Finite-size, chemical potential and magnetic effects on the phase transition in a four-fermion interacting model

A. P. C. Malbouisson

[1] CBPF/MCTI - Rio de Janeiro

Manaus, August 2015

Resumé

- Fundamentals: the formulation of Quantum Field Theories in toroidal topologies, such as presented in the recent developments:

  F. C. Khanna, APCM , J. M. C. Malbouisson and A. E. Santana, *Quantum field theory on toroidal topology: Algebraic structure and applications*, Phys. Rep., **539**, 135-224 (2014);

  F. C. Khanna, APCM , J. M. C. Malbouisson and A. E. Santana, *Ann. Phys. (N.Y.)* ***326****, 2364 (2011)*;

  F. C. Khanna, APCM, J. M. C. Malbouisson, A. E. Santana, *Thermal quantum field theory: algebraic aspects and applications*, World Scientific, Singapore (2009)

- This allows to treat jointly the effects of temperature and spatial boundaries. This sets forth grounds for an analysis of finite-size effects in phase transitions

Effective models

- Rigorous QCD calculations, both at zero and finite temperature, have been worked out, but mainly treating the asymptotically free domain at high energies or high temperatures, where perturbation theory is applicable. That is why simplified, phenomenological models have been implemented along the years, particularly four-fermion interacting models, considered as effective theories for QCD. They have proved to be an enlightening approaches in describing properties of hadronic matter
- Among them, the Gross-Neveu (GN) model provides the simplest effective theory which may be considered as describing quark interactions, as a direct four-fermion coupling, where gluon fields and color degrees of freedom are integrated out, resembling the Fermi model of the weak interaction. As such, one needs not to be worried about performing perturbative summations, nor with renormalizability.

Magnetic effects on the model

- We consider a massive version of the GN model in a D-dimensional Euclidean space, $\mathbb{R}^D$, at zero chemical potential, described by the Hamiltonian

$$H = \int d^D x \left\{ \psi^\dagger(x)(i\gamma_\mu D_\mu - m_0)\psi(x) + \frac{\lambda_0}{2}\left[\psi^\dagger(x)\psi(x)\right]^2 \right\}, \tag{1}$$

  where m_0 and λ_0 are respectively the physical mass and coupling constant at zero temperature and zero chemical potential in the absence of boundaries and of an applied magnetic field.
- D_μ is the covariant derivative, $D_\mu = \partial_\mu - ieA_\mu$, and the gauge $A_\mu = (0, 0, Bx_1, \ldots, 0)$ is used. A constant and uniform magnetic field B is applied along the x_3 -direction.

Magnetic effects on the model

- Introduce finite temperature β^{-1} and chemical potential μ corrections to the mass, considering one spatial dimension compactified with a compactification length L (the size of the system). We define the temperature-, chemical-potential-, magnetic-field-, and size-dependent mass, $m(D, \beta, L, \mu, \omega)$, by

$$m(D, \beta, L, \mu, \omega) = m_0 + \Sigma(D, \beta, L, \mu, \omega), \tag{2}$$

  where $\omega = eB$, is the so-called cyclotron frequency.

- Then we may write down a free-energy density of the Ginzburg–Landau type,

$$\mathcal{F} = a - m(D,\, \beta, L, \mu, \omega)\phi^2(x) + \lambda_0\, \phi^4(x); \tag{3}$$

  where $\phi(x) = \sqrt{\langle \psi^\dagger(x)\psi(x)\rangle}$, where $\langle \cdot \rangle$ means thermal average in the grand-canonical ensemble

Magnetic effects on the model

- Finite-temperature and density (chemical potential) corrections to the self-energy, $\Sigma(D, \beta, L, \mu, \omega)$, together with the compactification of one of the spatial dimensions, are taken into account by using the appropriate generalized Matsubara formalism, *i.e.*, the Feynman rules are modified accordingly to

$$\int \frac{d\bar{p}_0}{2\pi} \to \frac{1}{\beta} \sum_{n_1=-\infty}^{+\infty} , \qquad \bar{p}_0 \to \omega_{n_1} - i\mu \ ;$$

$$\int \frac{d\bar{p}_3}{2\pi} \to \frac{1}{L} \sum_{n_2=-\infty}^{+\infty} , \qquad \bar{p}_3 \to \omega_{n_2} \ ,$$

where $\omega_{n_1} = (2n_1 + 1)\pi/\beta$ and $\omega_{n_2} = (2n_2 + 1)\pi/L$ are generalized Matsubara frequencies

Magnetic effects on the model

- dimensionless parameters defined by

$$\lambda = \lambda_0 m_0^2, \; t = T/m_0, \; \xi = L^{-1}/m_0, \; \delta = \omega/m_0^2, \; \gamma = \mu/m_0, \quad (4)$$

  and where

$$c_\ell^2 = c_\ell^2(\delta, \sigma) = [\delta(2\ell + 1 - \sigma) + 1]/4\pi^2, (5)$$

  $a_1 = (m_0\beta)^{-2} = t^2$, $a_2 = (m_0 L)^{-2} = \xi^2$,
  $b_1 = i\beta\mu/2\pi - 1/2 = i\gamma/2\pi t - 1/2$ and $b_2 = -1/2$.

- we get a finite correction to the mass

$$\Sigma_R(t, \xi, \gamma, \delta) = \frac{\lambda\delta}{\pi} \sum_{\sigma=\pm 1} \sum_{\ell=0}^{\infty} \mathcal{R}_{c_\ell(\delta,\sigma)}(1; t, \xi, \gamma), \quad (6)$$

  where $\mathcal{R}_{c_\ell(\delta,\sigma)}(1; t, \xi, \gamma)$ is finite; $\sum_{\sigma=\pm 1}$ and $\sum_{\ell=0}^{\infty}$: respectively sums over the spin polarizations and Landau levels.

Magnetic effects on the model

- We get the corrected mass,

$$\frac{m(t,\xi,\gamma,\delta)}{m_0} = 1 + \frac{\lambda\delta}{\pi}\left[\mathcal{F}_0(t,\xi,\gamma,\delta) + 2\sum_{\ell=1}^{\infty}\mathcal{F}_\ell(t,\xi,\gamma,\delta)\right] \tag{7}$$

-

$$\begin{aligned}\mathcal{F}_\ell(t,\xi,\gamma,\delta) = \sum_{n_1=1}^{\infty}(-1)^{n_1}\cosh\left(\frac{n_1\gamma}{t}\right)K_0\left(\frac{\sqrt{2\delta\ell+1}\,n_1}{t}\right) \\ + \sum_{n_2=1}^{\infty}(-1)^{n_2}K_0\left(\frac{\sqrt{2\delta\ell+1}\,n_2}{\xi}\right) \\ +2\sum_{n_1,n_2=1}^{\infty}(-1)^{n_1+n_2}\cosh\left(\frac{n_1\gamma}{t}\right)K_0\left(\sqrt{2\delta\ell+1}\sqrt{\frac{n_1^2}{t^2}+\frac{n_2^2}{\xi^2}}\right).\end{aligned} \tag{8}$$

Magnetic effects on the model

- Criticality is attained when the corrected mass, $m(t, \xi, \gamma, \delta)$, vanishes. The solutions of $m(t, \xi, \gamma, \delta) = 0$ provide the size-dependent critical temperatures as a function the applied magnetic field.
- An entirely analogous (and simpler) calculation leads to the critical equation in the absence of an applied field but, in this case, an integral over two momentum variables (those whose symmetry over the corresponding coordinates is broken by the magnetic field) should be evaluated using dimensional regularization methods; one obtains

$$\frac{m(t, \xi)}{m_0} = 1 + \frac{2\lambda}{\pi^2} \mathcal{R}_c(0; t, \xi, \gamma), \tag{9}$$

where $c = 1/2\pi$

Magnetic effects on the model

- We adopt a *heuristic* approach; we think of the model as a simplified description (a “toy model") of a system of *fermion-antifermion* pairs of size L at temperature β^{-1}, under the influence of a magnetic field δ. The transition temperature under these conditions is interpreted as the temperature at which the pairs dissociate.

General features

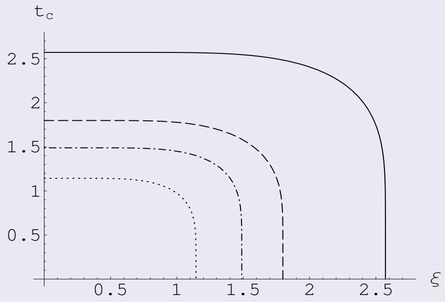

Figure: . Reduced critical temperature, t_c, as a function of the reduced inverse size of the system, ξ, for vanishing chemical potential, $\gamma = 0.0$, fixed $\lambda = 1.0$, and some values of the magnetic field: $\delta = 0.0$; 0.2; 2.0; and 7.0, full-, dashed-, dotdashed- and dotted-lines, respectively. We consider the system in the absence of an applied field. Using the definition of *reduced inverse size*, we find a minimal size of the system of $L_0 = 1/m_0\xi_0$. In this case, we get from the figure that at zero temperature and with $\gamma = 0$ and $\lambda = 1$, the reduced inverse size is roughly $\xi_0 \approx 2.60$. Taking for m_0 the effective quark mass of a light meson [*arXiv:hep-ph/0611084*] of $\sim 70\,\mathrm{MeV}$, we obtain, using the conversion $\mathrm{MeV}^{-1} \approx 196.9\,\mathrm{fm}$, $L_0 \approx 1.08\,\mathrm{fm}$; this is of the order of magnitude of the estimated size of a meson. On the other hand, we see from the full line in the figure, that for reduced inverse sizes such that $\xi \lesssim 1.5$, the reduced temperature is almost constant and has a value of $t \approx 2.60$. This gives a transition temperature of $T_c \approx 182\,\mathrm{MeV}$ for all sizes $L \gtrsim 1.89\,\mathrm{fm}$, much larger that the zero-temperature minimal size, which we think as being the size of a *fermion-antifermion bound state*. The transition temperature that we have is very close to the estimated deconfining temperature for hadrons in the absence of an applied field.

General features

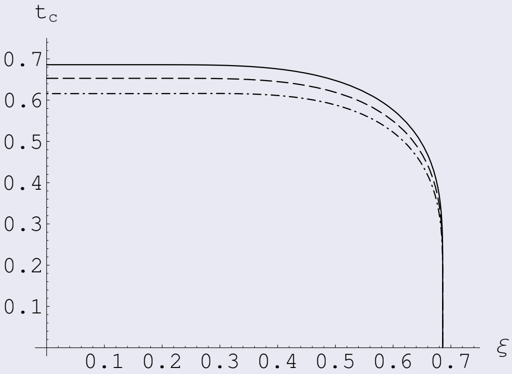

Figure: . Phase diagram ($\xi \times t_c$) for fixed values of the magnetic field, $\delta = 8.0$, and the coupling constant, $\lambda = 2.0$, and three values of the chemical potential: $\gamma = 0.0$; 0.35; and 0.5, corresponding to the full-, dashed-, dotdashed-lines respectively. This set of curves give the reduced critical temperature, t_c, as a function of the reduced inverse size of the system ξ, for three values of the reduced chemical potential: $\gamma = 0.0,\ 0.35,$ and 0.5, corresponding to the full-, dashed-, dotdashed-lines respectively; we take a fixed value of the reduced applied field, $\delta = 8.0$ and the dimensionless coupling constant $\lambda = 2.0$. For each value of γ the broken phase is at the interior of the the corresponding curve. We see that the system presents a minimal size below which there is no transition. This minimal size is independent of the chemical potential

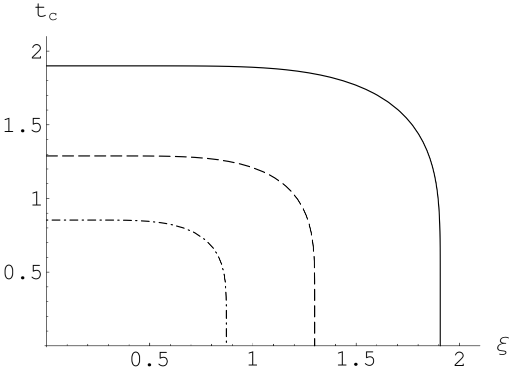

Figure: . Reduced transition temperature as a function of the reduced inverse size, for fixed $\gamma = 0.3$ and $\delta = 5.0$, and distinct values of the quartic self-coupling: $\lambda = 0.5$; 1.0; and 2.0, full-, dashed- and dotdashed-lines, respectively. Also, dissociation of the system is favored for higher values of the coupling constant; as already mentioned above, larger values of the quartic coupling constant leads to larger values of the minimal allowed size L_0 and lower values of the critical temperature.

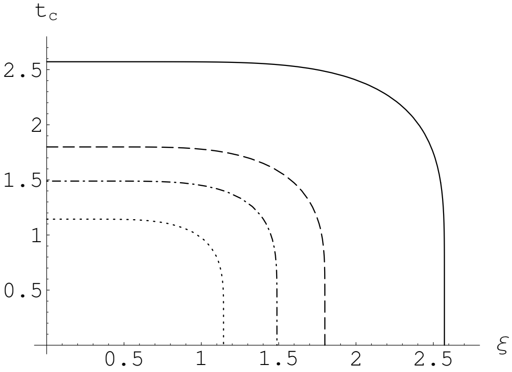

Figure: . Reduced critical temperature, t_c, as a function of the reduced inverse size of the system, ξ, for vanishing chemical potential, $\gamma = 0.0$, fixed $\lambda = 1.0$, and some values of the magnetic field: $\delta = 0.0$; 0.2; 2.0; and 7.0, full-, dashed-, dotdashed- and dotted-lines, respectively. When an applied field is present, for instance, $\delta = 2.0$, we find from the figure, $\xi_0(\delta = 2.0) \approx 1.5$, corresponding to a minimal size of $L_0(\delta = 2.0) \approx 1.9\,\mathrm{fm}$. Comparing with the value $L_0(\delta = 0.0) \approx 1.1\,\mathrm{fm}$, we see that, even at zero temperature, the action of the magnetic field tends to dissociate the system. This effect is more important for stronger magnetic fields and higher temperatures.

Finite size and magnetic effects on phase transitions

"Though this be madness, yet there is method in 't. Will you walk out of the air, my lord?"
(Hamlet, Act 1. Scene V)