The Top Mass and Monte Carlos

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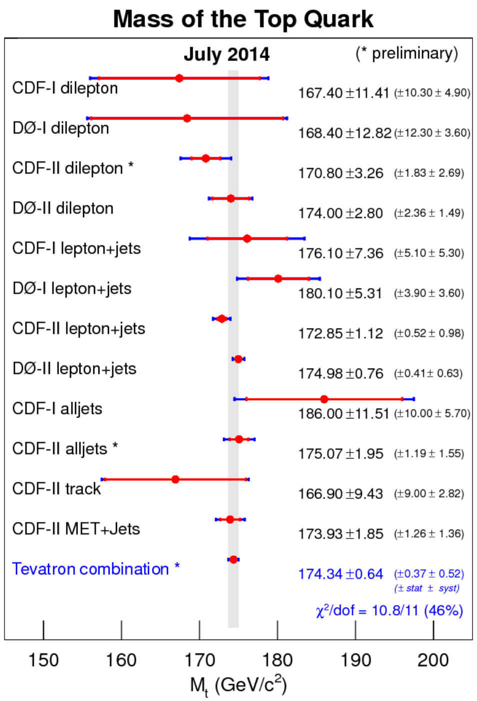
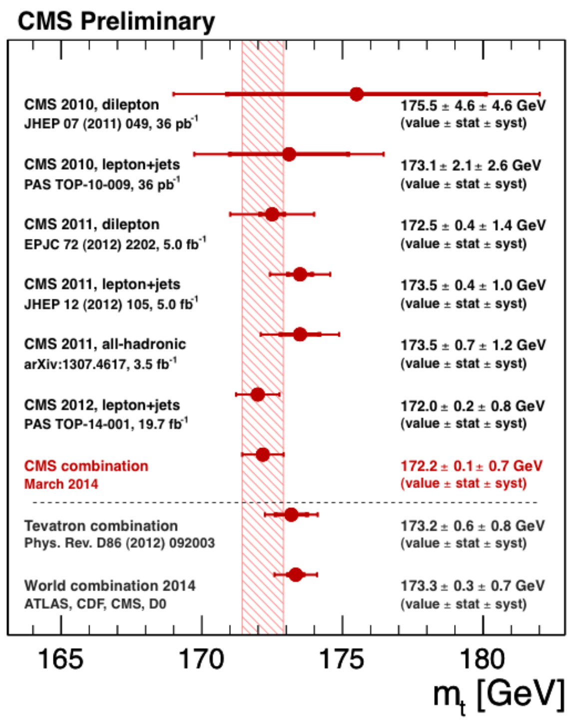
Motivation





→ Most precise mass from direct reconstruction: $m_t^{\mathrm{MC}} = 173.34 \pm 0.76\,\mathrm{GeV}$

→ m_t^{MC} cannot be used as direct input into NLO/NNLO calculations since it is not a field theoretic mass.

Outline

Part 1: $\rightarrow$ Theoretical thoughts on m_t^{MC}

- How m_t^{MC} is related to field theoretic masses.

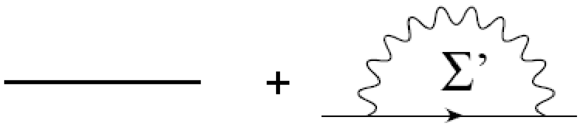
See: "The Top Mass: Interpretation and Theoretical Uncertainties", arXiv:1412.3649

Same conclusions: AH, Stewart: arXive:0808.0222

Part 2: $\rightarrow$ Towards a determination of m_t^{MC}

- Variable Flavor Number Scheme for final state jets.
  Full massive event shape distribution
- First encouraging preliminary results

Top Quark Mass



$$= p - m^0 - \Sigma(p, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \dots \right] + \Sigma^{\rm fin}(m^0, m^0, \mu)$$

$\overline{\text{MS}}$ scheme: $$m^0 = \overline{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \dots \right]$$

→ $\overline{m}(\mu)$ is pure UV-object without IR-sensitivity

→ Useful scheme for $\mu > m$

→ Far away from a kinematic mass of the quark

Pole scheme: $$m^0 = m^{\rm pole} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \dots \right] - \Sigma^{\rm fin}(m^{\rm pole}, m^{\rm pole}, \mu)$$

→ Absorbes all self energy corrections into the mass parameter

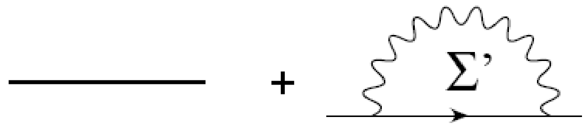
→ Close to the notion of the quark rest mass (kinematic mass)

→ Renormalon problem: infrared-sensitive contributions from < 1 GeV that cancel between self-energy and all other diagrams cannot cancel.

→ Has perturbative instabilities due to sensitivity to momenta < 1 GeV (Λ_{QCD})

Should not be used if uncertainties are below 1 GeV !

Heavy Quark Mass



$$= p\!\!\!/ - m^0 - \Sigma(p\!\!\!/, m^0, \mu)$$

$$\Sigma(m^0, m^0, \mu) = m^0 \left[\frac{\alpha_s}{\pi\epsilon} + \ldots \right] + \Sigma^{\rm fin}(m^0, m^0, \mu)$$

MS scheme: $m^0 = \overline{m}(\mu) \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right]$

Pole scheme: $m^0 = m^{\rm pole} \left[1 - \frac{\alpha_s}{\pi\epsilon} + \ldots \right] - \Sigma^{\rm fin}(m^{\rm pole}, m^{\rm pole}, \mu)$

MSR scheme: $m^{\rm MSR}(R) = m^{\rm pole} - \Sigma^{\rm fin}(R, R, \mu)$

Jain, AH, Scimemi, Stewart (2008)

→ Like pole mass, but self-energy correction from <R are not absorbed into mass

→ Interpolates between MSbar and pole mass scheme

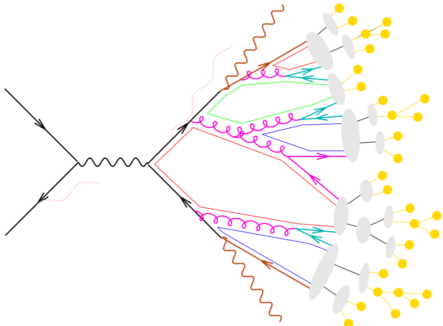
$$m_t^{\rm MSR}(R = 0) = m^{\rm pole}$$

$$m_t^{\rm MSR}(R = \overline{m}(\overline{m})) = \overline{m}(\overline{m})$$

→ More stable in perturbation theory.

→ $m_t^{\rm MSR}(R = 1\,{\rm GeV})$ close to the notion of a kinematic mass, but without renormalon problem.

Heavy Quark Mass in the MC



Monte-Carlo event generator:

- Hard matrix element:

  Initial parton annihilation and top production plus additional hard partons from pQCD.

- Parton shower evolution:

  Splitting into higher-multiplicity partonic states (plus top decay) with subsequently lower virtualities until shower cut Λ_s. NO top self-energy contributions.

  Splitting probabilities from pQCD (approx LL accuracy, soft-collinear limit).

  Can be viewed as a way to sum dominant perturbative corrections down to Λ_s = 1 GeV.

- Hadronization model:

  Turns partons into hadrons.

  Tune strongly dependent on parton shower implementation.

  Description of data (frequently) much better than the conceptual (LL) precision of parton evolution part.

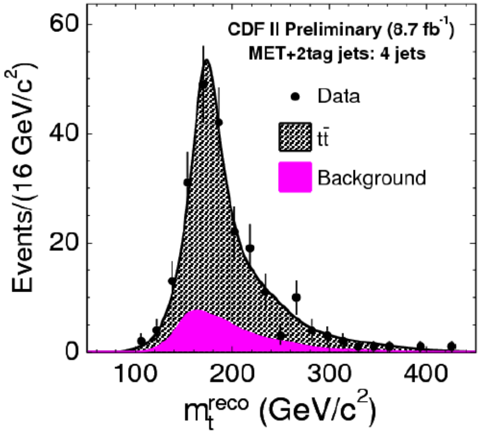
- MC mass:

  Mass of top propagator prior to top decay.

→ Interpretation of m_t^{MC} dependent on view whether MC is more model or or more first principles QCD.

Heavy Quark Mass in the MC

Let's take the reconstructed top invariant mass distribution as a concrete example to see how the MC components enter the templates and the MC mass fitting.



- Hard matrix element:

  Essentially only affects the norm

- MC mass:

  Determines overall location of mass range where distribution is peaked.

- Parton shower evolution + Hadronization model:

  Modify shape and distribution further.
  PS: perturbative part - self-energy contributions absorbed into mass above Λ_s
  HM: non-perturbative part below Λ_s

$$m_t^{\rm MC} = m_t^{\rm MSR}(R = 1\,{\rm GeV}) + \Delta_{t,{\rm MC}}(R = 1\,{\rm GeV})$$

$$\Delta_{t,{\rm MC}}(1\,{\rm GeV}) \simeq \mathcal{O}(1\,{\rm GeV})$$

Contains perturbative and non-perturbative contributions.
Conceptual reliability related to how precisely $\Delta_{t,{\rm MC}}$ can be determined.

Heavy Quark Mass in the MC

Analogy: Meson masses

$$m_B = m_b^{\rm MSR}(1\,{\rm GeV}) + \Delta_{b,B}(R = 1\,{\rm GeV})$$

$$\Delta_{b,B}(1\,{\rm GeV}) \simeq \mathcal{O}(1\,{\rm GeV})$$

Table 1. Some B mesons masses, MSR masses $m_b^{\rm MSR}(1\,{\rm GeV})$ and $m_b^{\rm MSR}(2\,{\rm GeV})$ from $m_b^{\rm 1S} = 4780 \pm 66\,{\rm MeV}$ [18], and corresponding values for $\Delta_{b,B}$. All in units of MeV, $\alpha_s(m_Z) = 0.1184$.

$m_b^{\rm MSR}(1\,{\rm GeV})$	$m_b^{\rm MSR}(2\,{\rm GeV})$	$m(B^0)$	$m(B^*)$	$m(B_1^0)$	$m(B_2^*)$
4795 ± 69	4571 ± 69	5279.58 ± 0.17	5325.2 ± 0.4	5724 ± 2	5743 ± 5
$\Delta_{b,B}(1\,{\rm GeV})$		485 ± 69	530 ± 69	929 ± 69	948 ± 69
	$\Delta_{b,B}(2\,{\rm GeV})$	709 ± 69	754 ± 69	1153 ± 69	1172 ± 69

Additional Comments

- Using NLO vs. LO matrix elements does not affect the interpretation of the MC mass
- Different parton evolution implies in principle a different MC mass.
- Relation of MC to MSR mass can be used to deal with mass dependent efficiencies for total cross section measurements.
- MC mass should be independent of the process and kinematic region used for fitting.

Theory Tools to Measure the MC mass

Part 2

The relation between MC mass and field theoretical mass can be made more precise by measuring the MC mass using a completely independent hadron level QCD prediction of a mass-dependent observable.

Need:

- Accurate analytic QCD predictions beyond LL/LO with full control over the quark mass dependence
- Theoretical description at the hadron level for comparison with MC at the hadron level

- Implementation of massive quarks into the SCET framework
- **VFNS for final state jets (with massive quarks)***

* In collaboration with: P. Pietrulewicz, V. Mateu, I. Jemos, S. Gritschacher

arXiv:1302.4743 (PRD 88, 034021 (2013))
arXiv:1309.6251 (PRD 89, 014035 (2013))
arXiv:1405.4860 (PRD 90 114001 (2014))
More to come …

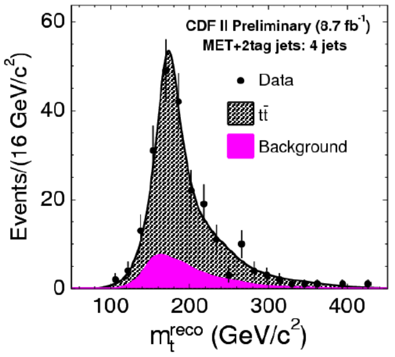
Theory Tools to Measure the MC mass

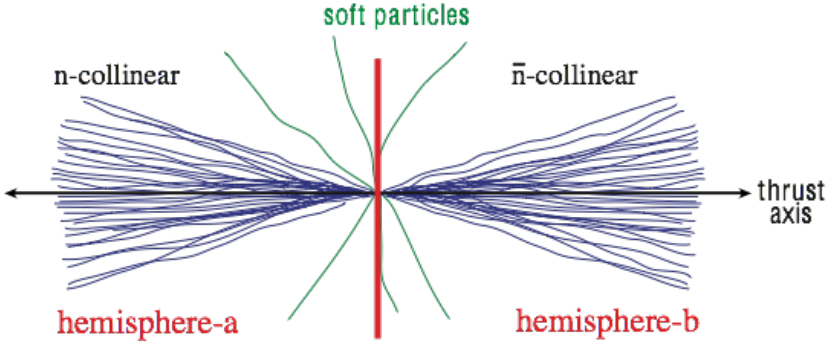
Observable: Thust in e+e-

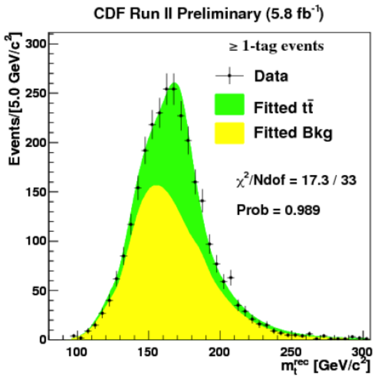
$$\tau \;=\; 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n}\cdot\vec{p}_i|}{Q}$$

$$\overset{\tau\to 0}{\approx} \;\; \frac{M_1^2 + M_2^2}{Q^2}$$

Invariant mass distribution in the resonance region !



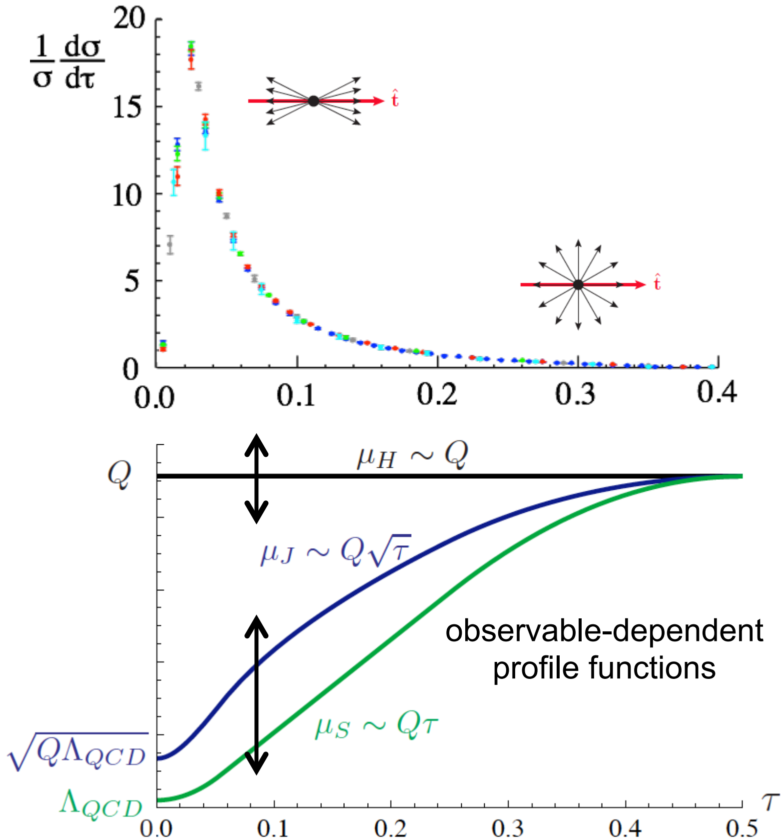




Factorization for Massless Quarks

$$\frac{d\sigma}{d\tau} = Q^2\sigma_0 H_0(Q,\mu) \int d\ell\, J_0(Q\ell,\mu)\, S_0\,(Q\tau-\ell,\mu)$$

Korshemski, Sterman
Schwartz
Fleming, AH, Mantry, Stewart
Bauer, Fleming, Lee, Sterman



Abbate, AH, Fickinger, Mateu, Stewart

$$\left(\frac{d\sigma}{d\tau}\right)^{\rm sing}_{\rm part} \sim \sigma_0\, H(Q,\mu_Q) U_H(Q,\mu_Q,\mu_s) \int d\ell d\ell'\, U_J(Q\tau-\ell-\ell',\mu_Q,\mu_s)\, J_T(Q\ell',\mu_j)\, S_T(\ell-\Delta,\mu_s)$$

VFN Scheme for Final State Jets

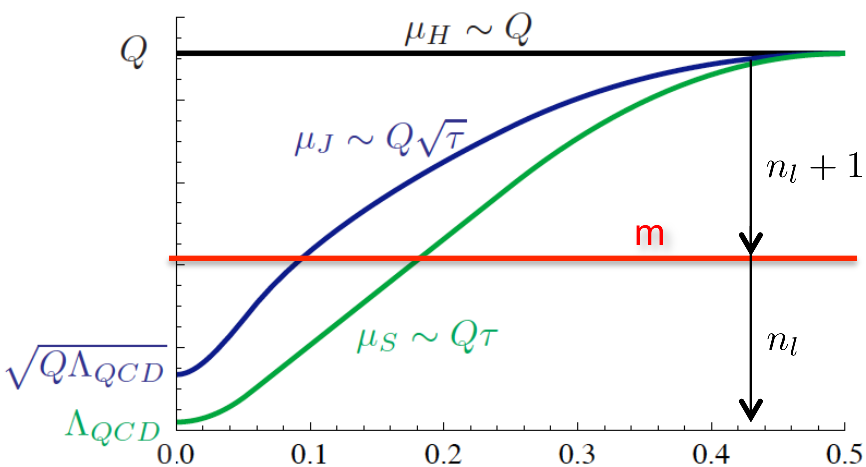
Gritschacher, AH,
Jemos, Pietrulewicz

→ consider: dijet in e^+e^- annihilation, n_l light quarks ⊕ one massive quark

→ obvious: (n_l+1)-evolution for $\mu \gtrsim m$ and (n_l)-evolution for $\mu \lesssim m$

→ obvious: different EFT scenarios w.r. to mass vs. Q – J – S scales

"profile functions"



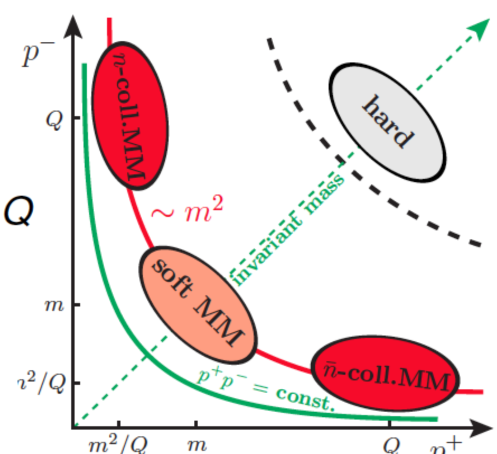
Aims:

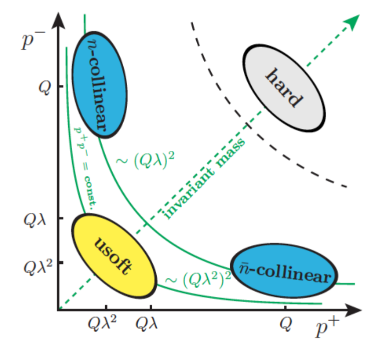
- Full mass dependence (little room for any strong hierarchies): decoupling, massless limit
- Smooth connections between different EFTs
- Determination of flavor matching for current-, jet- and soft-evolution
- Reconcile problem of $SCET_2$-type rapidity divergences

→ Deal with collinear and soft "mass modes"

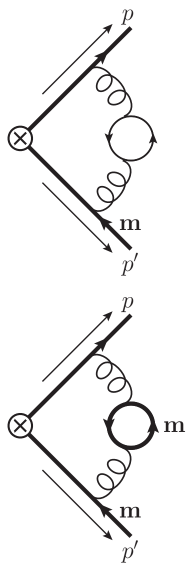
→ Additional power counting parameter $\lambda_m = m/Q$

mode	$p^\mu = (+, -, \perp)$	p^2
n-coll MM	$Q(\lambda_m^2, 1, \lambda_m)$	m^2
soft MM	$Q(\lambda_m, \lambda_m, \lambda_m)$	m^2





VFN Scheme: Primary Massive Quarks

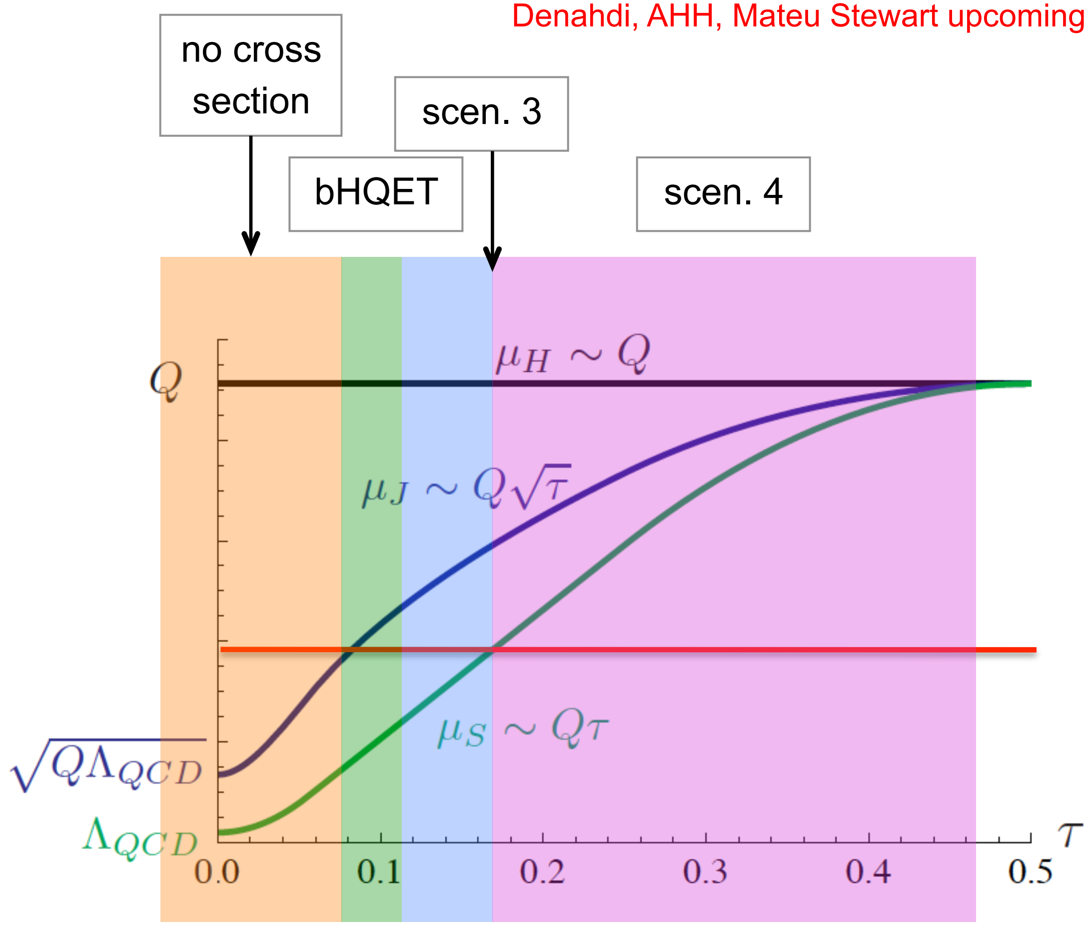


→ bHQET-type theory when the jet scale approaches the quark mass

Fleming, AHH, Mantry, Stewart 2007

→ two SCET-type theories

Denahdi, AHH, Mateu Stewart upcoming



MC vs. SCET: Primary Bottom Production

Preliminary !!

Denahdi, AHH, V. Mateu

Compare MC with SCET (pQCD, summation, hadronization effects) @ NNLL for Thrust

- Take central values for α_s and Ω_1 from our earlier NNLL thrust analysis for data on all-flavor production (=massless quarks)

$$\alpha_s(M_Z) = 0.1192 \pm 0.006$$
$$\Omega_1 = 0.276 \pm 0.155$$

- Compare with Pythia (m_b^{Pythia}=4.8 GeV) for consistency and mass sensitivity

- Which mass does m_b^{Pythia}=4.8 GeV correspond to for a field theoretic bottom mass?

Abbate,Fickinger, AHH, Mateu, Stewart 2010

order	$\bar{\Omega}_1$ ($\overline{\text{MS}}$)	Ω_1 (R-gap)
NLL′	0.264 ± 0.213	0.293 ± 0.203
NNLL	0.256 ± 0.197	0.276 ± 0.155
NNLL′	0.283 ± 0.097	0.316 ± 0.072
N^{3}LL	0.274 ± 0.098	0.313 ± 0.071
N^{3}LL′ (full)	0.252 ± 0.069	$\mathbf{0.323 \pm 0.045}$
N^{3}LL′$_{(\text{QCD}+m_b)}$	0.238 ± 0.070	0.310 ± 0.049
N^{3}LL′$_{(\text{pure QCD})}$	0.254 ± 0.070	0.332 ± 0.045

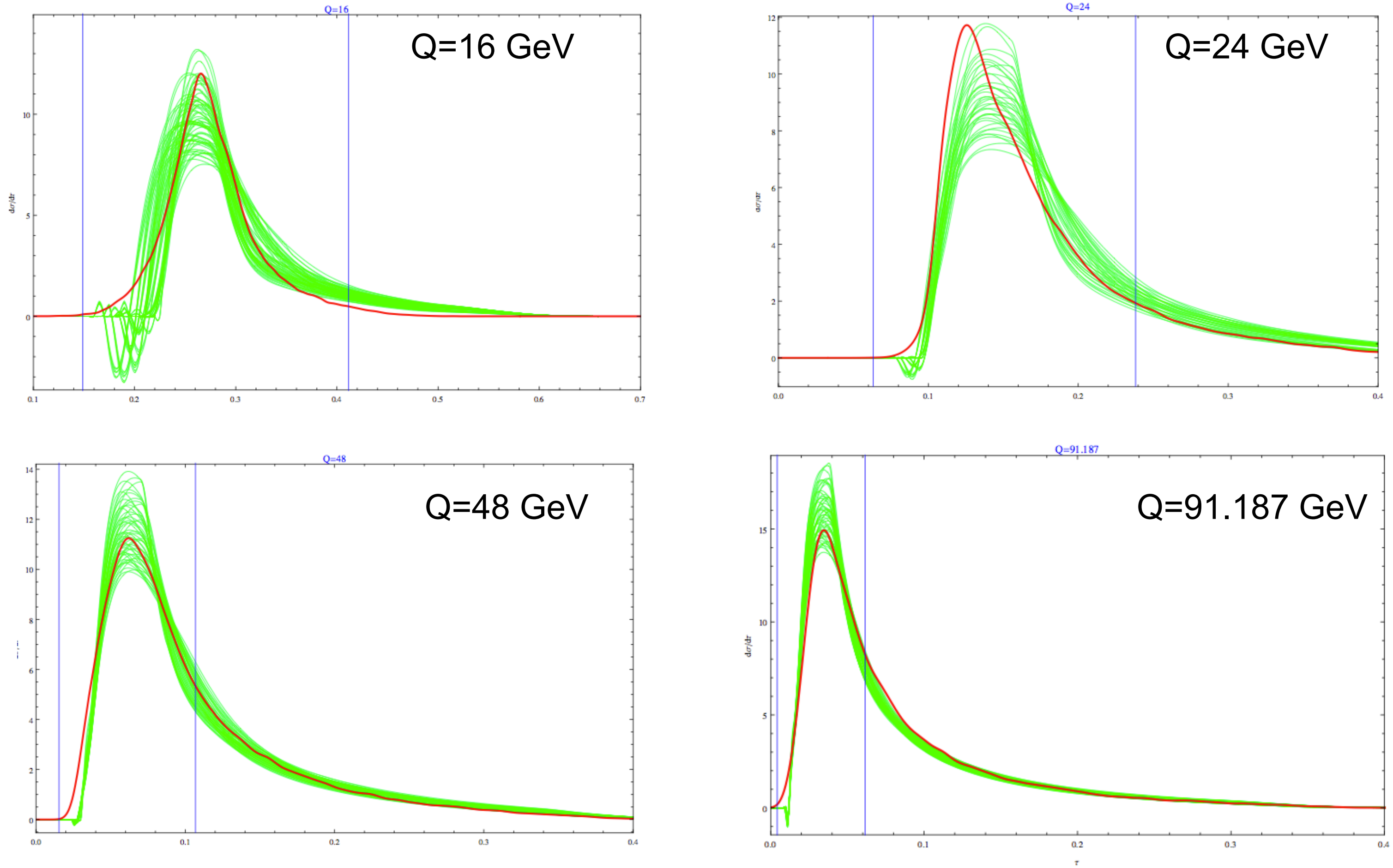
order	$\alpha_s(m_Z)$ (with $\bar{\Omega}_1^{\overline{\text{MS}}}$)	$\alpha_s(m_Z)$ (with Ω_1^{Rgap})
NLL′	0.1203 ± 0.0079	0.1191 ± 0.0089
NNLL	0.1222 ± 0.0097	0.1192 ± 0.0060
NNLL′	0.1161 ± 0.0038	0.1143 ± 0.0022
N^{3}LL	0.1165 ± 0.0046	0.1143 ± 0.0022
N^{3}LL′ (full)	0.1146 ± 0.0021	$\mathbf{0.1135 \pm 0.0009}$
N^{3}LL′$_{(\text{QCD}+m_b)}$	0.1153 ± 0.0022	0.1141 ± 0.0009
N^{3}LL′$_{(\text{pure QCD})}$	0.1152 ± 0.0021	0.1140 ± 0.0008

MC vs. SCET: Primary Bottom Production

Preliminary !! (no fit yet) all NNLL+NLO

Pythia: $m_b^{\text{Pythia}} = 4.8\ \text{GeV}$

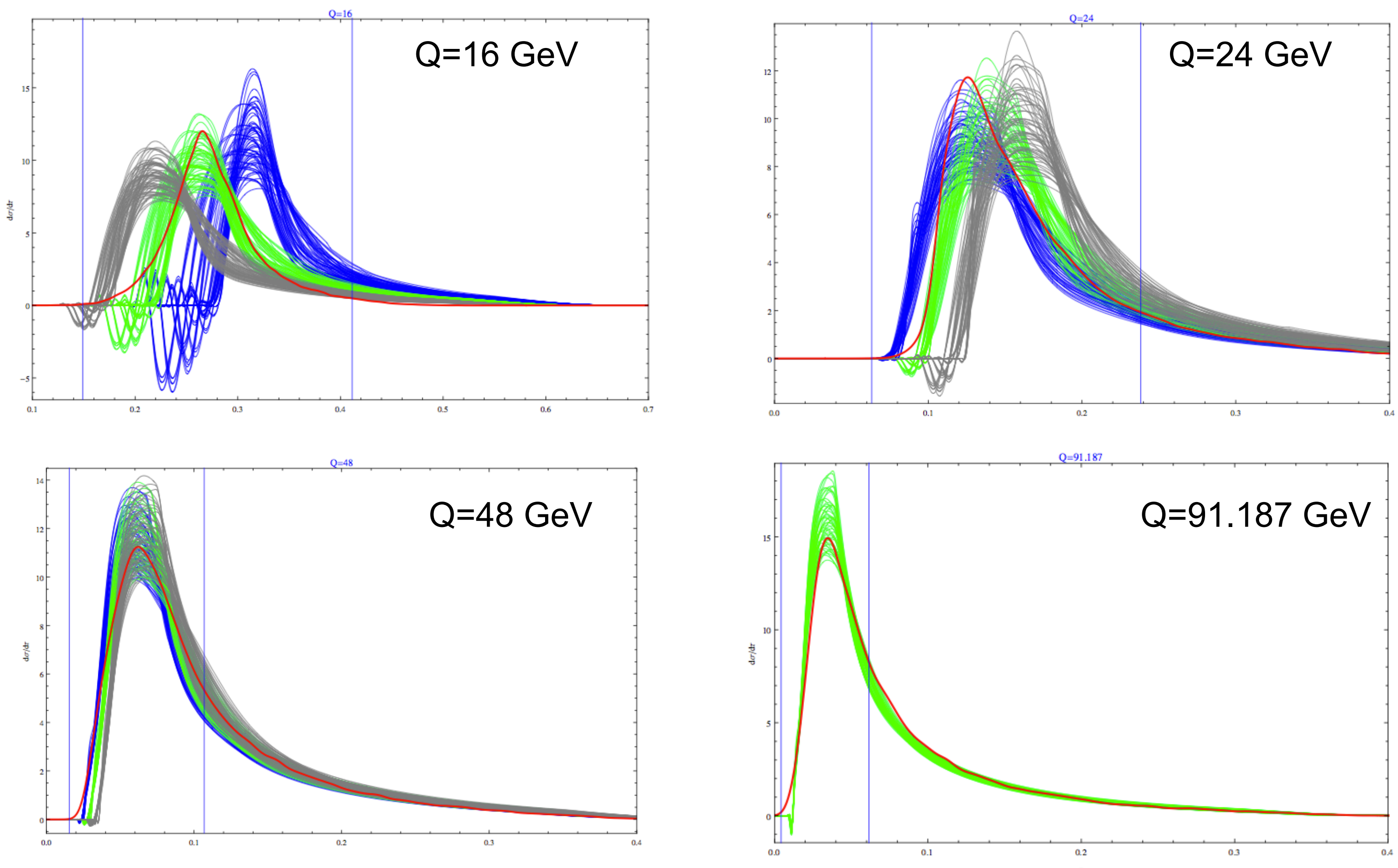
QCD calc.: $\overline{m}_b(\overline{m}_b) = 4.2\ \text{GeV}$ $\alpha_s(M_Z) = 0.1192$ $\Omega_1 = 0.276\,\text{GeV}$



MC vs. SCET: Primary Bottom Production

Preliminary !! (No fit yet)

$$\overline{m}_b(\overline{m}_b) = 3.7, 4.2, 4.7 \text{ GeV} \qquad \alpha_s(M_Z) = 0.1192 \qquad \Omega_1 = 0.276\,\text{GeV}$$



Conclusions

→ The MC top mass parameter has the status of a hadronic parameter (comparable to a meson mass) and is therefore not a field theoretic mass definition

→ The issue is becomes relevant when uncertainties in the MC top mass are becoming smaller than 1 GeV.

→ Ignoring the issue means that there is a conceptual uncertainty of about 1 GeV one needs to account for when relating the MC mass to a field theory mass.

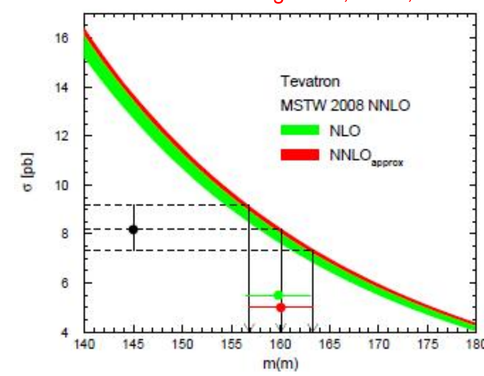
→ Suitable field theory mass definition in this context: e.g. MSR mass (R=1-3 GeV)

$$m_t^{\mathrm{MC}} = m_t^{\mathrm{MSR}}(R = 1\,\mathrm{GeV}) + \Delta_{t,\mathrm{MC}}(R = 1\,\mathrm{GeV})$$

→ It is possible to relate the MS top mass to a field theoretic mass by fits of QCD calculations at the hadron level to MC output for very mass sensitive quantities.

Masses Loop-Theorists Like to use

Langenfeld, Moch, Uwer



Total cross section (LHC/Tev):

$$m_t^{\rm MSR}(R=m_t) = \overline{m}_t(\overline{m}_t)$$

$$M_t \,=\, M_t^{(O)} \,+\, M_t(0)\alpha_s \,+\, \ldots$$

- more inclusive
- sensitive to top production mechanism (pdf, hard scale)
- indirect top mass sensitivity
- large scale radiative corrections

Threshold cross section (ILC):

$$m_t^{\rm MSR}(R\sim 20~{\rm GeV})\,,\;m_t^{\rm 1S}\,,\;m_t^{\rm PS}(R)$$

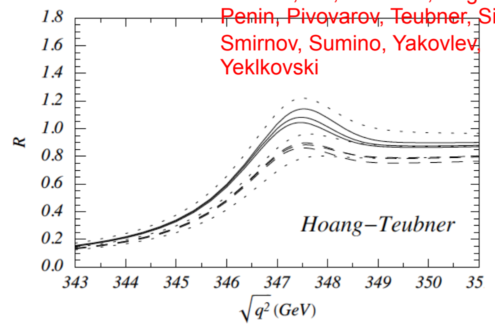
$$M_t \,=\, M_t^{(O)} \,+\, \langle p_{\rm Bohr}\rangle\alpha_s \,+\, \ldots$$

$$\langle p_{\rm Bohr}\rangle \,=\, 20\,{\rm GeV}$$

Mass schemes related to different computational methods

Relations computable in perturbation theory

Beneke, AH, Melnikov, Nagano, Penin, Pivovarov, Teubner, Signer, Smirnov, Sumino, Yakovlev, Yeklkovski



Inv. mass reconstruction (ILC/LHC):

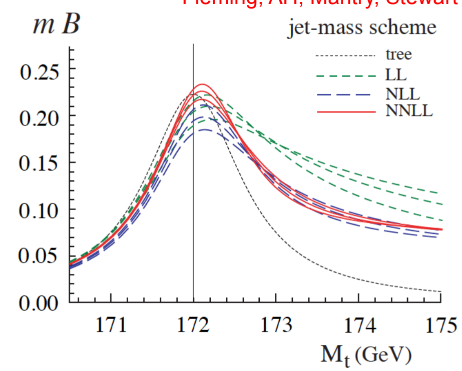
$$m_t^{\rm MSR}(R\sim\Gamma_t)\,,\;m_t^{\rm jet}(R)$$

$$M_t \,=\, M_t^{(O)} \,+\, \Gamma_t\alpha_s \,+\, \ldots$$

$$\Gamma_t \,=\, 1.3\,{\rm GeV}$$

- more exclusive
- sensitive to top final state interactions (low scale)
- direct top mass sensitivity
- small scale radiative corrections

Fleming, AH, Mantry, Stewart



MSR Mass Definition

MSbar Scheme: $(\mu > \overline{m}(\overline{m}))$

$$\overline{m}(\overline{m}) - m^{\rm pole} = -\overline{m}(\overline{m})\,[\,0.42441\,\alpha_s(\overline{m}) + 0.8345\,\alpha_s^2(\overline{m}) + 2.368\,\alpha_s^3(\overline{m}) + \ldots]$$

MSR Scheme: $(R < \overline{m}(\overline{m}))$

$$m_{\rm MSR}(R) - m^{\rm pole} = -R\,[\,0.42441\,\alpha_s(R) + 0.8345\,\alpha_s^2(R) + 2.368\,\alpha_s^3(R) + \ldots]$$

$$m_{\rm MSR}(m_{\rm MSR}) = \overline{m}(\overline{m})$$

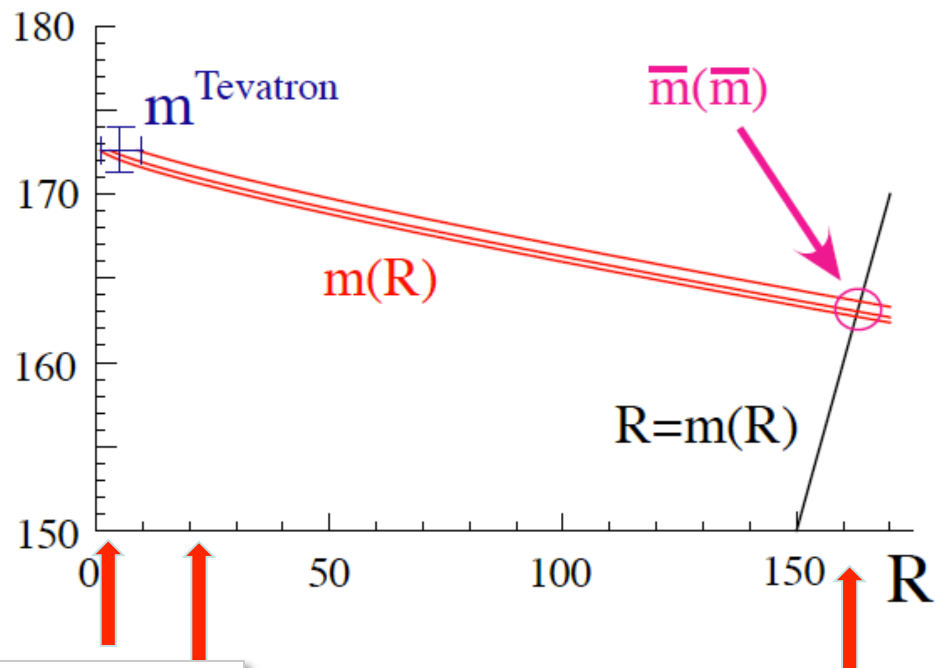
⟹ $m_{\rm MSR}(R)$ Short-distance mass that smoothly interpolates all R scales

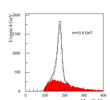
- Excellent convergence of relation between MSR masses at different R values
- Excellent convergence of relation between MSR masses and other short-distance masses
- Smoothy interpolates to the MSbar mass.

MSR Mass Definition

AH, Stewart: arXive:0808.0222

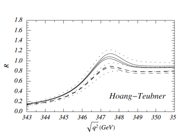
$$m_t^{\rm MC} = m_t^{\rm MSR}(3^{+6}_{-2}\ {\rm GeV}) = m_t^{\rm MSR}(3\ {\rm GeV})^{+0.6}_{-0.3}$$





Peak of invariant mass distribution, endpoints

Top-antitop threshold at the ILC



Total cross section, e.w.precsion obs., Unification, MSbar mass

Series with a Renormalon

→ Behavior depends on the typical scale R of the observable ?

→ Series for large R converge longer, but size of corrections at lower orders are large

→ Formal ambiguity always the same: $\Lambda_{\rm QCD} \approx 0.5~{\rm GeV}$