Cosmological bounces in spatially flat FRW spacetimes in metric $f(R)$ gravity

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Talk outline

- Brief introduction to cosmological bounce, $f(R)$ gravity.
- Motivations to discuss bounce in $f(R)$ gravity.
- Friedmann equations and bouncing conditions in $f(R)$ gravity.
- Analyzing a typical bouncing scenario in an $R + R^2$ gravity.
- Analyzing the evolution of the scalar perturbation through bounce in such a scenario.

Introductions

- Cosmological bounce is a paradigm proposed to avoid the singularity at the beginning of the universe.
    - Scale factor decreases,reaches a certain nonvanishing minimum,and then increase again.
    - $H_b = 0, \dot{H}_b > 0$.
- $f(R)$ theories are modified gravity theories which include corrections to GR for high or low values of R.
    - $f(R)$ action : $S = \frac{1}{2\kappa}\int d^4x\sqrt{-g}f(R) + S_m$
    - $G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{\kappa}{f'(R)}(T_{\mu\nu} + g_{\mu\nu}\frac{f(R)-Rf'(R)}{2\kappa} + \frac{\nabla_\mu\nabla_\nu f'(R) - g_{\mu\nu}\Box f'(R)}{\kappa})$
    - $f'(R) > 0$ for positive gravitational coupling
    - Unlike GR, here $T = 0 \nRightarrow R = 0$(in general); **A hidden d.o.f. is in play**!

Motivations and main focus

- GR : Bounce possible only for k=+1 Friedmann universe. $f(R)$: Bounce possible for both k=+1,0.
- At early times R was high, so corrections to GR are likely.
- I will focus on $R + \alpha R^2$ gravity with $\alpha < 0$.
- I will resort to radiation background($\omega = \frac{1}{3}$) and flat spatial section($k = 0$ Friedmann universe).

The metric and the equations

- Maximally symmetric FLRW spacetime
  $ds^2 = -dt^2 + a^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2]$

- For an ideal fluid $T^{\mu}_{\nu} = diag(-\rho, p, p, p)$, FLRW equations for $k = 0$:
  - $3H^2 = \frac{\kappa}{f'(R)}(\rho + \rho_{eff})$, $\rho_{eff} \equiv \frac{Rf'-f}{2\kappa} - \frac{3H\dot{R}f''(R)}{\kappa}$
  - $2\dot{H} + 3H^2 = \frac{-\kappa}{f'(R)}(p + p_{eff})$, $P_{eff} \equiv \frac{\dot{R}^2 f''' + 2H\dot{R}f'' + \ddot{R}f''}{\kappa} - \frac{Rf'-f}{2\kappa}$

- $\rho_{eff}\,, p_{eff}$; Originates NOT from some other type of matter component, but from the **modified geometry of space-time itself**!!!

Bounce Conditions in f(R) gravity

- We assume validity of the **WEC** in the matter sector : $\rho_M \geq 0, \rho + p > 0$
- $\rho_b + \frac{R_b f_b' - f_b}{2\kappa} = 0 \qquad \text{(for } k = 0\text{)}$
- For $k = 0$ **both matter bounce and matterless bounce is possible** depending on the form of $f(R)$.
- **Matterless bounce is possible iff** $(Rf' - f)$ **has a positive root**(e.g. $f(R) = R + \alpha R + \beta R^2; \alpha < 0, 0 < \alpha^2 < 3\beta$). Also then f''' is not identically zero.
- $f(R) = R + \alpha R^n$ (for any $n \geq 2$) : **Only matter bounce possible** and that too for $\alpha < 0$.

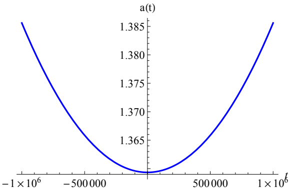
Einstein frame picture of $f(R)$ gravity

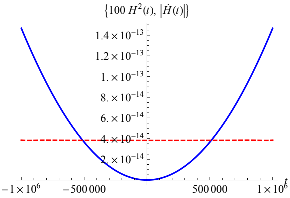
- $\tilde{g}_{\mu\nu} = f'(R) g_{\mu\nu}$; $\varphi = \sqrt{\frac{3}{2\kappa}} \ln f'(R)$; $V(\varphi) = \frac{Rf' - f}{2\kappa f'^2}$
    - The extra d.o.f. recast as a **scalar field directly coupled to matter**.
    - $\tilde{t} = \int \sqrt{f'} dt$, $\tilde{a} = \sqrt{f'} a$
      $\tilde{\rho} = \frac{\rho}{f'^2}$, $\tilde{p} = \frac{p}{f'^2}$
- In Einstein frame the theory becomes **GR with the matter field and the scalar field**.
- The dynamical equations are usual **GR Friedmann equations** and the **KG equation** for the scalr field.

Solving for bounce in Einstein frame

- Dynamical equations :
  - $\varphi^{''} + 3\tilde{H}\varphi^{'} + V_{,\varphi} = \sqrt{\frac{\kappa}{6}}(1-3\omega)\tilde{\rho}$
  - $\tilde{\rho}^{'} + \sqrt{\frac{\kappa}{6}}(1-3\omega)\varphi^{'}\tilde{\rho} + 3\tilde{H}\tilde{\rho}(1+\omega) = 0$
  - $\tilde{H}^{'} = \frac{k}{\tilde{a}^2} - \frac{\kappa}{2}(\varphi^{'2} + \tilde{\rho}(1+\omega))$
- Equations for initial conditions :
  - $H = \sqrt{F}(\tilde{H} - \sqrt{\frac{\kappa}{6}}\varphi^{'}) \longrightarrow \tilde{H}_b = \sqrt{\frac{\kappa}{6}}\varphi^{'}_b$
  - $\tilde{H}^2 = \frac{\kappa}{3}(\frac{1}{2}\varphi^{'2} + V(\varphi) + \tilde{\rho}) \longrightarrow \tilde{\rho}_b = -V(\varphi)_b$
- To solve the system for $k = 0$, we need to put by hand only $\varphi_b, \varphi^{\prime}_b$.

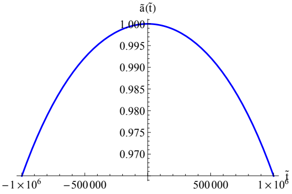
A typical symmetric bounce for $f(R) = R + \alpha R^2$, $(\alpha = -10^{12}$ in Planck units) ,$k = 0$,$\omega = \frac{1}{3}$: Jordan frame

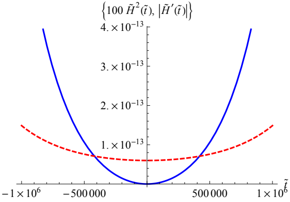




- **Bounce** in Jordan frame.
- The era before and after the bounce can be approximated by a '**deflationary**' and '**inflationary**' era.
- A comparison: $\alpha > 0$(Starobinsky model)$\Rightarrow$ Vacuum dominated ; $\alpha < 0 \Rightarrow$ Matter driven inflation

A typical symmetric bounce for $f(R) = R + \alpha R^2$, ($\alpha = -10^{12}$ in Planck units) ,$k = 0$,$\omega = \frac{1}{3}$: Einstein frame





- **No bounce** in the Einstein frame!
- For $k = 0$, considering there is a bounce in the Jordan frame, there **can never be an analogous bounce in the Einstein frame**.
- Interestingly, for $k = +1$, **there can be simultaneous bounce** in both frames iff $\dot{F}(t = 0) = 0$, $\ddot{F}(t = 0) > 0$.

Scalar metric perturbations for $k = 0$

- We use **conformal time** η : $d\eta = dt/a = d\tilde{t}/\tilde{a}$

- Scalar perturbed FRW metric in Jordan frame
$ds^2 = a^2(\eta)[-(1+2\Phi)d\eta + (1-2\Psi)\delta_{ij}dx^i dx^j]$
  - Φ, Ψ **: 2 gauge invariant Bardeen potentials in Jordan frame.**

- Scalar perturbed FRW metric in Einstein frame
$d\tilde{s}^2 = \tilde{a}^2(\eta)[-(1+2\tilde{\Phi})d\eta + (1-2\tilde{\Phi})\delta_{ij}dx^i dx^j]$
  - $\tilde{\Phi}$ **: 1 gauge invariant Bardeen potential in Einstein frame.**

- $\Phi = -\frac{2}{3}(F^2/F'a)[(a/F)\tilde{\Phi}]'$
$\Psi = \frac{2}{3}(1/FF'a)(aF^2\tilde{\Phi})'$

Perturbation equations for $k = 0$ in the Einstein frame

- Perturbation equation in GR, but for **both scalar field and hydrodynamic matter** together!!

- $c_s^2(\tilde{\Phi}'' - \nabla^2\tilde{\Phi}) + [2c_s^2(\tilde{\mathcal{H}} - \frac{\varphi_0''}{\varphi_0'}) + \frac{\tilde{a}^2}{\varphi_0'}\sqrt{\frac{\kappa}{6}}\tilde{\rho}_0(1+c_s^2)(1-3c_s^2)]\tilde{\Phi}' +$ $[2(\tilde{\mathcal{H}}' - \tilde{\mathcal{H}}\frac{\varphi_0''}{\varphi_0'})c_s^2 + \tilde{a}^2\tilde{\rho}_0(1+c_s^2)\sqrt{\frac{\kappa}{6}}\frac{\tilde{\mathcal{H}}}{\varphi_0'}(1-3c_s^2) - \frac{\kappa}{2}(1-c_s^2)]\tilde{\Phi} = 0$

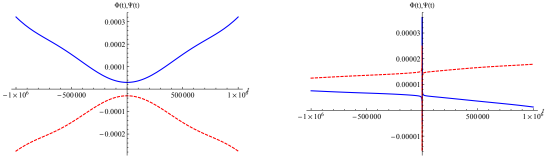
- For background comprised of **only one type of hydrodynamic matter, adiabaticity of the perturbations are preserved** from Jordan frame to Einstein frame.

- The equation involves the term $\frac{\varphi''}{\varphi'}$; $\phi'(0) = 0$; **Is the equation singular at $\eta = 0$?? ; Nope!** it is a removable singularity!! So the equation is well defined throughout.

Some special cases of the perturbation equation

- No matter :
  - $\tilde{\Phi}'' - \nabla^2\tilde{\Phi} + 2\left(\tilde{\mathcal{H}} - \frac{\varphi_0''}{\varphi_0'}\right)\tilde{\Phi}' + \left[2\left(\tilde{\mathcal{H}}' - \tilde{\mathcal{H}}\frac{\varphi_0''}{\varphi_0'}\right)\right]\tilde{\Phi} = 0$
  - Usual perturbation equation in presence of a scalar field
- $\omega = \frac{1}{3}$
  - $\tilde{\Phi}'' - \nabla^2\tilde{\Phi} + 2(\tilde{\mathcal{H}} - \frac{\varphi_0''}{\varphi_0'})\tilde{\Phi}' + [2(\tilde{\mathcal{H}}' - \tilde{\mathcal{H}}\frac{\varphi_0''}{\varphi_0'}) - \frac{4\kappa}{3}\tilde{a}^2\tilde{\rho}_0]\tilde{\Phi} = 0$
  - Only the coefficient of the $0th$ order term modified by a single term
- $c_s^2 = \omega = 0$
  - $\tilde{\Phi}' + \left(\tilde{\mathcal{H}} - \varphi_0'\sqrt{\frac{3}{2\kappa}}\right)\tilde{\Phi} = 0$
  - No info about the matter content!!
  - Usually one takes $c_s^2 \longrightarrow 0$ but $\neq 0$

Scalar perturbation evolution for a typical symmetric bounce in $f(R) = R + \alpha R^2$, ($\alpha = -10^{12}$ in Planck units) ,$k = 0$,$\omega = \frac{1}{3}$: Jordan frame



- The perturbations are **very slowly varying with time.**
- The kink in the 2nd picture is NOT a physical divergence. It is an artefact of the relations $\Phi(\tilde{\Phi}), \Psi(\tilde{\Phi})$ becoming singular at $\eta = 0$. So **Einstein frame description is not well defined at the bounce point**.
- However, only in the special case when the **perturbation evolution is also symmetric, it poses no problem.**

Conclusions

- Some $f(R)$ theories allow **cosmic bounce for** $k = 0$. Simplest is $R + \alpha R^2$ $(\alpha < 0)$.
- These type of bounce models are **preceded by deflationary epoch and followed by inflationary epoch**.
- In **Jordan frame**, analysis of background scenario and perturbations are **pretty complicated**.
- So we adopt a **'bypass': Einstein frame**; the theory is essentially GR and we also have many results to guide us forward.

Conclusions

- We analyze background evolution and perturbations in Einstein frame using GR(which is, off course, easier!!). **The two picture might look very different! We then return to Jordan frame using well defined prescriptions**.
- **Einstein frame description is otherwise very helpful in describing the perturbation evolution, except only at the bounce point**. Iff both the background and the perturbation evolution is symmetric, it poses no problem.

THANK YOU