Transverse-momentum resummation for heavy quark production at hadrom colliders

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Outline

- Motivation
- q_T-resummation
- q_T-subtraction
- Results
- Summary

Top quark

Mass of the top quark obtained through combining the measurements at the Tevatron and LHC colliders is $m_t = 173.34 \pm 0.27\,(stat)\ \pm 0.71\,(syst)\,\mathrm{GeV}$ [ATLAS and CDF and CMS and D0 Collaborations (2014)].

- Strong coupling to the Higgs boson
- Crucial to the hierarchy problem

Top quark pair production

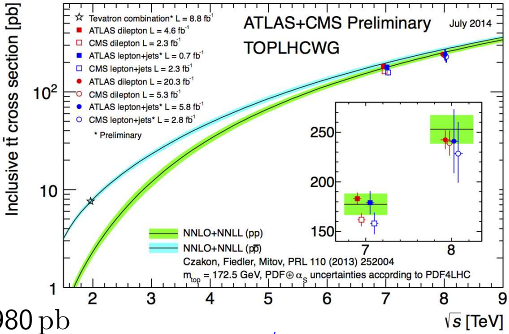
- The top quark pair production is the main source of the top quark events in the Standard Model (SM).
- Many New Physics models involve heavy top partners which then decay into a top quark pair.

The study of the $t\bar{t}$ pair production at hadron colliders can

- shed light on the electroweak symmetry breaking mechanism.
- provide information on the backgrounds of many NP models.

Top quark pair production

- Because of its large mass the top quark decay before hadronization, allowing for a better experimental measurements.



- $\sigma_{t\bar{t}}\,(14\,\mathrm{TeV}) \sim 980\,\mathrm{pb}$
  $\mathcal{L} \sim 10^{32}\,\mathrm{cm}^2\mathrm{s}^{-1}$ $\longrightarrow$ 1 event/10 s

More precise calculations are needed from the theory side

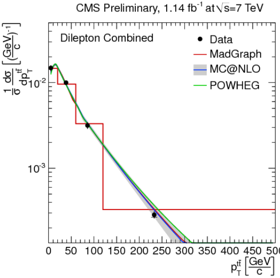
Transverse momentum of the $t\bar{t}$ pair

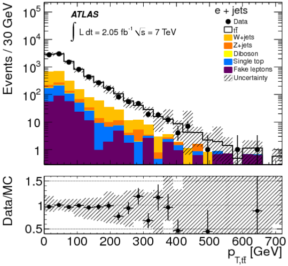
The q_T of the $t\bar{t}$ pair is one of the most important observables.

- D0 and CDF collaborations have shown that top quark charge asymmetry exhibits a strong dependence on the q_T of the top quark pair. V. M. Abazov et al. [D0 Collaboration], 2011, T. Aaltonen et al. [CDF Collaboration], 2013
- Restriction to the low q_T region enhances the sensitivity of the invariant mass distribution of the $t\bar{t}$ pair to NP contributions. E. Alvarez, 2012

q_T spectrum of the $t\bar{t}$ pair

Last year both CMS and ATLAS experiments measured the q_T distribution of the $t\bar{t}$ pair at the LHC. CMS-PAS-TOP-11-013, G. Aad et al. [ATLAS Collaboration], 2013.





QCD corrections

Theoretical efforts for obtaining precision predictions for the $t\bar{t}$ production at hadron colliders started almost 3 decades ago

- NLO QCD corrections are calculated by [Nason, Dawson and Ellis (1988), Beenakker, Kuijf, van Neerven and Smith (1989), Beenakker, van Neerven, Meng, Schuler and Smith (1989)].
- Recently the calculation of the full NNLO QCD corrections was completed for the total cross section (Not fully analytic though). [Barnreuther, Czakon, Mitov (2012), Czakon, Mitov (2012), Czakon, Mitov (2013), Czakon, Fiedler, Mitov (2013)].
- Precision result for the invariant mass distribution is worked out in [Ahrens, Ferroglia, Neubert, Pecjak, Yang].
- Computation of differential distribution underway [Abelof, Gehrmann-De Ridder, Maierhofer (2014), Abelof, Gerhrmann-de Ridder (2014)].

q_T distribution

- When $q_T^2 \sim M^2$, $\alpha_S(M^2)$ is small, and the standard fixed order expansion is theoretically justified.
- When $q_T^2 \ll M^2$ large logarithms of the form $\alpha_S^n \log(M^2/q_T^2)$ appear, due to soft and collinear gluon emissions. Effective expansion variable is the $\alpha_S^n \log(M^2/q_T^2)$, which can be ~ 1 even for small α_S. These large logarithms need to be resummed to all orders in α_S, in order to get reliable predictions over the whole range of the transverse momenta.

The resummation of large logs results in exponentiating these large logarithmic terms

$$\sigma^{(res)} \sim \sigma^{(0)} C(\alpha_S) \left(L g_1(\alpha_S L) + g_2(\alpha_S(L)) + \alpha_S g_3(\alpha_S L) \right) .$$

hard-virtual LL NLL NNLL

Resummation

q_T-resummation has been succesfully applied to the hadron-hadron scattering processes with 2 hard partons at LO, both in QCD and soft-collinear effective theory (SCET) [Parisi et al. (1979), Curci et al. (1979), Dokshitzer et al. (1980), Bassetto et al. (1980), Kodaira et al. (1982), Davies et al. (1984), Collins et al. (1985), Catani et al. (1988), de Florian et al. (2001), Catani et al. (2001), Bozzi et al. (2006), Catani et al. (2011), Becher et al. (2011)].

Resummation for the $t\bar{t}$ production

Production of coloured particles imposes additional complications compared to the production of a colourless system.

- Soft and collinear QCD radiation from the final state particles
- Colour flow between initial and final state particles leading to non-trivial colour correlations

The top quark is massive

- The collinear limit is not singular $\longrightarrow$ LL structure unaffected
- Additional NLL from large-angle soft radiation

Resummation for the $t\bar{t}$ production

- The first attempt to develop q_T-resummation formalism at next-to-leading logarithmic (NLL) accuracy for $t\bar{t}$ production was done in [Berger, Meng (1994), Mrenna, Yuan (1997)]. However, they did not consider color mixing between singlet and oktet final states and missed the initial-final gluon exchange.
- Recently resummation of $t\bar{t}$ q_T spectrum, based on soft collinear effective theory (SCET), was performed at NNLL+NLO. [Zhu, Li, Li, Shao, Yang (2013)]. This work is limited to the study of the q_T cross section after integration over the azimuthal angles of the produced heavy quarks.
- This year the q_T-resummation in QCD was performed at the fully-differential level with respect to the kinematics of the produced heavy quarks. [Catani, Grazzini, Torre (2014)].

The resummation procedure at small q_T

$$h_1(P_1) + h_2(P_2) \to Q(p_3) + \bar{Q}(p_4) + X.$$

- Consider the most general fully-differential cross section

$$\frac{d\sigma(P_1, P_2; \mathbf{q_T}, M, y, \mathbf{\Omega})}{d^2\mathbf{q_T} dM^2 dy d\mathbf{\Omega}}, \qquad (1)$$

  where P_1 and P_2 are the momenta of incoming hadrons, $\mathbf{q_T}$, M and y are the transverse momentum vector, invariant mass and rapidity of the $Q\bar{Q}$ pair, $\mathbf{\Omega}$ is a set of two additional independent kinematical variables that specify the angular distribution of heavy quarks with respect to the momentum q of the $Q\bar{Q}$ pair. For instance $\mathbf{\Omega} = \{y_3, \phi_3\}$.
- Decompose the cross section in a singular and a regular part

$$d\sigma = d\sigma^{(\mathrm{sing})} + d\sigma^{(\mathrm{reg})}.$$

- $d\sigma^{(\mathrm{sing})}$ embodies all the singular terms in the limit $q_T \to 0$.
- $d\sigma^{(\mathrm{reg})}$ includes the remaining non-singular terms.

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- Decompose the cross section in a singular and a regular part

should be replaced by $d\sigma^{res}$

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- $d\sigma^{(\mathrm{reg})}$ includes the remaining non-singular terms.

The singular component

- Factorization formula

$$\frac{d\sigma_{h_1h_2\to Q\bar{Q}X}}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{a_1,a_2'} f^{(\mu_F)}_{a_1/h_1}\otimes f^{(\mu_F)}_{a_2/h_2}\otimes \frac{d\hat{\sigma}_{a_1a_2\to Q\bar{Q}X}}{dx_T^2\,dz_1\,dz_2\,d\mathbf{\Omega}}\,.$$

$$x_T^2 = \mathbf{q_T}^2/M^2\,, \qquad z_{1,2} = \sqrt{\frac{M^2}{\hat{s}}}e^{\pm\hat{y}}\,.$$

$$\frac{d\hat{\sigma}^{\rm sing}_{a_1a_2\to Q\bar{Q}X}}{dx_T^2\,dz_1\,dz_2\,d\mathbf{\Omega}} = \frac{\pi}{M^2}\sum_{c=q,\bar{q},g}\langle\mathcal{M}^{\rm fin}_{c\bar{c}\to Q\bar{Q}}|\mathbf{\Delta}_{c\bar{c};a_1a_2}(x_T^2;z_1,z_2)|\mathcal{M}^{\rm fin}_{c\bar{c}\to Q\bar{Q}}\rangle\,.$$

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Hard-virtual amplitude:

$$|\mathcal{M}^{\mathrm{fin}}_{c\bar{c}\to Q\bar{Q}}\rangle = \left[1 - \tilde{I}^{\mathrm{DY}}_{c\bar{c}}(\epsilon) - \tilde{I}_{\mathrm{HQ}}(\epsilon)\right] |\mathcal{M}_{c\bar{c}\to Q\bar{Q}}\rangle .$$

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DY-like soft and collinear emission + emission from $Q\bar{Q}$:

$$\mathbf{\Delta}_{c\bar{c};a_1a_2}(x_T^2;z_1,z_2) = \mathbf{\Delta}^{\mathrm{DY}}_{c\bar{c};a_1a_2}(x_T^2;z_1,z_2)+\mathbf{\Delta}_{\mathrm{HQ}}(x_T^2)\delta^{1-z_1}_{ca_1}\delta^{1-z_2}_{\bar{c}a_2}+\text{c.c.}$$

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- Factorization formula

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$$x_T^2 = \mathbf{q_T}^2/M^2 \, , \qquad z_{1,2} = \sqrt{\frac{M^2}{\hat{s}}} e^{\pm\hat{y}} \, .$$

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At NLO, with $\left(\frac{\alpha_S}{4\pi}\right)$ normalization:

Fully computed at NLO

$$\boldsymbol{\Delta}^{(1)}_{\mathrm{HQ}}(x_T^2) - \tilde{I}^{(1)}_{\mathrm{HQ}}(\epsilon)\delta x_T^2 = \frac{1}{\epsilon}\left(\frac{M^2}{\mu_R^2}\right)^{-\epsilon} \delta(x_T^2)\mathbf{\Gamma}^{(1)}_{\mathbf{T}} - \left(\frac{1}{x_T^2}\right)_+ \mathbf{\Gamma}^{(1)}_{\mathbf{T}} + \delta(x_T^2)\mathbf{F}^{(1)}_{\mathbf{T}}$$

The all-order resummation formula

- Is obtained by working in impact parameter **b** space.

$$\frac{d\sigma^{(\mathrm{res})}}{d^2\mathbf{q_T}\,dM^2\,dy\,d\mathbf{\Omega}} = \frac{M^2}{s}\sum_{c=q,\bar{q},g}\left[d\sigma^{(0)}_{cc}\right]\int\frac{d^2\mathbf{b}}{(2\pi)^2}e^{i\mathbf{b}\mathbf{q_T}}S_c(M,b)$$

$$\times\sum_{a_1,a_2}\int_{x_1}^{1}\frac{dz_1}{z_1}\int_{x_2}^{1}\frac{dz_2}{z_2}\left[(\mathbf{H\Delta})C_1C_2\right]_{c\bar{c};a_1a_2}\times$$

$$f_{a_1/h_1}(x_1/z_1,b_0^2/b^2)f_{a_2/h_2}(x_2/z_2,b_0^2/b^2)\,.$$

$b_0 = 2e^{-\gamma_E}$ (γ_E is the Euler number).

$$x_1 = \frac{M}{\sqrt{s}}e^{+y} \qquad x_2 = \frac{M}{\sqrt{s}}e^{-y}\,.$$

$$\ln S_c M, b = \int_{M^2}^{b_0^2/b^2}\frac{dq^2}{q^2}\left[A_c(\alpha_S(q^2))\ln\frac{M^2}{q^2}+B_c(\alpha_S(q^2))\right]\,.$$

LL: $A_c^{(1)}$, NLL: $A_c^{(2)}, B_c^{(1)}$.

The all-order resummation formula

- For simplicity consider the $q\bar{q}$ channel.

$$[(\mathbf{H\Delta})C_1 C_2]_{q\bar{q};a_1 a_2} = \mathbf{H\Delta}_{q\bar{q}} C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) C_{\bar{q}a_2}(z_2; \alpha_S(b_0^2/b^2)) .$$

$$\mathbf{H\Delta}_{q\bar{q}} = \frac{\langle \tilde{\mathcal{M}}_{q\bar{q}\to Q\bar{Q}} | \mathbf{\Delta} | \tilde{\mathcal{M}}_{q\bar{q}\to Q\bar{Q}} \rangle}{\alpha_S^2(M^2) \left| \mathcal{M}^{(0)}_{q\bar{q}\to Q\bar{Q}}(p_1, p_2, p_3, p_4) \right|^2} .$$

$$\mathbf{\Delta}(b, M; y_{34}, \phi_3) = \mathbf{V}^\dagger(b, M; y_{34}) \mathbf{D}(\alpha_S(b_0^2/b^2); \phi_{3b}, y_{34}) \mathbf{V}(b, M; y_{34}) .$$

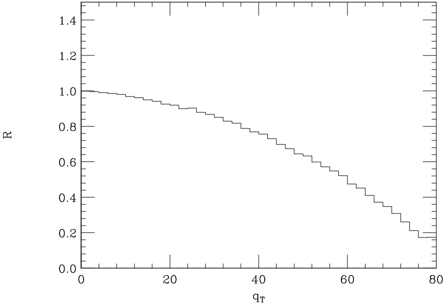
$$\mathbf{V}(b, M; y_{34}) = \bar{P}_q \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \mathbf{\Gamma}_t(\alpha_S(q^2); y34) \right\} ,$$

- $\mathbf{\Gamma}_t$ is the soft anomalous dimension matrix and computable order by order in perturbation theory.
- $\mathbf{D}$ is the azimuthal-correlation matrix and computable order by order in perturbation theory.
- All the perturbative coefficients have been computed at NLO

Subtraction

- Check that the subtraction works!

$$\frac{\left[d\sigma^{\rm res}_{t\bar{t}+X}/d^2\mathbf{q_T}\right]_{\rm NLO}}{\left[d\sigma_{t\bar{t}+X}/d^2\mathbf{q_T}\right]_{\rm NLO}}$$



q_T-subtraction

Knowledge of the low q_T limit is essential also for the fixed order calculation in the q_T-subtraction formalism.
q_T-subtraction formalism has been originally proposed for the production of colourless high-mass systems in hadron collisions. [Catani, Grazzini (2007)].
This subtraction formalism has been successfully applied to number of important processes of this class.

- $pp \to H$ [Catani, Grazzini (2007)].
- $pp \to V$. [Catani, Cieri, Ferrera, de Florian, Grazzini (2009)].
- $pp \to \gamma\gamma$. [Catani, Cieri, Ferrera, de Florian, Grazzini (2011)].
- $pp \to WH$. [Ferrera, Grazzini, Tramontano (2011)].
- $pp \to Z\gamma$. [Grazzini, Kallweit, Rathlev, Torre (2013)].
- $pp \to ZZ$. [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)].
- $pp \to W^+W^-$. [Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi (2014)].
- $pp \to ZH$. [Ferrera, Grazzini, Tramontano (2014)].

q_T-subtraction for $t\bar{t}$

- The fully differential cross section at N(NLO):

$$d\sigma^{t\bar{t}}_{\mathrm{N(NLO)}} = \mathcal{H}^{t\bar{t}}_{\mathrm{N(NLO)}} \otimes d\sigma^{t\bar{t}}_{\mathrm{LO}} + \left[d\sigma^{t\bar{t}+\mathrm{jet}}_{\mathrm{N(LO)}} - d\sigma^{\mathrm{CT}}_{\mathrm{N(LO)}} \right] .$$

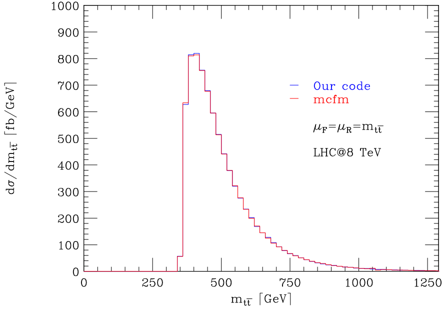
Regular as $q_T \to 0$

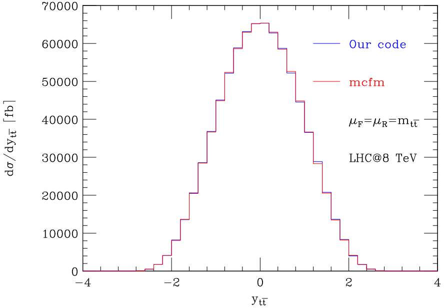
- $\mathcal{H}^{t\bar{t}}_{\mathrm{N(NLO)}}$ is the hard factor, which contains information on the virtual corrections to the LO process.
- $d\sigma^{t\bar{t}}_{\mathrm{LO}}$ is the Born cross section.
- $d\sigma^{t\bar{t}+\mathrm{jet}}_{\mathrm{N(LO)}}$ is the N(LO) cross section of $t\bar{t}$+jet(s) process.
- $d\sigma^{\mathrm{CT}}_{\mathrm{N(LO)}}$ is the counterterm, which can be derived by expanding the resummation formula.

Results

We have implemented the results in a numerical program up to NLO and compared the results with the general purpose NLO generator MCFM.

Preliminary!





- Very good agreement!

Ongoing works/future prospects

- Work in progress:
  We are working on implementation of the resummed cross section up to NLL+some part of NNLL accuracy in a fully-differential Monte Carlo program on the base of HRes program. [Grazzini, HS (2013)].
- Future prospects:
  The extension of q_T-subtraction formalism for $t\bar{t}$ production up to NNLO in α_S is feasible.

Summary

- I have briefly discussed the all-order q_T-resummation for the heavy-quark production at hadron colliders, worked out in [Catani, Grazzini, Torre (2014)].
- We have used the knowledge of the low q_T behaviour of the amplitudes to extend the q_T subtraction method for the $t\bar{t}$ production at hadron colliders.
- We have implemented the results in a fully-differential Monte Carlo program and found good agreement with the known results.
- The implementation of the fully-differential resummed cross section at NLO+NNLL aaccuracy is ongoing.