bottom quark, can be a parton of proton.
- Top parton will take away some of the momentum of proton, mostly, from gluon (at NLO).
- Need to use s-ACOT scheme to calculate hard part matrix elements, to be consistent with CT10 PDFs.

# Momentum fraction inside proton



# CT10 Top PDFs

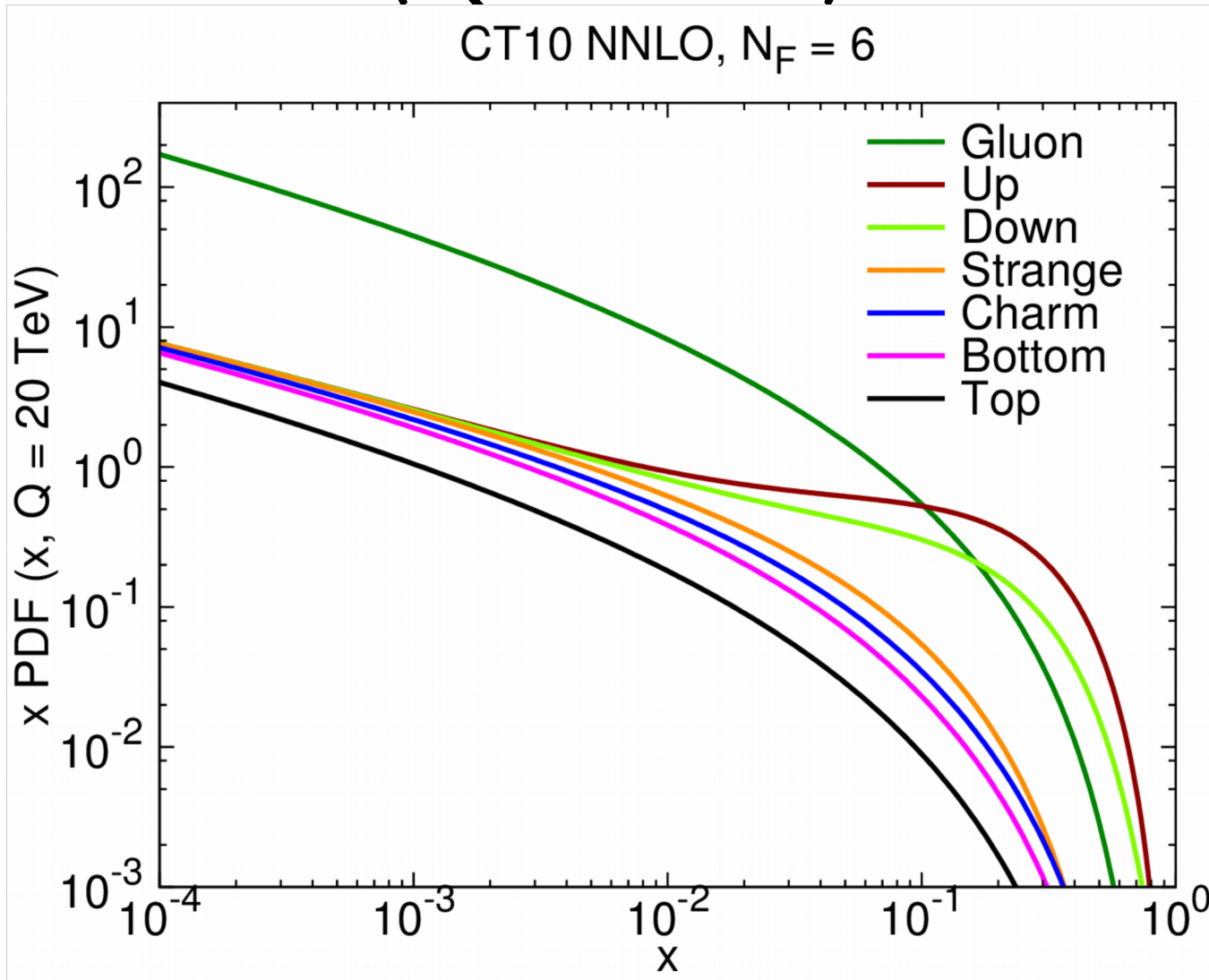
( $Q=2$  TeV)



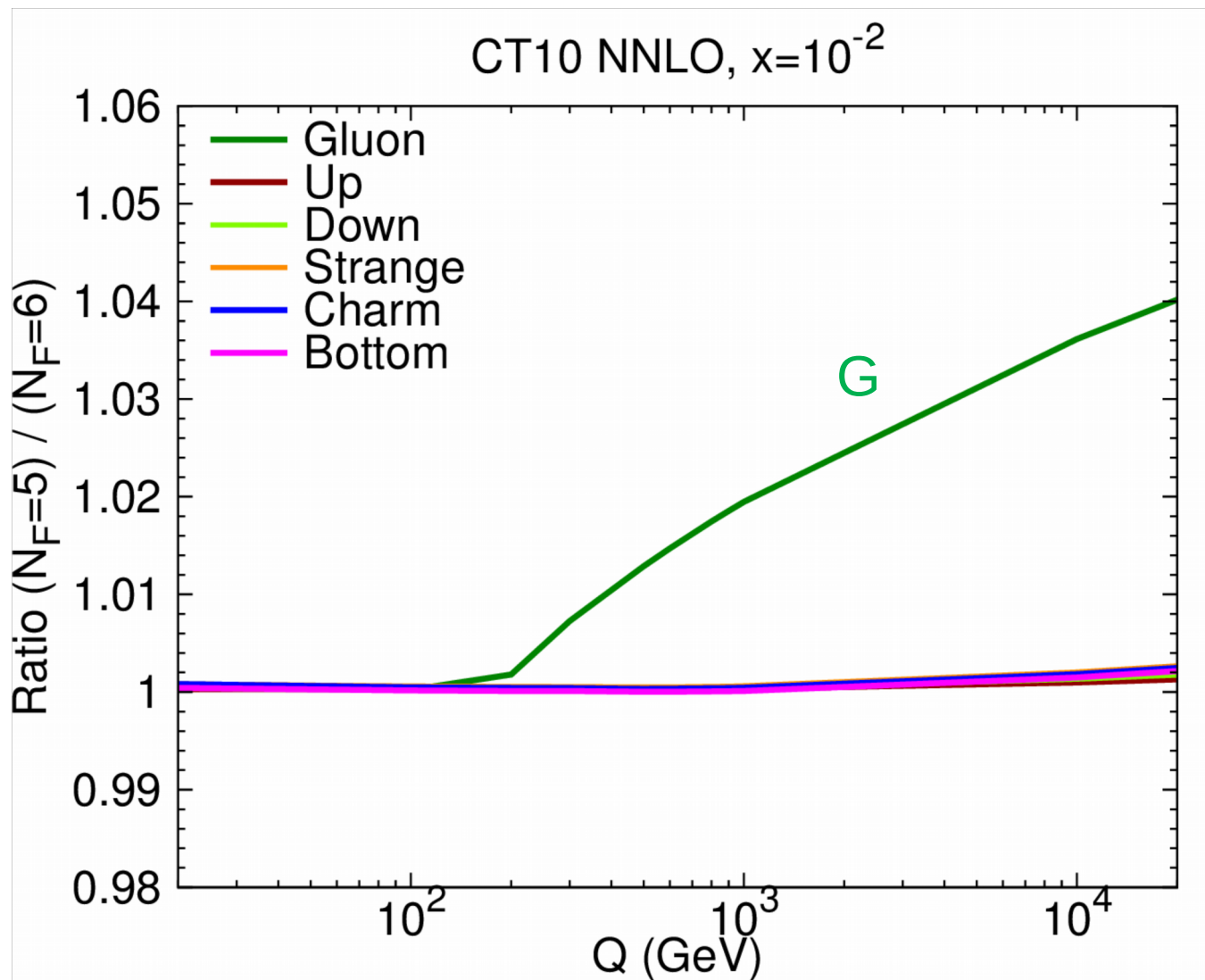
Top PDF is only a factor of 2 smaller than b PDF

# CT10 Top PDFs

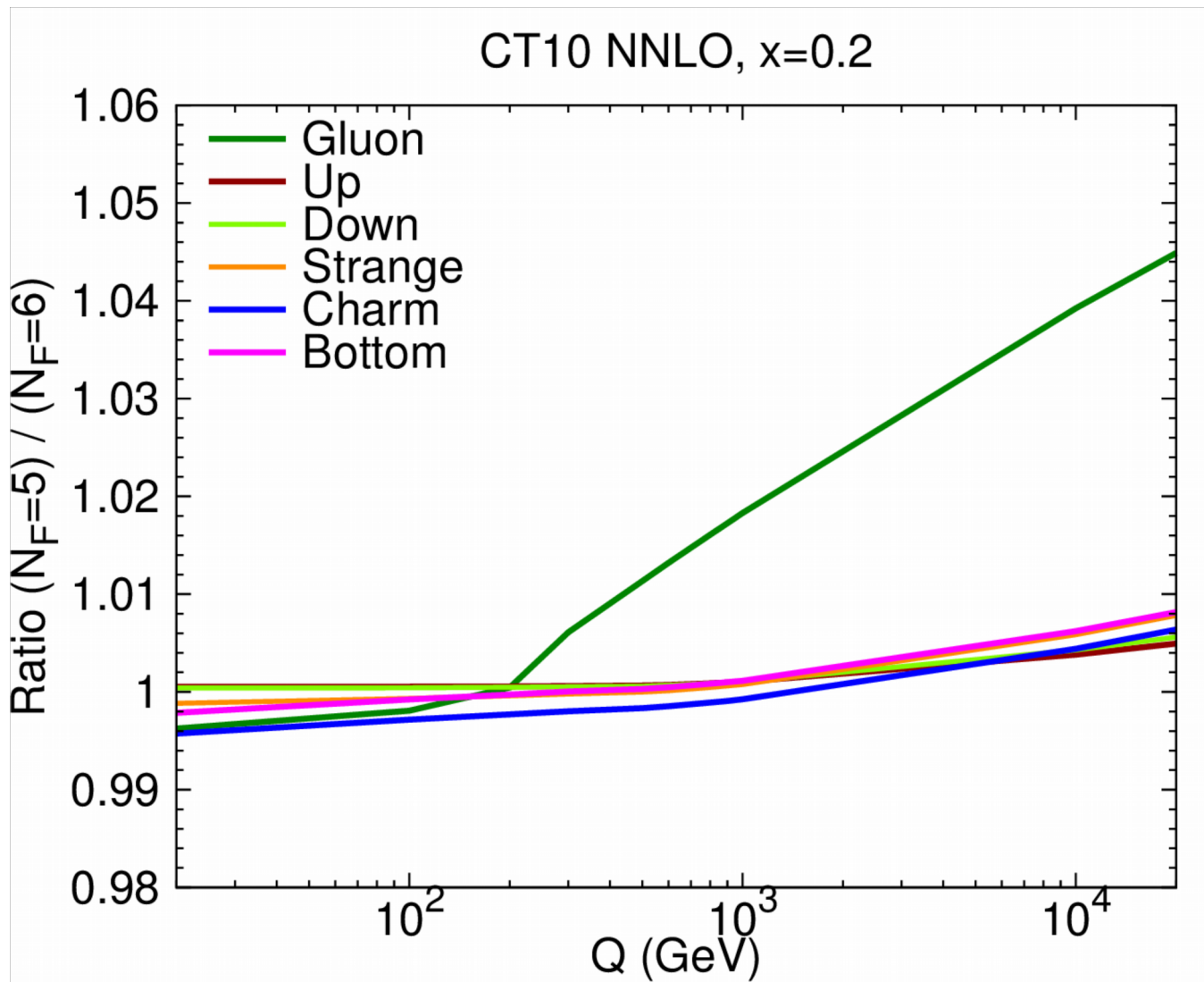
( $Q=20$  TeV)



# CT10Top PDFs

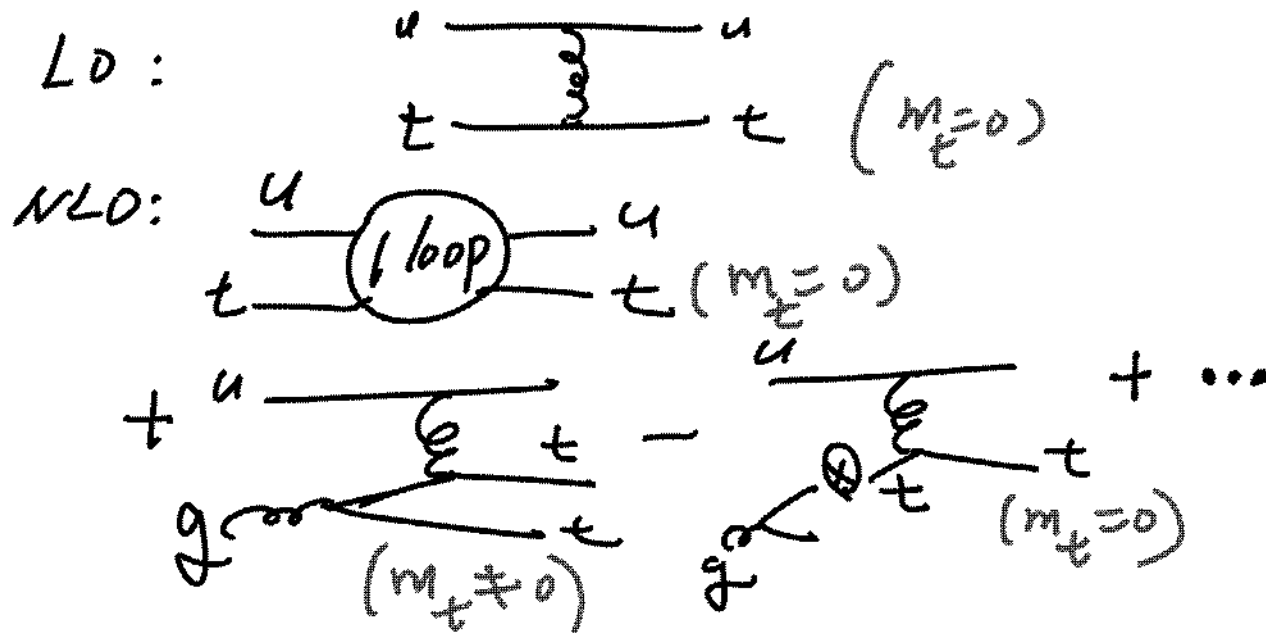


# CT10Top PDFs



# Hard part calculation

- S-ACOT scheme
- Example: single-top production





# Summary

- PDFs have larger uncertainties in both small  $x$  and large  $x$  regions.
- PDFs will be further determined by LHC data.
- In a 100TeV SppC, top quark can be a parton of proton, consistent hard part calculations are needed.