EVOLUTION OF COSMOLOGICAL PARAMETERS EXHIBITED BY THE STAROBINSKY f(R) GRAVITY MODEL IN EINSTEIN FRAME

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Outlines

- Introduction
- Basis formalism of the f(R) gravity theory in the Jordan frame
- Transformation from Jordan to Einstein frame(using conformal transformation)
- Field equation of scalar degree of freedom
- Scalar field dynamics of Starobinsky model
- Result and observations
- Conclusions

Introduction
(failure of general relativity)

There are two main conceptual approaches that have been led to the modification of GR.

Apporach I: A new scalar degree of freedom is incorporated in the energy-momentum tensor on the right hand side of the Einstein's equation.

Apporach II: The left hand side of the Einstein's equation can be modified by considering that the late time cosmic accelerated expansion is due to the large scale modification of gravity.

Introduction (Modified gravity theories)

The simplest class of modified gravity theories is the f(R) gravity theories in which Einstein gravity is modified by replacing the Ricci curvature scalar R by an arbitrary curvature function f(R)

Many f(R) gravity models which could produce the late time cosmic acceleration, are not cosmologically viable.

Many of them suffered from the singularity and stability problem.

So certain restrictions have to be imposed on f(R) gravity model to be linearly stable and cosmologically viable.

Introduction (appearence of scalaron)

- In f(R) gravity theory a new scalar degree of freedom appears due to redefinition of model variables.
- The scalar field is coupled to non-relativistic matter(dark matter,baryons) with a universal coupling constant($-1/\sqrt{6}$).
- We consider the Einstein frame for the study because a canonical scalar field is coupled to non-relativistic matter in this frame.

Basis of formalism of the f(R) gravity in the Jordan frame

The f(R) gravity action in Jordan frame is given by--

$$S = \frac{1}{2k^2}\int d^4x\sqrt{-g}f(R) + \int d^4\mathcal{L}_m(g_{\mu\nu},\Psi_m)$$

the variation of this action with respect to the metric leads to the equation of motion

$$f'R_{\mu\vartheta} - \nabla_{\mu\vartheta}f' + \left(\blacksquare f' - \frac{1}{2}f\right)g_{\mu\vartheta} = k^2T_{\mu\vartheta}$$

the non venishing term □f' indicates the presence of propagating scalar degree of freedom.

Basis of formalism of the f(r) gravity in the Jordan frame

the dynamics of the field is govern by the trace equation of motion

$$3\Box f' + f'R - 2f = k^2 T$$

Transformation from Jordan to Einstein frame

Using the Ricci scalar in Einstein frame and the relation

$$\sqrt{-g} = \Omega^{-4}\sqrt{-\tilde{g}}$$

we get the action,

$$S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{1}{2k^2}f'\Omega^{-2}\left(\tilde{R}+6\tilde{\Box}\omega-6\tilde{g}^{\mu\vartheta}\partial_\mu\omega\partial_\vartheta\omega\right)-\Omega^{-4}U\right]+\int d^4x\mathcal{L}_m\left(\Omega^{-2}\tilde{g}_{\mu\nu},\psi_m\right)$$

where the potential of the field is

$$V(\phi) = \frac{U}{f'^2} = \frac{f'R-f}{2k^2f'^2}$$

here the field dependent conformal factor is

$$\Omega^2 = f' = exp\left(\sqrt{2/_3}\,k\phi\right)$$

Field equation of scalar degree of freedom

The lagrangian of the scalar field is

$$\mathcal{L}_\phi = -\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

The variation of action in Einstein frame with respect to field , leads to the following field equations (without presence of matter.)

$$\ddot{\phi} + 3\tilde{H}\dot{\phi} + V_{,\phi} = 0$$

$$\tilde{H}^2 = \frac{k^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

Finally we define the energy density and pressure of the scalar field as

$$\tilde{\rho}_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$\tilde{P}_\phi = \dot{\phi}^2 - V(\phi)$$

Scalar field dynamics of Starobinsky model

We consider the Starobinsky f(R) gravity model with disappearing cosmological constant for the study.

$$f(R) = R + \lambda R_0\left[\left(1 + \frac{R^2}{{R_0}^2}\right)^{-n} - 1\right]$$

where n, R_0 and λ are free parameters.

The scalar field embodied in this model is

$$\phi = \sqrt{\frac{3}{2}}\frac{1}{k}\ln f' = \sqrt{\frac{3}{2}}\frac{1}{k}\ln\left[1 - 2\lambda n\frac{R}{R_0}\left(1 + \frac{R^2}{{R_0}^2}\right)^{-(n+1)}\right]$$

It is observed from this equation that when R apporaches infinity, the field tends to zero, which is the point of singularity.

Scalar field dynamics of Starobinsky model

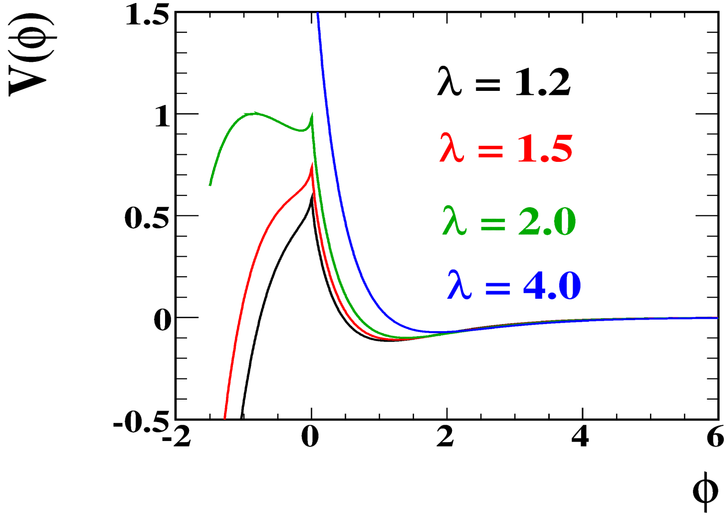
Using the field in a region $R/R_0 >> 1$, the field potential for the said model becomes,

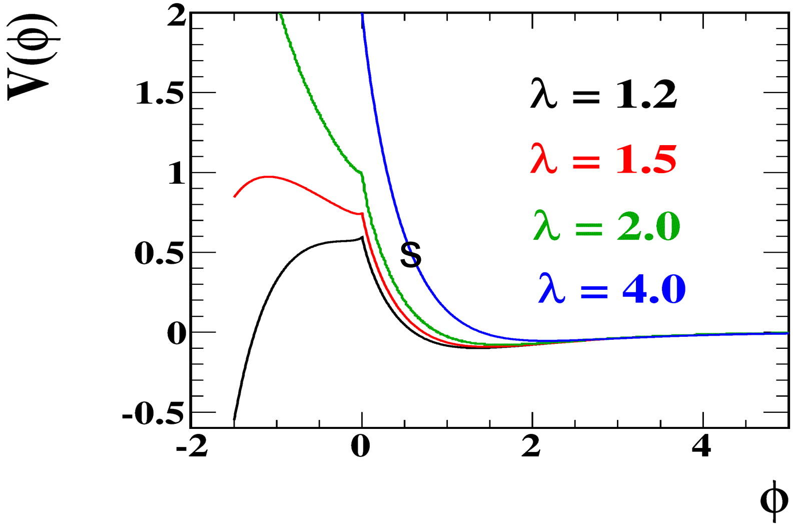
$$V(\phi) = -\lambda \frac{\left[(2n+1)\left(\frac{1-exp\left(\sqrt{2/3}\,k\phi\right)}{}\right)^{\frac{2n}{2n+1}} - 1\right]}{2k^2 exp\left(\sqrt{8/3}\,k\phi\right)}$$

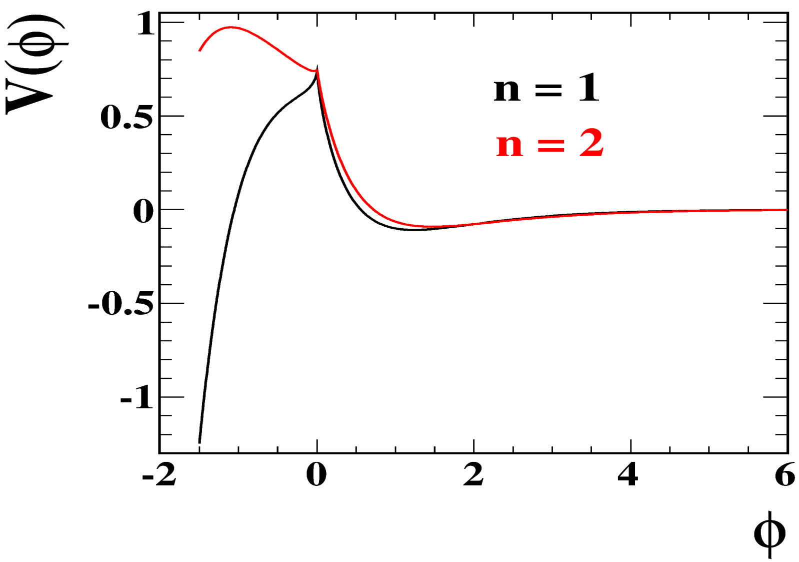
for validity of Starobinsky model the value of n and λ should be greater than zero.

Observations and results

- We have solved the field equations for scalar degree of freedom using the field potential for Starobinsky model numerically and different parameter are plotted, shwoing the following results.
- For numerical work we have taken n=1 and 2 with the values of λ = 1.2,1.5,2.0 and 4.0.

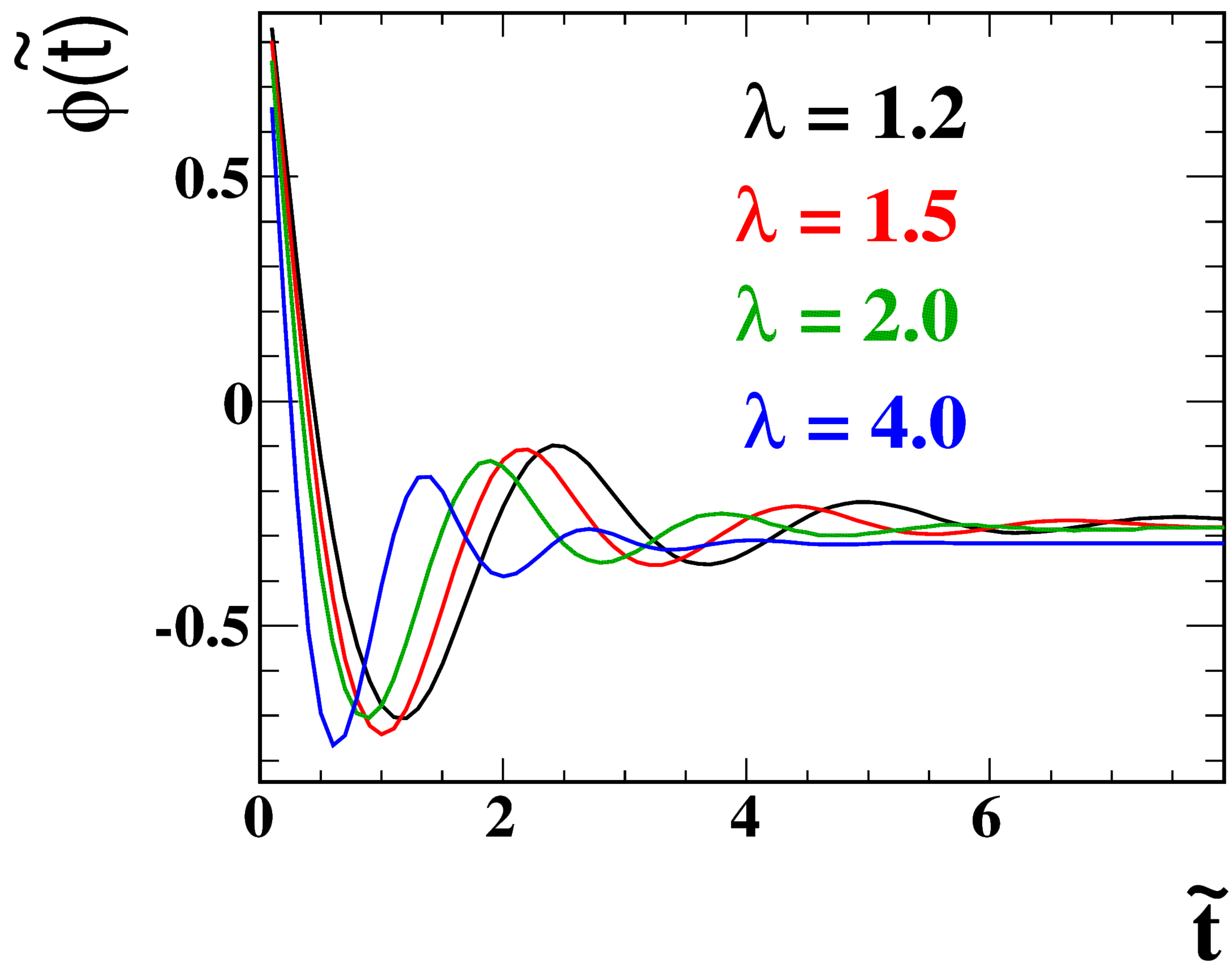

V(ϕ)
1.5
1
0.5
0
-0.5
-2
0
2
4
6
ϕ
λ = 1.2
λ = 1.5
λ = 2.0
λ = 4.0

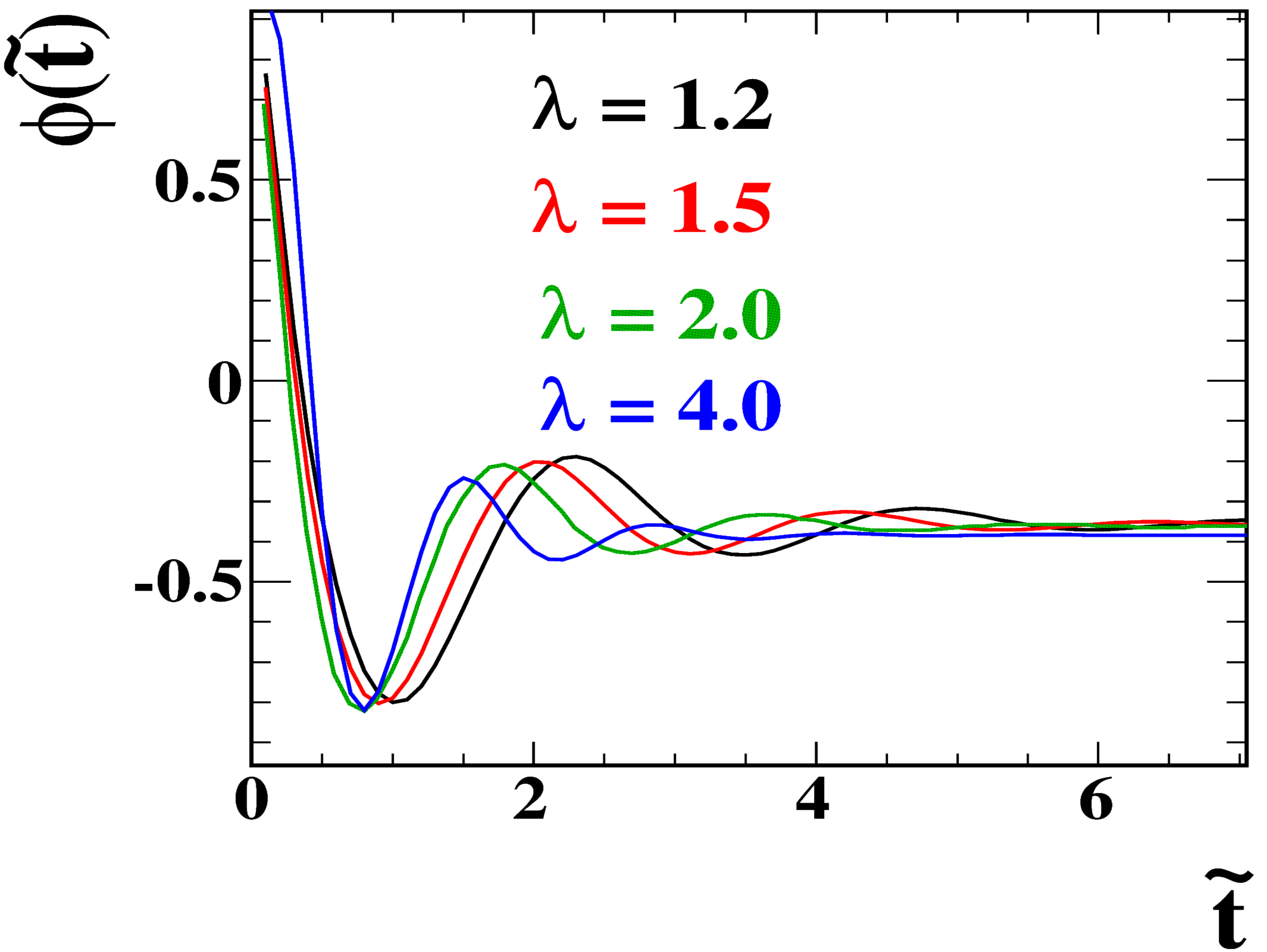

V(ϕ)
2
1.5
1
0.5
0
-0.5
-2
0
2
4
ϕ
S
λ = 1.2
λ = 1.5
λ = 2.0
λ = 4.0


V(ϕ)
1
0.5
0
-0.5
-1
-2
0
2
4
6
ϕ
n = 1
n = 2

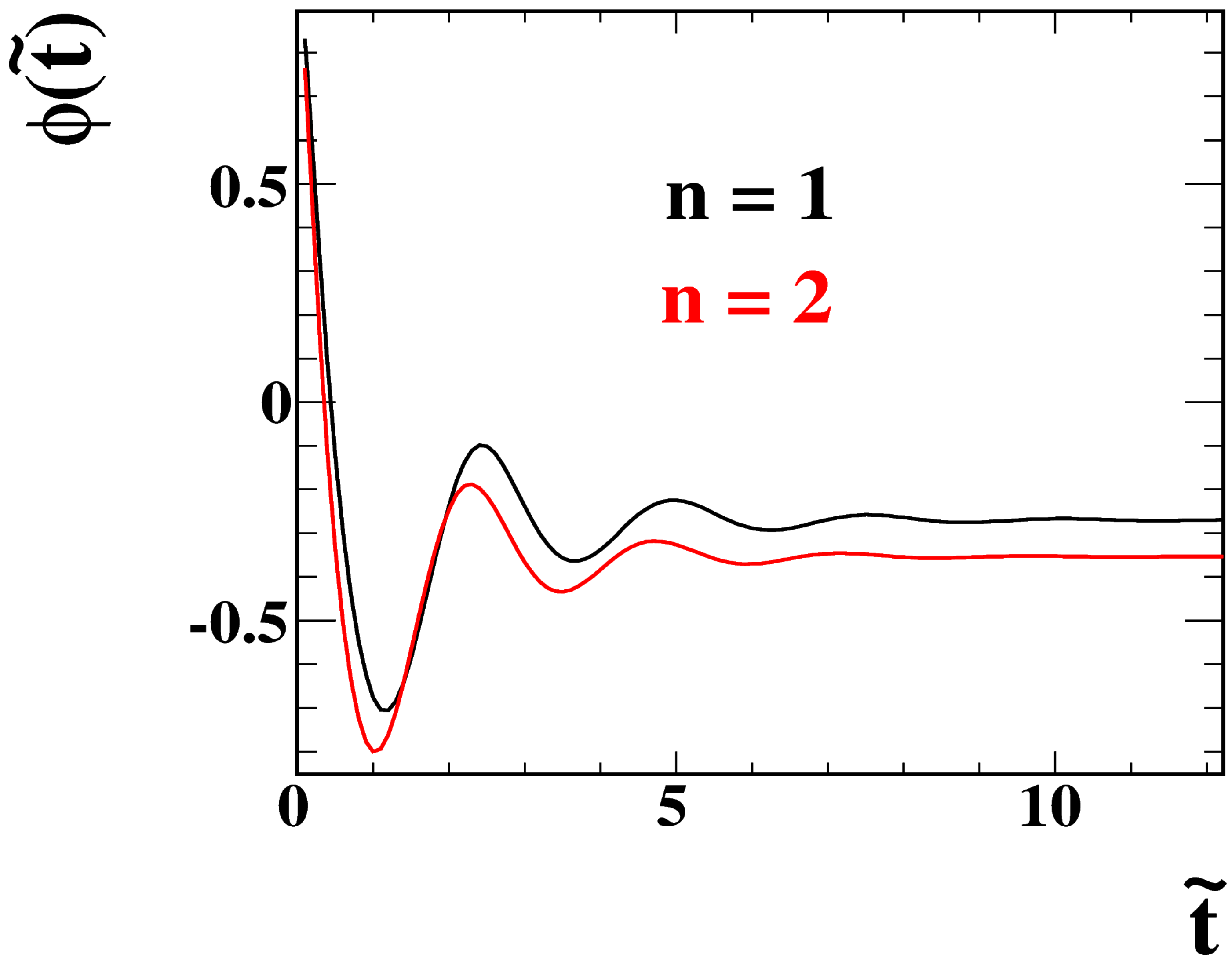
Observations

- For smaller value of λ, when field is rolling from -ve to zero, the potential increases very fast from -ve to its maximum and then falls off rapidly to a flat region near its zero value when the field increases again from its zero value.
- λ-pattern vanishes for higher value λ,
- Higher the value of n, the smaller is the range of λ for which the λ-patter exist.

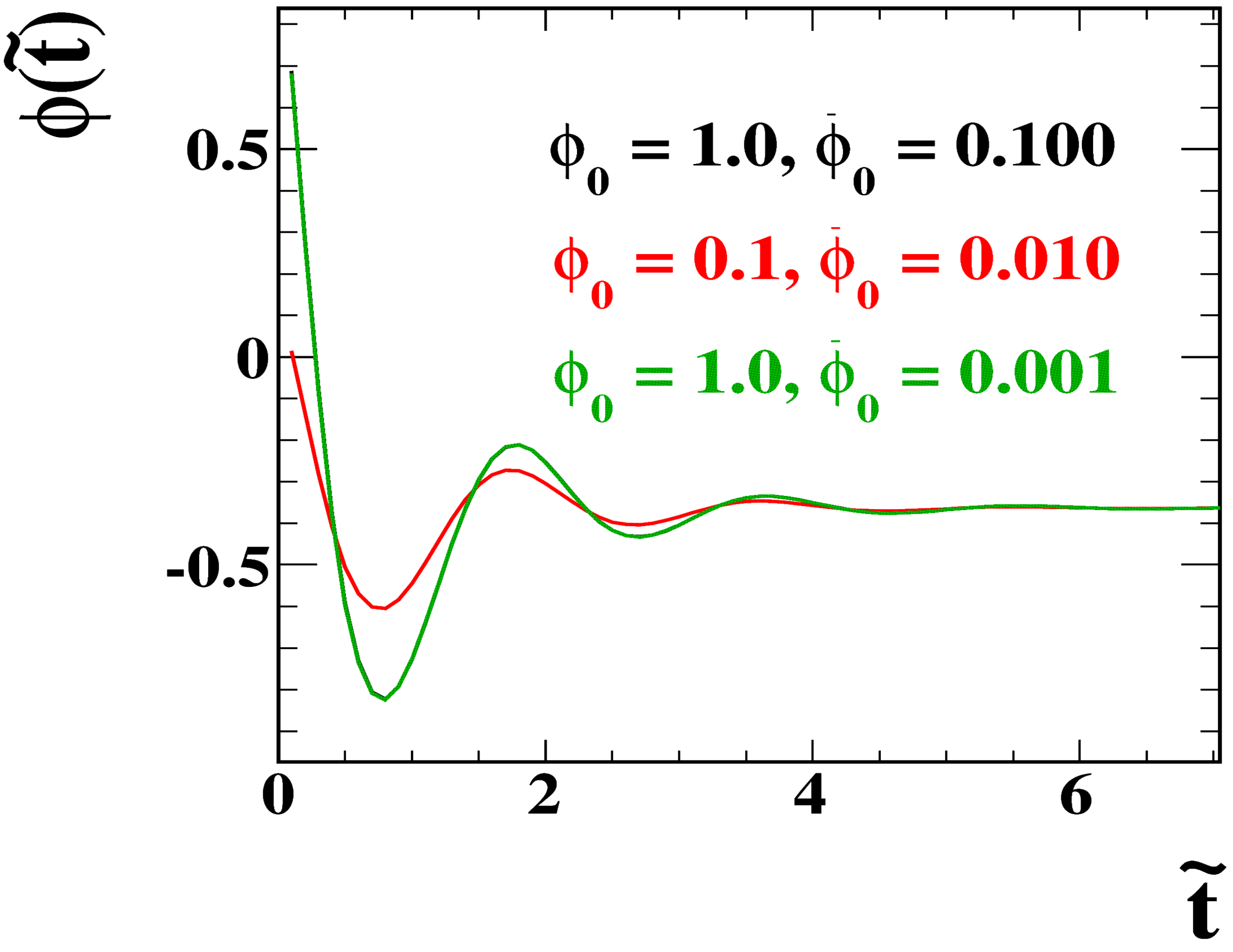

$\tilde{\phi}(t)$
0.5
0
-0.5
0
2
4
6
$\tilde{t}$
λ = 1.2
λ = 1.5
λ = 2.0
λ = 4.0



$\phi(\tilde{t})$
$\lambda = 1.2$
$\lambda = 1.5$
$\lambda = 2.0$
$\lambda = 4.0$
0.5
0
-0.5
0
2
4
6
$\tilde{t}$



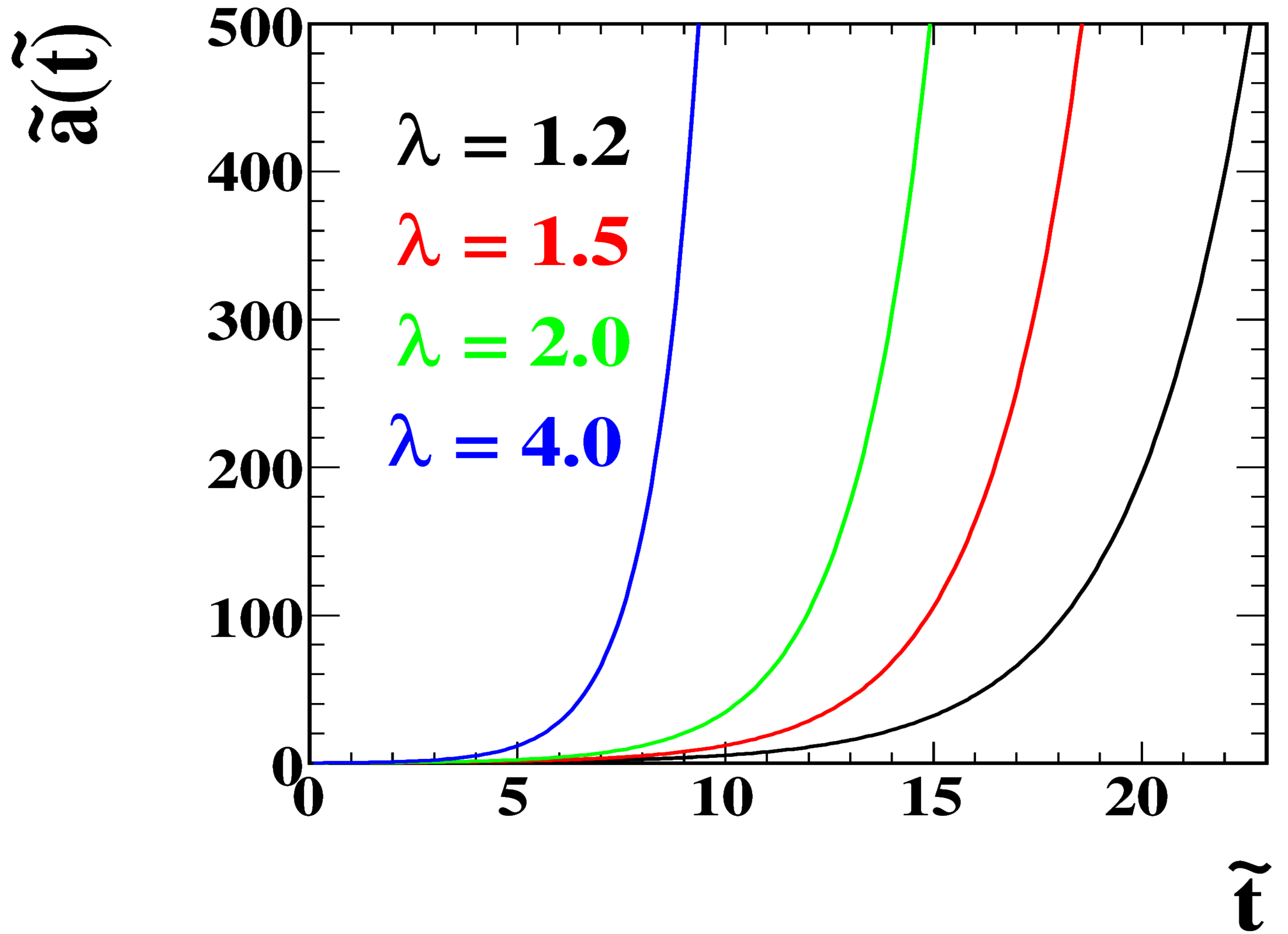
$\phi(\tilde{t})$
0.5
0
-0.5
n = 1
n = 2
0
5
10
$\tilde{t}$



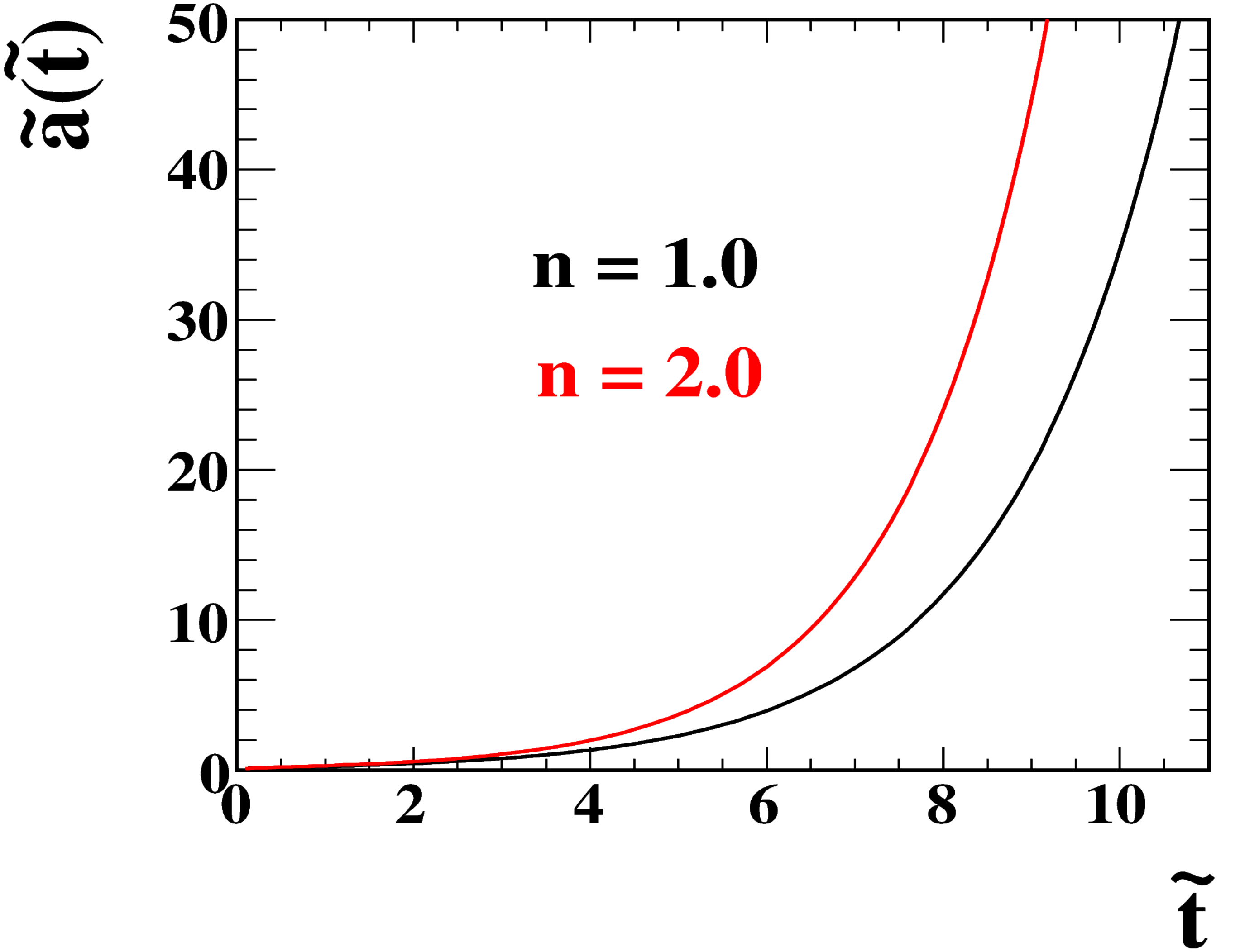
$\phi(\tilde{t})$
0.5
0
-0.5
0
2
4
6
$\tilde{t}$
$\phi_0 = 1.0, \dot{\phi}_0 = 0.100$
$\phi_0 = 0.1, \dot{\phi}_0 = 0.010$
$\phi_0 = 1.0, \dot{\phi}_0 = 0.001$

Observations

- The field Φ falls off rapidly from its initial positive value to its minimum negative value and the oscillate for sometimes with a decreasing amplitude
- For higher values of λ and n the field is legging behind the field for lower values of these parameters by shifting its minimum towards more negative side.



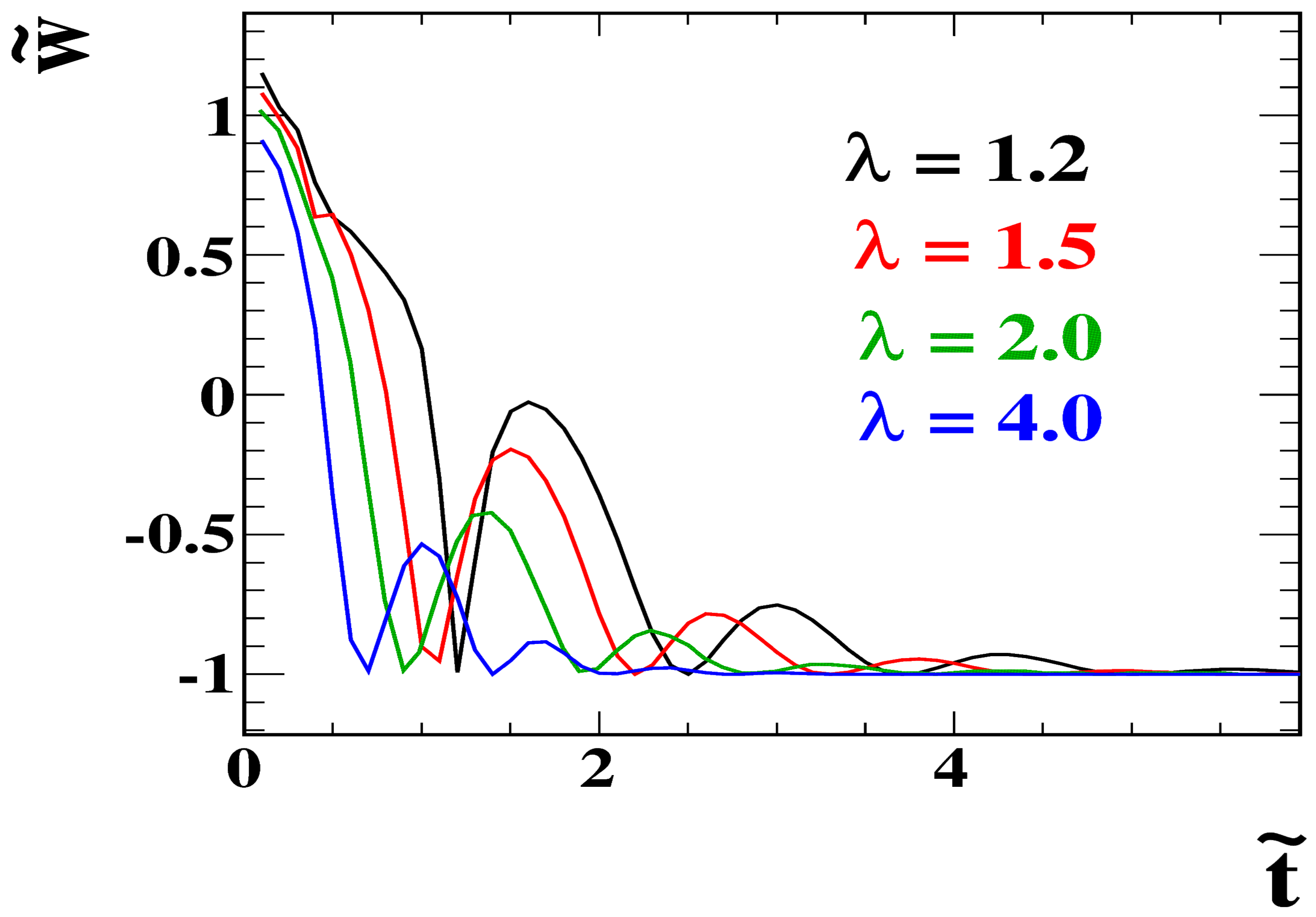
$\tilde{a}(\tilde{t})$
500
400
300
200
100
0
0
5
10
15
20
$\lambda = 1.2$
$\lambda = 1.5$
$\lambda = 2.0$
$\lambda = 4.0$
$\tilde{t}$



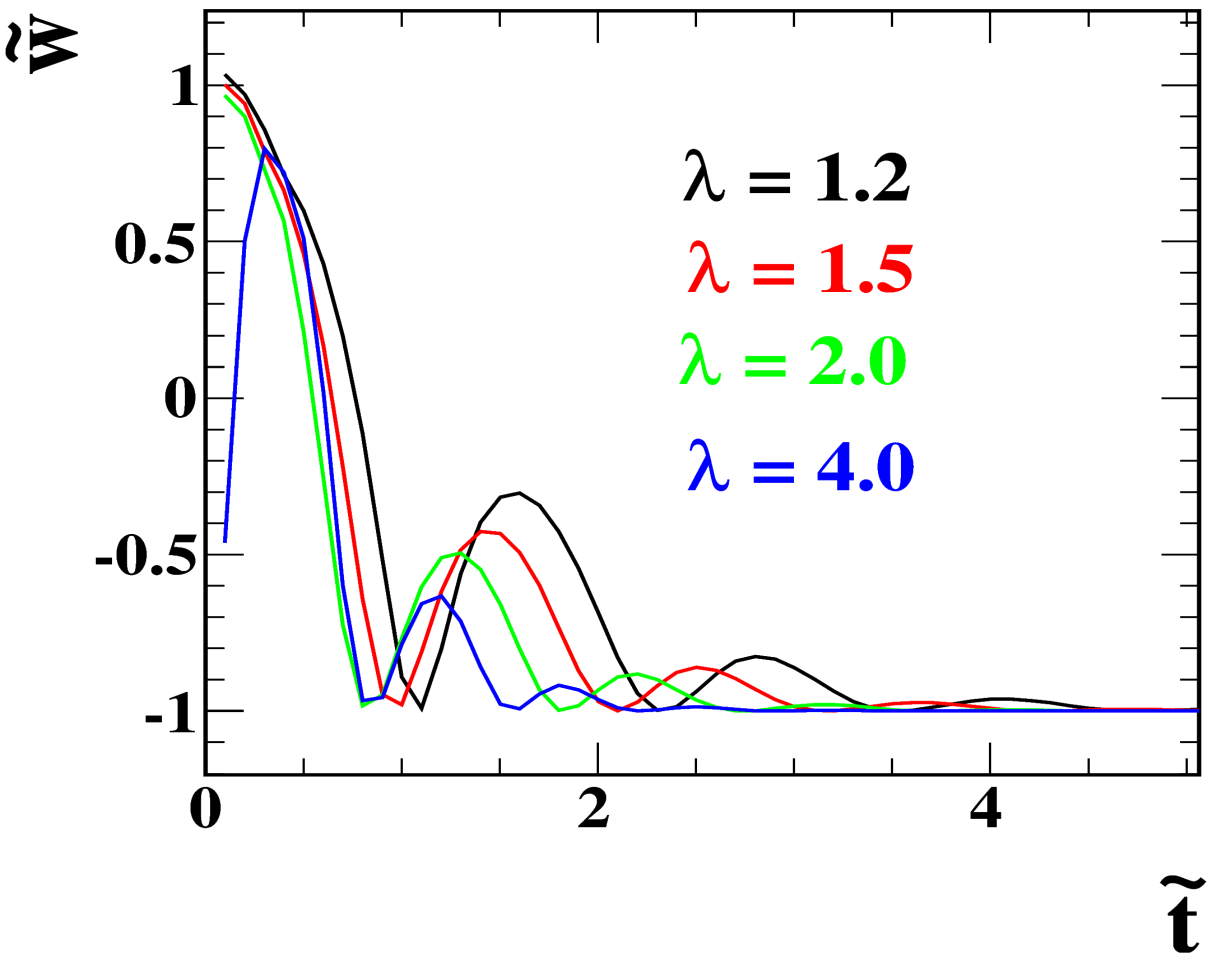
ã(t̃)
50
40
30
20
10
0
0
2
4
6
8
10
t̃
n = 1.0
n = 2.0

Observations

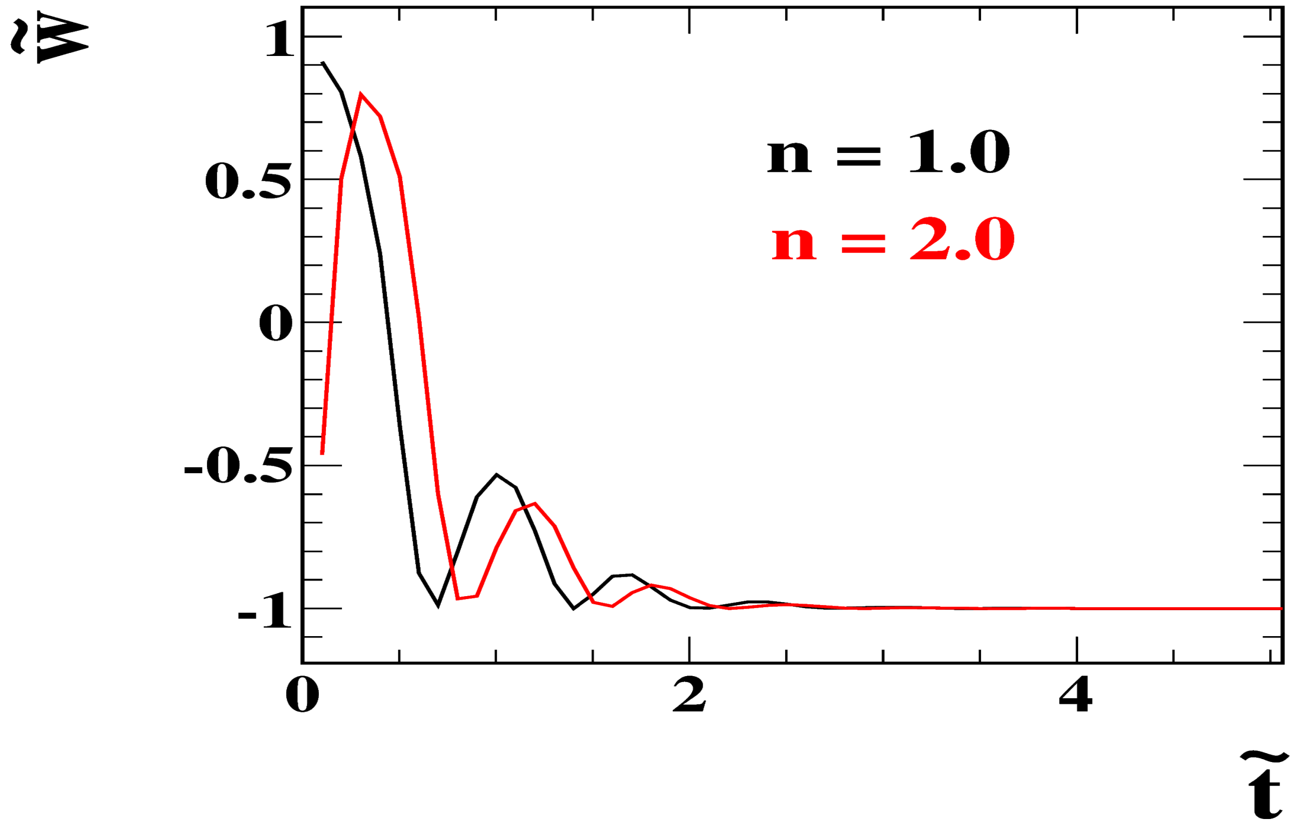
- During a considerable initial period the expansion of the universe is slower and after that period the Universe experiences very fast accelerated expansion.
- The initial period becomes shorter as the value of λ increases.
- The higher the value of n and λ, the earlier the period of accelerated expansion of the universe.



w̃
1
0.5
0
-0.5
-1
0
2
4
t̃
λ = 1.2
λ = 1.5
λ = 2.0
λ = 4.0



w̃
1
0.5
0
-0.5
-1
0
2
4
t̃
λ = 1.2
λ = 1.5
λ = 2.0
λ = 4.0



w̃
1
0.5
0
-0.5
-1
0
2
4
t̃
n = 1.0
n = 2.0

Observations

- The equation of state oscillate with decreasing amplitude during a period after falling from its initial positive value .
- After that period it remains steady with time.
- During a initial period depending on the value of λ and n the equation of state falls from 1 to -1.
- The scalar field make a transition from free scalar field through matter and dark energy to cosmological constant.
- For n=2 and λ=4.0,the equation of state initially make transit from the dark energy state to other state and then make transition to cosmological constant state.

Conclusions

- Field potential shows a peculiar behaviour with respect to field(Φ).
- The λ- pattern gradually vanishes with increasing values of λ and n.
- The unusual behaviour of the field potential around the zero value of Φ is the exhibition of the kind of singularity behaviour of the field.(however this singularity problem can be cured by adding a term proportional to R^2)

Conclusions

- The magnitued of the field is smaller for higher values of λ and n in comparision to the field of their smaller values.
- Time evolution of scale factor in Einstein frame shows that the Universe is accelerating very fast.
- This model with the suitable model parameters λ and n is very effective to give the late time accelerated expansion of the universe.

Conclusions

- The equation of state oscillates with a decreasing amplitude with time for a certain period after falling from its maximum value around 1 to its minimum value at -1.
- The scalar field related with the Starobinsky disappearing cosmological constant f(R) gravity model passages through different phases before finally behaves as the cosmological constant.

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Thank you