

# Unruh Effect and Planck Scale Physics

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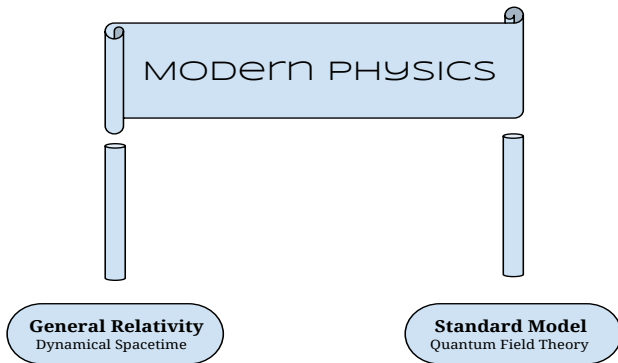
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## References

- G. M. Hossain and Gopal Sardar, [arXiv:1411.1935 [gr-qc]].
- G. M. Hossain and Gopal Sardar, (in preparation)
- G. M. Hossain, V. Husain and S. S. Seahra, Phys. Rev. D **82**, 124032 (2010)

# Pillars of Modern Physics



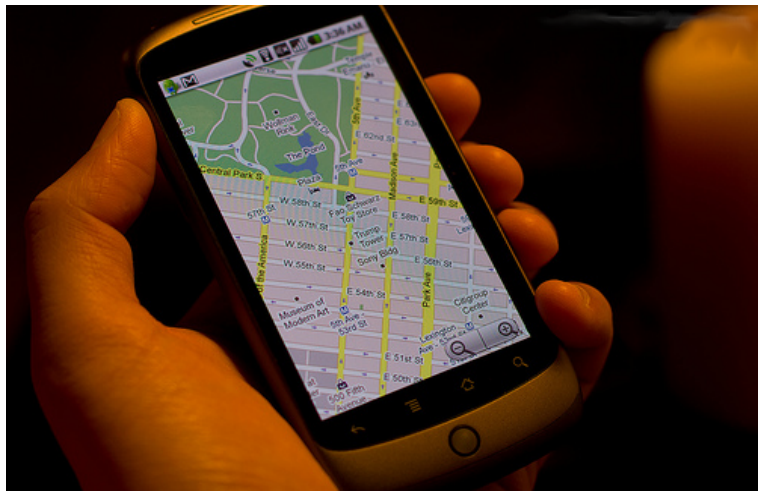
# Pillars of Modern Physics

- Quantum Field Theory is an extremely well-tested framework
  - constructed in flat Minkowski spacetime
  - unique vacuum state
- LHC validates it up to  $\sim 10^4 GeV$

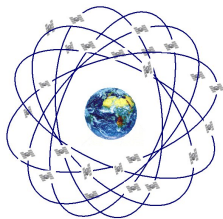
# Pillars of Modern Physics

- What about GR?

# Hand-full of General Relativity



# Hand-full of General Relativity



Schwarzschild geometry around the Earth :

$$ds^2 = - \left( 1 - \frac{r_s}{r} \right) dt^2 + d\vec{l}^2$$

$$r_s = 2GM \approx 9 \text{ mm}$$

$$r_{GPS} \approx 26600 \text{ km}$$

- GPS accuracy of say  $6m$  on ground requires time accuracy of 20 nano-seconds.
- GPS device will give wrong reading in about 2 minutes if **time dilation** effect from GR is not included.

# Quantum Gravity?

- Gravitation: Is it classical or quantum?

*"... Because of the intra-atomic movement of electrons, the atom must radiate not only electromagnetic but also gravitational energy, if only in minute amounts. Since, in reality, this cannot be the case in nature, then it appears that the quantum theory must modify not only Maxwell's electrodynamics but also the new theory of gravitation."* – Albert Einstein (1916, p. 696).

- **Quantum Gravity:** String theory, Loop gravity, ???



## Probe of Planck Scale Physics

- Scale of Quantum Gravity  $\sim \sqrt{\frac{\hbar c}{G}} \approx 10^{19}$  GeV (Planck mass)
- Direct observations in Lab is too far!! (Energy scale at LHC  $\sim 10^4 GeV$ )
- “QFT in Curved Spacetime” may be used to probe of some aspect of **Planck scale physics!**
  - Cosmological inflationary power spectrum
  - Hawking radiation
  - Unruh effect

# Uniformly Accelerating Observer

- **Unruh effect:** *uniformly accelerated* observer feels a **black-body radiation in vacuum state** of Minkowski spacetime whereas an inertial observer sees it empty.

Stephen Fulling (1973), Paul Davies (1975), W. G. Unruh (1976)

- Position 4-vector of the accelerating observer  $x^\mu$ :

$$u^\mu = \frac{dx^\mu}{ds} ; a^\mu = \frac{du^\mu}{ds} ; a^2 = a_\mu a^\mu$$

where  $s$  is *proper time* and acceleration parameter  $a$  is *constant*.

# Rindler Spacetime

- Inertial (Minkowski) metric (in 1+1 dim):

$$ds^2 = -dt^2 + dx^2$$

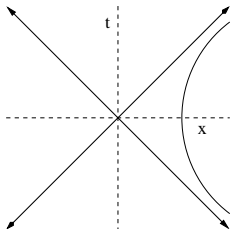
- Metric as seen by an *uniformly accelerating* observer, known as Rindler metric (conformally flat)

$$ds^2 = e^{2a\xi} (-d\tau^2 + d\xi^2)$$

- The coordinates of the Minkowski observer and Rindler observer are related to each other as

$$t = \frac{e^{a\xi}}{a} \sinh a\tau, \quad x = \frac{e^{a\xi}}{a} \cosh a\tau.$$

# Rindler Spacetime



Rindler Wedges:

$$x^2 - t^2 = \frac{e^{2a\xi}}{a^2} > 0$$

$$x > |t| \quad (\text{Right Wedge})$$

$$-x > |t| \quad (\text{Left Wedge})$$

## Light-cone Mode Expansion

- Mode expansion in Minkowski ( $u = t - x, v = t + x$ )

$$\hat{\phi} = \int_0^\infty \frac{d\omega}{\sqrt{4\pi\omega}} \left[ e^{-i\omega u} \hat{a}_\omega + e^{i\omega u} \hat{a}_\omega^\dagger + e^{-i\omega v} \hat{a}_{-\omega} + e^{i\omega v} \hat{a}_{-\omega}^\dagger \right]$$

where  $[\hat{a}_\omega, \hat{a}_{\omega'}^\dagger] = \delta(\omega - \omega')$  and vacuum state  $\hat{a}_\omega |0^M\rangle = 0$

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where  $[\hat{a}_\omega, \hat{a}_{\omega'}^\dagger] = \delta(\omega - \omega')$  and vacuum state  $\hat{a}_\omega |0^M\rangle = 0$

- Similarly for Rindler spacetime ( $\bar{u} = \tau - \xi, \bar{v} = \tau + \xi$ )

$$\hat{\phi} = \int_0^\infty \frac{d\bar{\omega}}{\sqrt{4\pi\bar{\omega}}} \left[ e^{-i\bar{\omega}\bar{u}} \hat{b}_{\bar{\omega}} + e^{i\bar{\omega}\bar{u}} \hat{b}_{\bar{\omega}}^\dagger + e^{-i\bar{\omega}\bar{v}} \hat{b}_{-\bar{\omega}} + e^{i\bar{\omega}\bar{v}} \hat{b}_{-\bar{\omega}}^\dagger \right]$$

where  $[\hat{b}_{\bar{\omega}}, \hat{b}_{\bar{\omega}'}^\dagger] = \delta(\bar{\omega} - \bar{\omega}')$  and vacuum state  $\hat{b}_{\bar{\omega}} |0^R\rangle = 0$

# Bogoliubov Transformations

- Transformations between creation and annihilation operators of Minkowski and Rindler observers

$$\hat{b}_{\bar{\omega}} = \int_0^{\infty} d\omega \left[ \alpha(\omega, \bar{\omega}) \hat{a}_{\omega} + \beta(\omega, \bar{\omega}) \hat{a}_{\omega}^{\dagger} \right]$$

where coefficients  $\alpha(\omega, \bar{\omega})$ ,  $\beta(\omega, \bar{\omega})$  are known as **Bogoliubov coefficients**

# Unruh Temperature

- Expectation value of the number operator in the Minkowski vacuum as seen by a Rindler observer is

$$\bar{N}_{\bar{\omega}} \equiv \langle 0^M | \hat{N}_{\bar{\omega}}^R | 0^M \rangle = \frac{1}{e^{2\pi\bar{\omega}/a} - 1} \quad , \quad (\bar{\omega} = |\bar{k}|)$$

- Recall number density in black-body radiation of Bosons

$$N_{\omega} = \frac{1}{e^{\hbar\omega/\kappa_B T} - 1}$$

- Unruh temperature

$$T = \frac{\hbar a}{2\pi c \kappa_B}$$



# Planck Scale Physics

- Where is Planck scale physics here?
- Bogoliubov Transformation

$$\hat{b}_{\bar{\omega}} = \int_0^{\infty} d\omega \left[ \alpha(\omega, \bar{\omega}) \hat{a}_{\omega} + \beta(\omega, \bar{\omega}) \hat{a}_{\omega}^{\dagger} \right]$$

→ depends on **all modes** of frequency  $\omega$  between 0 to  $\infty$ !!

- In the context of Hawking radiation, it was shown
  - effects of degrees of freedom at ultra-high energies/momenta could lead to strong deviation from Hawking's results
  - could give non-trivial information about Planckian physics.

W. G. Unruh, Ralf Schutzhold (2005)



## Free Fields as Harmonic Oscillators

- Hamiltonian for a massless scalar field

$$H_\phi = \int dx \left[ \frac{\pi^2}{2\sqrt{q}} + \frac{\sqrt{q}}{2} q^{ab} \nabla_a \phi \nabla_b \phi \right], \{ \phi(x), \pi(x') \} = \delta(x-x'),$$

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- Fourier modes (in a finite spatial volume  $V$ )

$$\phi(t, x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \tilde{\phi}_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad \pi(t, x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \sqrt{q} \tilde{\pi}_{\mathbf{k}}(t) e^{i\mathbf{k}\cdot\mathbf{x}}$$

## Free Fields as Harmonic Oscillators

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- Hamiltonian density for Fourier modes

$$H_\phi = \sum_{\mathbf{k} \in K} H_{\mathbf{k}}; \quad H_{\mathbf{k}} = \left[ \frac{\pi_{\mathbf{k}}^2}{2} + \frac{1}{2} k^2 \phi_{\mathbf{k}}^2 \right]; \quad \{ \phi_{\mathbf{k}}, \pi_{\mathbf{k}'} \} = \delta_{\mathbf{k}, \mathbf{k}'}$$

→ free scalar fields  $\sim$  **decoupled harmonic oscillators**

# Bogoliubov Transformations

- Similarly, for Rindler spacetime:  $\{\phi_\kappa, \pi_{\kappa'}\} = \delta_{\kappa, \kappa'}$
- Bogoliubov transformation

$$\phi_\kappa = \sum_{k>0} \phi_k F(k, -\kappa) + \sum_{k>0} \phi_{-k} F(-k, -\kappa) ,$$

$$\pi_\kappa = \sum_{k>0} \pi_k F_1(k, -\kappa) + \sum_{k>0} \pi_{-k} F_1(-k, -\kappa)$$

restricted at  $\tau = t = 0$  hyper-surface

- Bogoliubov coefficients

$$F(k, \kappa) = \int d\xi e^{ik \cdot x + i\kappa \cdot \xi} , \quad F_1(k, \kappa) = \int d\xi e^{a\xi} e^{ik \cdot x + i\kappa \cdot \xi} .$$

# Number Operator

- One may define number density operator for Unruh particles of positive frequency  $\bar{\omega} = \kappa > 0$  as follows

$$\hat{N}_{\bar{\omega}} \equiv \left[ \hat{\mathcal{H}}_{\kappa} - \lim_{a \rightarrow 0} \hat{\mathcal{H}}_{\kappa} \right] |\kappa|^{-1} .$$

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- Expectation value of the Rindler Hamiltonian operator in the Minkowski vacuum

$$\langle 0^M | \hat{\mathcal{H}}_{\kappa}^R | 0^M \rangle = \frac{\kappa e^{2\pi\kappa/a} + 1}{2 e^{2\pi\kappa/a} - 1} \sum_{\mathbf{k} > 0} \frac{2\pi}{ka} \left( \frac{2E_{\mathbf{k}}^0}{k} \right)$$

where ground state energy  $E_{\mathbf{k}}^0 = \langle 0^M | \hat{H}_{\mathbf{k}}^M | 0^M \rangle$

- **Planck scale physics** does play a role!

## Expectation value of Number Operator

- Vacuum expectation value of the operator

$$\bar{N}_{\bar{\omega}} \equiv \langle 0^\Theta | \hat{N}_{\bar{\omega}} | 0^\Theta \rangle = \frac{1 - \gamma_\star}{e^{2\pi\bar{\omega}/a} - 1},$$

where  $E_{\mathbf{k}}^0 = \langle 0^\Theta | \hat{\mathcal{H}}_{\mathbf{k}} | 0^\Theta \rangle$  and  $\gamma_\star = \lim_{\delta \rightarrow 0} \gamma_\star^\delta$  with

$$1 - \gamma_\star^\delta \equiv \frac{1}{\zeta(1 + 2\delta)} \sum_{r=1}^{\infty} \frac{\epsilon_r}{r^{1+2\delta}}, \quad \epsilon_r \equiv \frac{2E_{\mathbf{k}_r}^0}{|\mathbf{k}_r|}.$$

- In Fock quantization,  $\hat{\mathcal{H}}_{\mathbf{k}} |n_{\mathbf{k}}\rangle = E_{\mathbf{k}}^n |n_{\mathbf{k}}\rangle$  with

$$E_{\mathbf{k}}^n = \left( n + \frac{1}{2} \right) |\mathbf{k}| ; \quad \epsilon_r = 1 ; \quad \gamma_\star^\delta = 0$$



# Planck Scale Theories!

## String Theory:

- *Fundamental building blocks of our universe are extended objects such as strings, branes*
- *aims to unify the forces of nature*

## Loop Quantum Gravity:

- *a non-perturbative approach to quantum gravity*
- *uses a background-independent quantization method known as **polymer quantization** or loop quantization*

???:

→ ???

# Basic Variables

## Simple Harmonic Oscillator

- Hamiltonian and Poisson bracket

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 \quad , \quad \{x, p\} = 1 \quad ; \quad \dot{x} = \{x, H\}$$

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- Hamiltonian and Poisson bracket

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$$\{x, p\} = 1 \quad \rightarrow \quad [\hat{x}, \hat{p}] = i\hbar$$

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- Hamiltonian and Poisson bracket

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- Schrödinger quantization:

$$\{x, p\} = 1 \quad \rightarrow \quad [\hat{x}, \hat{p}] = i\hbar$$

- Loop (polymer) quantization:  $U_\lambda = e^{i\lambda p}$

$$\{x, U_\lambda\} = i\lambda U_\lambda \quad \rightarrow \quad [\hat{x}, \hat{U}_\lambda] = -\hbar\lambda \hat{U}_\lambda$$

$\lambda$  is a dimension-full parameter.

# Elementary Operators

- The basis states:  $\psi_\mu(p) = e^{i\mu p}$  ( $\mu \in \mathbb{R}$ )

Ashtekar, Fairhurst, Willis (2002); Halvorson (2001)

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- Basic actions:

$$\hat{x}\psi_\mu = i\hbar \frac{\partial}{\partial p} e^{i\mu p} = -\hbar\mu\psi_\mu$$

$$\hat{U}_\lambda\psi_\mu = e^{i\lambda p} e^{i\mu p} = \psi_{\mu+\lambda}$$

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- In Dirac notation:

$$\hat{x}|\mu\rangle = -\hbar\mu|\mu\rangle ; \hat{U}_\lambda|\mu\rangle = |\mu + \lambda\rangle$$

# Inner Product

- Inner product:

$$(\psi_{\mu'}, \psi_{\mu}) := \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T dp \psi_{\mu'}^* \psi_{\mu}$$

$$\langle \mu' | \mu \rangle = \delta_{\mu', \mu}$$

- *rhs* is the Kronecker delta
- even position eigenstates are **normalizable**.
- position eigenvalues are “discrete”



# Momentum Operator

How to define momentum operator?

One possible way to define  $\hat{p}$  could be to use classical relation  $U_\lambda = e^{i\lambda p}$  as

$$\hat{p} = -i \left( \frac{d\hat{U}_\lambda}{d\lambda} \right)_{\lambda=0}$$

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→  $\hat{p}$  does not exist

## Actions of elementary operators



# Actions of elementary operators

## Polymer Quantization

## Schrodinger Quantization

$$\hat{U}_\lambda |x\rangle = |x + \lambda\rangle$$



# Polymer Momentum Operator

- Using the classical relation  $U_\lambda = e^{i\lambda p}$  one can define

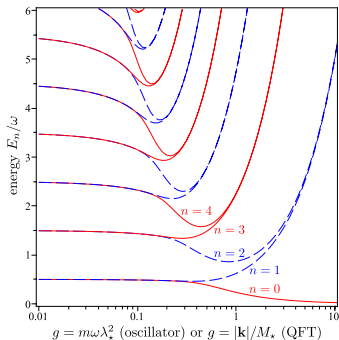
$$p_\star := \frac{1}{2i\lambda_\star} \left( U_{\lambda_\star} - U_{\lambda_\star}^\dagger \right)$$

- In the limit  $\lambda_\star \rightarrow 0$ ,  $p_\star \rightarrow p$
- In polymer quantization, this limit doesn't exist. So  $\lambda_\star$  is taken to be a **small but finite** scale

# Energy Spectrum

Energy eigenvalue equation:  $\hat{H}\psi = E\psi$

Hossain, Husain, Seahra (2010)



For small  $g$  limit

$$E_{2n} \approx E_{2n+1}$$

$$= \left[ \left( n + \frac{1}{2} \right) - \mathcal{O}(g) \right] \omega$$



# Unruh effect in Polymer quantization

- In polymer quantization

$$\sum_{r=1}^{\infty} \frac{\epsilon_r}{r^{1+2\delta}} = \zeta_{r_\star}(1+2\delta) - \frac{\zeta_{r_\star}(2\delta)}{\nu_1 r_\star} + \frac{r_\star \zeta(2+2\delta, r_\star)}{\nu_2},$$

where  $r_\star = (L/l_\star)(2 - \sqrt{2})/2\pi$ ,  $\nu_1 = 2(2 + \sqrt{2})$ ,  
 $\nu_2 = 2(2 - \sqrt{2})$

- *Truncated zeta function*:  $\zeta_{r_\star}(s) = \sum_{r=1}^{r_\star} r^{-s}$ ,
- *Hurwitz zeta function*:  $\zeta(s, r_\star) = \sum_{r=r_\star}^{\infty} r^{-s}$ .

## Expectation value of Number Operator

- *Zeta function identity*:  $\lim_{s \rightarrow 0} [s \zeta(1 + s)] = 1$

$$\lim_{\delta \rightarrow 0} \frac{1}{\zeta(1 + 2\delta)} \sum_{r=1}^{\infty} \frac{\epsilon_r}{r^{1+2\delta}} = 0$$

- Vacuum expectation value:

$$\bar{N}_{\bar{\omega}} \equiv \langle 0^{\Theta} | \hat{N}_{\bar{\omega}} | 0^{\Theta} \rangle = 0 ,$$

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- Vacuum expectation value:

$$\bar{N}_{\bar{\omega}} \equiv \langle 0^{\Theta} | \hat{N}_{\bar{\omega}} | 0^{\Theta} \rangle = 0 ,$$

- Contrary to **Fock quantization**:  $E_{\mathbf{k}}^n = (n + \frac{1}{2})|\mathbf{k}|$ ,  
 $\epsilon_r = 2E_{\mathbf{k}_r}^0 / |\mathbf{k}_r| = 1$

$$\lim_{\delta \rightarrow 0} \frac{1}{\zeta(1 + 2\delta)} \sum_{r=1}^{\infty} \frac{\epsilon_r}{r^{1+2\delta}} = 1 , \quad \bar{N}_{\bar{\omega}} = \frac{1}{e^{2\pi\bar{\omega}/a} - 1} ,$$

## Two-point function and KMS Condition

- Two-point function:  $G(\tau) \equiv \langle 0 | \phi(x(\tau), t(\tau)) \phi(x(0), t(0)) | 0 \rangle$
- KMS condition (“periodicity with a twist”) for Unruh effect

$$G(-\tau) = G(\tau - i\beta) \quad ; \quad k_B T = 1/\beta$$

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- What about two-point function in polymer quantization?

Rovelli, arXiv:1412.7827v1

- With leading order perturbative correction in  $l_*$ :

$$G_{poly} = \frac{1}{\Delta s^2} \left[ 1 - 4i \frac{l_* \Delta t}{\Delta s^2} \left( 1 + \frac{\Delta t^2}{\Delta s^2} \right) \right] \quad ; \quad \Delta s^2 = -\Delta t^2 + \Delta x^2$$

- Rindler trajectory:  $\Delta t = t(\tau) - t(0) = \sinh(a\tau)/a$ ,  
 $\Delta x = (\cosh(a\tau) - 1)/a$

$$\Delta s^2 = 2(1 - \cosh(a\tau))/a^2$$

# Discussions

- In polymer quantization, **Unruh effect appears to be absent!**
- Unruh effect can be used as a potential probe of Planck scale physics
- Several experimental proposals have been made to verify Unruh effect in laboratory!

Schutzhold, Schaller, Habs, PRL (2006,2008); Aspachs, Adesso, Fuentes, PRL (2010)

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Thank you