# Neutron radiography of a helium gas density gradient at cryogenic temperatures for a novel muon beam line

## Zurich PhD seminar 2014

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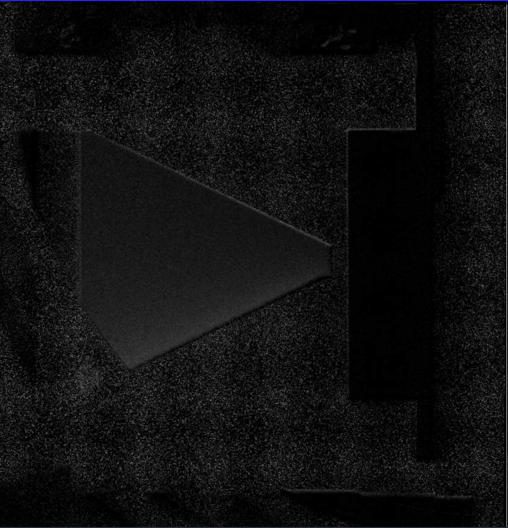
September 11, 2014




# Outline

# Motivation

Slow muons ( $< 1$ eV)

$\rightarrow$ Pure leptonic spectroscopy ($\mu^+ e^-$, ...)

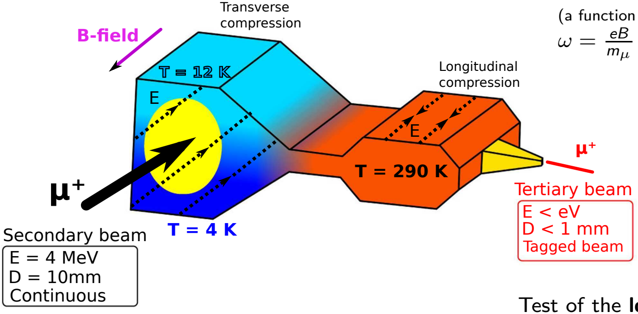
$\rightarrow$ ...

# How-to ...

Manipulating the $\mu^+$ **drift** in helium gas:
*D.Taqqu, PRL* **97**, *194801 (2006)*

$$v_D = \frac{|\vec{E}|}{1+(\omega\tau)^2}\left[\hat{\vec{E}} + \omega\tau\,\hat{\vec{E}}x\hat{\vec{B}} + \omega^2\tau^2\left(\hat{\vec{E}}\cdot\hat{\vec{B}}\right)\hat{\vec{B}}\right]$$

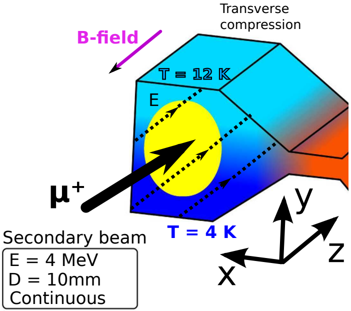
$\tau\,(\varrho)$ the **mean free path**.
(a function of the particle density $\varrho$)
$\omega = \frac{eB}{m_\mu}$ the cyclotron frequency.




Test of the **longitudinal** compression:

*Y.Bao et al., PRL* **112**, *224801 (2014)*

# Density Distribution $\varrho(y)$




Ideal gas equation:

$$\varrho(y) = \frac{n}{V} = \frac{p}{R \cdot T(y)}$$

$$\frac{\varrho(y=0)}{\varrho(y=h)} = \frac{T(y=h)}{T(y=0)}$$

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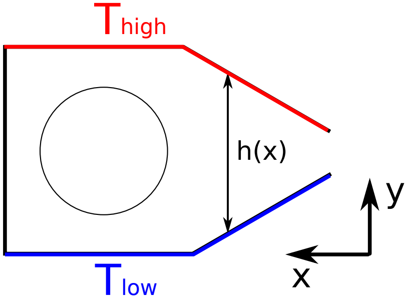


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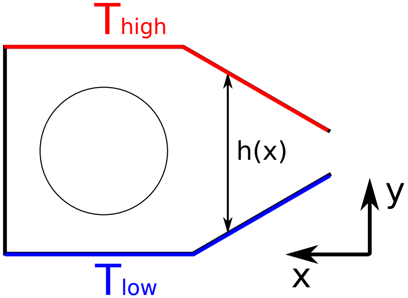


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$$\nabla \cdot (k(T) \nabla T) + \dot{q} = 0 = \left(\rho c \frac{\partial T}{\partial t}\right)$$

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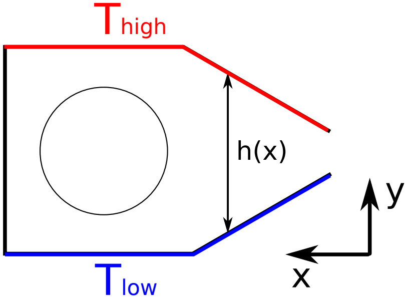


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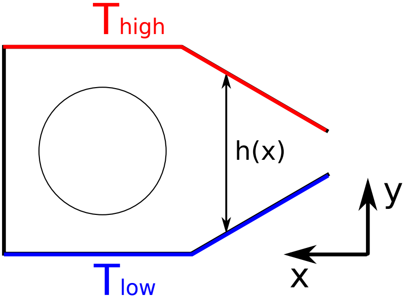


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$$\rightarrow T(y; h(x)) = (T_{high} - T_{low}) \left(\frac{y}{h(x)}\right)^{\frac{1}{1+n}} + T_{low}$$

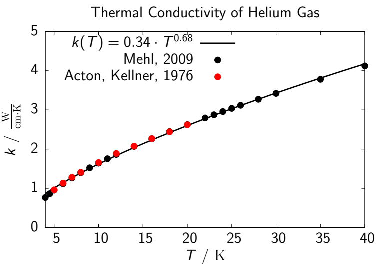
# Simplifications

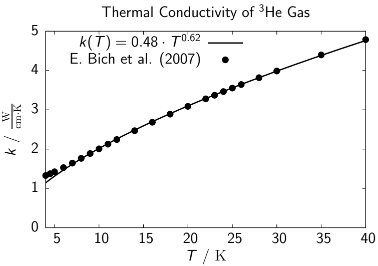
- Ideal gas equation.
- Stationary temperature distribution.
- One dimensional heat diffusion equation.
- Parametrization of the thermal conductivity:

$$k(T) = a \cdot T^n$$

$$\rightarrow T(y; h(x)) = (T_{high} - T_{low}) \left( \frac{y}{h(x)} \right)^{\frac{1}{1+n}} + T_{low}$$

# Thermal Conductivity $k(T) = a \cdot T^n$




$^4$He gas for $4\,\mathrm{K}$ to $40\,\mathrm{K}$:

$$a = 0.34(1)\,\frac{\mathrm{W}}{\mathrm{cm{\cdot}K^{1+n}}}$$

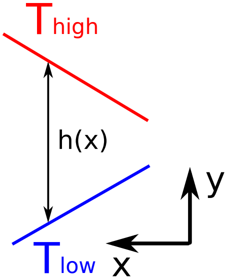
$$n = 0.683(6)$$

$^3$He gas for $6\,\mathrm{K}$ to $40\,\mathrm{K}$:

$$a = 0.483(4)\,\frac{\mathrm{W}}{\mathrm{cm{\cdot}K^{1+n}}}$$

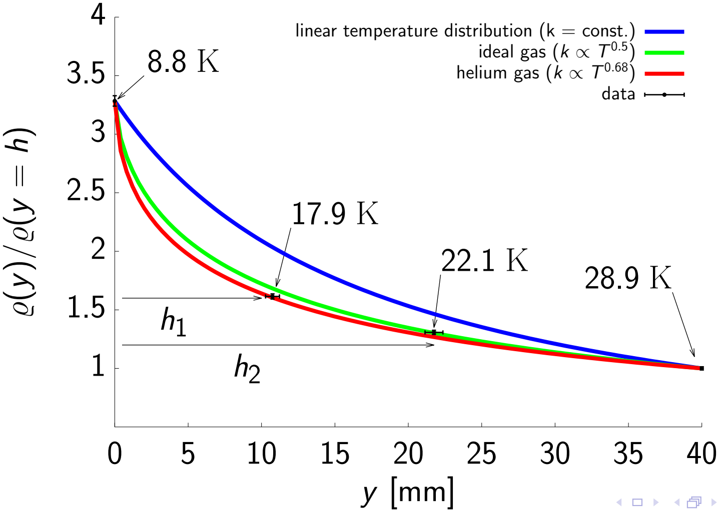
$$n = 0.620(3)$$

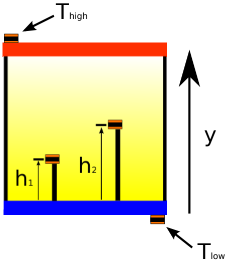
# Density Distribution $\varrho(y; h(x))$

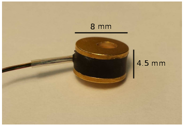



$$\to \varrho(y; h(x)) = \frac{p}{R \cdot \left( (T_{high} - T_{low}) \left( \frac{y}{h(x)} \right)^{\frac{1}{1+n}} + T_{low} \right)}$$

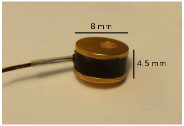
# Density Distribution $\varrho(y; h = 40\,\mathrm{mm})$



# Limitation of the temperature measurement




- Possible self heating (thermal contact within the gas).
- Thermal influence from the cable.
- Large size (8 mm) impractical for a wedge shaped gas cell or small cell heights $h(x)$.

# Neutron Radiography

- Using $^3\mathrm{He}$ gas instead of $^4\mathrm{He}$ gas because of neutron absorption cross section.

  (absorption: $\sigma_{abs} = 5333\,\mathrm{b}$, scattering: $\sigma_{scatt} = 6\,\mathrm{b}$ for $v_{neutron} = 2200\,\frac{m}{s}$)

- Attenuation: $I = I_0 \cdot \exp\left(-\sigma_{abs} \cdot \frac{v_0}{v} \cdot L \cdot \varrho_{^3\mathrm{He}}\right)$

$$= I_0 \cdot \exp\left(-14800\,\mathrm{b} \cdot L \cdot \varrho_{^3\mathrm{He}}\right)$$

# Neutron Radiography

- Neutrons transmitted through the gas cells are recorded with an image plate.
- The density $\varrho = \varrho\,(x, y)$ is extracted by normalizing to an empty cell measurement.

$$\frac{I(x,y)_{\mathrm{gas}}}{I(x,y)_{\mathrm{no\,gas}}} = e^{-\sigma_{\mathrm{abs}} \cdot L \cdot \varrho(x,y)}$$

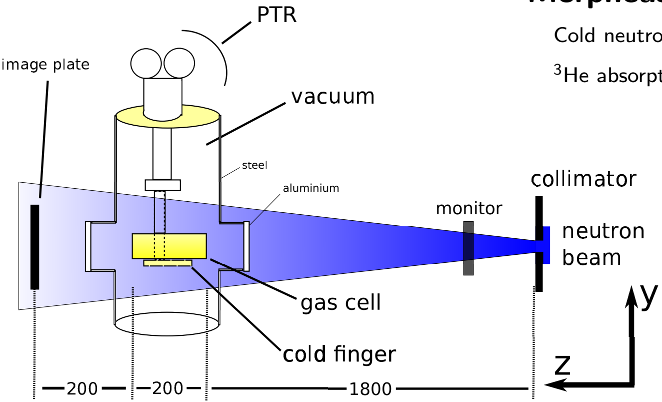
$$\varrho\,(x,y) = -\ln\left(\frac{I_{\mathrm{gas}}}{I_{\mathrm{no\,gas}}}\right) \cdot \frac{1}{\sigma_{\mathrm{abs}} \cdot L}$$

# The Experimental Setup

## Morpheus, SINQ (PSI):

Cold neutrons: $\mathrm{E} = 3.3\,\mathrm{meV} \quad (\lambda = 5\,\mathrm{\AA})$

$^{3}$He absorption: $\sigma_{\mathrm{abs}} \approx 14800\,\mathrm{b}$



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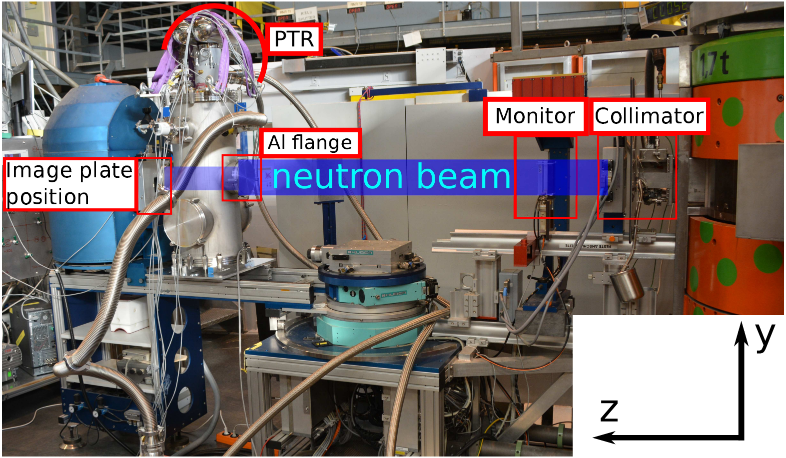
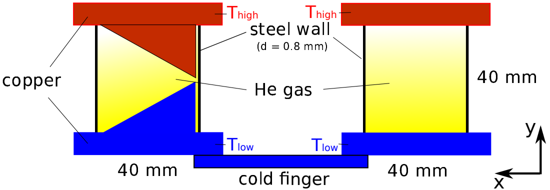
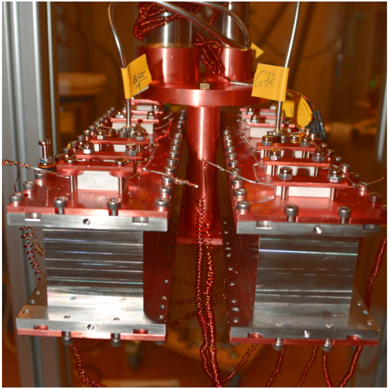
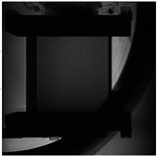
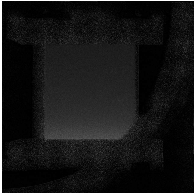


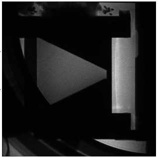
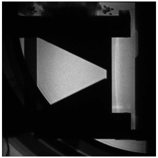
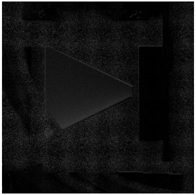
# The Gas Cells



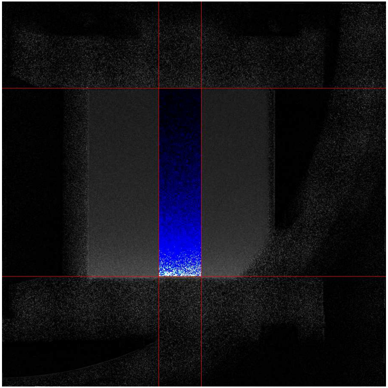



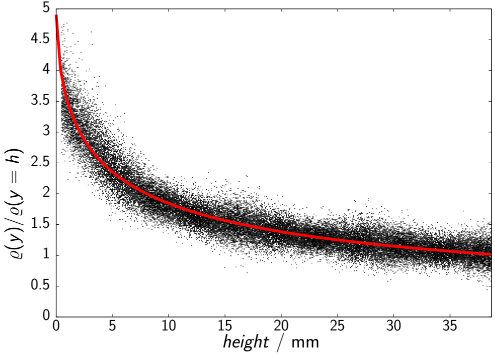
# Neutron Radiographies

-log(  /  ) = 

-log(  /  ) = 

# Result: Rectangular Shaped Gas Cell






$$\varrho\,(x,y) = -\ln\left(\frac{I_{\mathrm{gas}}}{I_{\mathrm{no\ gas}}}\right)\cdot\frac{1}{\sigma_{\mathrm{abs}}\cdot L}$$

$$\varrho(y;h(x)) = \frac{p}{R\cdot\left(\left(T_{high}-T_{low}\right)\left(\frac{y}{h(x)}\right)^{\frac{1}{1+n}}+T_{low}\right)}$$

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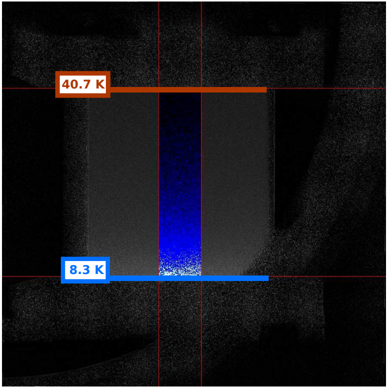
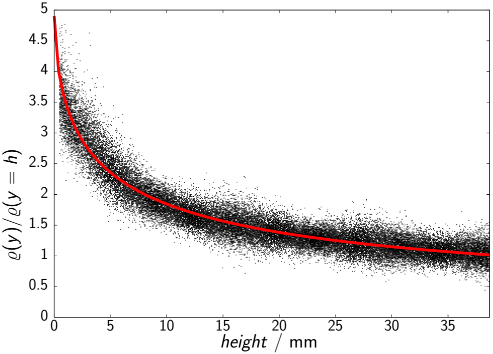


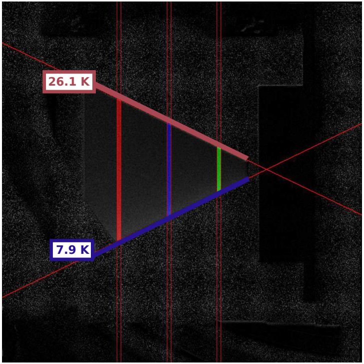



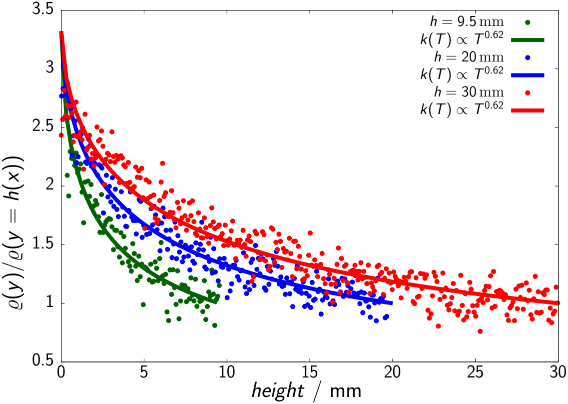

Measured gas density gradient for the rectangular shaped cell.

$T_{high} = 40.7$ K, $T_{low} = 8.3$ K and $^3$He pressure 15 mbar.

red line: $\varrho(y; h = 40\,\mathrm{mm})$ fit to data $\Rightarrow n = 0.584$ (theoretical parametrization: $n = 0.62$).

# Result: Wedge Shaped Gas Cell

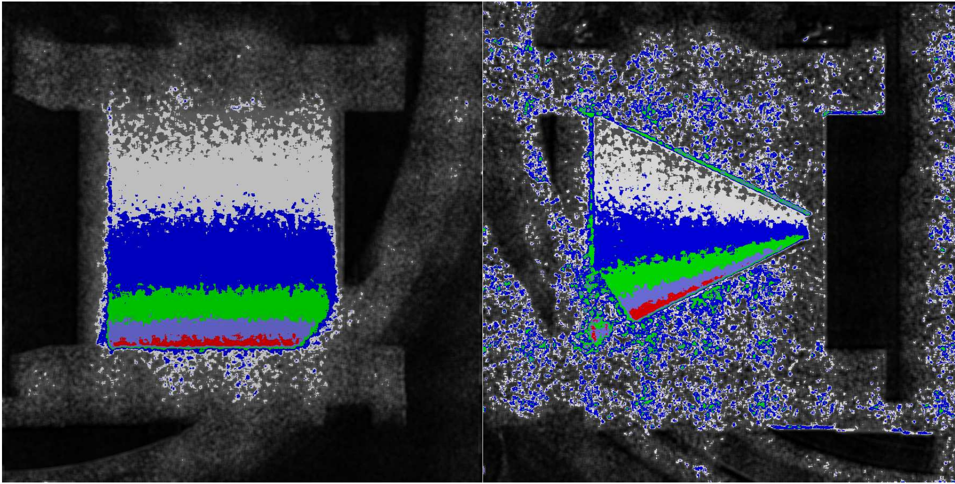



Wedge shaped cell: $T_{high} = 26.1$ K, $T_{low} = 7.86$ K and $^3$He pressure 9 mbar.

- red line: $h = 30$ mm
- blue line: $h = 20$ mm
- green line: $h = 9.5$ mm

# Helium Gas Density Gradient

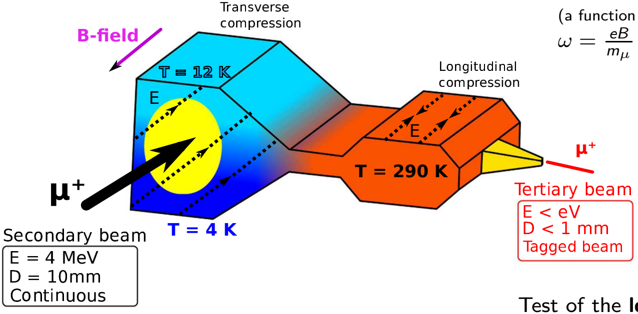


# Conclusion

Manipulating the $\mu^+$ **drift** in helium gas:
*D.Taqqu, PRL* **97**, *194801 (2006)*

$$v_D = \frac{|\vec{E}|}{1+(\omega\tau)^2}\left[\hat{\vec{E}} + \omega\tau\, \hat{\vec{E}} x \hat{\vec{B}} + \omega^2\tau^2 \left(\hat{\vec{E}}\cdot\hat{\vec{B}}\right)\hat{\vec{B}}\right]$$

$\tau(\varrho)$ the **mean free path**.
(a function of the particle density $\varrho$)
$\omega = \frac{eB}{m_\mu}$ the cyclotron frequency.




Test of the **longitudinal** compression:

*Y.Bao et al., PRL* **112**, *224801 (2014)*

# Conclusion

- The **gas density gradient** has been **measured** for rectangular and wedge shaped gas cells.
- The gas density ratio (bottom / top) is exceeding a factor of **3** over less than **15 mm** height.
- No fundamental difference between rectangular shaped and wedge shaped gas region was found.
  $\rightarrow$ no indication of turbulences.

# Acknowledgment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