Regularization scheme dependence of two-loop amplitudes

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Contents

Introduction and motivation

Physical cross section at 1-loop (all well known!):

$$\sigma = \sigma_V + \sigma_R = \int d\Phi_V |M_V|^2 + \int d\Phi_R |M_R|^2$$

In all dimensional schemes space-time is continued from 4 to $D = 4 - 2\epsilon$ dimensions
$\Rightarrow$ momentum integrals become well-defined and UV and IR singularities appear as $1/\epsilon^k$-poles.

Introduction and motivation

$$\Rightarrow \sigma^{R.S.} = \underbrace{\int d\Phi_V |M_V(\ldots,[g],\ldots)|^2}_{\frac{a}{\epsilon^2}+\frac{b}{\epsilon}+c+\epsilon\, d+\epsilon^2 e+\ldots} + \underbrace{\int d\Phi_R |M_R(\ldots,[g],\ldots)|^2}_{-\frac{a}{\epsilon^2}-\frac{b}{\epsilon}+l+\epsilon\, m+\epsilon^2 n+\ldots}$$

$$= \sigma_{\text{finite}} \underbrace{+\epsilon\,\sigma_1 + \epsilon^2\sigma_2 + \ldots}_{\textbf{scheme dep.}}$$

and we get for the physical cross section

$$\sigma = \lim_{\epsilon\to 0} \sigma^{R.S.} = \sigma_{\text{finite}}$$

Introduction and motivation

- A different (but consistent) treatment of the gluon metric in the amplitude will modify the scheme dep. in the virtual and real contribution, keeping however the physical cross-section invariant [*Signer and Stöckinger*, 2008].

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  - $\Rightarrow$ The purely D-dim. treatment of all objects is conceptually simpler, but it breaks supersymmetry.
  - The 4-dim. treatment of the gluon is better compatible with supersymmetry and it is more amenable to helicity methods, which are commonly used to simplify QCD higher-order calculations.

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    - The 4-dim. treatment of the gluon is better compatible with supersymmetry and it is more amenable to helicity methods, which are commonly used to simplify QCD higher-order calculations.

- $\Rightarrow$ **At two-loop the scheme dependence is not well understood yet, but thanks to our calculations we have now the complete knowledge of the double virtual amplitude, which allow one to predict the difference between two schemes for any virtual amplitude in massless QCD up to two-loop.**

Schemes

Variants of dimensional regularization and dimensional reduction

In order to define new schemes one needs to distinguish three spaces:

- the original 4-dimensional space ($4S$) with metric tensor $\overline{g}^{\mu\nu}$
- the "quasi-D-dimensional space" (QDS) with metric tensor $\hat{g}^{\mu\nu}$
- the "quasi-4-dimensional space" ($Q4S$) with metric tensor $g^{\mu\nu}$

Schemes

Variants of dimensional regularization and dimensional reduction

The dimensionality of the spaces are expressed by the following equations [*Stöckinger*, 2005]:

$$(Q4S):\ g^{\mu\nu}g_{\mu\nu} = D_s = 4$$
$$(QDS):\ \hat{g}^{\mu\nu}\hat{g}_{\mu\nu} = D = 4 - 2\epsilon$$
$$(4S):\ \overline{g}^{\mu\nu}\overline{g}_{\mu\nu} = 4$$

and

$$g^{\mu\nu}\hat{g}^{\rho}_{\nu} = \hat{g}^{\mu\rho}, \quad g^{\mu\nu}\overline{g}^{\rho}_{\nu} = \overline{g}^{\mu\rho}, \quad \hat{g}^{\mu\nu}\overline{g}^{\rho}_{\nu} = \overline{g}^{\mu\rho}$$

$$Q4S \supset QDS \supset 4S$$

Schemes

Variants of dimensional regularization and dimensional reduction

Only gluons that appear inside a divergent loop or phase space integral need to be regularized

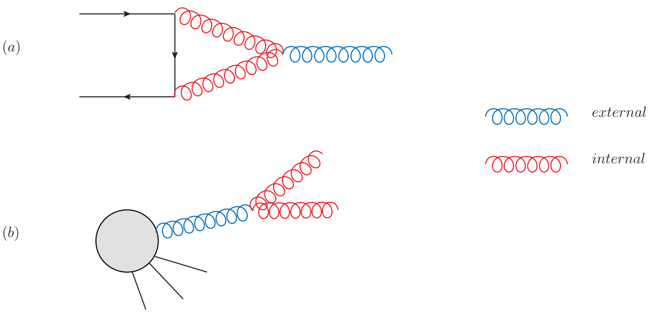

Figure : (a): loop-diagram. (b): gluon splitting into two collinear gluons.

Schemes

Variants of dimensional regularization and dimensional reduction

dimensional regularization

- CDR ("conventional dimensional regularization"): Here internal and external gluons (and other vector fields) are all treated as D-dimensional.
- HV ("'t Hooft Veltman scheme"): Internal gluons are treated as D-dimensional but external ones are treated as strictly 4-dimensional.

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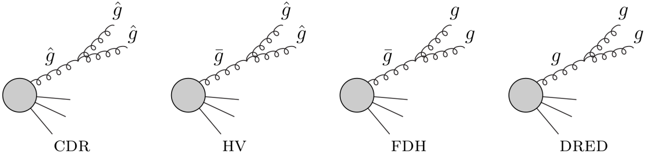
dimensional reduction

- DRED ("original/old dimensional reduction"): Internal and external gluons are all treated as quasi-4-dimensional.
- FDH ("four-dimensional helicity scheme"): Internal gluons are treated as quasi-4-dimensional but external ones are treated as strictly 4-dimensional.

	CDR	HV	FDH	DRED
internal gluon	$\hat{g}^{\mu\nu}$	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$	$g^{\mu\nu}$
external gluon	$\hat{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$g^{\mu\nu}$



Using DRED and FDH

The crucial step is to split quasi-4-dimensional gluons into D-component gauge fields and $N_\epsilon = 2\epsilon$ scalar fields, so called ϵ-scalars:

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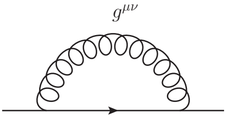
$\Rightarrow$ **During the renormalization process the couplings of the ϵ-scalars must be treated as independent, resulting in different renormalization constants and β-functions.**

Using DRED and FDH



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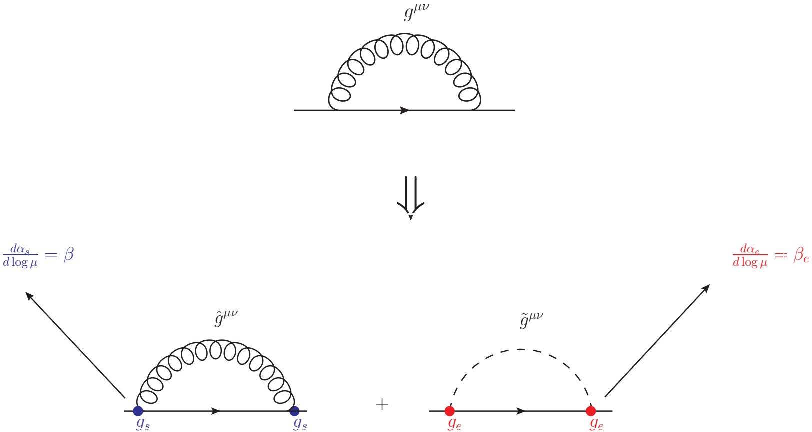

2-loop scheme dependence in massless QCD

The computation of a 2-loop cross-section is more complicated and the scheme dependence is not yet well understood:

$$\sigma^{2-loop} = \underbrace{\sigma_{VV}}_{\text{now clear!}} + \underbrace{\sigma_{VR} + \sigma_{RR}}_{\text{under investigation}}$$

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$$\sigma^{2-loop} = \underbrace{\sigma_{VV}}_{\text{now clear!}} + \underbrace{\sigma_{VR} + \sigma_{RR}}_{\text{under investigation}}$$

$\Rightarrow$ Thanks to our computation we have now the knowledge to convert any two-loop amplitude from one scheme to another, which was not clear so far.

2-loop scheme dependence in massless QCD

Infrared singularities of on-shell amplitudes in massless QCD

The IR poles can be subtracted by means of a multiplicative renormalization factor **Z**,

$$|M_n(\{\underline{p}\},\mu)\rangle = \lim_{\epsilon\to 0} \mathbf{Z}^{-1}(\epsilon,\{\underline{p}\},\mu)|M_n(\epsilon,\{\underline{p}\})\rangle$$

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$$\boxed{\mathbf{\Gamma}(\{\underline{p}\},\mu) = \sum_{(i,j)} \frac{\mathbf{T}_i\mathbf{T}_j}{2}\gamma_{\text{cusp}}(\alpha_s)\log\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)}$$

[*Becher and Neubert*, 2009]
[*Gardi and Magnea*, 2009]

2-loop scheme dependence in massless QCD

IR singularities in different schemes

All the scheme dependence is encoded in the two coefficients γ_{cusp} and γ^i.

- In order to obtain the IR structure in FDH and DRED we have to compute γ_{cusp} and γ^i in the two different schemes.
- The anomalous dimension generalised to

$$\boxed{\mathbf{\Gamma}(\{\underline{p}\},\mu) = \sum_{(i,j)} \frac{\mathbf{T}_i\mathbf{T}_j}{2}\gamma_{\text{cusp}}(\alpha_s,\alpha_e)\log\frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s,\alpha_e)}$$

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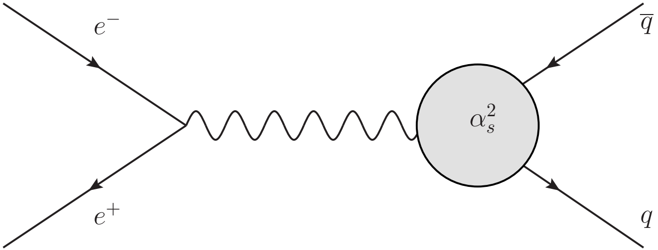
$\Rightarrow$ **How do we calculate γ_{cusp} and γ^i ?**

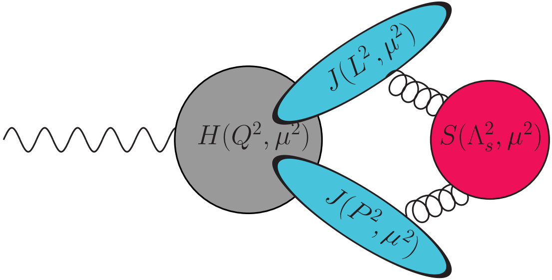
SCET approach

So far the anomalous dimensions have been extracted by the computation of the form factor, however due to the universality of the anomalous dimension, we can calculate γ_{cusp} and γ^i by computing a "simple" process in SCET



SCET approach

which factorized as [*Becher et al*, 2006]



$$\Rightarrow \boxed{F(Q^2,L^2,P^2) = H(Q^2,\mu^2)J(L^2,\mu^2)J(P^2,\mu^2)S(\Lambda_s^2,\mu^2)}$$

[HV jet function,*Becher et al*, 2006,2010],[HV soft function,*Monni et al*, 2011],[HV form factor,*Gonsalves*, 1983;*Harlander*, 2000]

SCET approach

Any function fulfil a similar RG equation,

$$\frac{d}{d\log\mu}H(Q^2,\mu^2) = \left[C_F\gamma_{\mathrm{cusp}}(\alpha_s)\log\frac{Q^2}{\mu^2} + \gamma_H(\alpha_s)\right]H(Q^2,\mu^2)$$
$$\frac{d}{d\log\mu}J(L^2,\mu^2) = \left[C_F\gamma_{\mathrm{cusp}}(\alpha_s)\log\frac{L^2}{\mu^2} + \gamma_J(\alpha_s)\right]J(Q^2,\mu^2)$$
$$\frac{d}{d\log\mu}S(\Lambda_s^2,\mu^2) = \left[C_F\gamma_{\mathrm{cusp}}(\alpha_s)\log\frac{\Lambda_s^2}{\mu^2} + \gamma_S(\alpha_s)\right]S(\Lambda_s^2,\mu^2)$$

and since the final physical result must be independent of μ^2

$$\frac{d}{d\log\mu}\left[H(Q^2,\mu^2)J(L^2,\mu^2)J(P^2,\mu^2)S(\Lambda_s^2,\mu^2)\right] = 0$$

$$\Rightarrow \boxed{\gamma_H + 2\gamma_J + \gamma_S = 0} \text{ with } \gamma_H = 2\gamma_q$$

Results

We have calculated the J and the S functions in FDH (and DRED...).
The form factor in FDH has been calculated by our collaborators [*Gnendiger et al*, 2014]
$\Rightarrow$ **This allowed us to check that $F(Q^2, L^2, P^2)$ is scheme independent**.

Results

As an example we give the result for the γ_{cusp}:

$$\gamma_{10} = 4$$
$$\gamma_{01} = 0$$
$$\gamma_{11} = 0$$
$$\gamma_{20} = \left(\frac{268}{9} - \frac{4\pi^2}{3}\right) C_A - \frac{80}{9} T_R n_f - \epsilon \, \frac{32}{9} C_A$$
$$\gamma_{02} = 0$$

Where

$$\gamma_{\text{cusp}}(\alpha_s, \alpha_e) = \sum_{n,m=0} \gamma_{nm} \left(\frac{\alpha_s}{4\pi}\right)^n \left(\frac{\alpha_e}{4\pi}\right)^m$$

Similar for γ_q and γ_g.

Results

We have cross-checked our results in the case of the Quark-Gluon and the Gluon-Gluon scattering at two loop order calculated in HV and FDH.
[in CDR, *Anastasiou et al*, 2001],[in HV and FDH, *Bern et al*, 2003]

Results

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$\Rightarrow$ **The difference between the two schemes is in complete agreement with our prediction and also with the results obtained by [*Kilgore*, 2012].**

Conclusion

- We have shown variants of dimensional regularisation and dimensional reduction.
- We stressed that in the case of FDH and DRED it is important to split the gluon $g^{\mu\nu}$ into a D-dimensional component and an ϵ-dimensional component.
  $\Rightarrow$ This splitting introduce a new non physical coupling, which has to be renormalized differently.
- We computed the infrared structure in different schemes up to two-loop
  $\Rightarrow$ This allow one to change from one scheme to another in any massless virtual QCD computation (up to 2-loop).
- The scheme dependence for σ_{VR} and σ_{RR} are under investigation.

Thank you!