Improved method for quark polarization in non-central nuclear collisions

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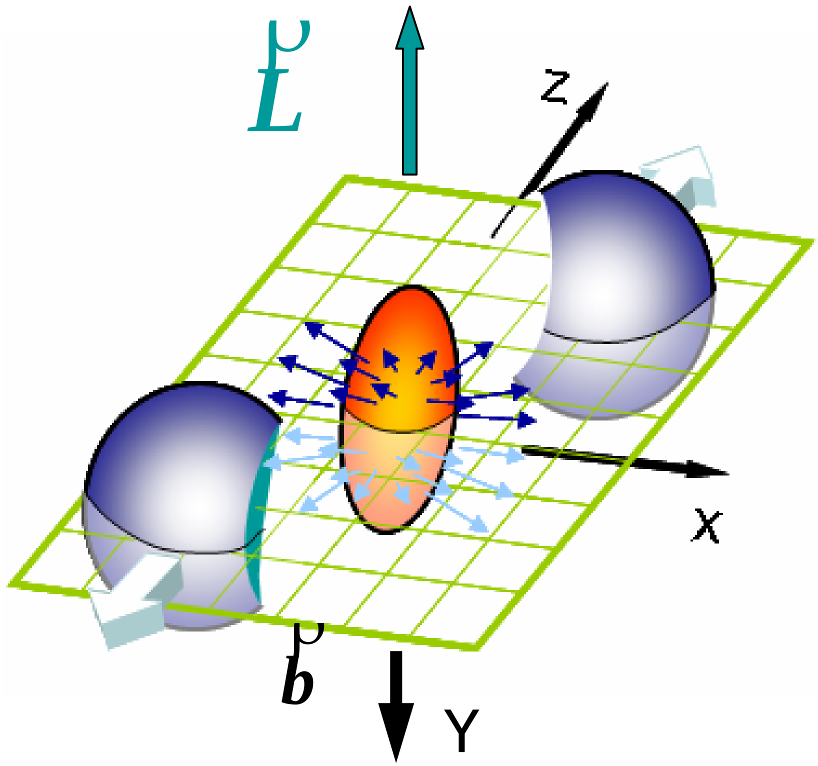
- ❖ Introduction
- ❖ Calculation method of global quark polarization
- ❖ Improved treatment
- ❖ Conclusion

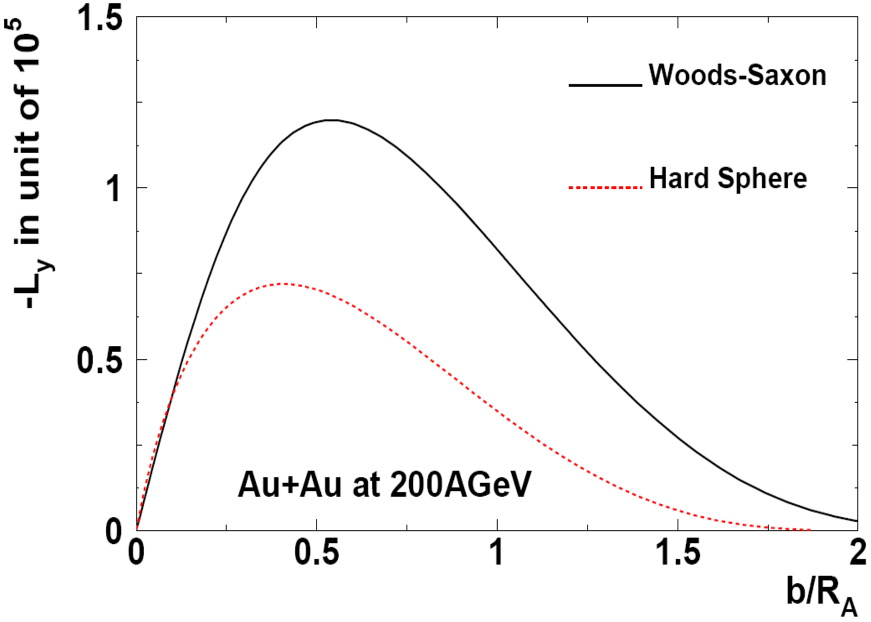
• J.H. Gao, S.W. Chen, W.T. Deng, Z.T. Liang, QW, X.N. Wang, Phys.Rev.C77:044902,2008.

• S.W. Chen, J. Deng, J.H. Gao, QW, arXiv:0803.4360

International workshop on heavy ion physics at LHC, May 21-24, 2008

Global Orbital Angular Momentum

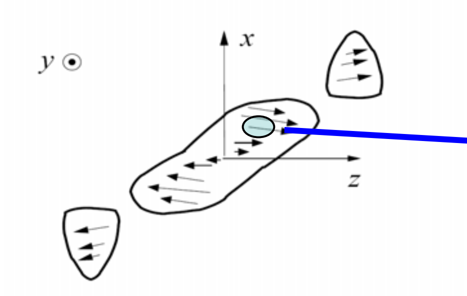


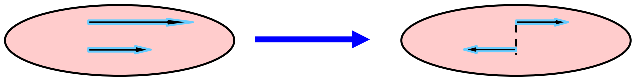


reaction plane $\mathbf{n}_b = \frac{\mathbf{p} \times \mathbf{b}}{|\mathbf{p} \times \mathbf{b}|}$

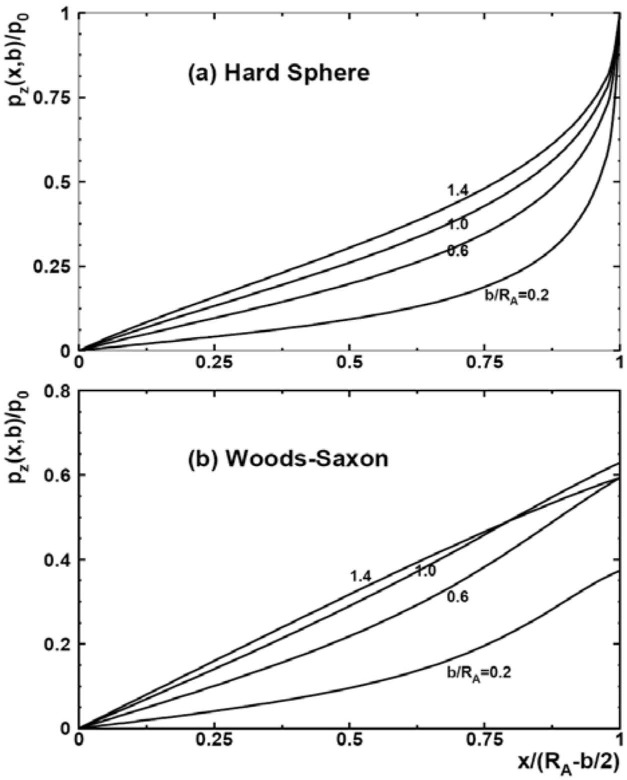
Z.T. Liang, X.N. Wang,
Phys. Rev. Lett. 94,102301(2005)

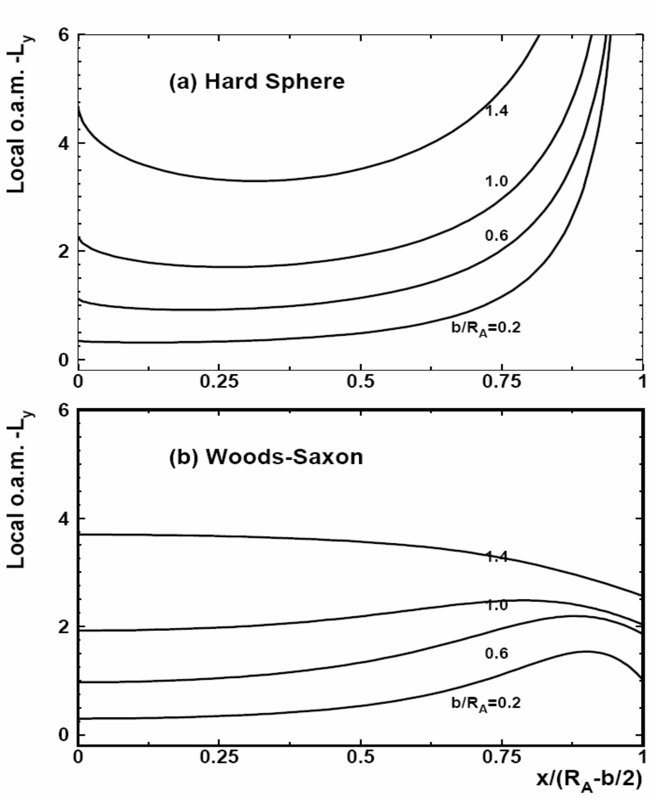
Longitudinal Fluid Shear



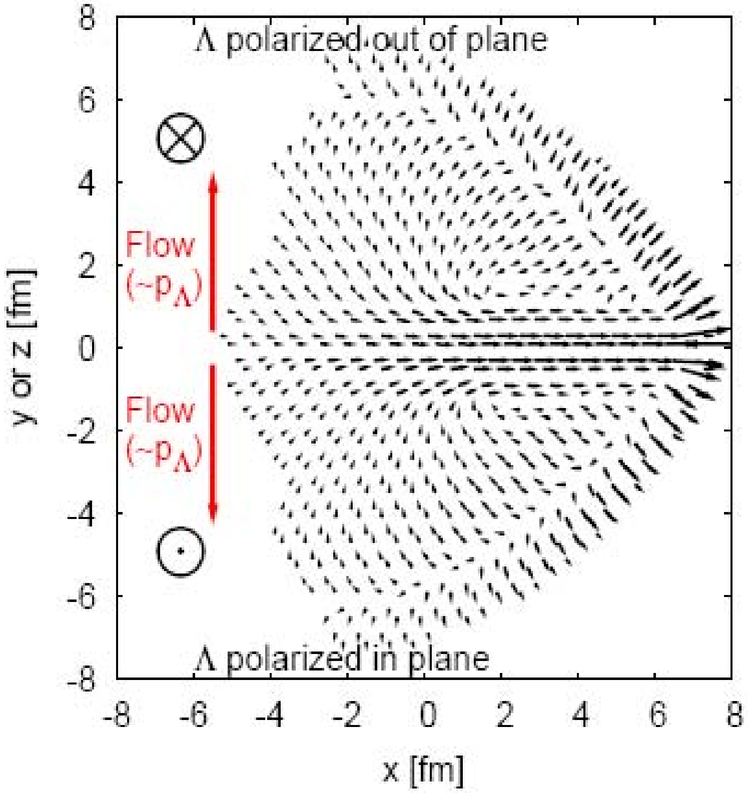


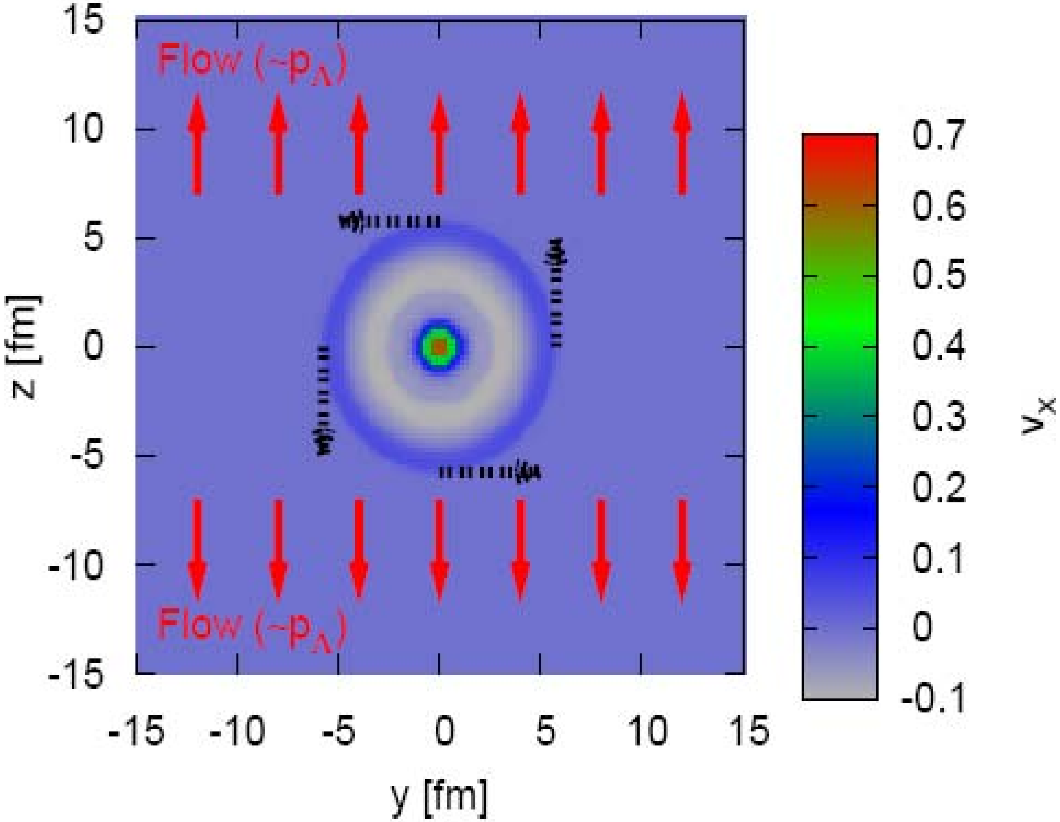
$\Delta x = 1\, fm$





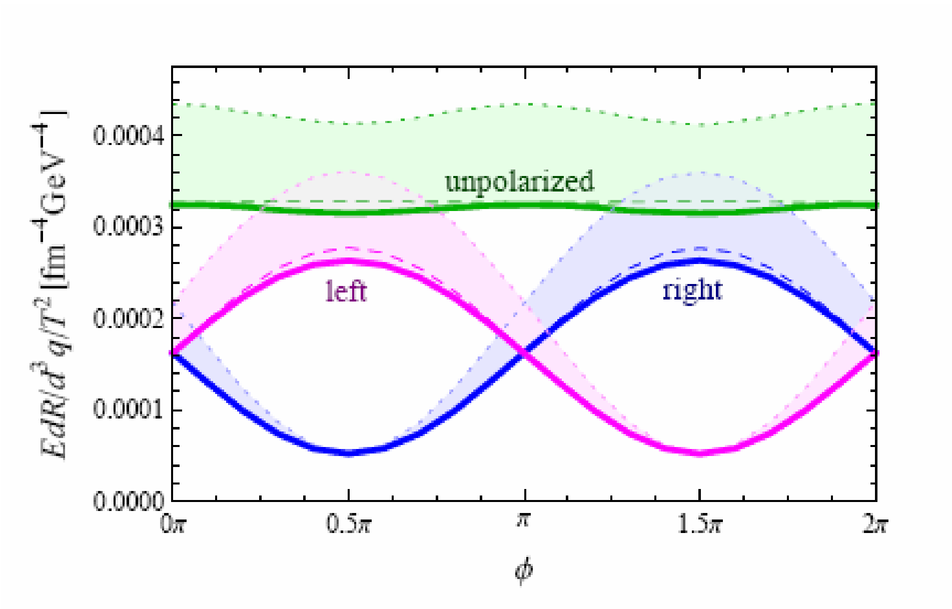
Related problem: Jet induced polarization

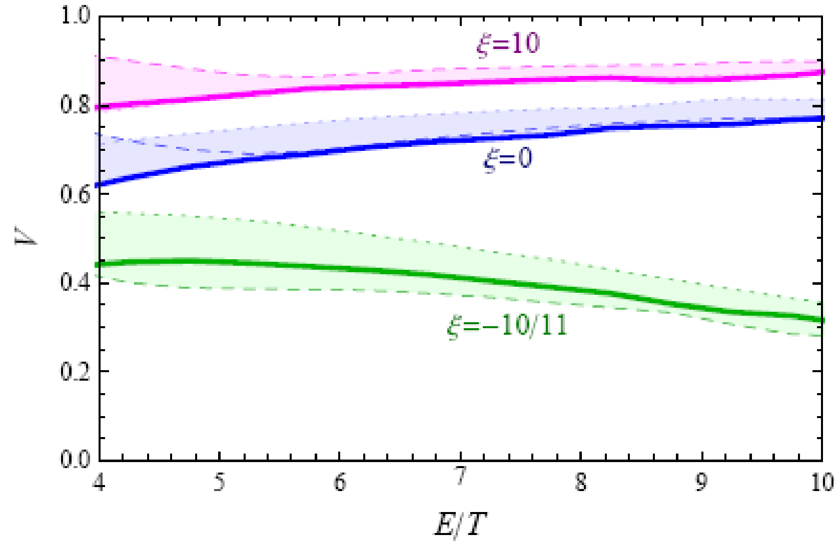


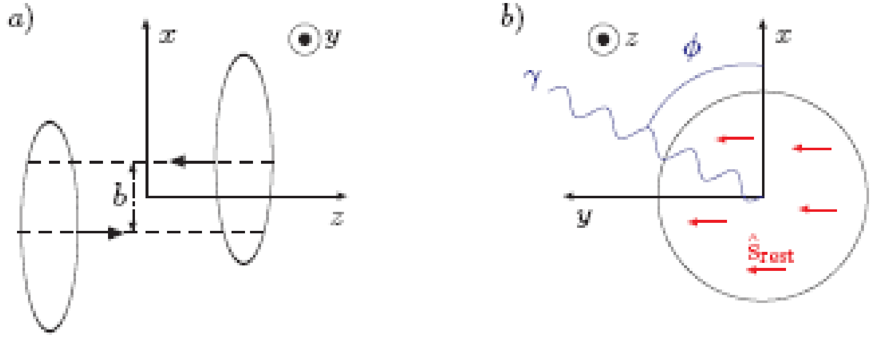


B.Betz, M.Gyulassy, G.Torrieri, Phys.Rev.C76:044901,2007

Induced photon Polarization





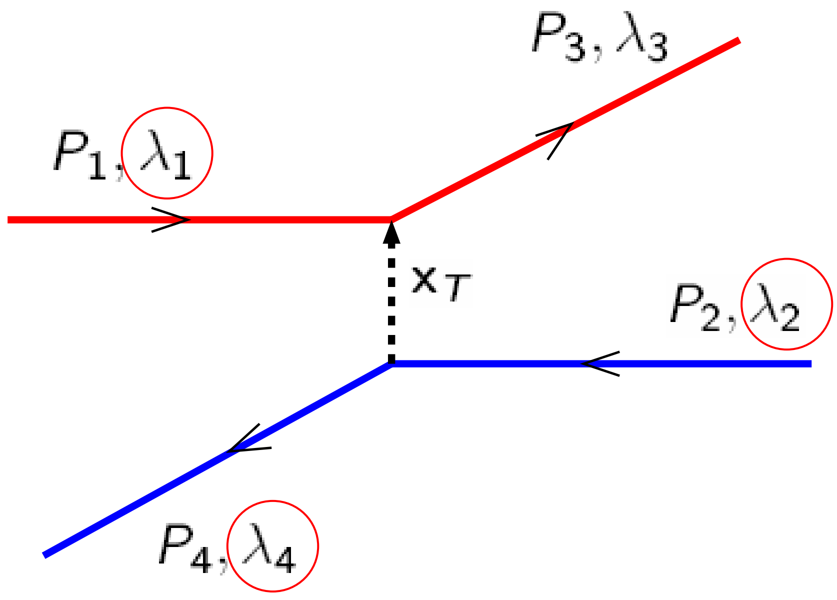


circularly polarized photon

$$qg \to q\gamma \ (\overline{q}g \to \overline{q}\gamma)$$

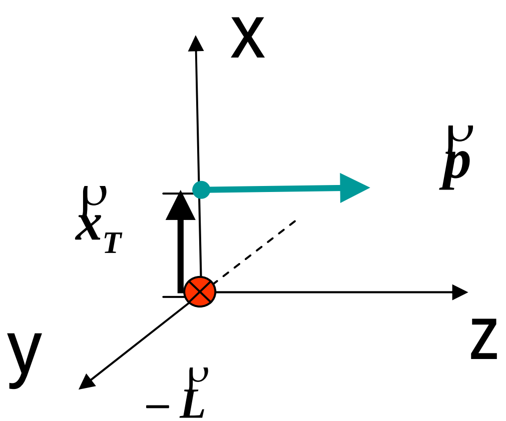
A.Ipp, A.D.Piazza, J.Evers, C.H.Keitel, arXiv:0710.5700

Collision configuration



$$
\begin{aligned}
P_1 &= (E_1, \mathbf{0}_T, p_{1z}) \\
P_2 &= (E_2, \mathbf{0}_T, p_{2z}) \\
P_3 &= (E_3, \mathbf{p}_T, p_{3z}) \\
P_4 &= (E_4, -\mathbf{p}_T, p_{4z})
\end{aligned}
$$

$$
\lambda_i = \begin{cases} +, & \odot \\ -, & \otimes \end{cases}
$$



Spin inclusive process

$\lambda_1, \lambda_2, \lambda_4$ → Summed

λ_3 → Kept

Calculation Formalism

cross section depends on polarization

$$\frac{d^2\sigma_\lambda}{d^2\mathbf{x}_T} = \frac{d^2\sigma}{d^2\mathbf{x}_T} + \lambda\frac{d^2\Delta\sigma}{d^2\mathbf{x}_T}$$

polarized cross section:

$$\frac{d^2\Delta\sigma}{d^2\mathbf{x}_T} = \frac{1}{2}\left(\frac{d^2\sigma_+}{d^2\mathbf{x}_T} - \frac{d^2\sigma_-}{d^2\mathbf{x}_T}\right)$$

un-polarized cross section:

$$\frac{d^2\sigma}{d^2\mathbf{x}_T} = \frac{1}{2}\left(\frac{d^2\sigma_+}{d^2\mathbf{x}_T} + \frac{d^2\sigma_-}{d^2\mathbf{x}_T}\right)$$

Total polarized and un-polarized cross section:

$$\sigma = \int_0^\infty dx \int_{-\infty}^\infty dy \frac{d^2\sigma}{d^2\mathbf{x}_T}$$

$$\Delta\sigma = \int_0^\infty dx \int_{-\infty}^\infty dy \frac{d^2\Delta\sigma}{d^2\mathbf{x}_T}$$

$$P_q = \frac{\Delta\sigma}{\sigma}$$

Ambiguity in conventional method

Diffrential cross section for $1(P_1\lambda_1) + 2(P_2\lambda_2) \longrightarrow 3(P_3\lambda_3) + 4(P_4\lambda_4)$

$$d\sigma = \frac{1}{16|P_1 \cdot P_2|} \sum_{\lambda_1,\lambda_2,\lambda_4} |M(P_1\lambda_1, P_2\lambda_2, P_3\lambda_3, P_4\lambda_4)|^2$$

$$\times (2\pi)^4 \delta^{(4)}(P_1 + P_2 - P_3 - P_4) \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3} \frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4}$$

Integrate out $dp_{3z} d^3\mathbf{p}_4$

$$\Lambda^2(p_T) \equiv \left| E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)} \right|$$

$$d\sigma = \frac{1}{64|P_1 \cdot P_2|} \sum_{i=1,2} \sum_{\lambda_1,\lambda_2,\lambda_4} |M(P_3\lambda_3, P_4\lambda_4)|^2 \frac{1}{\Lambda^2(p_T)} \frac{d^2\mathbf{p}_T}{(2\pi)^2}$$

Insert $\int d^2\mathbf{p}'_T \delta^{(2)}(\mathbf{p}'_T - \mathbf{p}_T) = \frac{1}{(2\pi)^2} \int d^2\mathbf{p}'_T d^2\mathbf{x}_T e^{i(\mathbf{p}'_T - \mathbf{p}_T)\cdot\mathbf{x}_T}$

Ambiguity in conventional method

Diffrential cross section at fixed impact parameter

$$\frac{d^2\sigma}{d^2\mathbf{x}_T} = \frac{1}{64|P_1\cdot P_2|}\int \frac{d^2\mathbf{p}_T}{(2\pi)^2}\frac{d^2\mathbf{p}'_T}{(2\pi)^2} e^{i(\mathbf{p}_T-\mathbf{p}'_T)\cdot\mathbf{x}_T} \times \sum_{i=1,2}\sum_{\lambda_1,\lambda_2,\lambda_4} \frac{M(P_3\lambda_3,P_4\lambda_4)M^*(P'_3\lambda_3,P'_4\lambda_4)}{\Lambda^2(p_T)}$$

We can replace $\frac{1}{\Lambda^2(p_T)} \rightarrow \frac{1}{\Lambda^{a_1}(p_T)}\frac{1}{\Lambda^{a_2}(p'_T)}$ with $a_1+a_2=2$

$$\frac{d^2\sigma}{d^2\mathbf{x}_T} = \frac{1}{64|P_1\cdot P_2|}\int \frac{d^2\mathbf{p}_T}{(2\pi)^2}\frac{d^2\mathbf{p}'_T}{(2\pi)^2} e^{i(\mathbf{p}_T-\mathbf{p}'_T)\cdot\mathbf{x}_T} \times \sum_{i=1,2}\sum_{\lambda_1,\lambda_2,\lambda_4} \frac{M(P_3\lambda_3,P_4\lambda_4)M^*(P'_3\lambda_3,P'_4\lambda_4)}{\Lambda^{a_1}(p_T)\Lambda^{a_2}(p'_T)}$$

Differential cross section is not unique,

but total one is same

Improved method

Mixed representation

$$|p_{3z}, \lambda_3, \mathbf{x}_{3T}\rangle = \int \frac{A_T d^2\mathbf{p}_{3T}}{(2\pi)^2} e^{i\mathbf{p}_{3T}\cdot\mathbf{x}_{3T}} |\mathbf{p}_3, \lambda_3\rangle$$

$$|p_{4z}, \lambda_4, \mathbf{x}_{4T}\rangle = \int \frac{A_T d^2\mathbf{p}_{4T}}{(2\pi)^2} e^{i\mathbf{p}_{4T}\cdot\mathbf{x}_{4T}} |\mathbf{p}_4, \lambda_4\rangle$$

Matrix element

$$\begin{aligned} S_{fi} &= \langle p_{3z}, \lambda_3, \mathbf{x}_{3T}; p_{4z}, \lambda_4, \mathbf{x}_{4T}| S |\mathbf{p}_1, \lambda_1, \mathbf{p}_2, \lambda_2\rangle \\ &= \int \frac{A_T d^2\mathbf{p}_{3T}}{(2\pi)^2} \frac{A_T d^2\mathbf{p}_{4T}}{(2\pi)^2} \frac{1}{\sqrt{2E_1V}} \frac{1}{\sqrt{2E_2V}} \frac{1}{\sqrt{2E_3V}} \frac{1}{\sqrt{2E_4V}} \\ &\quad \times e^{-i\mathbf{p}_{3T}\cdot\mathbf{x}_{3T}} e^{-i\mathbf{p}_{4T}\cdot\mathbf{x}_{4T}} (2\pi)^4 \delta^{(4)}(P_1 + P_2 - P_3 - P_4) \\ &\quad \times M(P_3\lambda_3, P_4\lambda_4) \end{aligned}$$

Matrix element

8 delta-functions and 2 amplitudes

$$
\begin{aligned}
|S_{fi}|^2 = & \int \frac{A_T d^2\mathbf{p}_{3T}}{(2\pi)^2} \frac{A_T d^2\mathbf{p}_{4T}}{(2\pi)^2} \frac{A_T d^2\mathbf{p}'_{3T}}{(2\pi)^2} \frac{A_T d^2\mathbf{p}'_{4T}}{(2\pi)^2} \\
& \times \frac{1}{V^4} \frac{1}{16 E_1 E_2 \sqrt{E_3 E_4 E'_3 E'_4}} \\
& \times e^{-i(\mathbf{p}_{3T} - \mathbf{p}'_{3T})\cdot \mathbf{x}_{3T}} e^{-i(\mathbf{p}_{4T} - \mathbf{p}'_{4T})\cdot \mathbf{x}_{4T}} \\
& \times (2\pi)^8 \delta^{(4)}(P_1 + P_2 - P_3 - P_4) \delta^{(4)}(P_1 + P_2 - P'_3 - P'_4) \\
& \times M(P_3\lambda_3, P_4\lambda_4) M^*(P'_3\lambda_3, P'_4\lambda_4)
\end{aligned}
$$

Momentum configuration

$$
\begin{aligned}
P_1 + P_2 & \rightarrow P_3(E_3, \mathbf{p}_{3T}, p_{3z}) + P_4(E_4, \mathbf{p}_{4T}, p_{4z}) \\
P_1 + P_2 & \rightarrow P'_3(E_3, \mathbf{p}'_{3T}, p_{3z}) + P'_4(E_4, \mathbf{p}'_{4T}, p_{4z})
\end{aligned}
$$

$\mathbf{p}_{3T} \rightarrow \mathbf{p}_T$, $\mathbf{p}_{4T} \rightarrow -\mathbf{p}_T$, $\mathbf{p}'_{3T} \rightarrow \mathbf{p}'_T$, $\mathbf{p}'_{4T} \rightarrow -\mathbf{p}'_T$

Differential cross section

Differential cross section at fixed impact parameter

$\mathbf{x}_T \equiv \mathbf{x}_{3T} - \mathbf{x}_{4T}$

$$d\sigma = \frac{d^2\mathbf{x}_T}{A_T} \frac{V}{4 v_{rel} \tau} \sum_{\lambda_1,\lambda_2,\lambda_4} \int \frac{L dp_{3z}}{2\pi} \frac{L dp_{4z}}{2\pi} |S_{fi}|^2$$

Integrating out $d^2\mathbf{p}_{4T} dp_{3z} d^2\mathbf{p}'_{4T} dp_{4z}$ and using two z-component delta function are identical, and using $2\pi\delta^{(z)}(0) = L$

$$\frac{d^2\sigma}{d^2\mathbf{x}_T} = \frac{1}{\tau} \frac{1}{64(2\pi)^3} \int d^2\mathbf{p}_T d^2\mathbf{p}'_T e^{-i(\mathbf{p}_T - \mathbf{p}'_T)\cdot \mathbf{x}_T} \delta(E_3 + E_4 - E'_3 - E'_4)$$
$$\times \sum_{i=1,2} \sum_{\lambda_1,\lambda_2,\lambda_4} \frac{1}{|P_1 \cdot P_2| \sqrt{E_3 E_4 E'_3 E'_4}} \frac{E_3 E_4}{\left| E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)} \right|}$$
$$\times M(P_3\lambda_3, P_4\lambda_4) M^*(P'_3\lambda_3, P'_4\lambda_4)$$

Treatment of last delta function

Using

Interaction time

$$2\pi\delta(E_3 + E_4 - E_3' - E_4') \approx \int_{-\tau/2}^{\tau/2} dt\, e^{i(E_3+E_4-E_3'-E_4')t}$$

Differential cross section can be written

$$\frac{d^2\sigma}{d^2\mathbf{x}_T} = \frac{1}{64(2\pi)^4|P_1 \cdot P_2|}\int d^2\mathbf{p}_T d^2\mathbf{p}_T' \Big\langle e^{-i(\mathbf{p}_T-\mathbf{p}_T')\cdot\mathbf{x}_T}$$
$$\times \sum_{i=1,2}\sum_{\lambda_1,\lambda_2,\lambda_4} \frac{1}{\sqrt{E_3E_4E_3'E_4'}} \frac{E_3E_4}{\left|E_3p_{4z}^{(i)} - E_4p_{3z}^{(i)}\right|}$$
$$\times M(P_3\lambda_3, P_4\lambda_4)M^*(P_3'\lambda_3, P_4'\lambda_4)\Big\rangle_t$$

With time average $\langle\cdots\rangle_t \equiv \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} dt\,(\cdots)\, e^{i(E_3+E_4-E_3'-E_4')t}$

Interaction time expansion

Expansion if $\left|(E_3 + E_4 - E_3' - E_4')t\right| \ll 1$

$$e^{i(E_3+E_4-E_3'-E_4')t} \approx 1 + i(E_3 + E_4 - E_3' - E_4')t - \frac{1}{2}(E_3 + E_4 - E_3' - E_4')^2 t^2 + \cdots$$

Leading order **~1** reproduces old result

$$\frac{d^2\sigma^{(0)}}{d^2\mathbf{x}_T} = \frac{1}{64(2\pi)^4|P_1 \cdot P_2|}\int d^2\mathbf{p}_T d^2\mathbf{p}_T' e^{-i(\mathbf{p}_T-\mathbf{p}_T')\cdot\mathbf{x}_T} \times \sum_{i=1,2}\sum_{\lambda_1,\lambda_2,\lambda_4}\frac{1}{\Lambda^2(p_T)}M(P_3\lambda_3, P_4\lambda_4)M^*(P_3'\lambda_3, P_4'\lambda_4)$$

$$= \frac{d^2\sigma^{(0)}_{upol}}{d^2\mathbf{x}_T} + \lambda_3\frac{d^2\sigma^{(0)}_{pol}}{d^2\mathbf{x}_T}$$

Leading order result

To leading order, the result is unique

$$\frac{1}{\Lambda^2(p_T)} \approx \frac{1}{\Lambda^{a_1}(p_T)} \frac{1}{\Lambda^{a_2}(p'_T)}$$

Here are polarized and unpolarized cross section

$$\frac{d^2\sigma^{(0)}_{upol}}{d^2\mathbf{x}_T} = \frac{\alpha_s^2}{9} A^2(x_T)$$

$$\frac{d^2\sigma^{(0)}_{pol}}{d^2\mathbf{x}_T} = \frac{\alpha_s^2}{9} \frac{2}{\sqrt{s}} \mathbf{n} \cdot (\mathbf{n}_1 \times \widehat{\mathbf{x}}_T) A(x_T) \frac{dA(x_T)}{dx_T}$$

$$A(x_T) = \int d^2\mathbf{p}_T e^{\pm i\mathbf{p}_T \cdot \mathbf{x}_T} \frac{1}{p_T^2 + \mu_m^2} = 2\pi \int_0^{p_T^{cut}} dp_T \frac{p_T J_0(p_T x_T)}{p_T^2 + \mu_m^2}$$

Next-to-leading order

Next-to-leading order $\sim t^2$

$$\begin{aligned}
\frac{d^2\sigma^{(1)}}{d^2\mathbf{x}_T} &= -\frac{\tau^2}{1536(2\pi)^4|P_1\cdot P_2|}\int d^2\mathbf{p}_T d^2\mathbf{p}'_T e^{-i(\mathbf{p}_T-\mathbf{p}'_T)\cdot\mathbf{x}_T} \\
&\times \sum_{i=1,2}\sum_{\lambda_1,\lambda_2,\lambda_4}\frac{(E_3+E_4-E'_3-E'_4)^2}{\sqrt{E_3E_4E'_3E'_4}}\frac{E_3E_4}{\left|E_3p_{4z}^{(i)}-E_4p_{3z}^{(i)}\right|} \\
&\times M(P_3\lambda_3,P_4\lambda_4)M^*(P'_3\lambda_3,P'_4\lambda_4) \\
&= \frac{d^2\sigma^{(1)}_{upol}}{d^2\mathbf{x}_T}+\lambda_3\frac{d^2\sigma^{(1)}_{pol}}{d^2\mathbf{x}_T}
\end{aligned}$$

Next-to-leading order

Here are polarized and unpolarized cross section

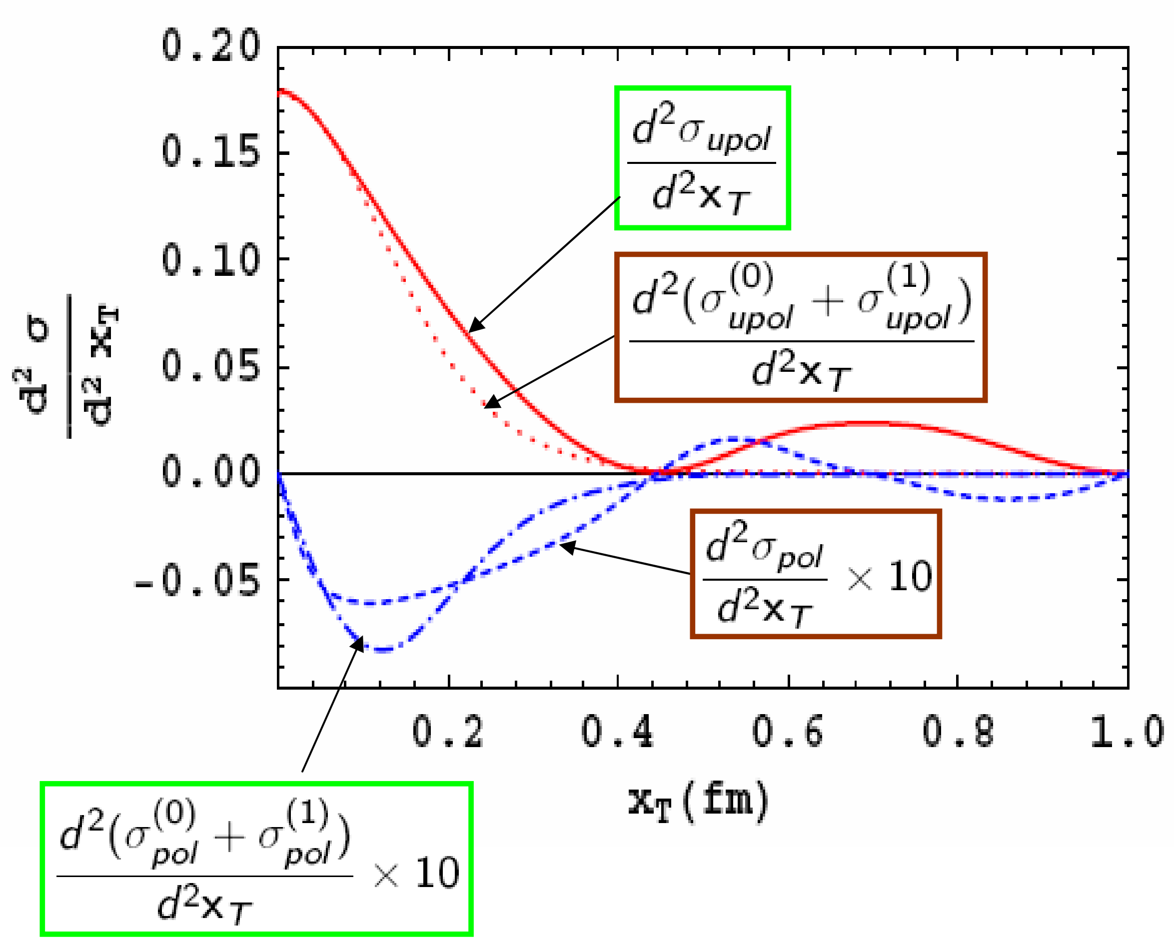
$$\frac{d^2\sigma^{(1)}_{upol}}{d^2\mathbf{x}_T} = -\frac{\tau^2\alpha_s^2}{27s}(A_0A_1 - A_2^2)$$

$$\frac{d^2\sigma^{(1)}_{pol}}{d^2\mathbf{x}_T} = -\frac{\tau^2\alpha_s^2}{27s^{3/2}}\mathbf{n}\cdot(\mathbf{n}_1 \times \widehat{\mathbf{x}}_T)\frac{d}{dx_T}(A_0A_1 - A_2^2)$$

$$A_i(x_T) \equiv \int_0^{p_T^{cut}} dp_T \frac{p_T^{n_i}}{p_T^2 + \mu_m^2} J_0(p_T x_T), \qquad \text{for } i = 0, 1, 2$$

Numerical result

Exact result and result with expansion



Conclusions

• A general approach to the differential cross section with impact parameters: well-defined and free of ambiguity existing in the conventional approach.

• The method is making use of the fact that the final state energy is not fixed at fixed impact parameter

• An expansion is proposed in terms of $\Delta E = E_f - E_i \sim 1/\tau$ the leading order result reproduces result from conventional method

• Applicable to many other parton-parton scatterings in non-central nuclear and proton-proton collisions