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A Singularity Free Cosmological Model in General Relativity

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Abstract

A singularity free cosmological model is obtained in a homogeneous and isotropic background with a specific form of the Hubble parameter in the presence of an interacting dark energy represented by a time-varying cosmological constant in general relativity. Different cases so arose have extensively been studied for different values of curvature parameter. Some interesting results have been found with this form of Hubble parameter to meet the possible negative value of the deceleration parameter $(-1/3 \leq q < 0)$ as the current observations reveal. For some particular values of these parameters, the model reduces to Berman's model (Berman 1983).

Keywords: Singularity, Time-dependent cosmological constant, Vacuum energy, Parameter.

Introduction

Even after the tremendous success of the standard cosmology, it suffers with the problem of the initial singularity or the big bang, where the theory breaks down. If we consider the homogeneous and isotropic Robertson-Walker space-time

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \qquad (1)$$

together with the perfect fluid distribution of matter represented by the energy-momentum tensor

$$T_{ij}^{M} = (\rho + p)U_i U_j + p g_{ij}, \qquad (2)$$

where 'ρ' is the energy density of the cosmic matter and 'p' is its isotropic pressure, then the Einstein field equations

$$R_{ij} - \frac{1}{2} R_k^k g_{ij} = -T_{ij}^M \quad \text{(with } 8\pi G = 1\text{)} \tag{3}$$

yields the following two independent equations

$$\rho = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2}, \tag{4}$$

$$p = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}. \tag{5}$$

where an overdot $(\cdot)$ represents ordinary derivative with respect to cosmic time 't' only. Equations (4) and (5) are two equations with three unknown functions R, ρ and p. If we assume the perfect fluid equation of state

$$p = w\rho, \tag{6}$$

where $0 \leq w \leq 1$ is a constant, then the system becomes closed and completely determines the dynamics of the Universe. Equations (4) and (5) together with equation (6) shows that, for normal matter ($\rho > 0$ and $p > 0$) the scale factor $R \to 0$ at some finite time in the past. At this point, the space-time becomes singular and $\rho \to \infty$ and $p \to \infty$.

Our aim in this paper is to obtain a non-singular bouncing solution by constraining the Hubble parameter 'H' (which regulates the dynamics of the Universe). We try to solve our problem within the framework of classical general relativity. As we can see that, the system becomes over-deterministic, if any extra condition (here we impose on the Hubble parameter) is assumed. We compensate this over-determinacy by inserting another entity into the field equations, the famous *dark energy* (DE). Now a days, the theory of dark energy has become very popular and is a well established theory in modern cosmology, which is responsible for the current observed accelerating expansion of the Universe (Reiss et al. 1998; Kowalski et al. 2008; Amanullah et al. 2010; Rubin et al. 2013).

In recent years, there has been spurt activity in discovering these accelerating models and are also supported by number of observations such as (Tegmark et al. 2004; Seljak et al. 2005; Wang & Mukherjee 2006; Bond et al. 1997; Eisenstein et al. 2005; Spergel et al. 2003; Spergel et al. 2007; Komatsu et al. 2009; Komatsu et al. 2011; Hinshaw et al. 2009; Ade et al. 2013; Jain & Taylor 2003).

Though much is not known, dark energy can be represented by a large-scale scalar field ϕ. For a scalar field, with Lagrangian density $\mathcal{L} = \partial_\mu \phi \partial^\mu \phi - V(\phi)$, the stress energy tensor takes the form

$$T_{ij}^{DE} = (\rho_\phi + p_\phi) U_i U_j + p_\phi g_{ij} \tag{7}$$

with its equation of state in the form $p_\phi = w_\phi \rho_\phi$, where w_ϕ is a function of time in general. Depending upon the dynamics of the field ϕ and its potential energy, this produces a number of candidates for dark energy.

As we know, the simplest and the most favored candidate of dark energy is the Einstein's cosmological constant Λ (supported by number of cosmological observations) for which w_ϕ reduces to the value -1 (potential energy dominated scalar field).

Dark energy can be introduced in Einstein's theory by replacing T_{ij}^{M} by T_{ij}^{Total} in equation (3), where

$$T_{ij}^{Total} = T_{ij}^{M} + T_{ij}^{DE} = (\rho_t + p_t)U_i U_j + p_t g_{ij} \tag{8}$$

with $\rho_t = \rho + \rho_\phi$ and $p_t = p + p_\phi$. Now, the modified Einstein's Field Equations are

$$\rho_t = 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2}, \tag{9}$$

$$p_t = -2\frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2}. \tag{10}$$

The Bianchi identities require that T_{ij}^{Total} has a vanishing divergence. We believe that the interaction between matter and dark energy is natural and is a fundamental principle (Vishwakarma 2007). Although there are a number of candidates of dark energy, we limit ourselves in the following to the case of cosmological constant only. We know that Λ can be represented as the intrinsic energy density of vacuum $\rho_v = \Lambda$ (as we have taken $8\pi G = 1$) arising from the zero point energy of quantum fluctuations. This however brings about the widely discussed cosmological constant problem, which is alleviated if we consider a dynamically decaying ρ_v. Due to its coupling with the other matter fields of the universe, a decaying ρ_v (with a large value in the early universe) can relax to its small observed value in course of the expansion of the universe by creating massive or massless particles (Vishwakarma 1996 & the references therein).

Dynamics of the Universe from the Hubble parameter

The observable parameters H (Hubble parameter) and q (deceleration parameter) defined as

$$H = \dot{R}/R, \qquad (11)$$

and

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right). \qquad (12)$$

Abdussattar and Prajapati (Abdussattar & Prajapati 2011) have obtained a class of non-singular and bouncing FRW cosmological models with a perfect fluid as the source of matter and an interacting dark energy represented by the time-varying cosmological constant by constraining the form of deceleration parameter q as $q = -\frac{\alpha}{t^2} + (\beta - 1)$.

Berman (Berman 1983) has considered a special form of Hubble parameter which leads to a constant deceleration parameter $q = m - 1$ and obtained a cosmological model with the variation of Λ (Berman 1991). In the quest for a negative value of deceleration parameter consistent with the observations, in the same line as that floated by Berman, here in this paper, we propose a specific form of the Hubble parameter given by

$$H = \frac{m}{\alpha t+\beta} \qquad (13)$$

which is the main ansatz of the paper. Here $m > 0, \alpha \neq 0$ and β are parameters. For $m = 1$ the model reduces to the model obtained by Berman. With the form of H given by equation (13), equation (11) can be integrated to give the time variation of the scale factor as

$$R(t) = A(\alpha t + \beta)^{\frac{m}{\alpha}} \qquad (14)$$

where A is a constant of integration. Obviously, the different values of m and α will give rise to different models. For the purpose of reference, the origin of the time coordinate is set at the bounce of these bouncing models.

It is easy to see from equation (14) that at $t = 0$, $R = R_0 \neq 0$ (say, Here and afterwards the suffix `zero' indicates the value of the parameter at $t = 0$). This imply

$$R = R_0 \beta^{-\frac{m}{\alpha}} (\alpha t + \beta)^{\frac{m}{\alpha}}. \tag{15}$$

The first and second derivatives of the scale factor R are given by

$$\dot{R} = R_0 \beta^{-\frac{m}{\alpha}} m (\alpha t + \beta)^{\frac{m}{\alpha}-1}, \tag{16}$$

and

$$\ddot{R} = R_0 \beta^{-\frac{m}{\alpha}} m (m - \alpha)(\alpha t + \beta)^{\frac{m}{\alpha}-2}, \tag{17}$$

indicating that at $t = 0$, $\dot{R} = R_0 \frac{m}{\beta}$ and $\ddot{R} = R_0 \frac{m(m-\alpha)}{\beta^2}$. This shows that the model is free from initial singularity and starts with a finite acceleration and also *finite velocity*. This is a significant deviation from the result obtained by Abdussattar and Prajapati (Abdussattar & Prajapati 2011). The deceleration parameter is obtained using equation (12) and (13) as

$$q = -1 + \frac{\alpha}{m}. \tag{18}$$

We observe that the deceleration parameter is independent of time. Again we see that the choice of α and m will suggest, whether the expansion of the Universe is accelerated or decelerated one.

With the help of equations (15), (16) and (17), equations (9), (10) give

$$\rho_t = \frac{3m^2}{(\alpha t+\beta)^2} + \frac{3k}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}, \tag{19}$$

$$p_t = \frac{m(2\alpha-3m)}{(\alpha t+\beta)^2} - \frac{k}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}. \tag{20}$$

yielding

$$\rho_t + p_t = \frac{2\alpha}{3m}(3H^2) + \frac{2k}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}. \tag{21}$$

This shows that for $\frac{\alpha}{m} = \frac{3}{2}$, the model would indicate $\rho = \rho_c$ at sufficiently large times ($p = 0$).

The total active gravitational mass is given by

$$(\rho_t + 3p_t)R^3 = 6R_0^3\beta^{-3\frac{m}{\alpha}}m(\alpha - m)(\alpha t + \beta)^{3\frac{m}{\alpha}-2} \qquad (22)$$

which is $-ve$, $zero$ or $+ve$ according as $\frac{\alpha}{m} \lesseqgtr 1$. Equation (19) suggests that at $t = 0$, $\rho_{t0} = \frac{3m^2}{\beta^2} + \frac{3k}{R_0^2}$ suggesting that $m > \frac{\beta}{R_0}$ for $k = -1$.

The age of the Universe is found to be $t_p = \left(\frac{m}{\alpha}\right) H_p^{-1} - \frac{\beta}{\alpha}$ and the Radius of the Universe is given by $R_p = \left(\frac{m}{\beta}\right)^{\frac{m}{\alpha}} \left(H_p^{-1}\right)^{\frac{m}{\alpha}} R_0$, where the suffix '$p$' represents the value at present time.

In the following sections, we study some properties of the model in the early radiation dominated era (RD) and mater dominated era (MD) for different values of the curvature parameter.

$k = 0$ (spatially flat Universe)

In the early pure radiation era, the equation of state is assumed to be $p = p_r = \frac{1}{3}\rho_r$. Equations (19) and (20) yield

$$\rho_r = \frac{3}{2}\frac{\alpha m}{(\alpha t+\beta)^2}, \tag{23}$$

$$\rho_v = \frac{3}{2}\frac{m(2m-\alpha)}{(\alpha t+\beta)^2}. \tag{24}$$

From equations (23) and (24) it is easy to see that at $t = 0$, we have $\rho_{r0} = \frac{3\alpha m}{2\beta^2}$ and $\rho_{v0} = \frac{3m(2m-\alpha)}{2\beta^2}$ suggesting that $\rho_{r0} > 0$ in the beginning and $\rho_{v0} > 0$ unless $\alpha > 2m$.

The differentiation of (23) and (24) with respect to cosmic time 't' yield

$$\dot{\rho}_r = -\frac{3\alpha^2 m}{(\alpha t+\beta)^3}, \tag{25}$$

$$\dot{\rho}_v = -\frac{3\alpha m(2m-\alpha)}{(\alpha t+\beta)^3}, \tag{26}$$

From equations (25) and (26), it follows that $\dot{\rho}_r$ and $\dot{\rho}_v$ are negative showing that ρ_r and ρ_v are decreasing functions of time. Furthermore the $\rho_r > 0$ and $\rho_v > 0$ at $t = 0$ implying that ρ_r and ρ_v are maximum initially and decreases rapidly by creating massive or massless particles.

The radiation temperature (T) is assumed to be related to radiation energy density by the relation

$$\rho_r = \frac{\pi^2}{30} N(T) T^4, \tag{27}$$

in the units with ħ$= c = k_B = 1$.

The effective number of spin degrees of freedom $N(T)$ at temperature T is given by $N(T) = N_b(T) + \frac{7}{8}N_f(T)$, where $N_b(T)$ and $N_f(T)$ correspond to bosons and fermions respectively. We assume $N(T)$ to be constant throughout this era. From equations (23) and (27) we obtain

$$T = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}} \left[\frac{\alpha m}{(\alpha t+\beta)^2}\right]^{\frac{1}{4}}. \tag{28}$$

From the equation (28), it is easy to see that like the radiation energy density the radiation temperature is also constant at $t = 0$ with $T_0 = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}} \left[\frac{\alpha m}{\beta^2}\right]^{\frac{1}{4}}$ and is maximum initially.

In the present matter dominated era, the matter pressure is negligible i.e. $p = p_m \approx 0$ and $\rho = \rho_m$.

Equations (19) and (20) give

$$\rho_m = \frac{2\alpha m}{(\alpha t+\beta)^2}, \quad (29)$$

$$\rho_v = \frac{m(3m-2\alpha)}{(\alpha t+\beta)^2}. \quad (30)$$

As $t \to \infty$, $\rho_m \to 0$ and $\rho_v \to 0$. Equations (29) and (30) can be written in terms of Hubble parameter as

$$\rho_{mp} = \left(\frac{2\alpha}{m}\right) H_p^2, \quad (31)$$

$$\rho_{vp} = \left(3 - \frac{2\alpha}{m}\right) H_p^2. \quad (32)$$

$k = 1$ (non-flat closed Universe)

RD Phase $\left(p = p_r = \frac{1}{3}\rho_r\right)$

In this phase of evolution of the Universe, the radiation and vacuum energy densities are obtained from equations (19) and (20) as

$$\rho_r = \frac{3}{2}\left[\frac{\alpha m}{(\alpha t+\beta)^2} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right], \qquad (33)$$

$$\rho_v = \frac{3}{2}\left[\frac{m(2m-\alpha)}{(\alpha t+\beta)^2} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right]. \qquad (34)$$

At $t = 0$, we have $\rho_{r0} = \frac{3}{2}\left[\frac{\alpha m}{\beta^2} + \frac{1}{R_0^2}\right]$ and $\rho_{v0} = \frac{3}{2}\left[\frac{m(2m-\alpha)}{\beta^2} + \frac{1}{R_0^2}\right]$. The differentiation of (33) and (34) with respect to cosmic time 't' yield

$$\dot{\rho}_r = -\frac{3\alpha^2 m}{(\alpha t+\beta)^3} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{3m}{(\alpha t+\beta)^{2\frac{m}{\alpha}+1}}, \tag{35}$$

$$\dot{\rho}_v = -\frac{3\alpha m(2m-\alpha)}{(\alpha t+\beta)^3} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{3m}{(\alpha t+\beta)^{2\frac{m}{\alpha}+1}}, \tag{36}$$

Equations (35) and (36) shows that $\dot{\rho}_r$ and $\dot{\rho}_v$ are negative implying that ρ_r and ρ_v are decreasing functions of time. Also $\ddot{\rho}_r > 0$ and $\ddot{\rho}_v > 0$ at $t = 0$ implying that ρ_r and ρ_v are maximum initially.

The radiation temperature (T) in this case is obtained from equations (27) and (33) as

$$T = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}} \left[\frac{\alpha m}{(\alpha t+\beta)^2} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}} \frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right]^{\frac{1}{4}}. \tag{37}$$

From the equation (37), at $t = 0$, we have $T_0 = \left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}} \left[\frac{\alpha m}{\beta^2}\right]^{\frac{1}{4}}$ which is maximum. As the Universe is geometrically closed in this case, it is possible to determine the time $t = t_{cau}$ when the whole Universe becomes causally connected. This is given by

$$\int_0^{t_{cau}} \frac{dt}{R(t)} = \int_0^1 \frac{dr}{\sqrt{1-r^2}} = \frac{\pi}{2} \tag{38}$$

This, by use of equation (15) yields

$$\int_0^{t_{cau}} \frac{dt}{(\alpha t+\beta)^{\frac{m}{\alpha}}} = \frac{\pi}{2} R_0 \beta^{-\frac{m}{\alpha}}, \tag{39}$$

which on integration yields

$$t_{cau} = \frac{1}{\alpha}\left[\frac{\pi}{2} R_0 \beta^{-\frac{m}{\alpha}}(\alpha - m) + \beta^{\frac{\alpha-m}{\alpha}}\right]^{\frac{\alpha}{\alpha-m}} - \frac{\beta}{\alpha}. \tag{40}$$

We find that the global causality is established at $t = t_{cau}$, where t_{cau} can be determined from (40) by giving the particular values of α, β and m.

MD Phase ($p = p_m \approx 0, \rho = \rho_m$)

In this phase of evolution of the Universe, we have

$$\rho_m = \left[\frac{2\alpha m}{(\alpha t+\beta)^2} + \frac{2}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right], \tag{41}$$

$$\rho_v = \left[\frac{m(3m-2\alpha)}{(\alpha t+\beta)^2} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right]. \tag{42}$$

As $t \to \infty$, $\rho_m \to 0$ and $\rho_v \to 0$. Equations (41) and (42) can be written in terms of Hubble parameter as

$$\rho_{mp} = \left(\frac{2\alpha}{m}\right) H_p^2 + \frac{2}{R_0^2}\left(\frac{\beta}{m}\right)^{2\frac{m}{\alpha}} H_p^{2\frac{m}{\alpha}}, \tag{43}$$

$$\rho_{vp} = \left(3 - \frac{2\alpha}{m}\right) H_p^2 + \frac{1}{R_0^2}\left(\frac{\beta}{m}\right)^{2\frac{m}{\alpha}} H_p^{2\frac{m}{\alpha}}. \tag{44}$$

k = -1 (non-flat open Universe)

RD Phase $(p = p_r = \frac{1}{3}\rho_r)$

Here, the radiation and vacuum energy densities are obtained as

$$\rho_r = \frac{3}{2}\left[\frac{\alpha m}{(\alpha t+\beta)^2} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right], \tag{45}$$

$$\rho_v = \frac{3}{2}\left[\frac{m(2m-\alpha)}{(\alpha t+\beta)^2} - \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right]. \tag{46}$$

At $t = 0$, equations (45) and (46) yield

$$\rho_{r0} = \frac{3}{2}\left[\frac{\alpha m}{\beta^2} - \frac{1}{R_0^2}\right] \tag{47}$$

$$\rho_{v0} = \frac{3}{2}\left[\frac{m(2m-\alpha)}{\beta^2} - \frac{1}{R_0^2}\right] \tag{48}$$

Equation (47) suggests that $\frac{\beta^2}{\alpha m} < R_0^2$. If $\frac{\beta^2}{\alpha m} = R_0^2$, we get $\rho_{r0} = 0$. From equations (45) and (46), we observe that $\rho_r \gtreqless 0$ for $t \gtreqless \frac{1}{\alpha}\left[\frac{\beta^{\frac{m}{\alpha}}}{\sqrt{\alpha m}} \frac{1}{R_0}\right]^{\frac{\alpha}{m-\alpha}} - \frac{\beta}{\alpha}$ and $\rho_v \gtreqless 0$ for $t \gtreqless \frac{1}{\alpha}\left[\frac{\beta^{\frac{m}{\alpha}}}{\sqrt{m(2m-\alpha)}} \frac{1}{R_0}\right]^{\frac{\alpha}{m-\alpha}} - \frac{\beta}{\alpha}$.

The differentiation of (45) and (46) with respect to cosmic time 't' yield

$$\dot{\rho}_r = -\frac{3\alpha^2 m}{(\alpha t+\beta)^3} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{3m}{(\alpha t+\beta)^{2\frac{m}{\alpha}+1}}, \tag{49}$$

$$\dot{\rho}_v = -\frac{3\alpha m(2m-\alpha)}{(\alpha t+\beta)^3} + \frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{3m}{(\alpha t+\beta)^{2\frac{m}{\alpha}+1}}. \tag{50}$$

$\dot{\rho}_r$ becomes zero at $t = \frac{1}{\alpha}\left[\frac{\beta^{\frac{m}{\alpha}}}{\alpha}\frac{1}{R_0}\right]^{\frac{\alpha}{m-\alpha}} - \frac{\beta}{\alpha}$. Also $\dot{\rho}_v$ becomes zero at $t = \frac{1}{\alpha}\left[\frac{\beta^{\frac{m}{\alpha}}}{\sqrt{\alpha(2m-\alpha)}}\frac{1}{R_0}\right]^{\frac{\alpha}{m-\alpha}} - \frac{\beta}{\alpha}$. At these points ρ_r and ρ_v are maximum.

In this case, the radiation temperature (T) is obtained from equations (27) and (45) as

$$T=\left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}}\left[\frac{\alpha m}{(\alpha t+\beta)^2}-\frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right]^{\frac{1}{4}}. \tag{51}$$

From the equation (51), at $t=0$, we have $T_0=\left(\frac{45}{\pi^2 N}\right)^{\frac{1}{4}}\left[\frac{\alpha m}{\beta^2}-\frac{1}{R_0^2}\right]^{\frac{1}{4}}$.

MD Phase ($p=p_m\approx 0, \rho=\rho_m$)

In this phase of evolution of the Universe, we have

$$\rho_m=\left[\frac{2\alpha m}{(\alpha t+\beta)^2}-\frac{2}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right], \tag{52}$$

$$\rho_v=\left[\frac{m(3m-2\alpha)}{(\alpha t+\beta)^2}-\frac{1}{R_0^2}\beta^{2\frac{m}{\alpha}}\frac{1}{(\alpha t+\beta)^{2\frac{m}{\alpha}}}\right]. \tag{53}$$

As $t \to \infty$, $\rho_m \to 0$ and $\rho_v \to 0$. Equations (52) and (53) can be written in terms of Hubble parameter as

$$\rho_{mp} = \left(\frac{2\alpha}{m}\right) H_p^2 - \frac{2}{R_0^2}\left(\frac{\beta}{m}\right)^{2\frac{m}{\alpha}} H_p^{2\frac{m}{\alpha}}, \tag{54}$$

$$\rho_{vp} = \left(3 - \frac{2\alpha}{m}\right) H_p^2 - \frac{1}{R_0^2}\left(\frac{\beta}{m}\right)^{2\frac{m}{\alpha}} H_p^{2\frac{m}{\alpha}}. \tag{55}$$

The evolution of the Universe in our obtained model heavily depends on the choice of the parameters α and m and β. In the next section we discuss the consequences of the choice of these parameters α, m and β.

The Parameters and the model

From equation (12), we observe that for

$\alpha = m$, we have $q = 0$ (Expanding Universe without acceleration)

$\alpha < m$, we have $q < 0$ (Accelerated expansion of the Universe)

$\alpha > m$, we have $q > 0$ (Decelerated expansion of the Universe).

A statistical observation is given in the following table for different values of α and m giving rise to different models.

Parameters	Exemplification	q	H	R
$\alpha = m$	$\alpha = 1, m = 1$	0	$\frac{1}{t+\beta}$	$R_0\beta^{-1}(t+\beta)$
	$\alpha = 2, m = 2$		$\frac{2}{2t+\beta}$	$R_0\beta^{-1}(2t+\beta)$
	$\alpha = 3, m = 3$		$\frac{1}{3t+\beta}$	$R_0\beta^{-1}(3t+\beta)$

Parameters	Exemplification	q	H	R
$\alpha < m$	$\alpha = 1, m = 2$	$-\frac{1}{2}$	$\frac{2}{t+\beta}$	$R_0\beta^{-2}(t+\beta)^2$
	$\alpha = 2, m = 3$	$-\frac{1}{3}$	$\frac{3}{2t+\beta}$	$R_0\beta^{-\frac{3}{2}}(2t+\beta)^{\frac{3}{2}}$
	$\alpha = 3, m = 4$	$-\frac{1}{4}$	$\frac{4}{3t+\beta}$	$R_0\beta^{-\frac{4}{3}}(3t+\beta)^{\frac{4}{3}}$

Parameters	Exemplification	q	H	R
$\alpha > m$	$\alpha = 2, m = 1$	1	$\frac{1}{2t+\beta}$	$R_0\beta^{-\frac{1}{2}}(2t+\beta)^{\frac{1}{2}}$
	$\alpha = 3, m = 2$	$\frac{1}{2}$	$\frac{2}{3t+\beta}$	$R_0\beta^{-\frac{2}{3}}(3t+\beta)^{\frac{2}{3}}$
	$\alpha = 4, m = 3$	$\frac{1}{3}$	$\frac{3}{4t+\beta}$	$R_0\beta^{-\frac{3}{4}}(4t+\beta)^{\frac{3}{4}}$

For the best fit value of the deceleration parameter as suggested by the observations, $-\frac{1}{3} \leq q < 0$, we must have $\frac{2}{3} \leq \frac{\alpha}{m} < 0$. From the table above, it is observed that for a model consistent with the observations, we should have $1.5 \leq \alpha \leq 3$ and $2.5 \leq m \leq 4$. The value of β to be constrained according to the curvature parameter. These values of α and m produce some interesting models with $0 < \beta < 2$, if the curvature parameter is $k = 0$ or $k = 1$, but is incompatible with $k = -1$ for higher values of β within this range as is clear from equations (45) and (46). If we consider the present value of the Hubble parameter to be $H_p = 72\ (km/s)/Mpc$, then equations (54) and (55) suggest that the value of β should be in the range $0 < \beta < 0.624$.

By taking, $\alpha = 2$, $m = 3$, $\beta = 0.2$ and with the present value of the Hubble parameter $H_p = 72\ (km/s)/Mpc$, we may obtain

Age of the Universe	(t_p)	$\approx 6 \times 10^{17}\ S$
Radius of the Universe	(R_p)	$\approx 1.7 \times 10^{28}\ R_0$

Curvature Parameter	**Matter energy density**	**Vacuum energy density**	
$k = 0$	$\approx 7.2594317 \times 10^{-30}\ S^{-2}$	$\approx 9.0742895 \times 10^{-30}\ S^{-2}$	$8\pi G = 1$
$k = 1$	$\approx 7.2600242 \times 10^{-30}\ S^{-2}$	$\approx 9.074882 \times 10^{-30}\ S^{-2}$	
$k = -1$	$\approx 7.2588392 \times 10^{-30}\ S^{-2}$	$\approx 9.0736970 \times 10^{-30}\ S^{-2}$	

Conclusion

In this paper we have obtained a class of non-singular and bouncing FRW cosmological models with a perfect fluid as the source of matter and an interacting dark energy represented by the time-varying cosmological constant by constraining the form of Hubble parameter. Here, we have a freedom with the parameters involved in it to obtain a suitable model of the Universe consistent with the observations. For some specific values of these parameters we have obtained the age and radius of the Universe which are slightly greater than the age and radius obtained in the standard model. In all the three cases of the curvature parameters the present values of the matter and vacuum energy densities are almost same. The model is a simple generalization of the model obtained by Berman (Berman 1983).

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