Generation of large scale magnetic fields driven by cosmic neutrinos

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Outline

- Introduction
- Calculation of the antisymmetric contribution to the photon polarization tensor, i.e. Chern-Simons (CS) parameter, in electroweak plasma
- CS parameter in relativistic plasmas
- CS term from the exact solution of Dirac equation
- Magnetic field evolution in the early universe
- Generation of strong magnetic fields in magnetars
- Discussion

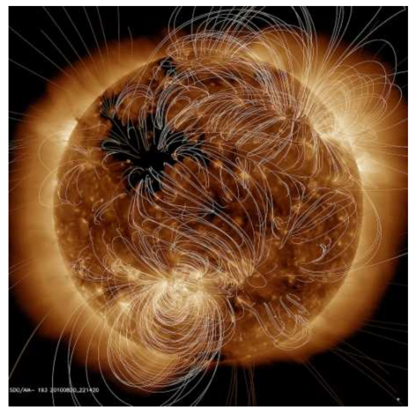
References

- M. Dvornikov and V.B. Semikoz, *Instability of magnetic fields in electroweak plasma driven by neutrino asymmetries*, JCAP 05 (2014) 002, arXiv:1311.5267 [hep-ph].
- M. Dvornikov, *Impossibility of the strong magnetic fields generation in an electron-positron plasma*, Phys. Rev. D 90, 041702 (2014), arXiv:1405.3059 [hep-ph].
- M. Dvornikov and V.B. Semikoz, *Magnetic field instability in a neutron star driven by electroweak electron-nucleon interaction versus chiral magnetic effect*, arXiv:1410.6676 [astro-ph.HE].

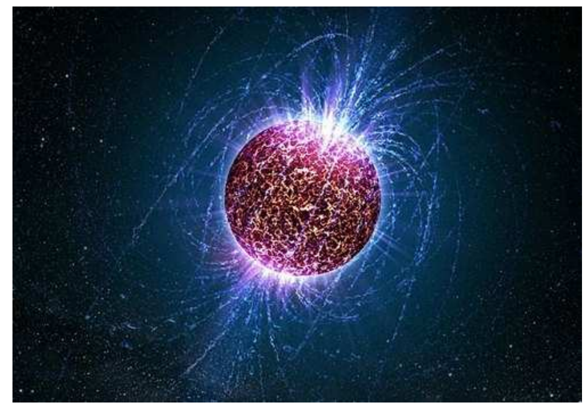
Cosmic magnetic fields: Overview

(1950's – 1960's) – Alfven, Biermann, Chandrasekhar, Parker etc – first studies on large-scale cosmic magnetic fields

Galactic and intergalactic magnetic fields ~ 1 μG



Stellar magnetic fields ~ (1 – 100) G



Magnetic fields in neutron stars (magnetars) > 10^{15} G

Recent reviews:

- Durrer & Neronov 2013 Astron. Astrophys. Rev. **21** 62 – 70 (arXiv:1303.7121)
- Charbonneau 2010 Living Rev. Solar Phys. **7** 3 – 91
- Mereghetti 2008 Astron. Astrophys. Rev. **15** 225 – 287 (arXiv:0804.0250)

Photon dispersion in background matter

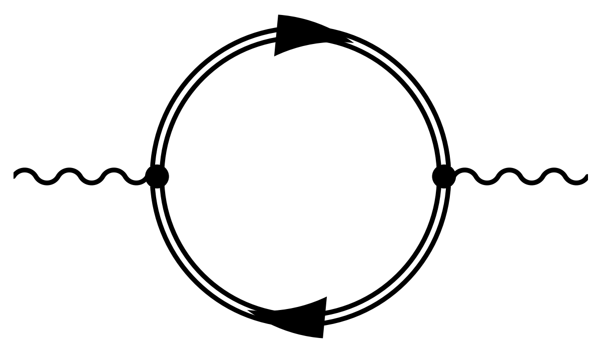
Background matter with parity violating interaction between particles

$$\Pi_{\mu\nu}(k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Pi_T + \frac{k_\mu k_\nu}{k^2} \Pi_L + i\varepsilon_{\mu\nu\alpha\beta} k^\alpha \left(V_L^\beta - V_R^\beta \right) \Pi_P$$

Π_T – vacuum polarization in QED
Π_L – plasma frequency or plasmon mass in QED plasma
$\Pi_2 = (V^0_L - V^0_R)\, \Pi_P$ – new form factor in parity violating isotropic medium

$$\Pi_{\mu\nu} = ie^2 \int \frac{d^4 p}{(2\pi)^4} tr \left\{ \gamma_\mu S_0(p+k) \gamma_\nu S_1(p) + \gamma_\nu S_0(p) \gamma_\mu S_1(p+k) \right\}$$

$$S(p) = S_0(p) + S_1(p) + \cdots$$



Electron propagator in presence of a neutrino gas

Dirac equation for an electron weakly interacting with a neutrino-antineutrino gas

$$\left[i\gamma^\mu\partial_\mu - m - \gamma_\mu\left(V_L^\mu P_L + V_R^\mu P_R\right)\right]\psi = 0$$

Effective potentials for left and right electrons

$$V_L = V_L^0 = 2\sqrt{2}G_F\left[\Delta n_{\nu_e} + \left(\sin^2\theta_W - \frac{1}{2}\right)\sum_\alpha \Delta n_{\nu_\alpha}\right]$$

$$V_R = V_R^0 = 2\sqrt{2}G_F \sin^2\theta_W \sum_\alpha \Delta n_{\nu_\alpha}$$

Neutrino and antineutrino densities

$$n_{\nu,\bar{\nu}} = \int \frac{d^3p}{(2\pi)^3}\left[\exp\left(\frac{|\mathbf{p}| \mp \mu_\nu}{T}\right) + 1\right]^{-1}$$

$$\left[\gamma^\mu p_\mu - m - \gamma_\mu\left(V_L^\mu P_L + V_R^\mu P_R\right)\right]S(p) = 1$$

Wave equation for the electron propagator

Series expansion of the propagator

$$S_0 = \frac{\gamma^\mu p_\mu + m}{p^2 - m^2}$$

$$S_1 = \frac{1}{p^2 - m^2}\left[\frac{i\sigma_{\alpha\beta}\gamma^5 p^\alpha\left(V_L^\beta - V_R^\beta\right)\left(\gamma^\mu p_\mu + m\right)}{p^2 - m^2} + \frac{1}{2}\gamma_\mu\gamma^5\left(V_L^\mu - V_R^\mu\right)\right]$$

Calculation of the Chern-Simons parameter

We expand $\Pi_2 = \Pi_2^{(\nu)} + \Pi_2^{(\nu e)}$, where $\Pi_2^{(\nu)}$ corresponds to only neutrino gas, i.e. electrons are virtual particles, and $\Pi_2^{(\nu e)}$ is the electron plasma contribution

Standard QFT methods

$$\Pi_2^{(\nu)} = -2\alpha_{em} V_5 \frac{k^2}{m^2}\int_0^1 dx \frac{x(1-x)}{1-\frac{k^2}{m^2}x(1-x)} \qquad V_5 = \frac{1}{2}(V_L - V_R)$$

Imaginary time perturbation theory (summation over Matsubara frequencies)

$$i\int \frac{dp_0}{2\pi} \to T\sum_n \ , \ p_0 = (2n+1)\pi T i + \mu, \ n = 0, \pm 1, \pm 2, \ldots$$

$$\Pi_2^{(\nu e)} = -V_5 e^2 \int_0^1 dx \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\mathcal{E}_{\mathbf{p}}^3}$$

$$\times \left\{ I_0^+ - (1-x)\left[\frac{1}{\mathcal{E}_{\mathbf{p}}^2}\left(\mathbf{p}^2[3-5x] - 3x\left[k^2 x(1-x) + m^2\right]\right)\left(J_0^+ + J_0^-\right) - \frac{\beta k_0}{2}\left(J_1^+ - J_1^-\right) - x\left(J_2^+ + J_2^-\right)\right]\right\}$$

$$I_0^+ = \frac{1}{\exp\left[\beta\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]+1} + \frac{\beta\mathcal{E}_{\mathbf{p}}}{2}\frac{1}{\cosh\left[\beta\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]+1} + (\mu_+ \to -\mu_+)$$

$$J_1^+ = \frac{1 + \beta\mathcal{E}_{\mathbf{p}} \tanh\left[\frac{\beta}{2}\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]}{\cosh\left[\beta\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]+1} - (\mu_+ \to -\mu_+)$$

$$J_0^+ = \frac{1}{\exp\left[\beta\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]+1} + \frac{\beta\mathcal{E}_{\mathbf{p}}}{2}\frac{1 + \frac{\beta\mathcal{E}_{\mathbf{p}}}{3}\tanh\left[\frac{\beta}{2}\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]}{\cosh\left[\beta\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]+1} + (\mu_+ \to -\mu_+)$$

$$J_2^+ = \frac{1}{\exp\left[\beta\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]+1} + \frac{\beta\mathcal{E}_{\mathbf{p}}}{2}\frac{1 - \beta\mathcal{E}_{\mathbf{p}}\tanh\left[\frac{\beta}{2}\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]}{\cosh\left[\beta\left(\mathcal{E}_{\mathbf{p}} + \mu_+\right)\right]+1} + (\mu_+ \to -\mu_+)$$

Gell-Mann theorem

- Gell-Mann (1961) showed that the $\nu\gamma$ interaction is vanishing in the first order in G_F
- Abbasabadi & Repko (2001) demonstrated that the $\nu\gamma$ interaction can be nonvanishing in two loops, i.e. $\Pi_2^{(\nu)} \sim \alpha_{em}\, G_F \,/\, M_W^{\,4}$
- We get that $\Pi_2^{(\nu)} = 0$ at $k^2 = 0$
- Our result is in agreement with Coleman & Glashow (1999)
- Note that Jackiw & Kostelecky (1999) claimed that the value of $\Pi_2^{(\nu)}$ can depend on the regularization scheme used

Chern-Simons parameter in relativistic plasmas

In presence of the antisymmetric part of the polarization tensor, the Lagrangian of the electromagnetic field acquires a term $L_{CS} = \Pi_2(\mathbf{AB})$. Such a contribution is called a Chern-Simons term

We shall calculate CS term for relativistic electrons: (i) hot plasma of the early universe and (ii) degenerate matter of a neutron star

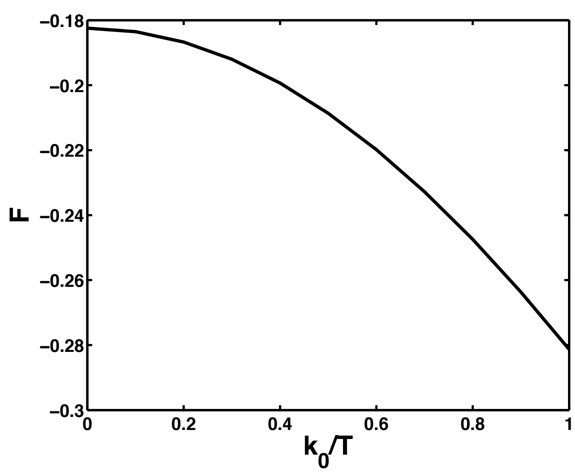
Dispersion relation for long waves in these media (Braaten & Segel, 1993)

$$k^2 = \omega_p^2 = \begin{cases} \frac{4\pi}{9}\alpha_{em}T^2, \text{hot relativistic plasma having } T \gg (m,\mu) \\ \frac{4}{3\pi}\alpha_{em}\mu^2, \text{degenerate relativistic plasma with } \mu \gg (m,T) \end{cases}$$

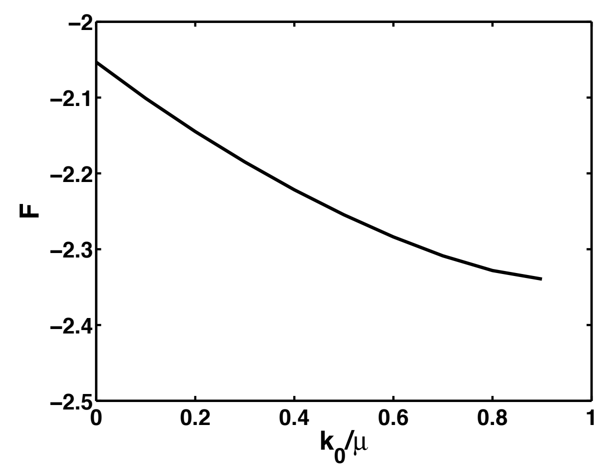
$$m^2 \to m_{eff}^2 = \frac{e^2}{8\pi^2}\left(\mu^2 + \pi^2 T^2\right)$$

Effective electron mass (Weldon, 1982)

$$\Pi_2 = \frac{2\alpha_{em}}{\pi} V_5 F$$



(a)



(b)

More elegant way to derive CS term: Exact solution of Dirac equation

Dirac equation for a massless electron in external magnetic field $A^{\mu} = (0,0,Bx,0)$

$$\left[\gamma^{\mu}\left(i\partial_{\mu} + eA_{\mu}\right) - \gamma^{0}\left(V_L P_L + V_R P_R\right)\right]\psi = 0$$

$$\psi = \psi_L + \psi_R\,,\;\; P_{L,R}\psi_{L,R} = \psi_{L,R}$$

$$\psi_{L,R} = \exp\left(-iE_{L,R}t + ip_y y + ip_z z\right)\psi_{L,R}(x)$$

Solution of this equation

Basis spinors

$$\psi_{L,R}^{(n>0)}(x) = \frac{1}{4\pi\sqrt{E_{L,R} - V_{L,R}}}\begin{pmatrix} \sqrt{E_{L,R} - V_{L,R} \mp p_z}\,u_{n-1} \\ \mp i\sqrt{E_{L,R} - V_{L,R} \pm p_z}\,u_n \\ \mp\sqrt{E_{L,R} - V_{L,R} \mp p_z}\,u_{n-1} \\ i\sqrt{E_{L,R} - V_{L,R} \pm p_z}\,u_n \end{pmatrix},\;\; \psi_{L,R}^{(0)}(x) = \frac{1}{2\pi\sqrt{2}}\begin{pmatrix} 0 \\ u_0 \\ 0 \\ \mp u_0 \end{pmatrix}$$

$$\left(E_{L,R} - V_{L,R}\right)^2 = p_z^2 + 2eBn$$

Energy levels

Here $u_n(\eta)$ is the Hermite function and

$$\eta = \sqrt{eB}\,x + \frac{p_y}{\sqrt{eB}}$$

Induced current in external magnetic field

- At zero Landau level n = 0 electrons are polarized against magnetic field
- Massless particles have a strong correlation between their momentum and helicity
- Left electrons have $p_z > 0$, whereas right ones have $p_z < 0$
- There is a nonzero current along the magnetic field direction

$$J_z^{(L,R)} = -e\sum_{n=0}^{\infty}\int_{-\infty}^{+\infty} dp_y \int dp_z \frac{\bar{\psi}\gamma^3\psi}{\exp\left[(E_{L,R}-\mu_{L,R})/T\right]+1} + \text{positron contribution}$$

Only zero Landau level contributes to the current

$$\mathbf{J} = \frac{2\alpha_{em}}{\pi}\left(\mu_5 + V_5\right)\mathbf{B} \qquad \mu_5 = \frac{1}{2}\left(\mu_R - \mu_L\right)$$

The first term μ_5 reproduces the chital magnetic effect (e.g., Kharzeev arXiv:1501.01336); the second one V_5 is the correction due electroweak interaction

Generalized MHD

In the presence of the CS term the Maxwell equations are modified

$$i\left(\mathbf{k}\times\mathbf{B}\right)=-i\omega\mathbf{E}+\mathbf{j}+\mathbf{j}_5,\; i\left(\mathbf{k}\times\mathbf{E}\right)=i\omega\mathbf{B},\; \left(\mathbf{k}\cdot\mathbf{B}\right)=0,$$

$$\mathbf{j}=\sigma\mathbf{E},\; \mathbf{j}_5=\Pi_2\mathbf{B}$$

MHD approximation $\sigma\gg\omega$

One gets the Faraday equation for the large scale magnetic field evolution

$$\frac{\partial\mathbf{B}}{\partial t}=\alpha\left(\nabla\times\mathbf{B}\right)+\eta\nabla^2\mathbf{B},\; \alpha=\frac{\Pi_2}{\sigma},\; \Pi_2\equiv\Pi_2(k_0=0),\; \eta=\frac{1}{\sigma}$$

The Faraday equation has the unstable solution

$$B(k,t)=B_0\exp\left[\int_{t_0}^{t}\left(|\alpha|k-\eta k^2\right)dt'\right]$$

If $k<|\alpha|/\eta$, this solution describes the exponential growth of a seed magnetic field B_0

Magnetic fields in the early universe

Galactic and intergalactic magnetic fields are supposed to evolve from a seed field, which can be of the cosmological origin

Recently Bonvin et al. (2014) claimed that there is an the indication on the existence of the inflationary magnetic field basing on the analysis of BICEP2 data

Various models of the cosmological magnetic fields generation are reviewed by Durrer & Neronov (2013)

In causal scenario the length scale should be less than the horizon

$$\Lambda_B \sim \frac{\eta}{|\alpha|} < l_H = H^{-1}, \ \alpha = \frac{\Pi_2}{\sigma}, \ \sigma \approx 10^2 T$$

$$\xi_\nu \sim \left|\xi_{\nu_e} - \xi_{\nu_e} - \xi_{\nu_e}\right| > \frac{1.1\times 10^{-6}\sqrt{g^*/106.75}}{(T/\mathrm{MeV})}$$

The lower bound on the neutrino asymmetries

$$\xi_\nu = \frac{\mu_\nu}{T} = \frac{6\Delta n_\nu}{T^3}$$

The obtained lower bound is consistent with the well known Big Bang nucleosynthesis constraint: $\xi_\nu < 10^{-1} - 10^{-2}$ (Dolgov et al. (2002); Mangano et al. (2012))

Magnetic fields in magnetars

Magnetic fields of pulsars $B_{NS} = 10^{12}$ G can be explained by the conservation of the magnetic flux $B_{NS} = B_{proto}(R_{proto}/R_{NS})^2$, where $B_{SUN} = (1 - 10^2)$ G is the magnetic fields of a protostar, $R_{proto} = R_{SUN}$ is the protostar raduis, and $R_{NS} = 10$ km is the pulsar raduis

Some NSs, called magnetars, have magnetic fields > 10^{15} G (Mereghetti, 2008), which is at least 3 orders of magnitude greater than B_{NS}

Even a popular model of Duncan and Thompson (1992), based on the turbulent dynamo, for the generation of magnetic fields in magnetars, has some tensions with observational data

During ~ 1 ms after the onset of the SN explosion, the regions outside the neutrinosphere deleptonize. The lepton number is carried away by ν_e, leading to the electron neutrino burst with $L_\nu = 10^{52}$ erg/s. Thus Δn_ν is nonvanishing

The magnetic diffusion time is

$$t_{diff} = \frac{\sigma}{\Pi_2^2} \approx 2.3\times10^{-2}\ \mathrm{s},\ \sigma = \frac{1.6\times10^{28}}{(T/10^8\ \mathrm{K})}\left(\frac{n_e}{10^{36}\ \mathrm{cm}^{-3}}\right)^{3/2}\mathrm{s}^{-1},\ \Pi_2 = \frac{\sqrt{2}}{\pi}\alpha_{em}G_F n_{\nu_e}F(0)$$

The disadvantage of the model for magnetic field growth driven by the neutrino asymmetry is that the magnetic field scale is small $\Lambda_B \sim 10^{-3}$ cm. However, at the subsequent moments of the PNS evolution, when $\Delta n_\nu = 5 \times 10^{27}\ \mathrm{cm}^{-3}$, Λ_B can be ~ 10 km

Discussion: Calculation of the Chern-Simons term

- We calculated the Chern-Simons parameter in electroweak plasma consisting of e^-e^+ and neutrino-antineutrino gas
- Our result exactly accounts for the plasma temperature and chemical potential
- We take into account the radiative corrections to the electron mass and the plasmon dispersion law
- Our calculation is consistent with the Gell-Mann theorem
- We considered particular cases of high temperature and degenerate relativistic plasmas. In both cases a nonzero Π_2 in the static limit $\omega=0$ was obtained

Discussion: Magnetic fields in the early universe

- We applied our results for the description of the magnetic field enhancement in a hot primordial plasma driven by neutrino asymmetries
- Boyarsky et al. (2012) suggested that a primordial magnetic field can grow owing to the chiral magnetic effect with nonzero $\mu_5 = (\mu_R - \mu_L)/2$. However, $\mu_5 \to 0$ in the cooling universe because of the ee collisions
- On the contrary, CS parameter, based on the parity violating interaction, is proportional to Δn_ν and becomes constant at the neutrino decoupling time corresponding to $T = (2 - 3)$ MeV
- Assuming the causal scenario, i.e. the magnetic field scale being less than the horizon, we obtained the lower bound on the neutrino asymmetries which is consistent with the Big Bang nucleosynthesis constraint

Discussion: Magnetic fields in magnetars

- We proposed a new model for the generation of strong magnetic fields in magnetars driven by neutrino asymmetries
- Short scale ~ 10^{-3} cm magnetic fields are generated in ~ 10^{-3} s during a neutrino burst
- The generation of large scale ~ 10 km magnetic fields is potentially possible
- The model by Ohnishi & Yamamoto (2014), based on chiral magnetic effect, is invalid since the chirality flip was underestimated (Grabowska et al., 2014; Dvornikov & Semikoz, 2014)
- The suggestion of Boyarsky et al. (2012) that a magnetic field in magnetars can be created in a self-interacting e^-e^+ plasma is incorrect since Dvornikov (2014) showed that $\Pi_2(\omega=0) = 0$
- Recently Dvornikov & Semikoz in arXiv:1410.6676 proposed a different model based on electron-nucleon parity violating interaction. This model can explain some the features of magnetic fields in magnetars

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