Very rare, exclusive decays of electroweak bosons as tests of the Standard Model

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erc
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An Effective Field Theory Assault on the Zeptometer Scale: Exploring the Origins of Flavor and Electroweak Symmetry Breaking

Introduction

Obtaining a rigorous control of strong-interaction phenomena in a regime where QCD is strongly coupled is still one of the main challenges to particle physics

- static properties of hadrons: lattice QCD
- inclusive processes such as $e^+e^-\to$hadrons, $B\to Xl\nu$: quark-hadron duality & local operator-product expansion
- deep-inelastic scattering, collider physics: factorization into partonic cross sections convoluted with **parton distribution functions**
- hard exclusive processes with individual final-state hadrons: **QCD factorization approach**, factorization into partonic rates convoluted with **light-cone distribution amplitudes** (LCDAs)

  Brodsky, Lepage (1979); Efremov, Radyushkin (1980)
- later generalized to the more complicated case of non-leptonic decays of B mesons

  Beneke, Buchalla, MN, Sachrajda (1999)

Introduction

All existing applications of QCD factorization suffer from the fact that the characteristic energy scales are not sufficiently large for power corrections to be negligible

- notoriously difficult to disentangle Λ_{QCD}/Q power corrections from uncertainties related to the LCDAs
- in many cases the power corrections are not well understood, e.g. because they are related to divergent convolution integrals
- this introduces poorly known model parameters and makes phenomenological predictions less precise
- no comprehensive program to determine the leading-twist LCDAs of the ground-state mesons and baryons can be devised

Introduction

In this work, we propose to use **exclusive radiative decays of Z and W bosons** into final states containing a single meson as a laboratory to test and study the QCD factorization approach in a context where power corrections are definitely under control

Price to pay is that the higher the energy release in the process, the smaller the probability for any particular final state is

The **enormous rates** of electroweak gauge bosons that will become available at future, high-luminosity machines (including large circular colliders) present us with a **new playground for precision electroweak and QCD physics**, which will make such studies possible:

- high-luminosity LHC (3000 fb^{-1}): ~10^{11} Z boson and ~$5\cdot10^{11}$ W bosons
- ILC or TLEP, dedicated run at Z pole: ~10^{12} Z boson per year
- large samples of W bosons in dedicated runs at WW or $t\bar{t}$ thresholds

Mangano, Melia (2014)

Introduction

Our work is motivated by recent investigations of exclusive Higgs decays $h \to V\gamma$, which were proposed as a way to probe for non-standard Yukawa couplings of the Higgs boson, both diagonal and non-diagonal ones

Isidori, Manohar, Trott (2013)
Bodwin, Petriello, Stoynev, Velasco (2013)
Kagan *et al.* (2014); Bodwin *et al.* (2014)

Such measurements are extremely challenging at the LHC and future colliders

Observing exclusive radiative decays of Z and W bosons would provide a proof-of-principle that such kind of searches can be performed

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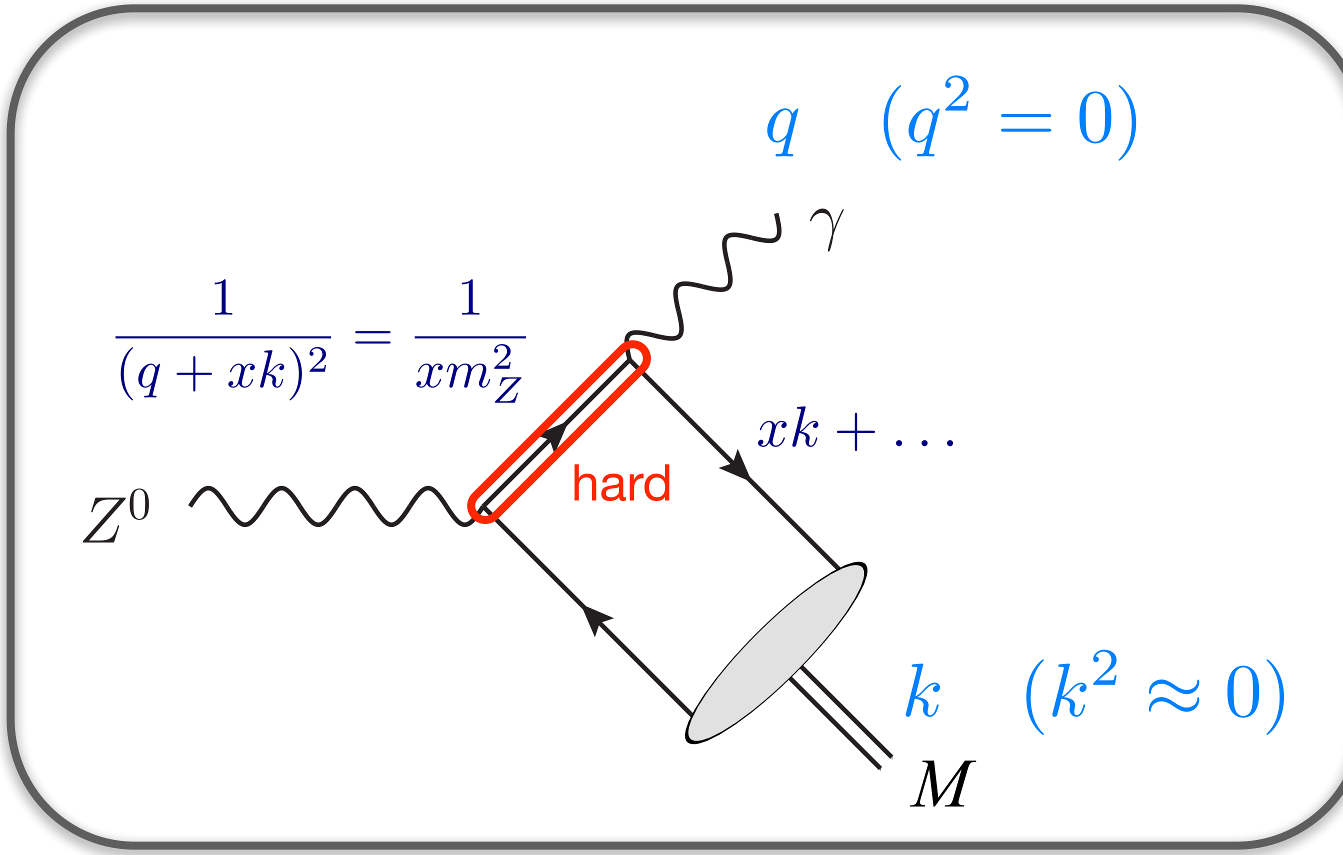
Based on:
"Exclusive radiative decays of W and Z bosons in QCD factorization"
Yuval Grossmann, Matthias König, MN (arXiv:1501.06569) — today!

Physical picture: Exclusive Z→Mγ decays

- the intermediate propagator is highly virtual ($q^2 \sim m_Z^2$) and can be "integrated out", giving rise to a **hard function** H(x)
- field operators for the external quark (and gluon) fields can be separated by **light-like distances**, since $k^2 \approx 0$
- soft-collinear effective theory (SCET) can be used to perform a systematic expansion of the decay amplitude in powers of $\lambda \sim \Lambda_{\rm QCD}/E_M$
- this **factorizes** short-distance physics (hard function) from long-distance hadronic dynamics



Bauer, Pirjol, Stewart (2001); Bauer *et al.* (2002)
Beneke, Chapovsky, Diehl, Feldmann (2002)

QCD factorization formula in SCET

At leading power in an expansion in $\Lambda_{\mathrm{QCD}}/m_Z$ only two operators are allowed, containing either a collinear quark-antiquark bilinear or a two-gluon bilinear, which only contributes to final states such as η or η’

For all other mesons, we find the **QCD factorization theorem**:

$$\mathcal{A} = -i f_M E \int_0^1 dx\, H_M(x,\mu)\, \phi_M(x,\mu) + \text{power corrections}$$

hard function:
calculable in PT

decay constant:
extractable from data

LCDA:
non-perturbative hadronic physics

Meson decay constants

Decay constants are the amplitudes for producing a meson out of the vacuum via a local current:

$$\langle P(k)|\, \bar{q}_1 \gamma^\mu \gamma_5 q_2 \,|0\rangle = -i f_P k^\mu \qquad \langle V(k,\varepsilon_V)|\, \bar{q}_1 \gamma^\mu q_2 \,|0\rangle = -i f_V m_V \varepsilon_V^{*\mu}$$

Meson M	f_M [MeV]
π	130.4 ± 0.2
K	156.2 ± 0.7
ρ	212 ± 4
ω	187 ± 5
K^*	203 ± 6
ϕ	210 ± 5

Meson M	f_M [MeV]
D	204.6 ± 5.0
D_s	257.5 ± 4.6
B	186 ± 9
B_s	224 ± 10
J/ψ	403 ± 5
$\Upsilon(1S)$	684 ± 5
$\Upsilon(4S)$	326 ± 17

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lattice QCD

Light-cone distribution amplitudes (LCDAs)

Momentum distribution of partons in a given Fock state of a meson(quark-antiquark, quark-antiquark-gluon, …):

$$\langle M(k)|\, \bar{q}(t\bar{n})\, \frac{\bar{\not{n}}}{2}\, (\gamma_5)\, [t\bar{n}, 0]\, q(0)|0\rangle = -i f_M E \int_0^1 dx\, e^{ixt\bar{n}\cdot k}\, \phi_M(x,\mu)$$

Expansion in **Gegenbauer polynomials** (diagonalizes evolution at LO):

$$\boxed{\phi_M(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu)\, C_n^{(3/2)}(2x-1)\right]}$$

- Gegenbauer moments fall off faster than 1/n for large n
- odd moments are SU(3)-violating effects
- all moments $a_n^M(\mu) \to 0$ (except $a_0^M \equiv 1$) in the limit $\mu \to \infty$

Light-cone distribution amplitudes (LCDAs)

Model predictions based on **QCD sum rules** & **lattice QCD** ($\mu_0 = 1\,\mathrm{GeV}$):

Ball, Braun (1996); Ball *et al.* (2006, 2007)
Arthur *et al.* (2010)

Meson M	f_M [MeV]	$a_1^M(\mu_0)$	$a_2^M(\mu_0)$
π	130.4 ± 0.2	0	0.29 ± 0.08
K	156.2 ± 0.7	-0.07 ± 0.04	0.24 ± 0.08
ρ	212 ± 4	0	0.17 ± 0.07
ω	187 ± 5	0	0.15 ± 0.12
K^*	203 ± 6	-0.06 ± 0.04	0.16 ± 0.09
ϕ	210 ± 5	0	0.23 ± 0.08

Model estimate suggest than higher moments (n=6 and higher) for light mesons are tiny; will use $a_4^M(\mu_0) \in [-0.15, 0.15]$ to estimate such effects

Bakulev, Passek-Kumericki, Schroers, Stefanis (2001)
Bakulev, Mikhailov, Stefanis (2003)

Light-cone distribution amplitudes (LCDAs)

Heavy quarkonia:

NRQCD matrix element
Braguta, Likhoded, Luchinsky (2006)

$$\int_0^1 dx\,(2x-1)^2\,\phi_M(x,\mu_0) = \frac{\langle v^2\rangle_M}{3} + \mathcal{O}(v^4)$$

- simple model function:

$$\phi_M(x,\mu_0) = N_\sigma\,\frac{4x(1-x)}{\sqrt{2\pi}\sigma}\,\exp\left[-\frac{(x-\frac{1}{2})^2}{2\sigma^2}\right]; \qquad \sigma^2 = \frac{\langle v^2\rangle_M}{12}$$

Heavy-light mesons:

$$\int_0^1 dx\,\frac{\phi_M(x,\mu_0)}{x} \equiv \frac{m_M}{\lambda_M(\mu_0)} + \dots$$

HQET matrix element
Grozin, MN (1996)

- simple mode function:

$$\phi_M(x,\mu_0) = N_\sigma\,\frac{x(1-x)}{\sigma^2}\,\exp\left(-\frac{x}{\sigma}\right); \qquad \sigma = \frac{\lambda_M(\mu_0)}{m_M}$$

Light-cone distribution amplitudes (LCDAs)

Input parameters for heavy mesons:

Meson M	f_M [MeV]	λ_M [MeV]	$\langle v^2 \rangle$	σ
D	204.6 ± 5.0	460 ± 110	–	0.246 ± 0.059
D_s	257.5 ± 4.6	550 ± 150	–	0.279 ± 0.076
B	186 ± 9	460 ± 110	–	0.087 ± 0.021
B_s	224 ± 10	550 ± 150	–	0.102 ± 0.028
J/ψ	403 ± 5	–	0.30 ± 0.15	0.158 ± 0.040
$\Upsilon(1S)$	684 ± 5	–	0.10 ± 0.05	0.091 ± 0.023
$\Upsilon(4S)$	326 ± 17	–	0.10 ± 0.05	0.091 ± 0.023

- first n~1/σ Gegenbauer moments are important for heavy mesons

RG evolution effects

RG evolution from μ_0 up to the **electroweak scale** changes the shapes of the LCDAs significantly, as they approach closer to the asymptotic form $\phi_M(x, \mu \to \infty) = 6x(1-x)$

positive and increasing with n

Evolution of moments:

$$a_n^M(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_n/2\beta_0} a_n^M(\mu_0)$$

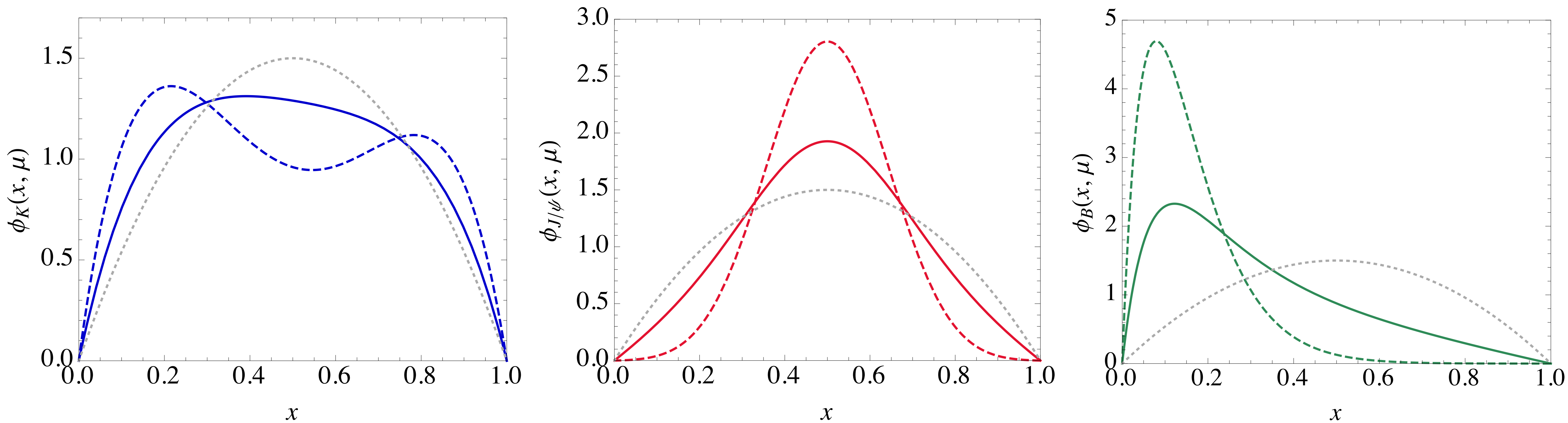

Figure 3: RG evolution of the LCDAs of the kaon (left), the J/ψ meson (middle) and the B meson (right) from a low scale $\mu_0 = 1\,\mathrm{GeV}$ (dashed lines) to a high scale $\mu = m_Z$ (solid lines). The dotted grey line shows the asymptotic form $6x(1-x)$ for comparison.

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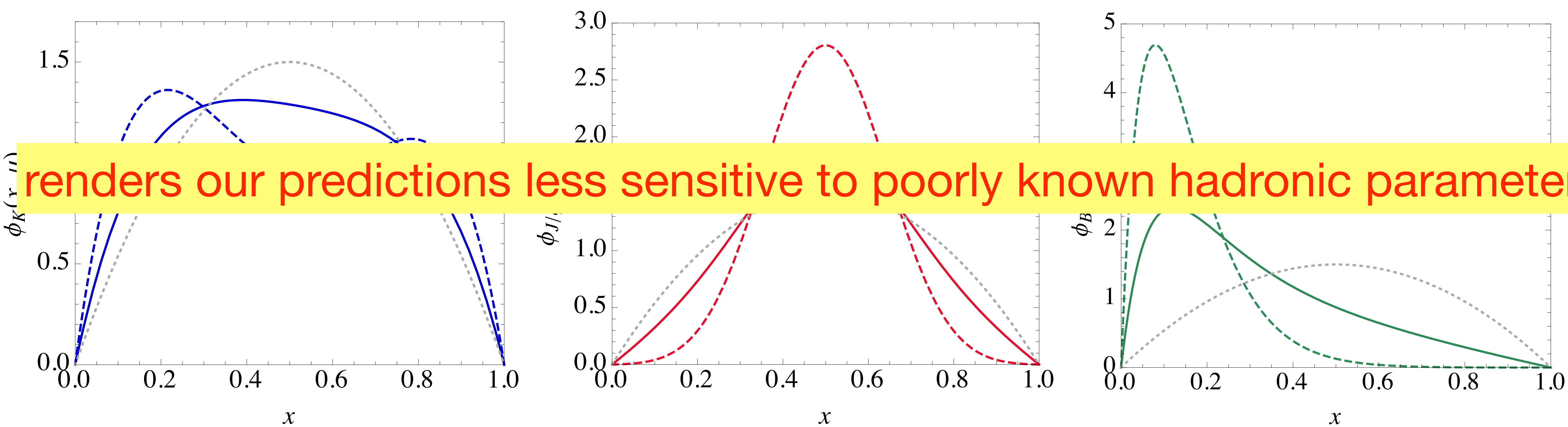

renders our predictions less sensitive to poorly known hadronic parameters

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Exclusive radiative decays $Z \to M\gamma$

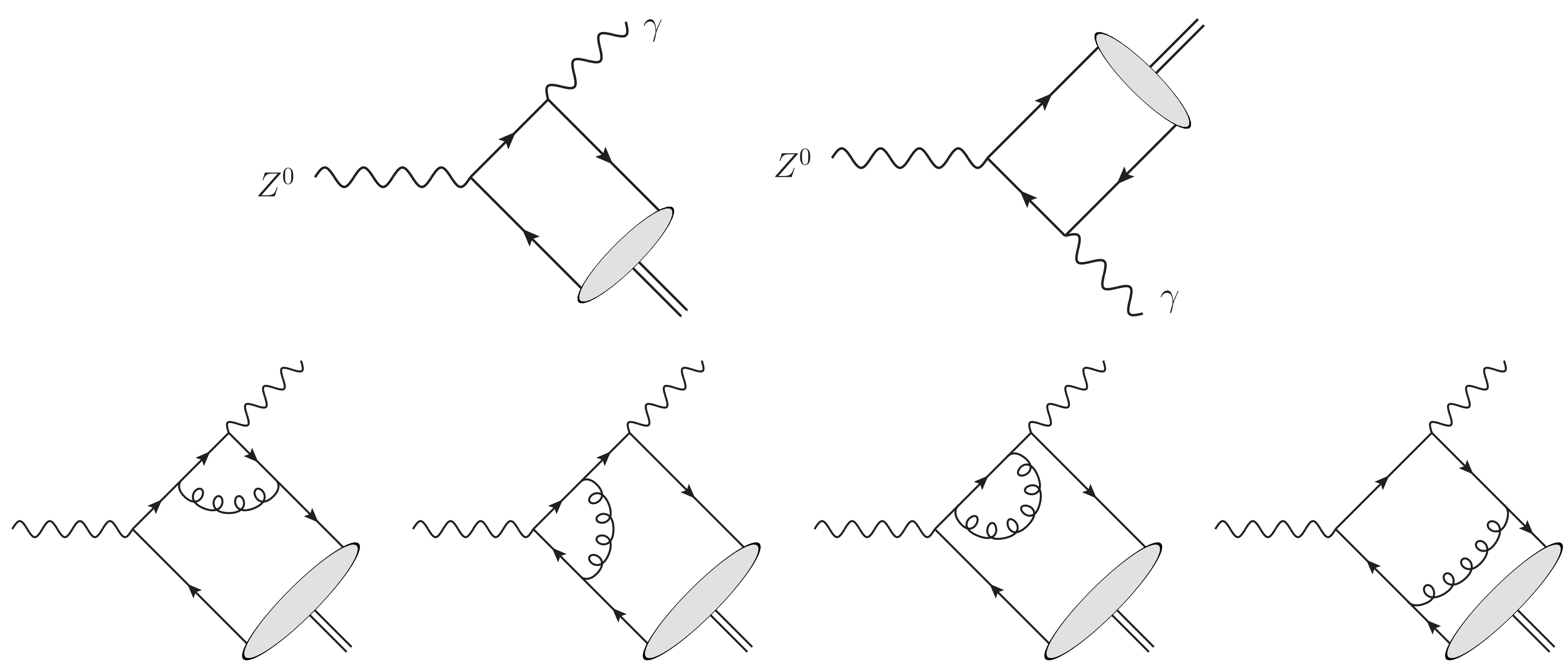
Factorization of the decay amplitude

Form-factor decomposition of the decay amplitude:

$$iA(Z \to M\gamma) = \pm \frac{egf_M}{2\cos\theta_W}\left[i\epsilon_{\mu\nu\alpha\beta}\,\frac{k^\mu q^\nu \varepsilon_Z^\alpha\,\varepsilon_\gamma^{*\beta}}{k\cdot q}\,F_1^M - \left(\varepsilon_Z\cdot\varepsilon_\gamma^* - \frac{q\cdot\varepsilon_Z\,k\cdot\varepsilon_\gamma^*}{k\cdot q}\right)F_2^M\right]$$

At leading power, the Z-boson (and the photon) have transverse polarization, while a final-state vector meson is longitudinally polarized

Diagrams at LO and NLO:



Factorization of the decay amplitude

Form factors are related to overlap integrals of hard functions with LCDAs:

$$F_1^M = \frac{\mathcal{Q}_M}{6}\left[I_+^M(m_Z) + \bar{I}_+^M(m_Z)\right] \qquad F_2^M = \frac{\mathcal{Q}'_M}{6}\left[I_-^M(m_Z) - \bar{I}_-^M(m_Z)\right]$$

depend on quark electric charges and Z-boson couplings

Master integrals:

$$I_\pm^M(m_V) = \int_0^1 dx\, H_\pm(x, m_V, \mu)\, \phi_M(x, \mu)$$

$$\bar{I}_\pm^M(m_V) = \int_0^1 dx\, H_\pm(1-x, m_V, \mu)\, \phi_M(x, \mu)$$

Hard functions:

$$H_\pm(x, m_V, \mu) = \frac{1}{x}\left[1 + \frac{C_F\alpha_s(\mu)}{4\pi}\, h_\pm(x, m_V, \mu) + \mathcal{O}(\alpha_s^2)\right]$$

with:

$$h_\pm(x, m_V, \mu) = (2\ln x + 3)\left(\boxed{\ln\frac{m_V^2}{\mu^2}} - i\pi\right) + \ln^2 x - 9 + (\pm 1 - 2)\,\frac{x\ln x}{1-x}$$

Factorization of the decay amplitude

Form factors expressed in terms of Gegenbauer moments:

$$F_1^M = \frac{\mathcal{Q}_M}{6}\left[I_+^M(m_Z) + \bar{I}_+^M(m_Z)\right] = \quad \mathcal{Q}_M \sum_{n=0}^{\infty} C_{2n}^{(+)}(m_Z,\mu)\, a_{2n}^M(\mu)$$ even moments

$$F_2^M = \frac{\mathcal{Q}'_M}{6}\left[I_-^M(m_Z) - \bar{I}_-^M(m_Z)\right] = -\mathcal{Q}'_M \sum_{n=0}^{\infty} C_{2n+1}^{(-)}(m_Z,\mu)\, a_{2n+1}^M(\mu)$$ odd moments

Hard functions in moment space:

$$C_n^{(\pm)}(m_V,\mu) = 1 + \frac{C_F\alpha_s(\mu)}{4\pi}\, c_n^{(\pm)}\left(\frac{m_V}{\mu}\right) + \mathcal{O}(\alpha_s^2)$$

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with:

$$c_n^{(\pm)}\left(\frac{m_V}{\mu}\right) = \left[\frac{2}{(n+1)(n+2)} - 4H_{n+1} + 3\right]\left(\boxed{\ln\frac{m_V^2}{\mu^2}} - i\pi\right) + 4H_{n+1}^2 - \frac{4(H_{n+1}-1)\pm 1}{(n+1)(n+2)} + \frac{2}{(n+1)^2(n+2)^2} - 9$$

→ large logs are resummed to all orders by choosing $\mu \sim m_Z$

Factorization of the decay amplitude

For flavor-diagonal neutral mesons all odd moments vanish:

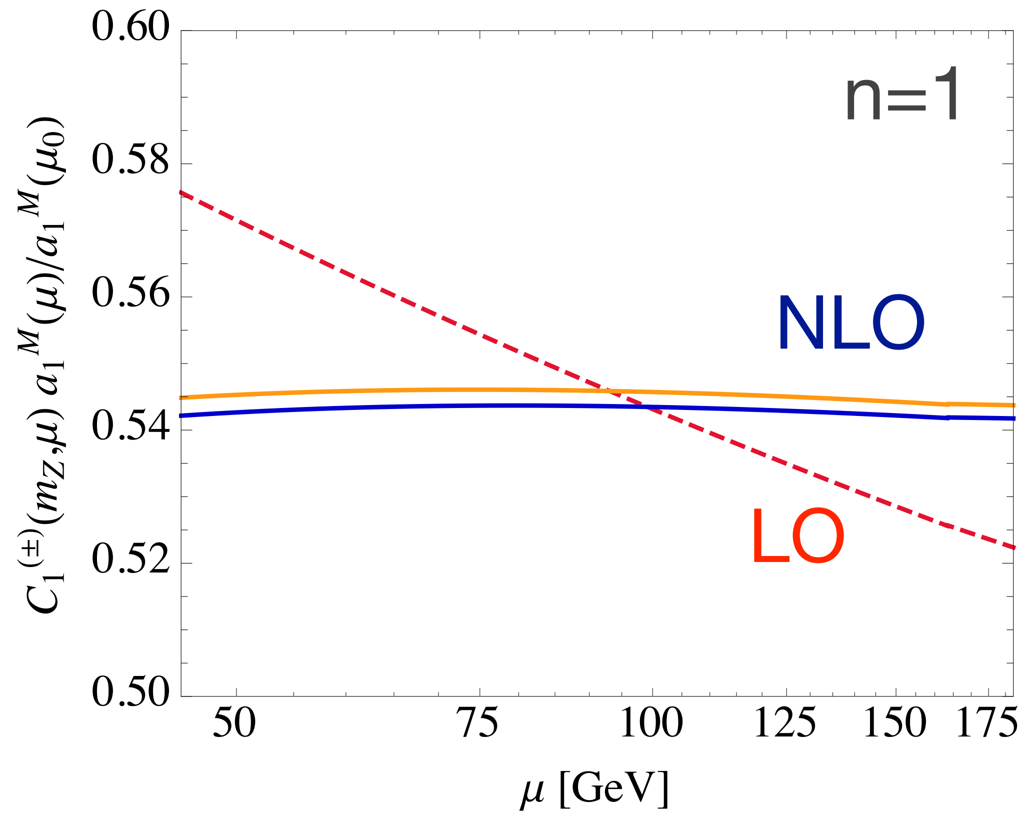
$$\mathrm{Re}\,F_1^M = \mathcal{Q}_M \left[0.94 + 1.05\,a_2^M(m_Z) + 1.15\,a_4^M(m_Z) + 1.22\,a_6^M(m_Z) + \dots\right]$$

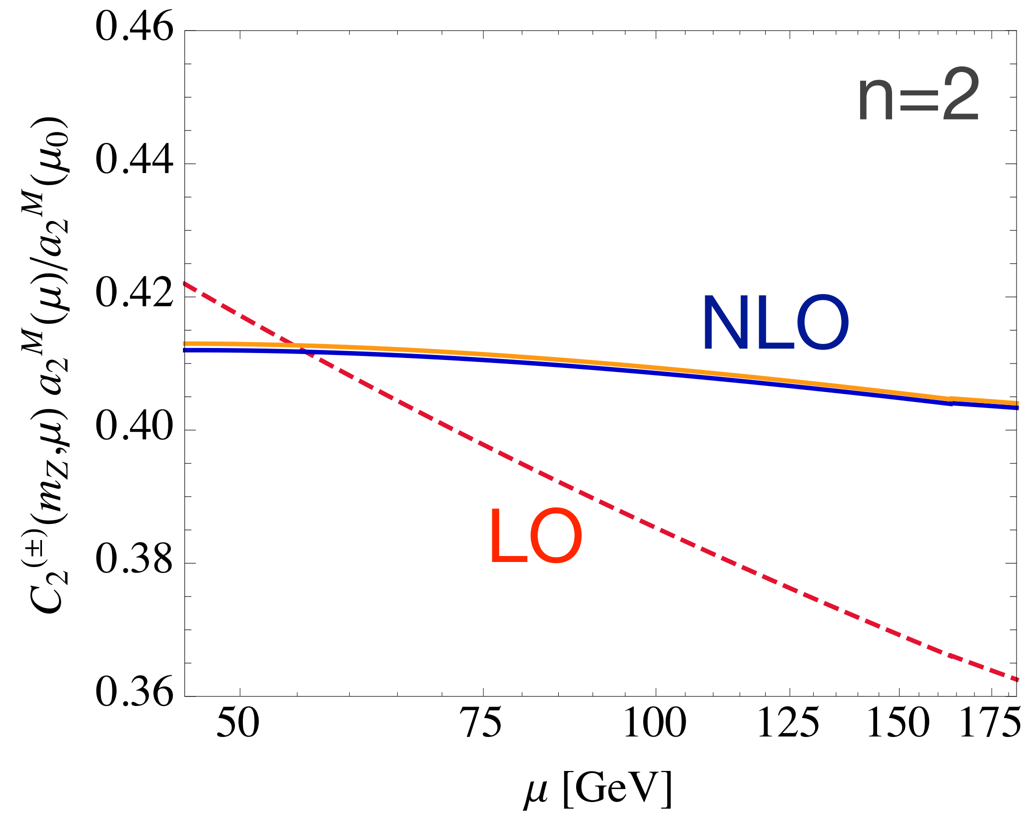
$$= \mathcal{Q}_M \left[0.94 + 0.41\,a_2^M(\mu_0) + 0.29\,a_4^M(\mu_0) + 0.23\,a_6^M(\mu_0) + \dots\right].$$

$$F_2^M = 0$$

strongly reduced sensitivity to hadronic parameters!

Each term in the sum is formally scale independent:





Power-suppressed corrections

Power-suppressed contributions to the decay amplitudes **with given helicities** are organized in an expansion in powers of $(\Lambda_{\mathrm{QCD}}/m_Z)^2$ for light mesons and $(m_M/m_Z)^2$ for mesons containing heavy quark

These corrections are **tiny**, of order 10^{-4} for light mesons and at most 1% for the heaviest meson we will consider — the Υ(1S)

QCD factorization approach thus allows for precise predictions, which are limited only by our incomplete knowledge of the LCDAs

This opens up the possibility for a beautiful program of electroweak precision physics and precisions tests of the SM and the QCD factorization approach!

Power-suppressed amplitudes

As an example for a power-suppressed contribution, consider decays into **transversely polarized** vector mesons, which arise at twist-3 order:

$$i\mathcal{A}(Z_\parallel \to V_\perp \gamma) = -\frac{egf_V}{2\cos\theta_W}\boxed{\frac{m_V}{m_Z}}\left(i\epsilon_{\mu\nu\alpha\beta}\,\frac{k^\mu q^\nu \varepsilon_V^{*\alpha}\varepsilon_\gamma^{*\beta}}{k\cdot q}F_1^\perp - \varepsilon_V^{\perp *}\cdot\varepsilon_\gamma^{\perp *}\,F_2^\perp \right)$$

with:

$$F_1^\perp = \mathcal{Q}_V\left[1+\sum_{n=1}^{\infty}\frac{a_{2n}^V(\mu)}{(n+1)(2n+1)}\right] \qquad F_2^\perp = 0$$

Up to corrections from Gegenbauer moments, this yields:

$$\frac{\Gamma(Z\to V_\perp\gamma)}{\Gamma(Z\to V_\parallel\gamma)} \approx \frac{m_V^2}{m_Z^2}$$

Exclusive radiative decays of Z bosons

Predictions for branching ratios including detailed error estimates:

Decay mode	Branching ratio	asymptotic	LO
$Z^0 \to \pi^0\gamma$	$(9.80\,^{+0.09}_{-0.14}\,{}_\mu \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	7.71	14.67
$Z^0 \to \rho^0\gamma$	$(4.19\,^{+0.04}_{-0.06}\,{}_\mu \pm 0.16_f \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63	5.68
$Z^0 \to \omega\gamma$	$(2.89\,^{+0.03}_{-0.05}\,{}_\mu \pm 0.15_f \pm 0.29_{a_2} \pm 0.25_{a_4}) \cdot 10^{-8}$	2.54	3.84
$Z^0 \to \phi\gamma$	$(8.63\,^{+0.08}_{-0.13}\,{}_\mu \pm 0.41_f \pm 0.55_{a_2} \pm 0.74_{a_4}) \cdot 10^{-9}$	7.12	12.31
$Z^0 \to J/\psi\,\gamma$	$(8.02\,^{+0.14}_{-0.15}\,{}_\mu \pm 0.20_f\,^{+0.39}_{-0.36}\,{}_\sigma) \cdot 10^{-8}$	10.48	6.55
$Z^0 \to \Upsilon(1S)\,\gamma$	$(5.39\,^{+0.10}_{-0.10}\,{}_\mu \pm 0.08_f\,^{+0.11}_{-0.08}\,{}_\sigma) \cdot 10^{-8}$	7.55	4.11
$Z^0 \to \Upsilon(4S)\,\gamma$	$(1.22\,^{+0.02}_{-0.02}\,{}_\mu \pm 0.13_f\,^{+0.02}_{-0.02}\,{}_\sigma) \cdot 10^{-8}$	1.71	0.93
$Z^0 \to \Upsilon(nS)\,\gamma$	$(9.96\,^{+0.18}_{-0.19}\,{}_\mu \pm 0.09_f\,^{+0.20}_{-0.15}\,{}_\sigma) \cdot 10^{-8}$	13.96	7.59

Table 4: Predicted branching fractions for various $Z \to M\gamma$ decays, including error estimates due to scale dependence (subscript "μ") and the uncertainties in the meson decay constants ("f"), the Gegenbauer moments of light mesons ("a_n"), and the width parameters of heavy mesons ("σ"). See text for further explanations.

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$Z^0 \to \Upsilon(nS)\,\gamma$	$(9.96\,^{+\,0.18}_{-\,0.19}{}_{\mu} \pm 0.09_f\,^{+\,0.20}_{-\,0.15}{}_{\sigma}) \cdot 10^{-8}$	13.96	7.59

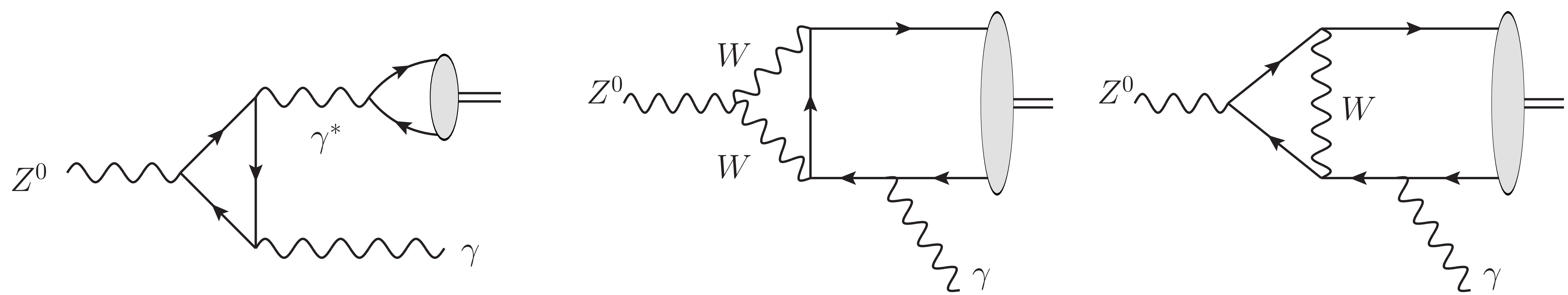
Table 4: Predicted branching fractions for various $Z \to M\gamma$ decays, including error estimates due to scale dependence (subscript "μ") and the uncertainties in the meson decay constants ("f"), the Gegenbauer moments of light mesons ("a_n"), and the width parameters of heavy mesons ("σ"). See text for further explanations.

LO PT @ $\mu_0 = 1\,\mathrm{GeV}$

asymptotic LCDAs $(a_N^M \to 0)$

Exclusive radiative decays of Z bosons

Neglected contributions (QED and EW radiative corrections):



→ estimated contributions $\alpha/\pi \sim 2 \cdot 10^{-3}$ relative to leading terms; confirmed by explicit calculation in Huang, Petriello (2014)

Large logarithms $\left(\alpha_s \ln(m_Z^2/\mu^2)\right)^n$ have been resummed to all orders by choosing the factorization scale $\mu \sim m_Z$

Two-loop QCD corrections are accounted for by error estimate based on scale variation

Comparison with existing predictions

When all Gegenbauer moments are neglected, i.e. $\phi_M(x) = 6x(1-x)$, we obtain for the decay rates:

$$\Gamma(Z^0 \to M^0\gamma)\big|_{\text{asymp}} = \frac{\alpha m_Z f_M^2}{6v^2}\, \mathcal{Q}_M^2 \left[1 - \frac{10}{3}\frac{\alpha_s(m_Z)}{\pi}\right]$$

→ agrees with a formula for $Z^0 \to P^0\gamma$ in Arnellos, Marciano, Parsa (1982)

Manohar obtained an estimate for the $Z^0 \to \pi^0\gamma$ rate using a **local OPE**, which is too small by a factor $(2/3)^2 = 4/9$ (understood ✓) Manohar (1990)

Huang and Petriello (2014) performed a calculation of some $Z^0 \to V^0\gamma$ decay rates using NRQCD and an approach similar to ours, finding:

$$B_{SM}(Z \to J/\psi + \gamma) = (9.96 \pm 1.86) \times 10^{-8} \quad (✓)$$

$$B_{SM}(Z \to \Upsilon(1S) + \gamma) = (4.93 \pm 0.51) \times 10^{-8} \quad (✓)$$

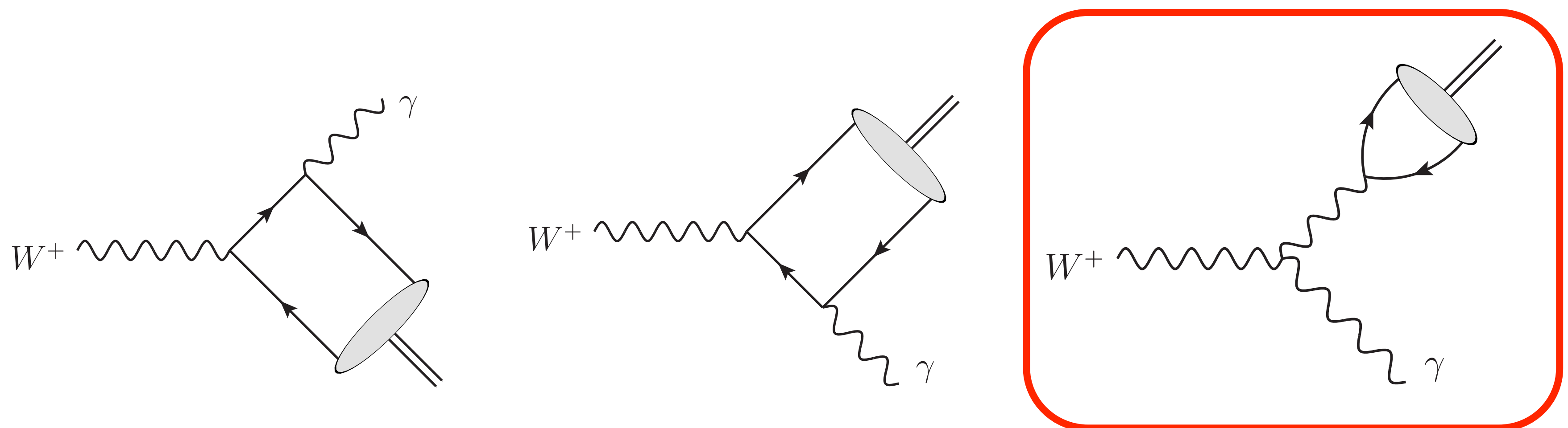
$$B_{SM}(Z \to \phi + \gamma) = (1.17 \pm 0.08) \times 10^{-8}$$

→ consistent when rescaled to up-to-date value of f_ϕ

Exclusive radiative decays $W \to M\gamma$

Factorization of the decay amplitude

Situation is analogous, but the trilinear WWγ vertex gives rise to an additional (local) contribution:



Form-factor decomposition:

$$i\mathcal{A}(W^+ \to M^+\gamma) = \pm \frac{e g f_M}{4\sqrt{2}}\, V_{ij} \left(i\epsilon_{\mu\nu\alpha\beta}\, \frac{k^\mu q^\nu \varepsilon_W^\alpha\, \varepsilon_\gamma^{*\beta}}{k \cdot q}\, F_1^M - \varepsilon_W^\perp \cdot \varepsilon_\gamma^{\perp *}\, F_2^M \right)$$

Explicit results:

$$F_1^M = \sum_{n=0}^{\infty} \left[C_{2n}^{(+)}(m_W, \mu)\, a_{2n}^M(\mu) - 3C_{2n+1}^{(+)}(m_W, \mu)\, a_{2n+1}^M(\mu) \right]$$

$$F_2^M = -2 + \sum_{n=0}^{\infty} \left[3C_{2n}^{(-)}(m_W, \mu)\, a_{2n}^M(\mu) - C_{2n+1}^{(-)}(m_W, \mu)\, a_{2n+1}^M(\mu) \right]$$

Exclusive radiative decays of W bosons (SM)

Predictions for branching ratios including detailed error estimates:

Decay mode	Branching ratio	asymptotic	LO
$W^\pm \to \pi^\pm\gamma$	$(4.00\,^{+\,0.06}_{-\,0.11}\,{}_\mu \pm 0.01_f \pm 0.49_{a_2} \pm 0.66_{a_4}) \cdot 10^{-9}$	2.45	8.09
$W^\pm \to \rho^\pm\gamma$	$(8.74\,^{+\,0.17}_{-\,0.26}\,{}_\mu \pm 0.33_f \pm 1.02_{a_2} \pm 1.57_{a_4}) \cdot 10^{-9}$	6.48	15.12
$W^\pm \to K^\pm\gamma$	$(3.25\,^{+\,0.05}_{-\,0.09}\,{}_\mu \pm 0.03_f \pm 0.24_{a_1} \pm 0.38_{a_2} \pm 0.51_{a_4}) \cdot 10^{-10}$	1.88	6.38
$W^\pm \to K^{*\pm}\gamma$	$(4.78\,^{+\,0.09}_{-\,0.14}\,{}_\mu \pm 0.28_f \pm 0.39_{a_1} \pm 0.66_{a_2} \pm 0.80_{a_4}) \cdot 10^{-10}$	3.18	8.47
$W^\pm \to D_s\gamma$	$(3.66\,^{+\,0.02}_{-\,0.07}\,{}_\mu \pm 0.12_{\rm CKM} \pm 0.13_f\,^{+\,1.47}_{-\,0.82}\,{}_\sigma) \cdot 10^{-8}$	0.98	8.59
$W^\pm \to D^\pm\gamma$	$(1.38\,^{+\,0.01}_{-\,0.02}\,{}_\mu \pm 0.10_{\rm CKM} \pm 0.07_f\,^{+\,0.50}_{-\,0.30}\,{}_\sigma) \cdot 10^{-9}$	0.32	3.42
$W^\pm \to B^\pm\gamma$	$(1.55\,^{+\,0.00}_{-\,0.03}\,{}_\mu \pm 0.37_{\rm CKM} \pm 0.15_f\,^{+\,0.68}_{-\,0.45}\,{}_\sigma) \cdot 10^{-12}$	0.09	6.44

Table 5: Predicted branching fractions for various $W \to M\gamma$ decays, including error estimates due to scale dependence and the uncertainties in the CKM matrix elements, the meson decay constants and the LCDAs. The notation is the same as in Table 4. See text for further explanations.

Exclusive radiative decays of W bosons (SM)

Predictions for branching ratios including detailed error estimates:

Decay mode	Branching ratio	asymptotic	LO
$W^\pm \to \pi^\pm\gamma$	$(4.00\,{}^{+\,0.06}_{-\,0.11}{}_\mu \pm 0.01_f \pm 0.49_{a_2} \pm 0.66_{a_4}) \cdot 10^{-9}$	2.45	8.09
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LO PT @ $\mu_0 = 1\,\text{GeV}$

asymptotic LCDAs $(a_N^M \to 0)$

Comparison with existing predictions

When all Gegenbauer moments are neglected, i.e. $\phi_M(x) = 6x(1-x)$, we obtain for the decay rates:

$$\Gamma(W^\pm \to M^\pm \gamma)\big|_{\text{asymp}} = \frac{\alpha m_W f_M^2}{24 v^2} |V_{ij}|^2 \left[1 - \frac{17}{3} \frac{\alpha_s(m_W)}{\pi}\right]$$

→ agrees with a formula for $W^\pm \to P^\pm \gamma$ in Arnellos, Marciano, Parsa (1982)

Using Manohar's approach, Mangano and Melia (2014) obtained an estimate for the $W^\pm \to \pi^\pm \gamma$ rate, which is too small by a factor 2/9 (understood ✓)

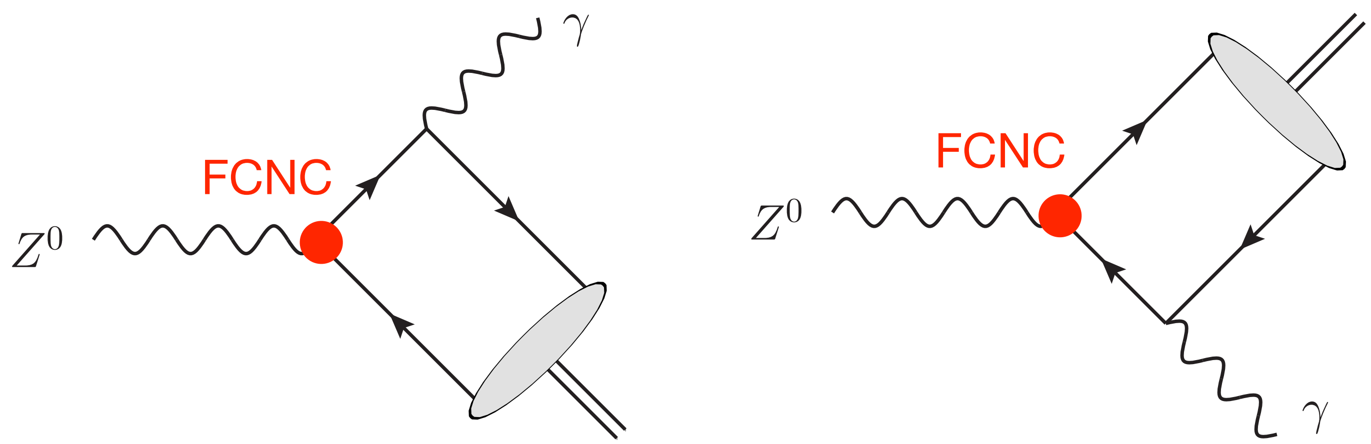
In some very old papers, the authors claimed that the $W, Z \to P\gamma$ rates are **enhanced by several orders of magnitude** due to an unsuppressed contribution $\sim 1/f_P$ from the axial anomaly. Jacob, Wu (1989); Keum, Pham (1993)

We find that such claims are false!

Exclusive radiative decays as BSM probes

Radiative Z decays as a BSM probe

Predictions for branching ratios with non-standard FCNC Z-couplings:

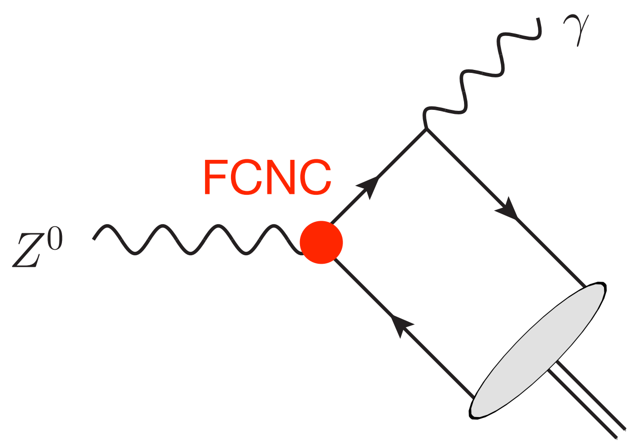


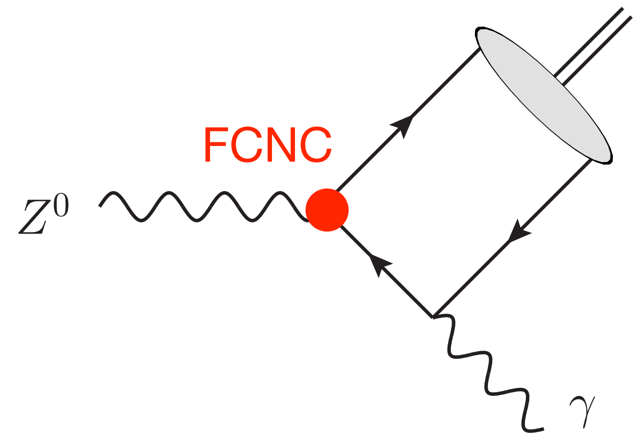
Decay mode	Branching ratio	SM background
$Z^0 \to K^0\gamma$	$\left[(7.70 \pm 0.83)\,\lvert v_{sd}\rvert^2 + (0.01 \pm 0.01)\,\lvert a_{sd}\rvert^2\right] \cdot 10^{-8}$	$\frac{\lambda}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \to D^0\gamma$	$\left[(5.30\,^{+0.67}_{-0.43})\,\lvert v_{cu}\rvert^2 + (0.62\,^{+0.36}_{-0.23})\,\lvert a_{cu}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \to B^0\gamma$	$\left[(2.08\,^{+0.59}_{-0.41})\,\lvert v_{bd}\rvert^2 + (0.77\,^{+0.38}_{-0.26})\,\lvert a_{bd}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda^3}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 8 \cdot 10^{-5}$
$Z^0 \to B_s\gamma$	$\left[(2.64\,^{+0.82}_{-0.52})\,\lvert v_{bs}\rvert^2 + (0.87\,^{+0.51}_{-0.33})\,\lvert a_{bs}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda^2}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 4 \cdot 10^{-4}$

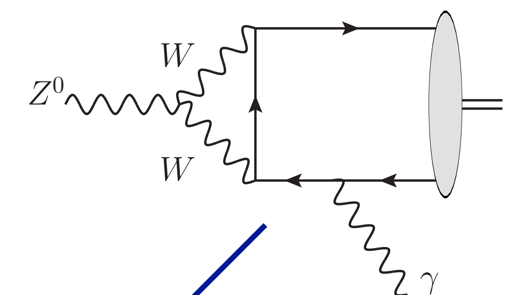
Table 6: Branching fractions for FCNC transitions $Z \to M\gamma$, which could arise from physics beyond the Standard Model. The different theoretical uncertainties have been added in quadrature. The last column shows our estimates for the irreducible Standard Model background up to which one can probe the flavor-changing couplings v_{ij} and a_{ij}. Here $\lambda \approx 0.2$ is the Wolfenstein parameter.

Radiative Z decays as a BSM probe

Predictions for branching ratios with non-standard FCNC Z-couplings:







Decay mode	Branching ratio	SM background
$Z^0 \to K^0\gamma$	$\left[(7.70 \pm 0.83)\,\lvert v_{sd}\rvert^2 + (0.01 \pm 0.01)\,\lvert a_{sd}\rvert^2\right] \cdot 10^{-8}$	$\frac{\lambda}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \to D^0\gamma$	$\left[(5.30\,^{+0.67}_{-0.43})\,\lvert v_{cu}\rvert^2 + (0.62\,^{+0.36}_{-0.23})\,\lvert a_{cu}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \to B^0\gamma$	$\left[(2.08\,^{+0.59}_{-0.41})\,\lvert v_{bd}\rvert^2 + (0.77\,^{+0.38}_{-0.26})\,\lvert a_{bd}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda^3}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 8 \cdot 10^{-5}$
$Z^0 \to B_s\gamma$	$\left[(2.64\,^{+0.82}_{-0.52})\,\lvert v_{bs}\rvert^2 + (0.87\,^{+0.51}_{-0.33})\,\lvert a_{bs}\rvert^2\right] \cdot 10^{-7}$	$\frac{\lambda^2}{\sin^2\theta_W}\,\frac{\alpha}{\pi} \sim 4 \cdot 10^{-4}$

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Radiative Z decays as a BSM probe

Indirect upper bounds on FCNC couplings from neutral-meson mixing:

$\left|\mathrm{Re}\left[(v_{sd} \pm a_{sd})^2\right]\right|$	$< 2.9 \cdot 10^{-8}$	$\left|\mathrm{Re}\left[(v_{sd})^2 - (a_{sd})^2\right]\right|$	$< 3.0 \cdot 10^{-10}$
$\left|\mathrm{Im}\left[(v_{sd} \pm a_{sd})^2\right]\right|$	$< 1.0 \cdot 10^{-10}$	$\left|\mathrm{Im}\left[(v_{sd})^2 - (a_{sd})^2\right]\right|$	$< 4.3 \cdot 10^{-13}$
$\left|(v_{cu} \pm a_{cu})^2\right|$	$< 2.2 \cdot 10^{-8}$	$\left|(v_{cu})^2 - (a_{cu})^2\right|$	$< 1.5 \cdot 10^{-8}$
$\left|(v_{bd} \pm a_{bd})^2\right|$	$< 4.3 \cdot 10^{-8}$	$\left|(v_{bd})^2 - (a_{bd})^2\right|$	$< 8.2 \cdot 10^{-9}$
$\left|(v_{bs} \pm a_{bs})^2\right|$	$< 5.5 \cdot 10^{-7}$	$\left|(v_{bs})^2 - (a_{bs})^2\right|$	$< 1.4 \cdot 10^{-7}$

Bona *et al.* (2007); Bertone *et al.* (2012)
Carrasco *et al.* (2013)

These imply:

$$|v_{sd}| < 8.5 \cdot 10^{-5}, \; |v_{cu}| < 7.4 \cdot 10^{-5}, \; |v_{bd}| < 1.0 \cdot 10^{-4}, \; |v_{bs}| < 3.7 \cdot 10^{-4}$$

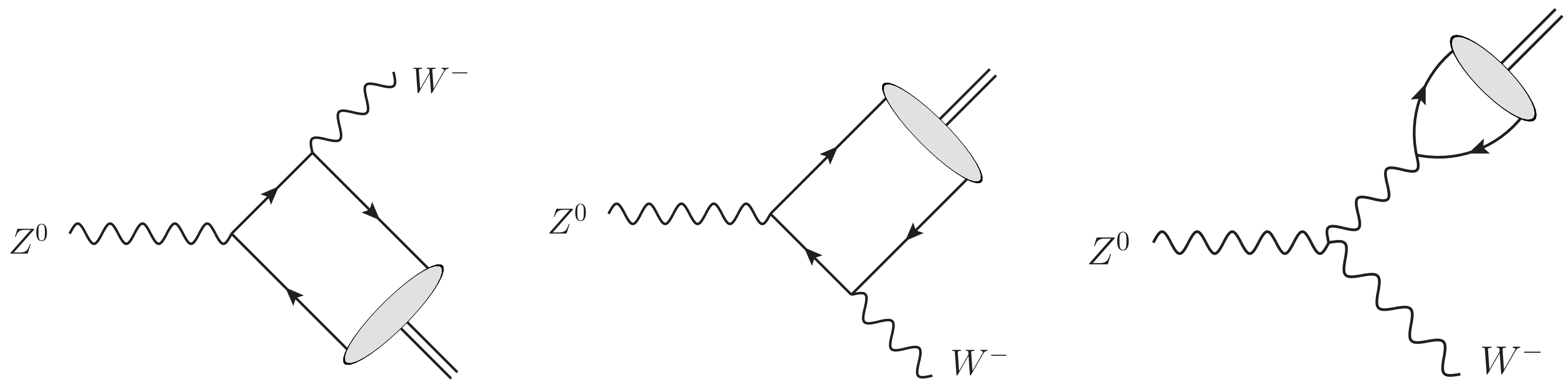
If these indirect bounds are used, the $Z \to P\gamma\big|_{\mathrm{FCNC}}$ branching ratios are pushed to below 10^{-14}, which makes them unobservable

However, the **direct bounds** obtainable using our method are **model independent** and should be seen as **complementary** to the indirect ones!

Weak radiative decays $Z \to MW$

Weak radiative Z decays to M+W

Situation is similar to $W \to M\gamma$ decays, but now also longitudinal polarization states of the final-state gauge boson (W) contribute



Also, due to smaller energy release, the QCD factorization approach can be tested at the lower scale $(m_Z - m_W) \approx 10\,\text{GeV}$, factor 2 above m_b

Form-factor decomposition:

$$i\mathcal{A}(Z \to M^+W^-) = \pm\frac{g^2 f_M}{4\sqrt{2}\cos\theta_W} V_{ij}\left(1 - \frac{m_W^2}{m_Z^2}\right)$$
$$\times\left(i\epsilon_{\mu\nu\alpha\beta}\,\frac{k^\mu q^\nu \varepsilon_Z^\alpha\,\varepsilon_W^{*\beta}}{k\cdot q}\,F_1^M - \varepsilon_Z\cdot\varepsilon_W^*\,F_2^M + \boxed{\frac{q\cdot\varepsilon_Z\,k\cdot\varepsilon_W^*}{k\cdot q}\,F_3^M}\right)$$

Weak radiative Z decays to M+W

Result for the decay rates:

very small sensitivity to LCDA

$$\Gamma(Z \to M^+W^-) = \underbrace{\frac{\pi\alpha^2(m_Z)\, f_M^2}{48 m_Z}\, |V_{ij}|^2\, \frac{s_W^2}{c_W^2} \left(\frac{3}{2} + \frac{3}{2}\, s_W^2\right.}_{\text{Manohar (1990)}} \left. + \frac{227}{180}\, s_W^4 + 0.003\, a_1^M + \dots \right)$$

Phenomenological predictions:

Decay mode	Branching ratio
$Z^0 \to \pi^\pm W^\mp$	$(1.51 \pm 0.005_f) \cdot 10^{-10}$
$Z^0 \to \rho^\pm W^\mp$	$(4.00 \pm 0.15_f) \cdot 10^{-10}$
$Z^0 \to K^\pm W^\mp$	$(1.16 \pm 0.01_f) \cdot 10^{-11}$
$Z^0 \to K^{*\pm} W^\mp$	$(1.96 \pm 0.12_f) \cdot 10^{-11}$
$Z^0 \to D_s W^\mp$	$(6.04 \pm 0.20_{\rm CKM} \pm 0.22_f) \cdot 10^{-10}$
$Z^0 \to D^\pm W^\mp$	$(1.99 \pm 0.14_{\rm CKM} \pm 0.10_f) \cdot 10^{-11}$

Summary

Predicted branching ratios with theory error added in quadrature:

Decay mode	Branching ratio	Decay mode	Branching ratio
$Z^0 \to \pi^0\gamma$	$(9.80 \pm 1.03) \cdot 10^{-12}$	$W^\pm \to \pi^\pm\gamma$	$(4.00 \pm 0.83) \cdot 10^{-9}$
$Z^0 \to \rho^0\gamma$	$(4.19 \pm 0.47) \cdot 10^{-9}$	$W^\pm \to \rho^\pm\gamma$	$(8.74 \pm 1.91) \cdot 10^{-9}$
$Z^0 \to \omega\gamma$	$(2.89 \pm 0.41) \cdot 10^{-8}$	$W^\pm \to K^\pm\gamma$	$(3.25 \pm 0.69) \cdot 10^{-10}$
$Z^0 \to \phi\gamma$	$(8.63 \pm 1.01) \cdot 10^{-9}$	$W^\pm \to K^{*\pm}\gamma$	$(4.78 \pm 1.15) \cdot 10^{-10}$
$Z^0 \to J/\psi\,\gamma$	$(8.02 \pm 0.45) \cdot 10^{-8}$	$W^\pm \to D_s\gamma$	$(3.66\,^{+\,1.49}_{-\,0.85}) \cdot 10^{-8}$
$Z^0 \to \Upsilon(1S)\,\gamma$	$(5.39 \pm 0.16) \cdot 10^{-8}$	$W^\pm \to D^\pm\gamma$	$(1.38\,^{+\,0.51}_{-\,0.33}) \cdot 10^{-9}$
$Z^0 \to \Upsilon(4S)\,\gamma$	$(1.22 \pm 0.13) \cdot 10^{-8}$	$W^\pm \to B^\pm\gamma$	$(1.55\,^{+\,0.79}_{-\,0.60}) \cdot 10^{-12}$

- for Z decays, can trigger on high-energy photon and muons
- estimate that one can get several hundreds of $J/\psi\,\gamma$ events at LHC
- ideas for reconstructing $(\rho, \omega, \phi) + \gamma$ exists Kagan *et al.* (2014)
- reconstructing W decays at LHC is more challenging Mangano, Melia (2014)
- a Z-factory could measure most modes with good precision!

Summary

With precise measurements of branching ratios, one can extract — in a model-independent way — information about LCDAs (sums over even and odd moments at scale $\mu \sim m_Z$)

It will also be possible to perform a series of novel new-physics searches

Possible generalizations:

- radiative Z decays into η and η' including the two-gluon LCDA
- exclusive decays of Higgs bosons, extending previous (tree-level) analyses
- purely hadronic decays such as $Z \to M_1 M_2$ or even 3-body modes

The physics case for studying these very rare decays is compelling!
The challenge is to make it possible to observe them!

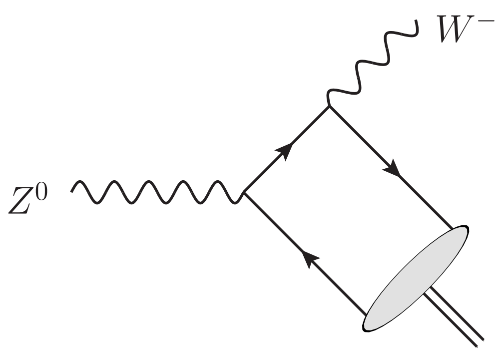

Let’s go and play!

BACKUP SLIDE

Manohar's approach (1990)

Expansion of the propagator in the tree-level diagrams for Z→MW decays in powers of a small parameter ω_0 yields a tower of **local** operator matrix elements, the first of which is given by f_M:

$$\frac{1}{xm_Z^2 + (1-x)m_W^2} = \frac{2}{m_Z^2 + m_W^2} \frac{1}{\left[1 - \left(\frac{1}{2} - x\right)\omega_0\right]} = \frac{2}{m_Z^2 + m_W^2} \left[1 + \sum_{n=0}^{\infty} \left(\frac{\omega_0}{2}\right)^n (1-2x)^n\right]$$



with: $$\frac{\omega_0}{2} = \frac{m_Z^2 - m_W^2}{m_Z^2 + m_W^2} \approx 0.125$$

However, applying this method to Z→Mγ decays leads to an uncontrolled expansion:

$$\frac{1}{xm_Z^2} = \frac{2}{m_Z^2} \left[1 + \sum_{n=0}^{\infty} (1-2x)^n\right]$$

Keeping the first term only makes an error of 2/3.