The Hunt for Heavy Neutrinos at the FCC

courtesy J. Wenninger

Alain Blondel University of Geneva
with many thanks to
S. Ganjour, M. Mitra, S. Pascoli, E. Graverini, P. Mermod, N. Serra, M. Shaposhnikov


Where is Everybody?

But Where Is Everybody?

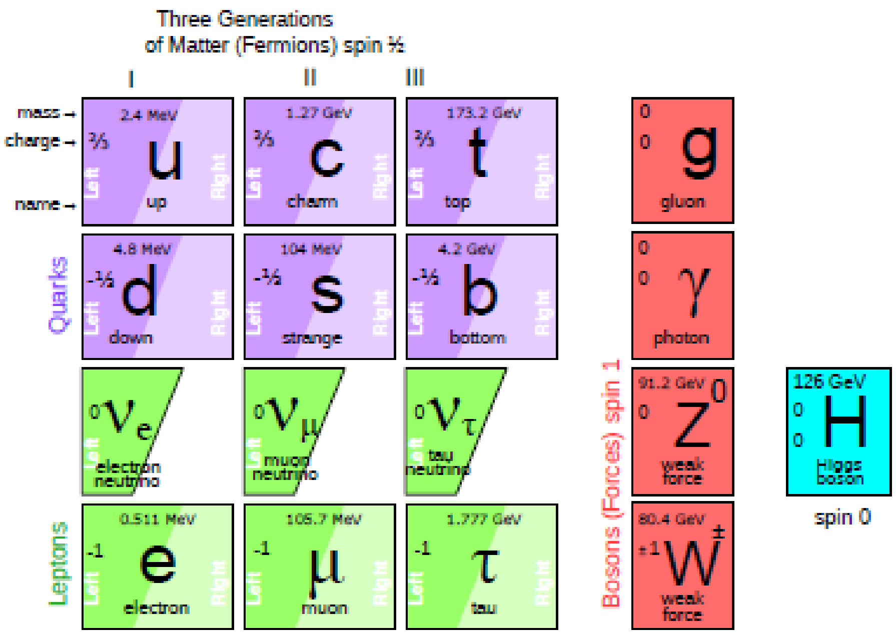
Nima

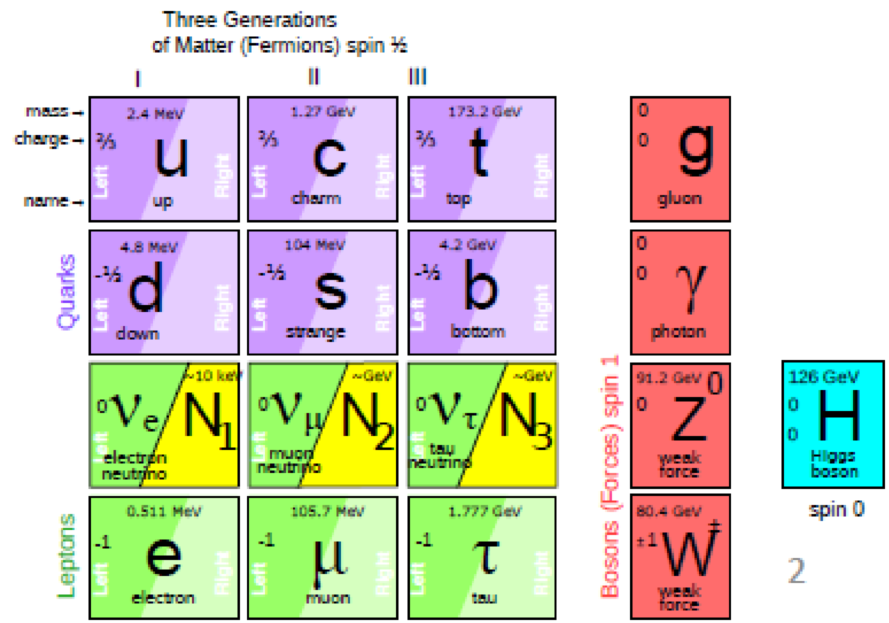
Nima

At higher masses -- or at smaller couplings?

THE STANDARD MODEL IS COMPLETE .....

But at least 3 pieces are still missing





neutrinos have mass...

and this very probably implies new degrees of freedom

➔ Right-Handed, Almost «Sterile» (*very* small couplings) Neutrinos completely unknown masses (meV to ZeV), nearly impossile to find.

.... but could perhaps explain all: DM, BAU,ν-masses

some REFERENCES

PHYSICAL REVIEW D VOLUME 29, NUMBER 11 1 JUNE 1984

Extending limits on neutral heavy leptons

Michael Gronau*
Department of Physics, Syracuse University, Syracuse, New York 132

FLAVOUR(267104)-ERC-23 TUM-HEP 850/12 SISSA 25/2012/EP CFTP/12-013

arxiv:1208.3654

Higgs Decays in the Low Scale Type I See-Saw Model

C. Garcia Cely[a)], A. Iba

theories of the ei
and mixings with

JHEP

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First look at the physics case of TLEP

arxiv:1308.6176

The TLEP Design Study Working Group
M. Bicer,[a] H. Duran Yildiz,[b] I. Yildiz,[c] G. Coignet,[d] M. Delmastro,[d] T. Alexopoulos,[e] C. Grojean,[f] S. Antusch,[g] T. Sen,[h] H.-J. He,[i] K. Potamianos,[j] S. Haug,[k] A. Moreno,[l] A. Heister,[m] V. Sanz,[n] G. Gomez-Ceballos,[o] M. Klute,[o] M. Zanetti,[o] L.-T. Wang,[p] M. Dam,[q] C. Boehm,[r] N. Glover,[r] F. Krauss,[r] A. Lenz,[r] M. Syphers,[s]

Search for Heavy Right Handed Neutrinos at the FCC-ee

A. Blondel (presenter)[a], E. Graverini[b], N. Serra[b], M. Shaposhnikov[c]

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[c]ITPP, EPFL, CH-1015 Lausanne, Switzerland

CERN–PPE/96–195
18 December 1996

Neutral Heavy Leptons
uced in Z Decays

DELPHI Collaboration

The Role of Sterile Ne
Astrophysics

Alexey Boyarsky*†, Oleg Ruchayskiy‡ and Mikhail Shaposhni

FCC design study and FCC-ee http://cern.ch/fcc-ee
and presentations at *FCC-ee physics workshops*
http://indico.cern.ch/category/5684/

The νMSM, Dark Matter and Neutrino Masses

Takehiko Asaka, Steve Blanchet, and Mikhail Shaposhnikov
Institut de Théorie des Phénomènes Physiques,
CH-1015 Lausanne, Switzerland
2005)

Phys.Lett.B631:151-156,2005
arXiv:hep-ph/0503065

Preprint typeset in JHEP style - HYPER VERSION
FERMILAB-PUB-08-086-T, NSF-KITP-08-54, MADPH–06–1466, DCPT/07/198, IPPP/07/99

The Search for Heavy Majorana Neutrinos

Anupama Atre[1,2], Tao Han[2,3,4], Silvia Pascoli[5], Bin Zhang[4*]

Electroweak eigenstates

I = 1/2	I = 0	
$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L \quad \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L \quad \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L$	$(e)_R \quad (\mu)_R \quad (\tau)_R$	Q= -1
	$(\nu_e)_R \quad (\nu_\mu)_R \quad (\nu_\tau)_R$	Q= 0

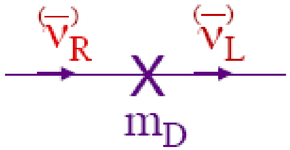
I = 1/2   I = 0

Right handed neutrinos are singlets
no weak interaction
no EM interaction
no strong interaction

can't produce them
can't detect them
-- so why bother? --

Adding masses to the Standard model neutrino 'simply' by adding a Dirac mass term (Yukawa coupling)

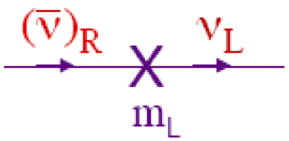
$$m_D \nu_L \overline{\nu_R}, \qquad m_D \overline{\nu_L}\, \nu_R$$



implies adding a right-handed neutrino (new particle)

<u>No SM symmetry</u> prevents adding then a term like

$$m_M \overline{\nu_R{}^c}\, \nu_R$$



and this simply means that a neutrino turns into a antineutrino
(the charge conjugate of a right handed antineutrino is a left handed neutrino!)

It is perfectly conceivable ('natural'?) that both terms are present ➔ 'see-saw'

B. Kayser, the physics of massive neutrinos (1989)

Mass eigenstates

See-saw in a general way :

$$\mathcal{L} = \frac{1}{2}(\bar{\nu}_L, \bar{N}_R^c) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix}$$

$M_R \neq 0$
$m_D \neq 0$
Dirac + Majorana mass terms

$$\tan 2\theta = \frac{2\, m_D}{M_R - 0} \qquad \ll 1$$

$$m_\nu = \tfrac{1}{2}\left[(0 + M_R) - \sqrt{(0 - M_R)^2 + 4\, m_D^2}\right] \qquad \simeq -m_D^2/M_R$$

$$M = \tfrac{1}{2}\left[(0 + M_R) + \sqrt{(0 - M_R)^2 + 4\, m_D^2}\right] \qquad \simeq M_R$$

general formula — if $m_D \ll M_R$

$M_R = 0$
$m_D \neq 0$
Dirac only, (like e- vs e+):

m ↑

	ν_L	ν_R	$\bar{\nu}_R$	$\bar{\nu}_L$
I_{weak}=	½	0	½	0

4 states of equal masses

Some have I=1/2 (active)
Some have I=0 (sterile)

$M_R \neq 0$
$m_D = 0$
Majorana only

m ↑

	ν_L	$\bar{\nu}_R$
I_{weak}=	½	½

2 states of equal masses

All have I=1/2 (active)

---

see-saw

$M_R \neq 0$
$m_D \neq 0$
Dirac + Majorana

m ↑

	ν_L	N_R	$\bar{\nu}_R$	$\bar{N}_L$
I_{weak}=	½	0	½	0

4 states , 2 mass levels

m_1 have ~I=1/2 (~active)
m_2 have ~I=0 (~sterile)

There even exists a scenario that claims to explain everything: the νMSM

Shaposhnikov et al

TeV

GeV

MeV

keV

eV

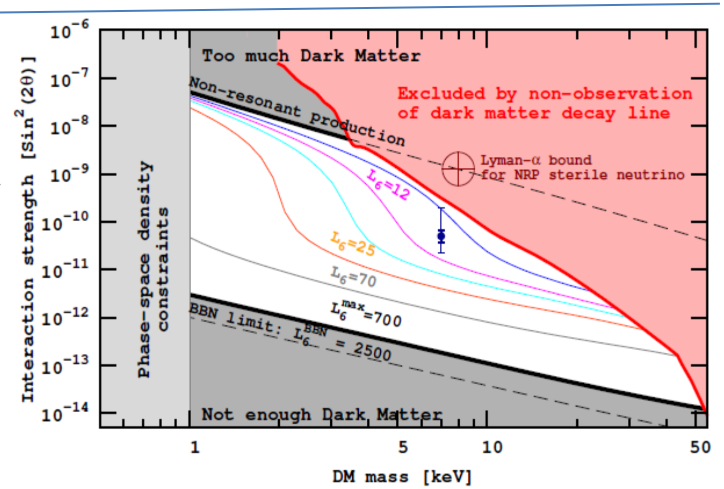
meV

N_2, N_3

can generate Baryon Asymmetry of Universe
if $m_{N2,N3}$ > 140 MeV

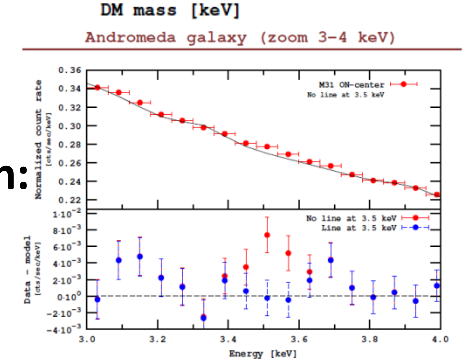
N_1

constrained:
mass: 1-50 keV
mixing :
10^{-7} to 10^{-13}
decay time:
$\tau_{N1} > \tau_{Universe}$



ν_3
ν_2
ν_1

$N_1 \rightarrow \nu\,\gamma$
may have been seen:
arxiv:1402:2301
arxiv:1402.4119



Manifestations of right handed neutrinos

one family see-saw :
$\theta \approx (m_D/M)$
$m_\nu \approx \frac{m_D^2}{M}$
$m_N \approx M$
$|U|^2 \propto \theta^2 \approx m_\nu / m_N$
can be larger with 3 families

$$\nu = \nu L \cos\theta - N^c{}_R \sin\theta$$

$$N = N_R \cos\theta + \nu_L{}^c \sin\theta$$

what is produced in W, Z decays is:

$$\nu_L = \nu \cos\theta + N \sin\theta$$

ν = light mass eigenstate
N = heavy mass eigenstate
$\neq \nu_L$, active neutrino
which couples to weak inter.
and $\neq N_R$, which does'nt.

-- mixing with active neutrinos leads to various observable consequences
-- if very light (eV) , possible effect on neutrino oscillations (short baseline)
-- if in keV region (dark matter), monochromatic photons from galaxies with $E=m_N/2$

-- possibly measurable effects at High Energy
If N is heavy it will decay in the detector (not invisible)
→ PMNS matrix unitarity violation and deficit in Z «invisible» width
→ Higgs, W, Z exotic decays H→ $\nu_i \bar{N}_i$ and Z→ $\nu_i \bar{N}_i$, W-> $l_i \bar{N}_i$
→ also in charm and b decays via W^*-> $l_i \bar{N}_i$
→ violation of unitarity and lepton universality in Z, W or τ decays
-- etc... etc...

-- Couplings are small (m_ν / m_N) *(but who knows?)* and generally out of reach of hadron colliders *(but this deserves to be revisited for detached vertices @LHC, HL-LHC, FCC-hh)*

Indirect effects

-- neutrino Majorana mass term can lead to lepton number violating processes by virtual neutrino exchange and to flavour violation

-- neutrinoless double beta decay (the most powerful one)
-- FCNC ($\mu \rightarrow e\gamma$) etc...

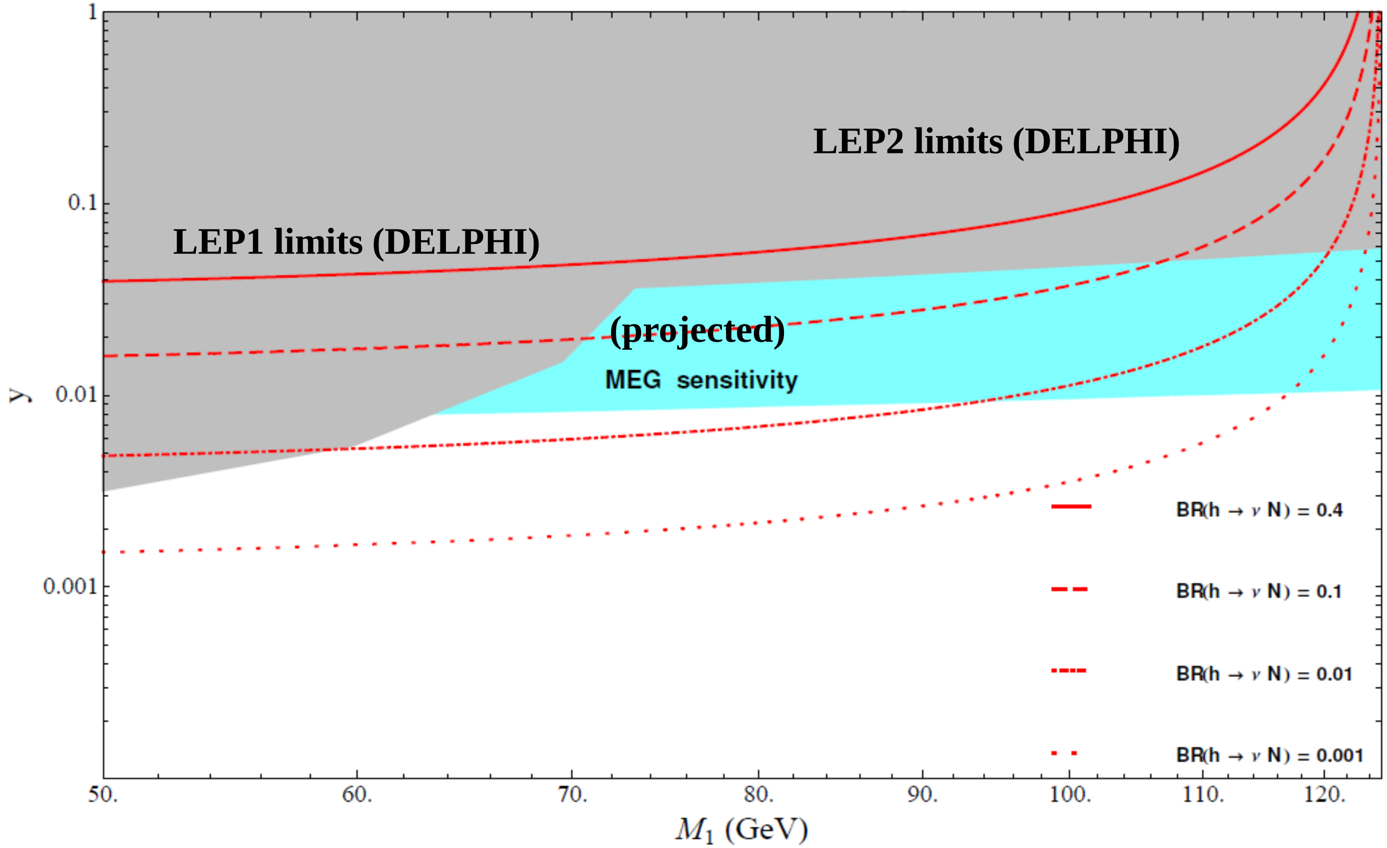
-- at a Z factory : $Z \rightarrow \tau\mu$ $Z \rightarrow \tau e$ $Z \rightarrow \tau\tau$, $\tau \rightarrow \mu\gamma$ $\tau \rightarrow e\gamma$ etc...

Indirect constraints from lepton flavour violation are weak

arxiv:1208.3654

Higgs Decays in the Low Scale Type I See-Saw Model

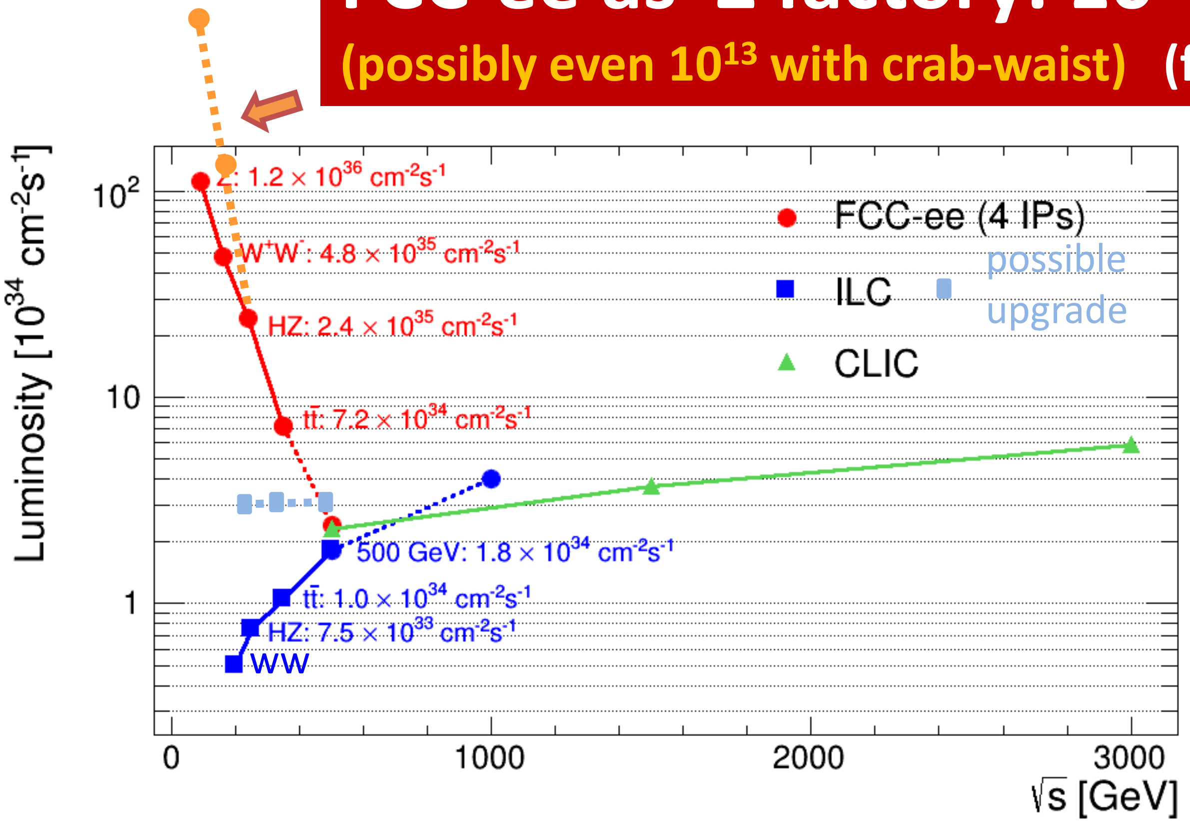
C. Garcia Cely[a)], A. Ibarra[a)], E. Molinaro[b)] and S. T. Petcov[c,d)] [1]



Goal performance of e+ e- colliders

FCC-ee as Z factory: 10^{12} Z

(possibly even 10^{13} with crab-waist) (few years)



complementarity

NB: ideas for lumi upgrades:
ILC arxiv:1308.3726 (not in TDR. Upgrade at 250GeV by reconfiguration after 500 GeV running; under discussion)
FCC-ee (crab waist)

-- invisible widths
-- FCNC
-- direct search of heavy neutrino decays

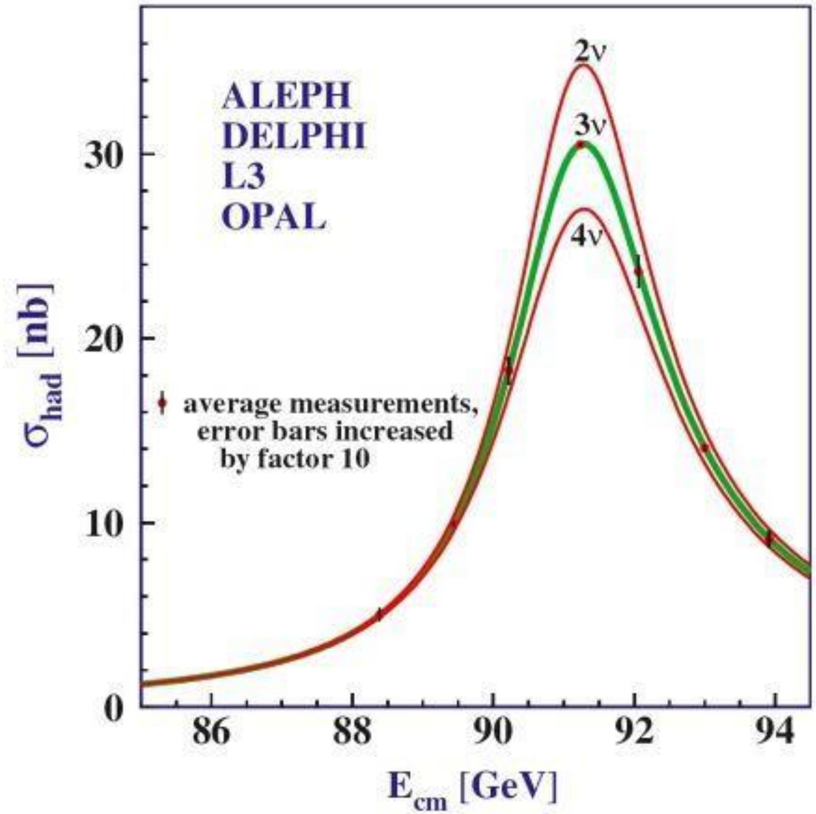
At the end of LEP:

Phys.Rept.427:257-454,2006

N_ν = 2.984 ±0.008

- 2 σ :^) !!

This is determined from the Z line shape scan and dominated by the measurement of the hadronic cross-section at the Z peak maximum ➔



The dominant systematic error is the theoretical uncertainty on the Bhabha cross-section (0.06%) which represents an error of ±0.0046 on N_ν

Improving on N_ν by more than a factor 2 would require a large effort to improve on the Bhabha cross-section calculation!

Volume 241, number 4 PHYSICS LETTERS B 24 May 1990

NEUTRINO COUNTING AT THE Z-PEAK AND RIGHT-HANDED NEUTRINOS

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Received 20 February 1990

We consider the implications of extending the minimal standard model, with n families of quarks and leptons, by introducing an arbitrary number of right-handed neutrinos, for neutrino-counting via the "invisible width" of the Z. It is shown that the effective number of neutrinos, $\langle n \rangle$, satisfies, the inequality $\langle n \rangle \leqslant n$, where $\langle n \rangle$ is defined by $\Gamma(Z \to \text{neutrinos}) \equiv \langle n \rangle \Gamma_0$ and Γ_0 is the standard width for one massless neutrino. Thus, in the case of three families, the neutrino-counting can give a result which is less than three, if there are right-handed neutrinos.

Theorem.

In the standard model, with n left-handed lepton doublets and $N-n$ right-handed neutrinos, the effective number of neutrinos, $\langle n \rangle$, defined by

$$\Gamma(Z \to \text{neutrinos}) \equiv \langle n \rangle \Gamma_0 ,$$

where Γ_0 is the standard width for one massless neutrino, satisfies the inequality

$$\langle n \rangle \leqslant n . \qquad (15)$$

Neutrino counting at TLEP

given the very high luminosity, the following measurement can be performed

$$N_{\nu} = \frac{\frac{\gamma Z(inv)}{\gamma Z \to ee, \mu\mu}}{\frac{\Gamma_{\nu}}{\Gamma e, \mu}\,(SM)}$$

The common γ tag allows cancellation of systematics due to photon selection, luminosity etc. The others are extremely well known due to the availability of O(10^{12}) Z decays.

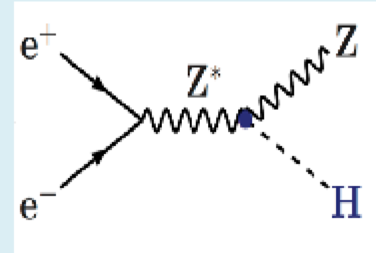
The full sensitivity to the number of neutrinos is restored , and the theory uncertainty on $\frac{\Gamma_{\nu}}{\Gamma e}\,(SM)$ is very very small.

A good measurement can be made from the data accumulated at the WW threshold where σ (γ Z(inv)) ~4 pb for $|\cos\theta_{\gamma}| < 0.95$

161 GeV (10^7 s) running at $1.6\text{x}10^{35}/\text{cm}^2/\text{s}$ x 4 exp → $3\text{x}10^7$ γ Z(inv) evts, ΔN_{ν}=0.0011
adding 5 yrs data at 240 and 350 GeV .... ΔN_{ν}=0.0008

A better point may be 105 GeV (20pb and higher luminosity) may allow ΔN_{ν}=0.0004?

Higgs factory



$2\ 10^6$ ZH events in 5 years

«A tagged Higgs beam».

(constrained fit including 'exotic')

	4 IPs	TLEP (2 IPs)
g_{HZZ}	0.05%	(0.06%)
g_{HWW}	0.09%	(0.11%)
g_{Hbb}	0.19%	(0.23%)
g_{Hcc}	0.68%	(0.84%)
g_{Hgg}	0.79%	(0.97%)
$g_{H\tau\tau}$	0.49%	(0.60%)
$g_{H\mu\mu}$	6.2%	(7.6%)
$g_{H\gamma\gamma}$	1.4%	(1.7%)
BR_{exo}	0.16%	(0.20%)

sensitive to new physics in loops

incl. invisible = (dark matter?)
NB will improve with inclusion of ZH-> qq H tagging

➔ *total width* <1%

HHH (best at FCC-hh) 28% → from HZ thresh

H*tt* (best at FCC-hh) 13% → from tt thresh

Lepton flavour violating Z decays with 10^{13} Z decays

A. Abada et al, arXiv:1412.6322 Indirect searches for sterile neutrinos at a high-luminosity Z-factory A. Abada, V. De Romeri, S. Monteil, J. Orloff, A. M. Teixeira

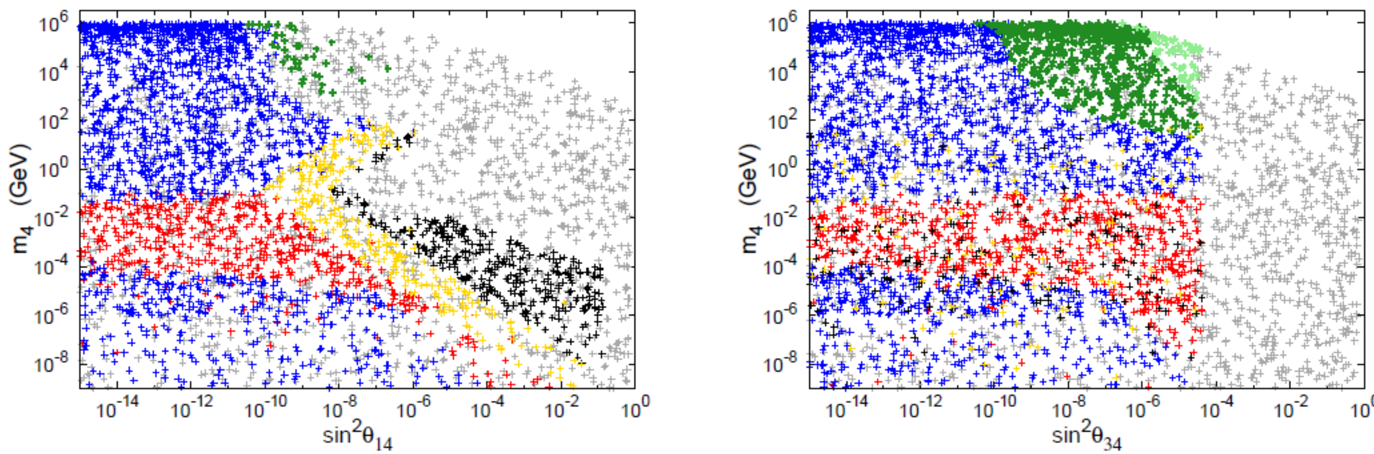


Figure 4: The "3+1 model": on the left $(\sin^2\theta_{14}, m_4)$ parameter space of the sterile state, displaying the regimes for $\mathrm{BR}(Z \to e\mu)$ for a NH light neutrino spectrum. Line and colour code as in Fig. 3 (dark green points are associated with $10^{-13} \lesssim \mathrm{BR}(Z \to e\mu) \lesssim 10^{-9}$, while light green ones correspond to $\mathrm{BR}(Z \to e\mu) \gtrsim 10^{-9}$). On the right, $(\sin^2\theta_{34}, m_4)$ displaying with the same colour code the corresponding regimes for $\mathrm{BR}(Z \to \mu\tau)$.

RHASnu's production in Z decays

Production:

$$BR\,(Z^0 \rightarrow \nu_m \overline{\nu}) = BR\,(Z^0 \rightarrow \nu \bar{\nu})\ |U|^2\ \left(1 - \frac{m_{\nu_m}^2}{{m_{Z^0}}^2}\right)^2 \left(1 + \frac{1}{2}\frac{m_{\nu_m}^2}{{m_{Z^0}}^2}\right)$$

multiply by 2 for anti neutrino and add contributions of 3 neutrino species (with different $|U|^2$)

Decay

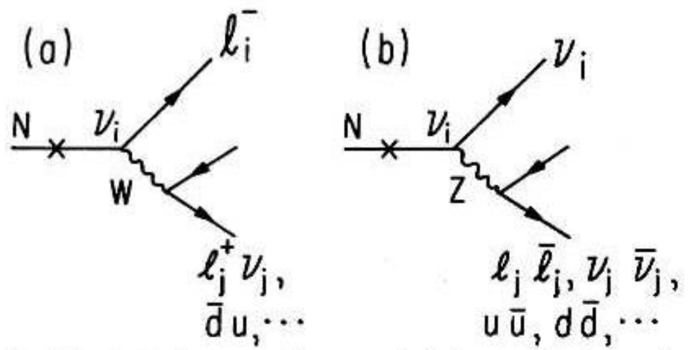


FIG. 2. Typical decays of a neutral heavy lepton via (a) charged current and (b) neutral current. Here the lepton l_i denotes e, μ, or τ.

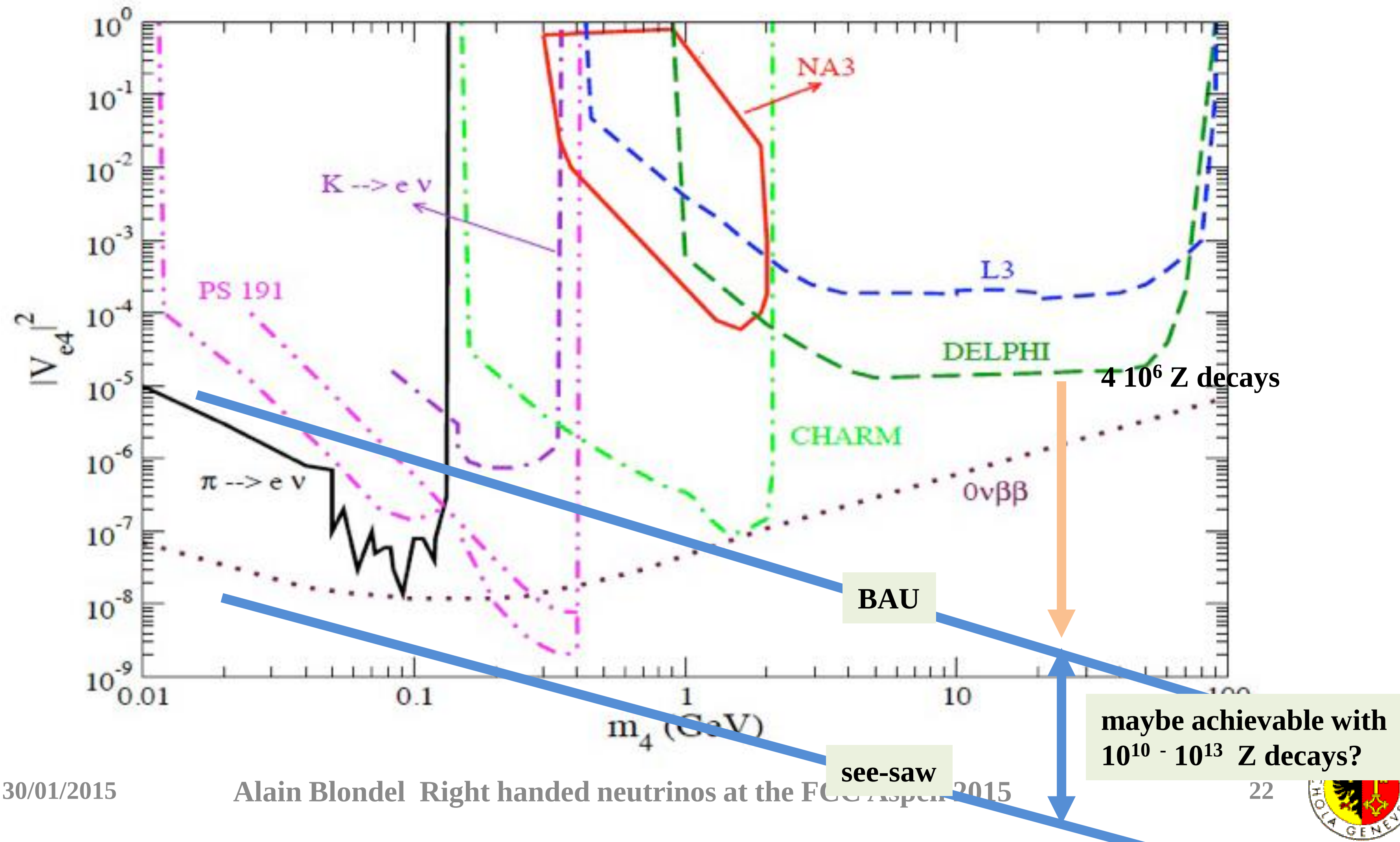
Decay length:

$$L \approx \frac{3 \text{ cm}}{|U|^2\ (m_{\nu_m}(\mathrm{GeV}/c^2))^6}$$

NB CC decay always leads to ≥ 2 charged tracks

Backgrounds : four fermion: e+e- → W^{*+} W^{*-} e+e- → Z*(vv) + (Z/γ)*

Order-of-magnitude extrapolation of existing limits



Search for heavy neutral leptons

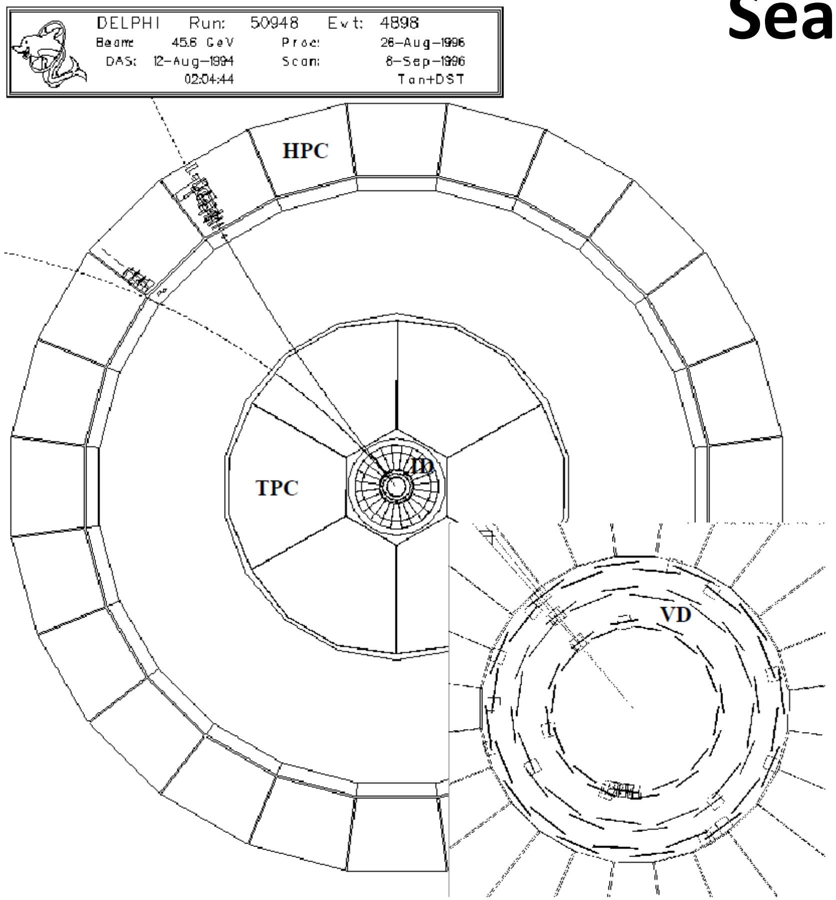


Fig. 3. Surviving event in the monojet search. It has an invariant mass of 300 MeV/c^2 and a missing p_t of 6 GeV/c and is probably an $e^+e^- \to e^+e^-\nu\bar{\nu}$ interaction

search $e^+ e^- \to \nu$ N

N $\to \nu(\gamma/Z)^* \to$ monojet

Find: one event in 4×10^6Z:

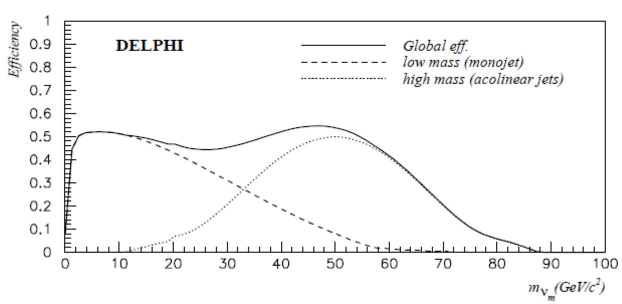
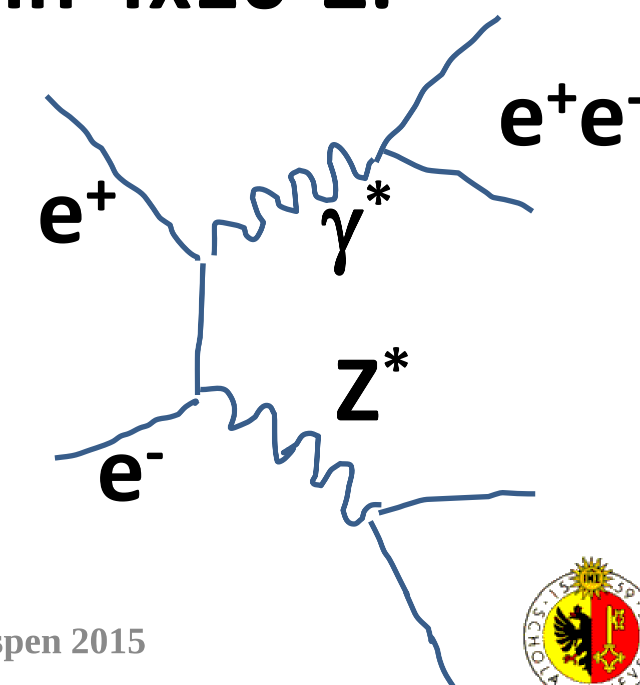
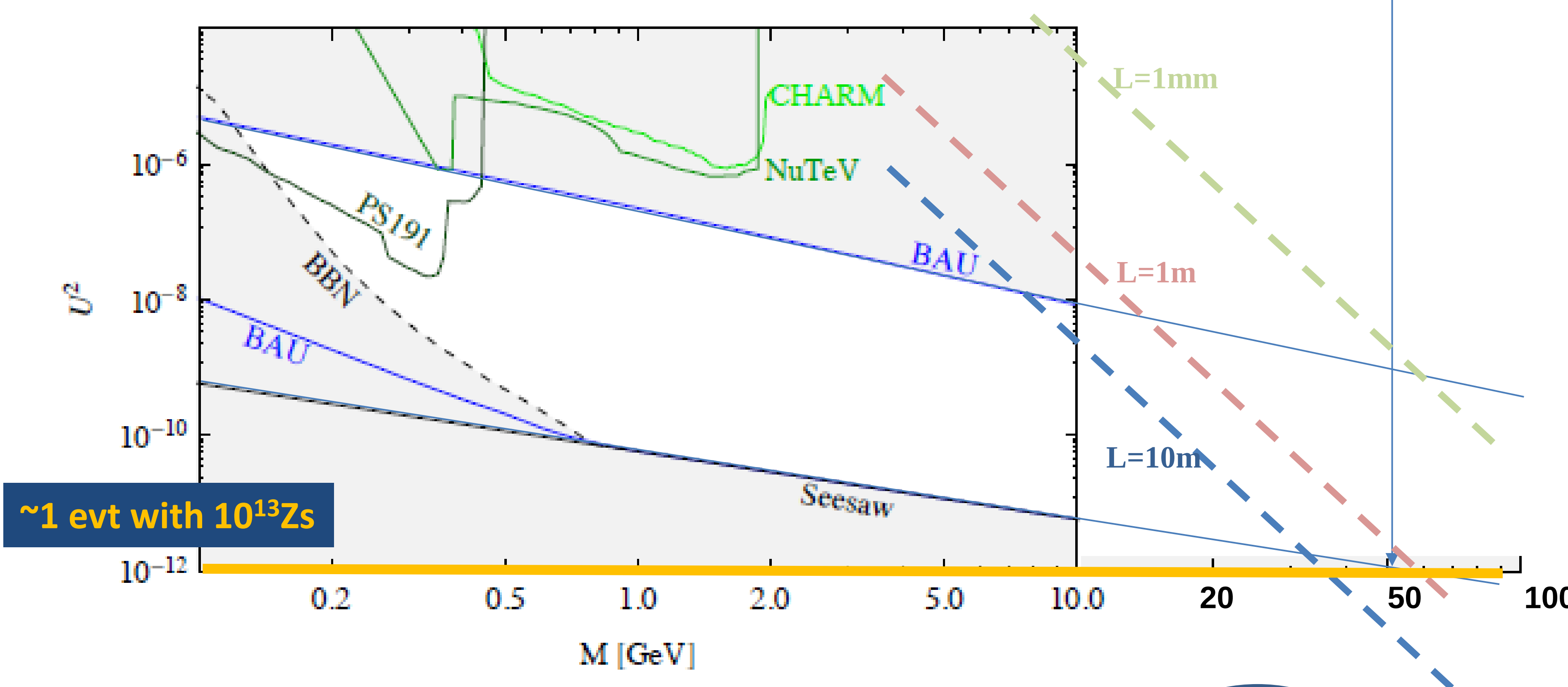


Fig. 4. Efficiency of the monojet search (Sect. 3) and the acollinear jets search (Sect. 4). The *full curve* shows the efficiency of the two searches combined



Decay length

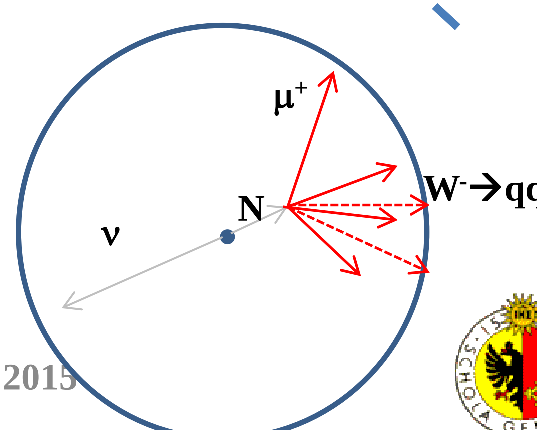
Interesting region
$|U|^2 \sim 10^{-9}$ to 10^{-12} @ 50 GeV



heavy neutrino mass ~ M

a large part of the interesting region will lead to detached vertices

... ➔very strong reduction of background!

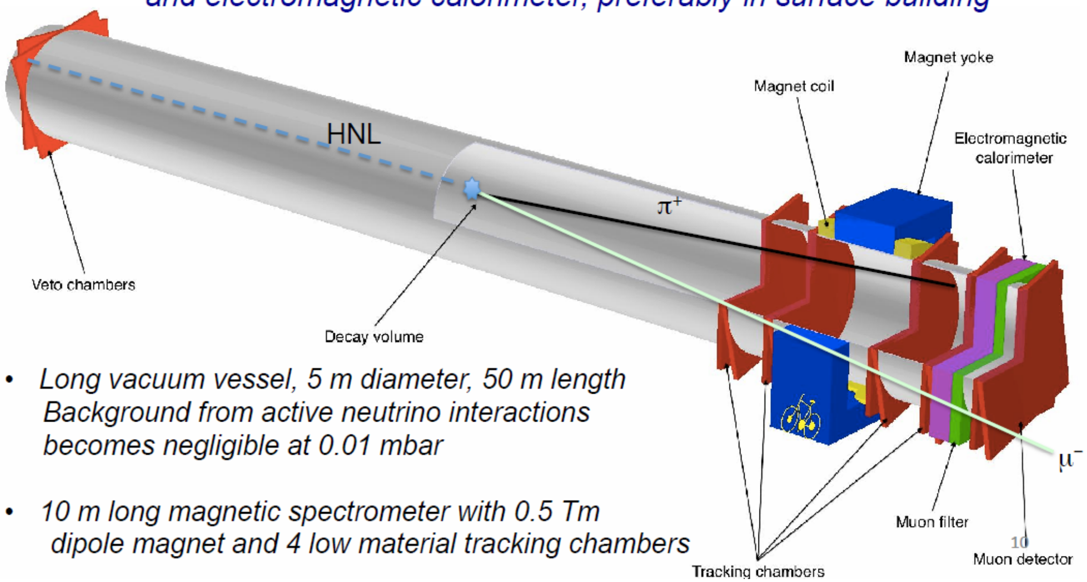


Exact reach domain will depend on detector size and details of displaced vertex efficiency & background

Detector concept
(based on existing technologies)

- Reconstruction of the HNL decays in the final states: $\mu^-\pi^+$, $\mu^-\rho^+$ & $e^-\pi^+$

→ Requires long decay volume, magnetic spectrometer, muon detector and electromagnetic calorimeter, preferably in surface building



- Long vacuum vessel, 5 m diameter, 50 m length. Background from active neutrino interactions becomes negligible at 0.01 mbar
- 10 m long magnetic spectrometer with 0.5 Tm dipole magnet and 4 low material tracking chambers

Proposal to search for Heavy Neutral Leptons at the SPS

(CERN-SPSC-2013-024 / SPSC-EOI-010)

Disclaimer: It is not a classical neutrino physics experiment

On behalf of:

W. Bonivento[1,2], A. Boyarsky[3], H. Dijkstra[2], U. Egede[4], M. Ferro-Luzzi[2], B. Goddard[2], A. Golutvin[4], D. Gorbunov[5], R. Jacobsson[2], J. Panman[2], M. Patel[4], O. Ruchayskiy[6], T. Ruf[2], N. Serra[7], M. Shaposhnikov[6], D. Treille[2 (‡)]

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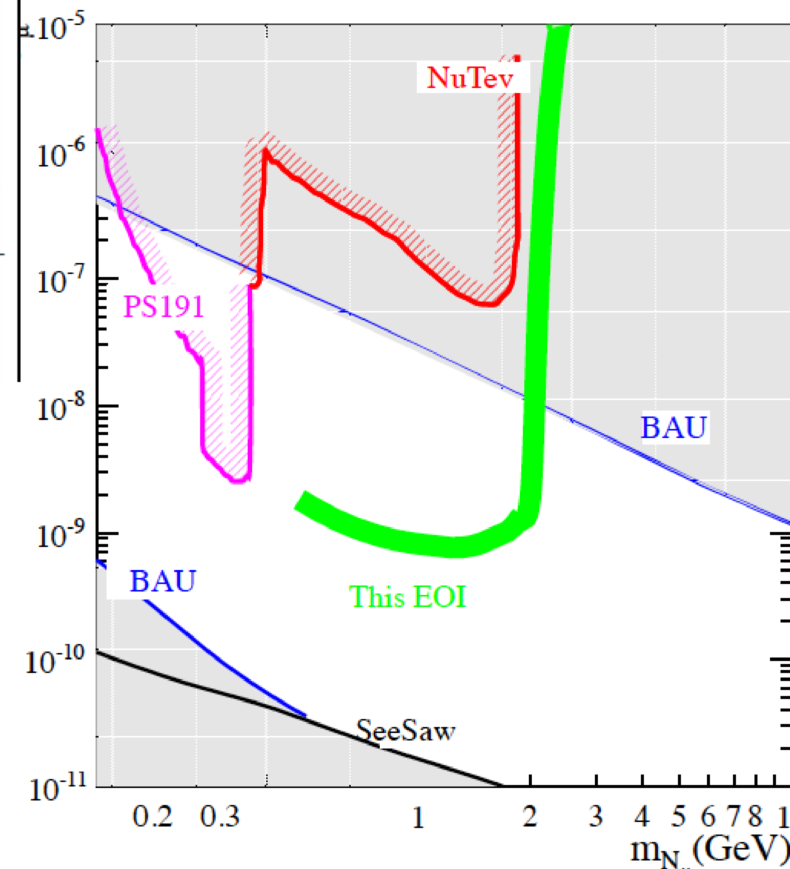
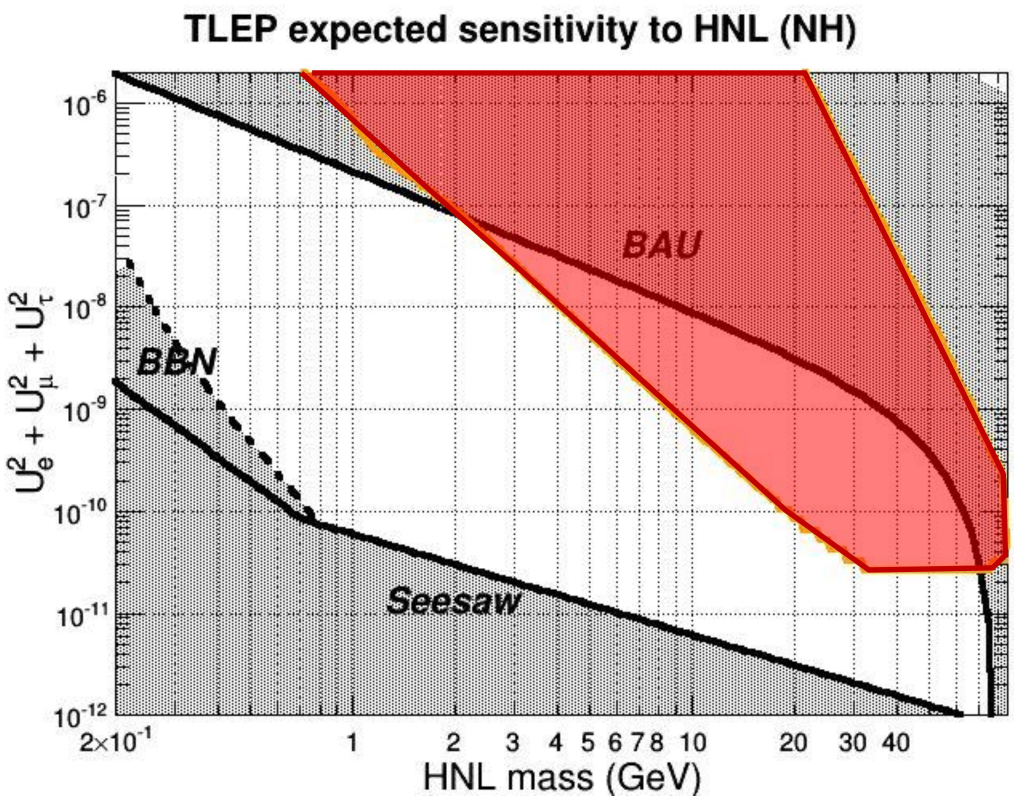


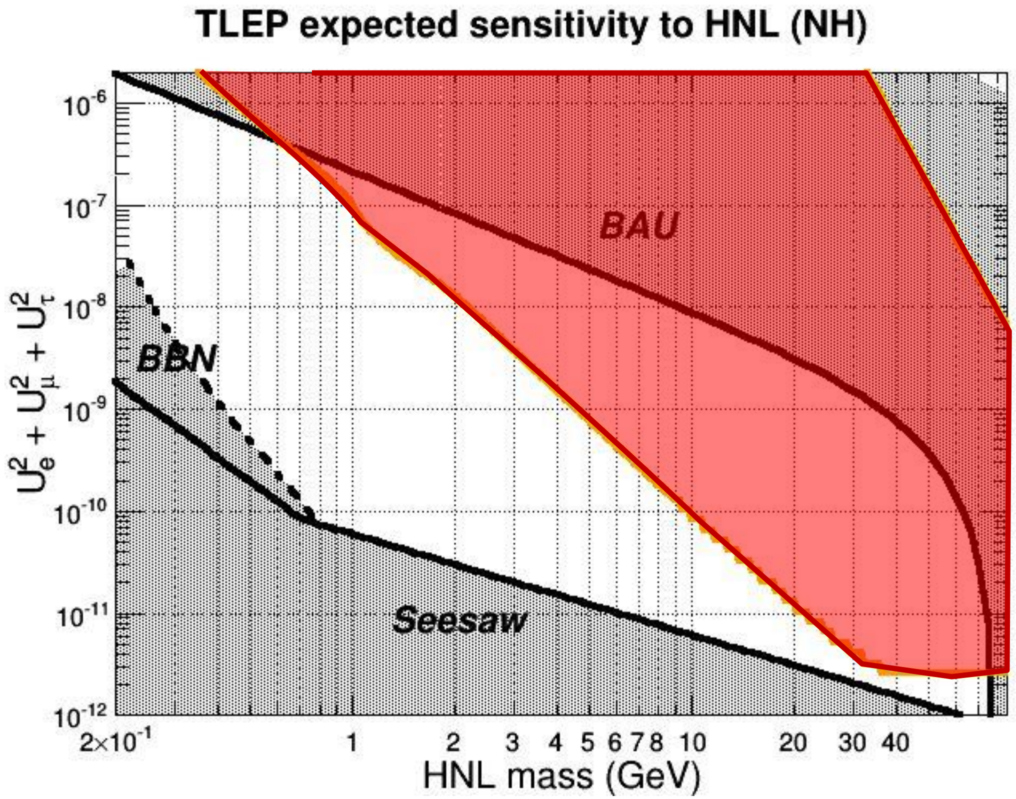


$N_Z = 10^{12}$ 1mm<L<1m

region of interest

FCC-ee sensitivity

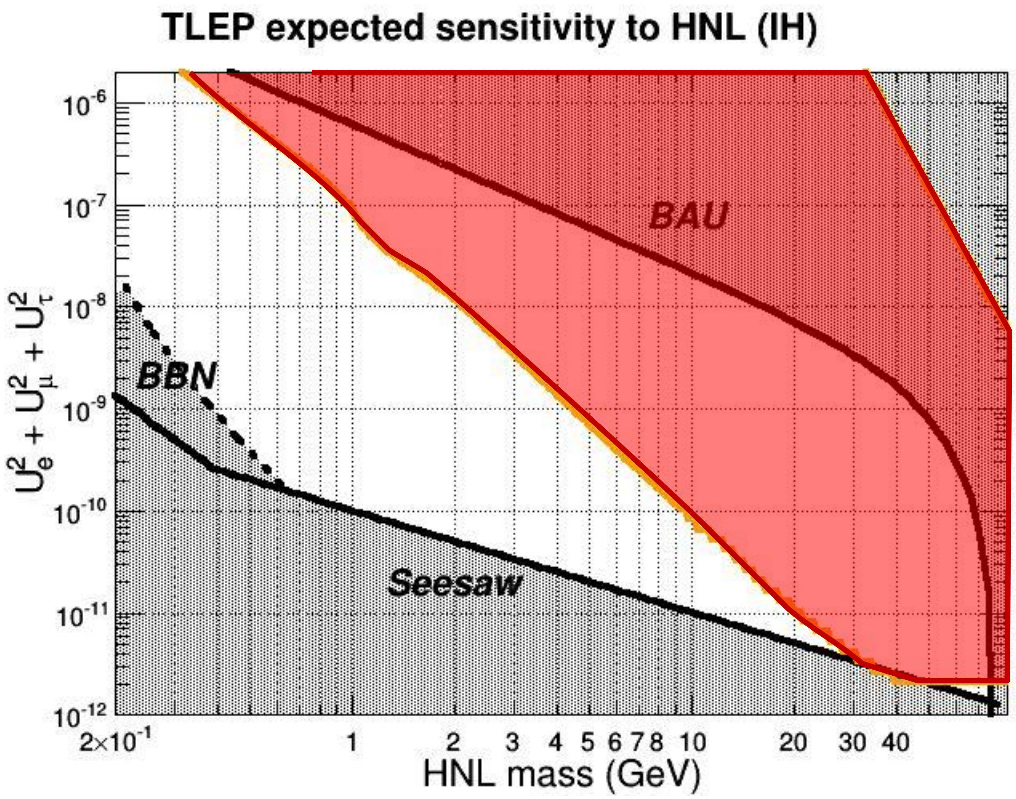
A.B, Elena Graverini, Nicola Serra, Misha Shaposhnikov



$N_Z = 10^{13}$ $100\mu m$ <L<5m

region of interest

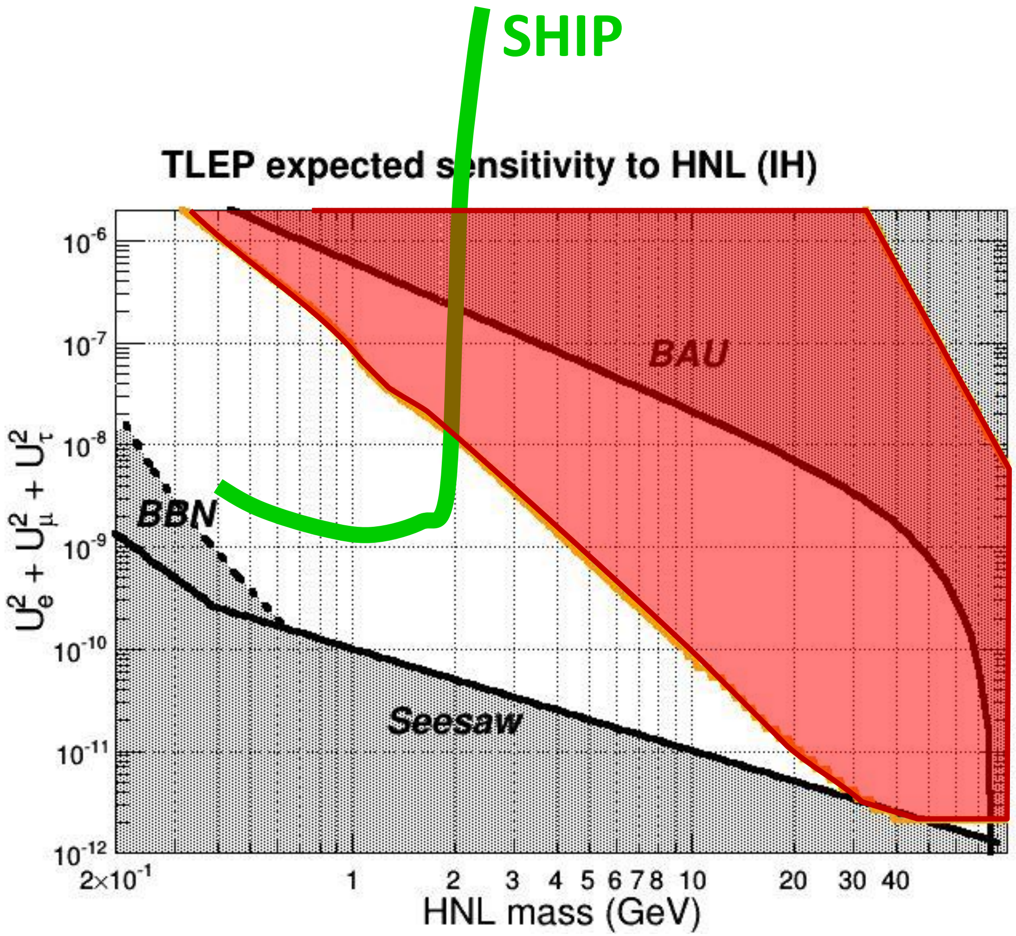
FCC-ee sensitivity



$N_Z = 10^{13}$ $100\mu m$ <L<5m

region of interest

FCC-ee sensitivity



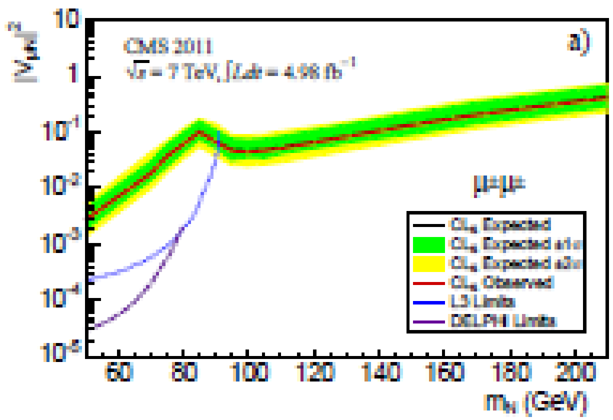
$N_Z = 10^{13}$ $100\mu m$ <L<5m

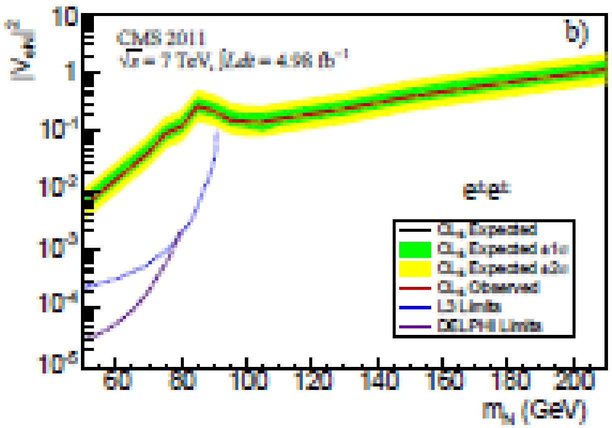
region of interest

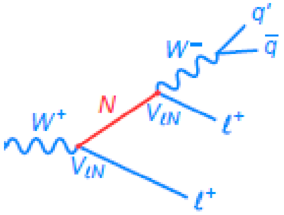
FCC-ee sensitivity

NB very large detector caverns for FCC-hh may allow very large FCC-ee detector (R=15m?) leading to improved reach at lower masses.

-- also search for same sign muons or electrons at the LHC
(e.g. CMS:)





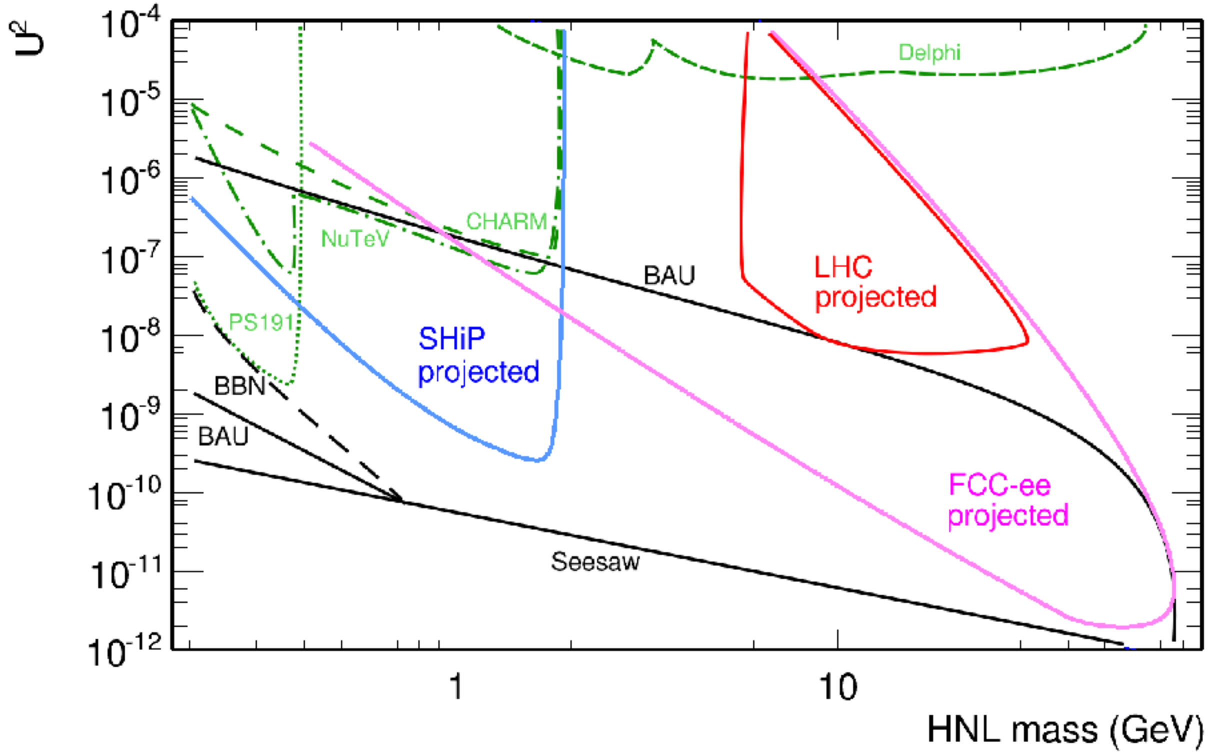


ATLAS arXiv:1203.5420.
CMS arXiv:1207.6079.

limits at $|U|^2 \sim 10^{-2-5}$ level

7 GeV run
papers on 8 GeV run
in preparation

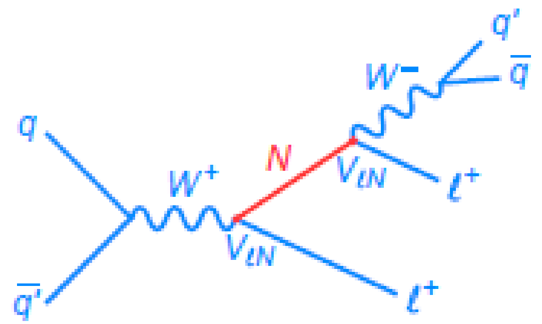
Preliminary projection for LHC (P. Mermod, very preliminary)



Comment/Outlook for FCC-hh

We have seen that the Z factory offers a clean method for detection of Heavy Right-Handed neutrinos
Ws are less abundant at the lepton colliders

At the 100 TeV hadron machine the W is the dominant particle.



There is a lot of /pile-up/backgrounds/lifetime/trigger issues which need to be investigated.
BUT.... in the regime of long lived HNLs the simultaneous presence of
-- the initial lepton from W decays
-- the detached vertex with kinematically constrained decay
allows for a significant background reduction.

But it allows also a characterization both in flavour and charge of the produced neutrino, thus information of the flavour sensitive mixing angles and a test of the fermion violating nature of the intermediate (Majorana) particle.

VERY interesting...

Conclusions

1. H, Z factory will investigate invisible widths with exp precision of 0.15% and 0.0004 Γ_ν

2. The quest for the
«Right-Handed-Almost-Sterile-See-saw-partners neutrinos»
(dextrinos? RHASnus? Heavy Neutral Leptons? Shaposhninos? heavinos?)

is not desperate at all

for Majorana mass at weak scale. In particular it may lead to spectacular 'detached vertex' signatures in a beam dump experiment (SHIP)
or in Z-> neutrino decays at a Tera-Z factory like FCC-ee,

Observation in W decays at the FCC-hh would be very interesting...

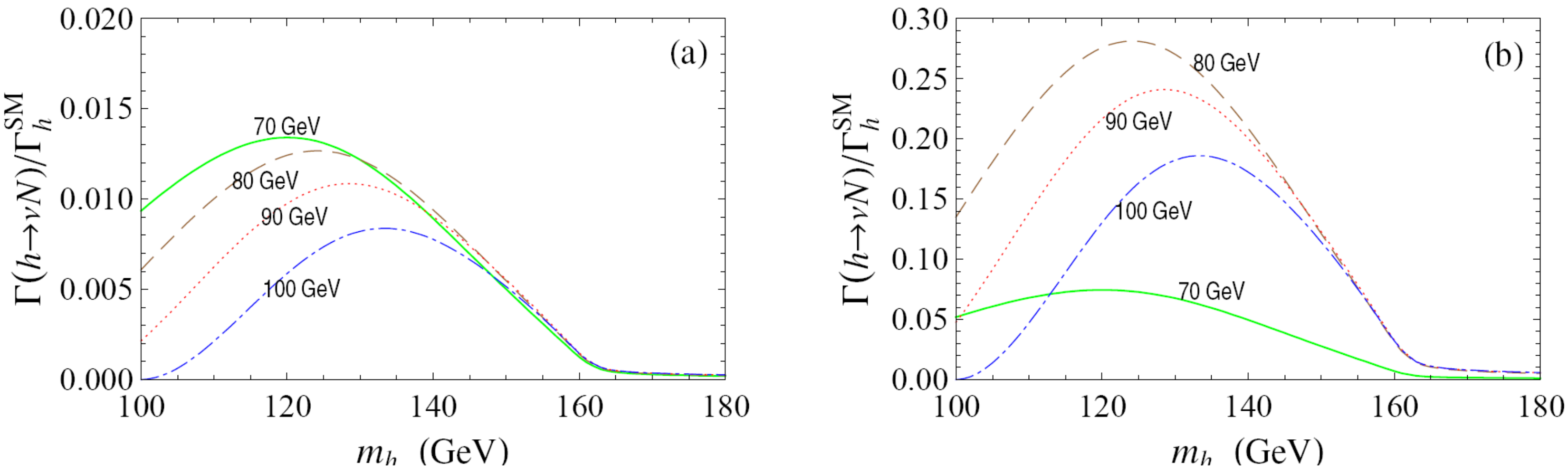
SPARES

Higgs Decay into nu + N

$h \rightarrow \nu N$ allowed if N is light enough

Lepton universaility constraints must be met

$\theta_{\nu_e - N} < 0.05, \quad \theta_{\nu_\mu - N} < 0.05, \quad \theta_{\nu_\tau - N} < 0.08$



Chen, He, Tandean, Tsai (2011)

FLAVOUR(267104)-ERC-23 TUM-HEP 850/12 SISSA 25/2012/EP CFTP/12-013

arxiv:1208.3654

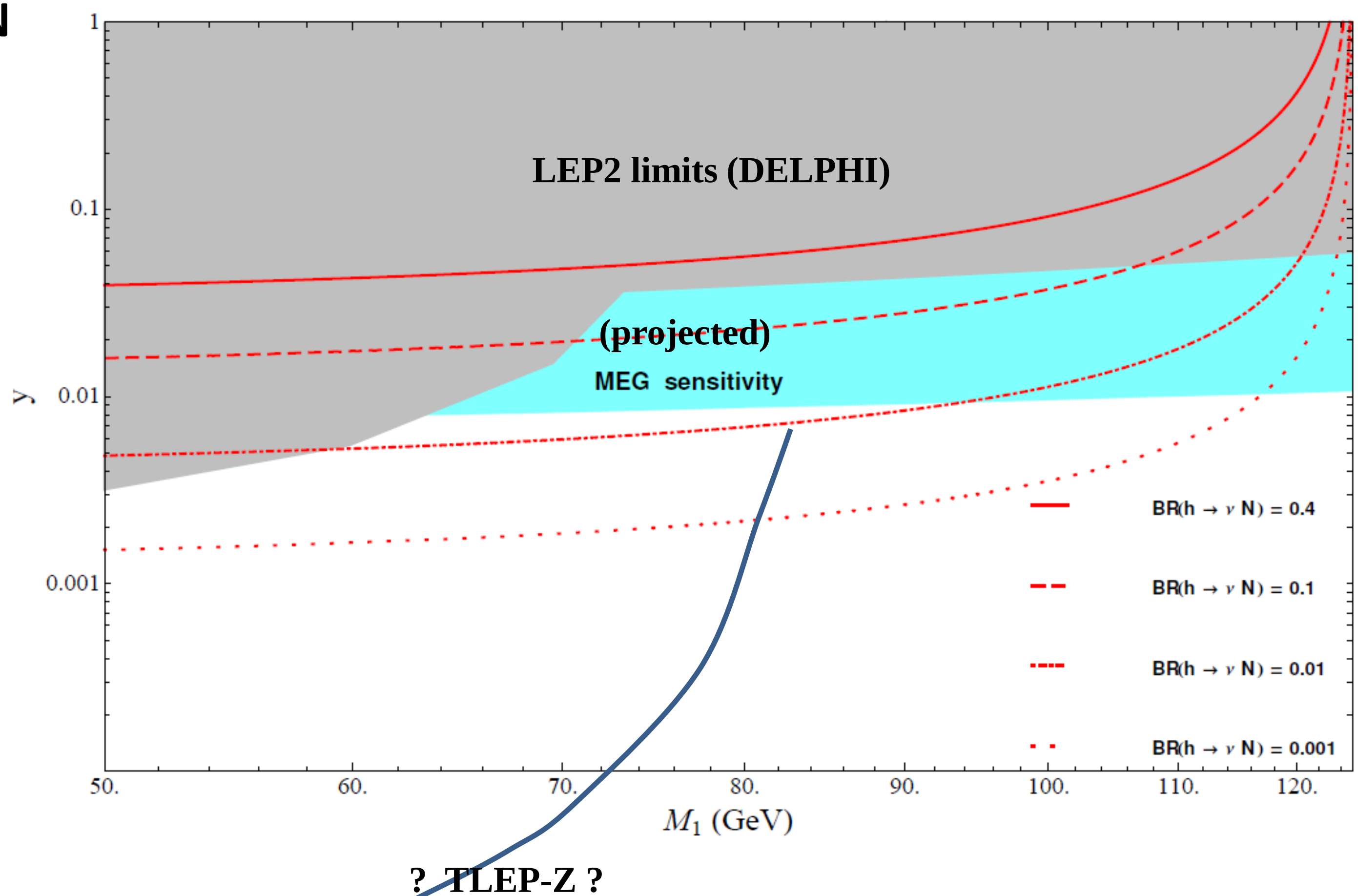
H-> vN

or

Z-> vN

Higgs Decays in the Low Scale Type I See-Saw Model

C. Garcia Cely[a)], A. Ibarra[a)], E. Molinaro[b)] and S. T. Petcov[c,d) 1]



Another solution:

determine the number of neutrinos from the radiative returns

$$e^+e^- \rightarrow \gamma\ Z\ (\rightarrow \nu\ \bar{\nu})$$

CERN-TH.5528/89

NEUTRINO COUNTING

G. Barbiellini[1], X. Berdugo[2], G. Bonvicini[3], P. Colas[4], L. Mirabito[4], C. Dionisi[5], D. Karlen[6], F. Linde[7], C. Luci[8], C. Mana[8], C. Matteuzzi[9], O. Nicrosini[10], R. Ragazzon[1], D. Schaile[11], F. Scuri[1] and L. Trentadue*),[12]

in its original form (Karlen) the method only counts the 'single photon' events and is actually less sensitive than claimed. It has poorer statistics and requires running ~10 GeV above the Z pole. Systematics on photon selection are not small.

present result: N_ν= 2.92±0.05

NEUTRINO COUNTING AT THE Z-PEAK AND RIGHT-HANDED NEUTRINOS

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Received 20 February 1990

We consider the implications of extending the minimal standard model, with n families of quarks and leptons, by introducing an arbitrary number of right-handed neutrinos, for neutrino-counting via the "invisible width" of the Z. It is shown that the effective number of neutrinos, $\langle n \rangle$, satisfies, the inequality $\langle n \rangle \leqslant n$, where $\langle n \rangle$ is defined by $\Gamma(Z \to \text{neutrinos}) \equiv \langle n \rangle \Gamma_0$ and Γ_0 is the standard width for one massless neutrino. Thus, in the case of three families, the neutrino-counting can give a result which is less than three, if there are right-handed neutrinos.

Theorem.

In the standard model, with n left-handed lepton doublets and $N-n$ right-handed neutrinos, the effective number of neutrinos, $\langle n \rangle$, defined by

$$\Gamma(Z \to \text{neutrinos}) \equiv \langle n \rangle \Gamma_0 ,$$

where Γ_0 is the standard width for one massless neutrino, satisfies the inequality

$$\langle n \rangle \leqslant n . \quad (15)$$