Drell-Yan transverse-momentum resummation at NNLL+NNLO

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In collaboration with:
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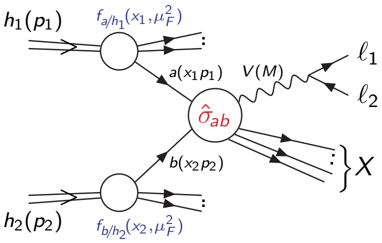
The Drell–Yan q_T distribution



$$\mathbf{h_1(p_1) + h_2(p_2) \;\rightarrow\; V(M) + X \;\rightarrow\; \ell_1 + \ell_2 + X}$$

where $\quad V = \gamma^*, Z^0, W^\pm \quad$ and $\quad \ell_1\ell_2 = \ell^+\ell^-, \ell\nu_\ell$

pQCD factorization formula:

$$\frac{d\sigma}{dq_T^2}(q_T,M,s) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2 \; f_{a/h_1}(x_1,\mu_F^2)\; f_{b/h_2}(x_2,\mu_F^2)\; \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T,M,\hat{s};\alpha_S,\mu_R^2,\mu_F^2).$$

Standard fixed-order perturbative expansions ($Q_T \ll 1$):

$$\int_0^{Q_T^2} dq_T^2 \frac{d\hat{\sigma}_{q\bar{q}}}{dq_T^2} \sim 1 + \alpha_S\left[c_{12}\log^2\frac{M^2}{Q_T^2} + c_{11}\log\frac{M^2}{Q_T^2} + c_{10}\right]$$
$$+\alpha_S^2\left[c_{24}\log^4\frac{M^2}{Q_T^2} + \cdots + c_{21}\log\frac{M^2}{Q_T^2} + c_{20}\right] + \mathcal{O}(\alpha_S^3)$$

Fixed order calculation reliable only for $q_T \sim M$

For $q_T \to 0$, $\alpha_S^n \log^m(M^2/q_T^2) \gg 1$:

need for resummation of logarithmic corrections.

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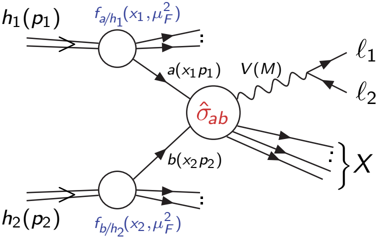

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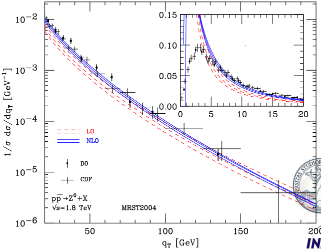

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Idea of (analytic) resummation

Idea of large logs (Sudakov) resummation: reorganize the perturbative expansion by all-order summation ($L = \log(M^2/q_T^2)$).

$\alpha_S L^2$	$\alpha_S L$	$\cdots$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_S)$
$\alpha_S^2 L^4$	$\alpha_S^2 L^3$	$\alpha_S^2 L^2$	$\alpha_S^2 L$	$\cdots$	$\mathcal{O}(\alpha_S^2)$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\alpha_S^n L^{2n}$	$\alpha_S^n L^{2n-1}$	$\alpha_S^n L^{2n-2}$	$\cdots$	$\cdots$	$\mathcal{O}(\alpha_S^n)$
dominant logs	next-to-dominant logs	$\cdots$	$\cdots$	$\cdots$	$\cdots$

- Ratio of two successive rows $\mathcal{O}(\alpha_S L^2)$: fixed order expansion valid when $\alpha_S L^2 \ll 1$.
- Ratio of two successive columns $\mathcal{O}(1/L)$: resummed expansion valid when $1/L \ll 1$.

Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions.

$$dw_n(q_1, \ldots, q_n) \simeq \frac{1}{n!} \prod_{i=1}^{n} dw_i(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution of DY process it holds in the impact parameter space (Fourier transform).

$$\int d^2\mathbf{q_T}\, \exp(-i\mathbf{b}\cdot\mathbf{q_T})\, \delta\left(\mathbf{q_T} - \sum_{j=1}^{n} \mathbf{q_{T_j}}\right) = \exp(-i\mathbf{b}\cdot\sum_{j=1}^{n} \mathbf{q_{T_j}}) = \prod_{j=1}^{n} \exp(-i\mathbf{b}\cdot\mathbf{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space.

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State of the art: transverse-momentum (q_T) resummation

- Method to resum large q_T logarithms is known [Dokshitzer,Diakonov,Troian('78)], [Parisi,Petronzio('79)],[Kodaira,Trentadue('82)],[Collins,Soper,Sterman('85)], [Altarelli et al.('84)],[Catani,de Florian,Grazzini('01)],[Catani,Grazzini('10)]
- Various phenomenological studies [ResBos:Balasz,Yuan,Nadolsky et al.('97,'02)], [Ellis et al.('97)],[Kulesza et al.('02)],[Guzzi,Nadolsky,Wang('13)].
- Results for q_T resummation in the framework of Effective Theories [Gao,Li,Liu('05)],[Idilbi,Ji,Yuan('05)],[Mantry,Petriello('10)],[Becher, Neubert('10)],[Echevarria,Idilbi,Scimemi('11)].
- Studies within transverse-momentum dependent (TMD) factorization and TMD parton densities [Roger,Mulders('10)],[Collins('11)],[D'Alesio,Echevarria, Melis,Scimemi('14)].
- Effective q_T-resummation can be obtained with Parton Shower algorithms [Corcella,Seymour('00)]. QCD/EW DY corrections implemented in POWHEG [Barze et al.('12,'13)]. Recent results for NNLO+PS DY predictions obtained [Hoeche,Li,Prestel('14)],[Karlberg,Re,Zanderighi('14)] (see E. Re talk).

Transverse momentum resummation in pQCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(res)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \sum_{n=0}\sum_{m=0}^{2n} c_{nm}\,\alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \mathcal{O}(\frac{Q_T^2}{M^2})$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1,\quad \log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

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$$\frac{d\hat\sigma^{(res)}}{dq_T^2} = \frac{M^2}{\hat s} \int \frac{d^2\mathbf{b}}{4\pi} e^{i\mathbf{b}\cdot\mathbf{q_T}}\, \mathcal{W}(b, M),$$

In the Mellin moments ($f_N \equiv \int_0^1 f(z) z^{N-1} dz$, with $z = M^2/\hat s$) space we have:

$$\mathcal{W}_N(b, M) = \mathcal{H}_N(\alpha_S) \times \exp\{\mathcal{G}_N(\alpha_S, L)\} \qquad \text{where} \qquad L \equiv \log(M^2 b^2)$$

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Resummed NLL(NNLL) result *matched* with corresponding fixed order "finite" part at α_S (α_S^2), to have a good control both for $q_T \ll M$ and for $q_T \sim M$.

Transverse momentum resummation in pQCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

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The q_T resummation formalism

Main distinctive features of the formalism [Catani,de Florian, Grazzini('01)], [Bozzi,Catani,de Florian, Grazzini('03,'06,'08)]:

- Resummation performed at partonic level: PDF evaluated at $\mu_F \sim M$: no PDF extrapolation in the non perturbative region, customary study of μ_R and μ_F dependence.
- Introduction of resummation scale $Q \sim M$: variations give an estimate of the uncertainty from uncalculated logarithmic corrections.

$$\ln\left(M^2 b^2\right) \to \ln\left(Q^2 b^2\right) + \ln\left(M^2/Q^2\right)$$

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DYqT: q_T-resummation at NNLL:

Bozzi,Catani,de Florian,G.F.,Grazzini('11)

- We have applied for Drell–Yan transverse-momentum distribution the resummation formalism developed by [Catani,de Florian, Grazzini('01)] already applied for the case of Higgs boson production [Bozzi,Catani,de Florian, Grazzini('03,'06,'08)].
- We have performed the resummation up to NNLL(+NNLO). It means that our complete formula includes:
  - NNLL logarithmic contributions to all orders;
  - NNLO corrections (i.e. $\mathcal{O}(\alpha_S^2)$) at small q_T;
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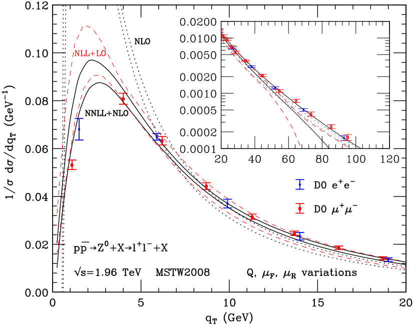
DYqT results: q_T spectrum of Z boson at the Tevatron



D0 data for the Z q_T spectrum compared with perturbative results.

- Uncertainty bands obtained varying μ_R, μ_F, Q independently:
$\frac{1}{2} \leq \{\mu_F/m_Z, \mu_R/m_Z, 2Q/m_Z, \mu_F/\mu_R, Q/\mu_R\} \leq 2$
- Significant reduction of scale dependence from NLL to NNLL for all q_T.
- Good convergence of resummed results: NNLL and NLL bands overlap (contrary to the fixed-order case).
- Good agreement between data and resummed predictions (without any model for non-perturbative effects).
The perturbative uncertainty of the NNLL results is comparable with the experimental errors.

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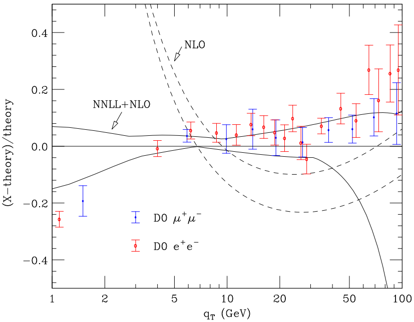
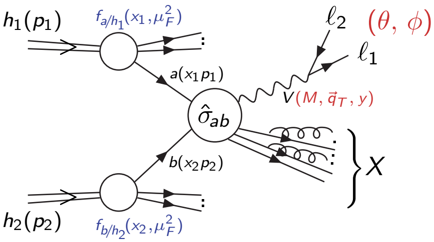


D0 data for the Z q_T spectrum: Fractional difference with respect to the reference result: NNLL, $\mu_R = \mu_F = 2Q = m_Z$.

- NNLL scale dependence is $\pm 6\%$ at the peak, $\pm 5\%$ at $q_T = 10\,GeV$ and $\pm 12\%$ at $q_T = 50\,GeV$. For $q_T \geq 60\,GeV$ the resummed result looses predictivity.
- At large values of q_T, the NLO and NNLL bands overlap.
  At intermediate values of transverse momenta the scale variation bands do not overlap.
- The resummation improves the agreement of the NLO results with the data.
  In the small-q_T region, the NLO result is theoretically unreliable and the NLO band deviates from the NNLL band.

DYRES: q_T-resummation with decay variables dependence



- Experiments have finite acceptance: important to provide exclusive theoretical predictions.
- Analytic resummation formalism inclusive over soft-gluon emission: not possible to apply selection cuts on final state partons.

- We have included the full dependence on the decay products variables: possible to apply cuts on vector boson and decay products.
- To construct the "finite" part we rely on the fully-differential NNLO result from the code DYNNLO [Catani,Cieri,de Florian,Ferrera,Grazzini('09)].
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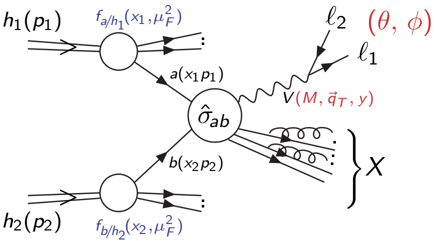

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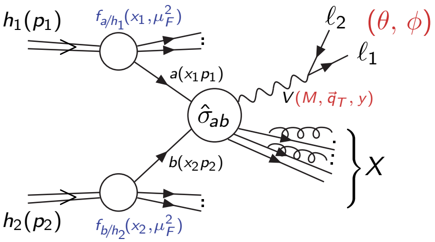

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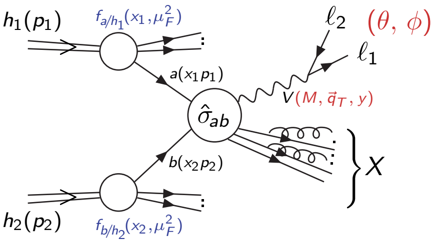

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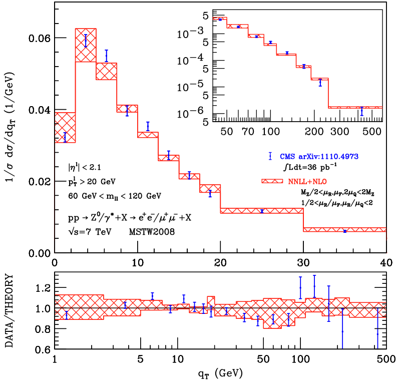
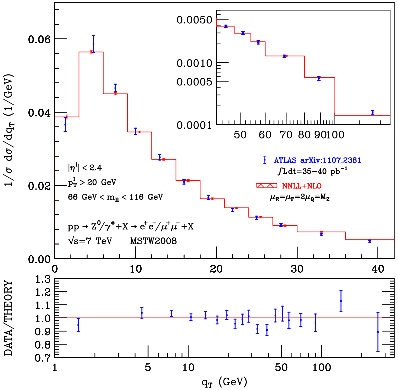
- Calculation implemented in a numerical program DYRES which includes spin correlations, $\gamma^* Z$ interference, finite-width effects and compute distributions in form of bin histograms: analogously to the HRES code.

DYRES results: q_T spectrum of Z boson at the LHC



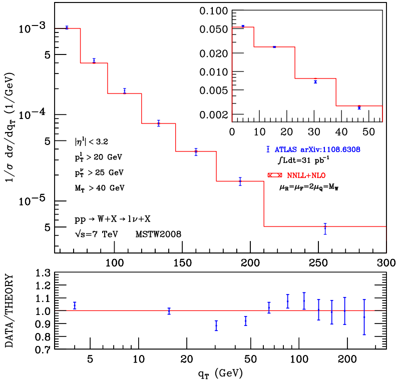


CMS data for the Z q_T spectrum compared with NNLL result. Scale variation:

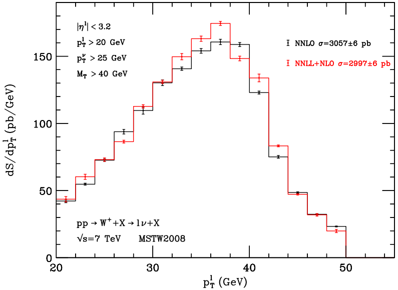
$$1/2 \leq \{\mu_F/m_Z, \mu_R/m_Z, \mu_F/\mu_R, 2Q/m_Z, Q/\mu_R\} \leq 2$$

ATLAS data for the Z q_T spectrum compared with NNLL result.

DYRES results: q_T spectrum of W boson at the LHC

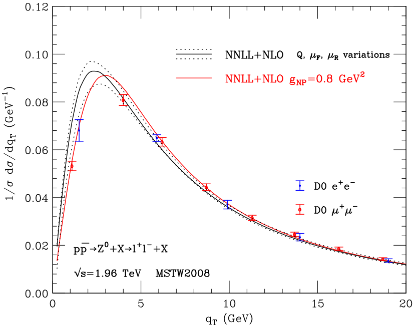


ATLAS data for the W q_T spectrum compared with NNLL result.



Lepton p_T spectrum from W^+ decay. NNLL result compared with the NNLO result.

Non perturbative Fermi motion effects



D0 data for the Z q_T spectrum.

- Up to now result in a complete perturbative framework (plus PDFs).
- Non perturbative *intrinsic* k_T effects can be parametrized by a NP form factor $S_{NP} = \exp\{-g_{NP} b^2\}$:

$$\exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\} \rightarrow \exp\{\mathcal{G}_N(\alpha_S, \widetilde{L})\}\, S_{NP}$$

$$g_{NP} \simeq 0.8\ GeV^2$$ [Kulesza et al.('02)]

- With NP effects the q_T spectrum is harder. Quantitative impact of intrinsic k_T effects is comparable with perturbative uncertainties and with non perturbative effects from PDFs.

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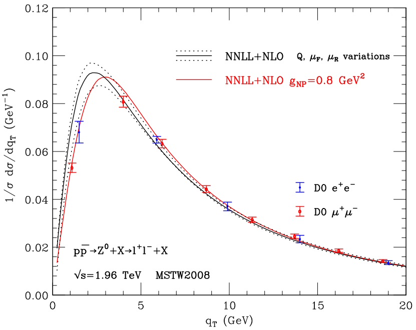

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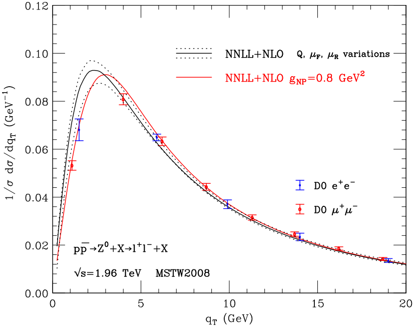

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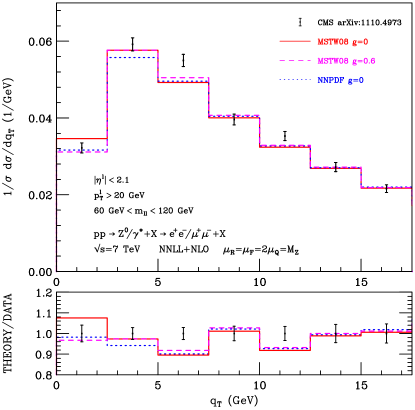

CMS data for the Z q_T spectrum.

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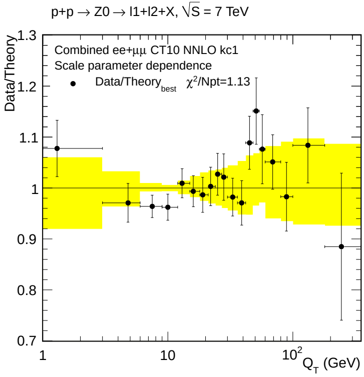
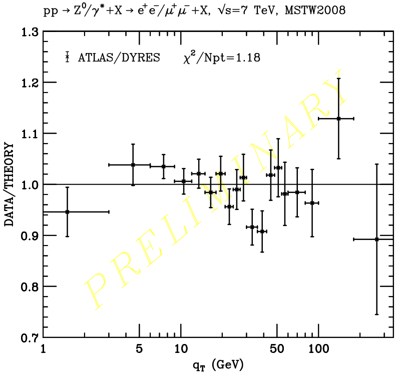
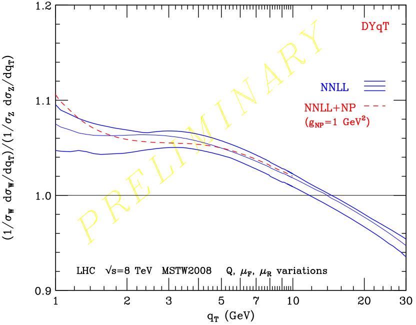




ATLAS ('11) data for the Z q_T spectrum compared with ResBos predictions with a Non Perturbative smearing parameter $g_{NP} = 1.1\,GeV^2$ [Guzzi,Nadolsky,Wang('13)].

ATLAS ('11) data for the Z q_T spectrum compared with DYRES predictions without Non Perturbative smearing ($g_{NP} = 0$).

W/Z ratio of observables: the q_T spectrum



DYqT resummed predictions for the ratio of W/Z normalized q_T spectra.

- The use of the W/Z ratio observables substantially reduces both the experimental and theoretical systematic uncertainties [Giele,Keller('97)].
- Resummed perturbative prediction for

$$\frac{\frac{1}{\sigma_W}\frac{d\sigma_W}{dq_T}}{\frac{1}{\sigma_Z}\frac{d\sigma_Z}{dq_T}}(\mu_R, \mu_F, Q)$$

  with the customary scale variation.
- NNLL perturbative uncertainty band very small: 2-5% for $1 < q_T < 2$ GeV, 1.5-2% for $2 < q_T < 30$ GeV.
- Non perturbative effects within 1% for $1.5 < q_T < 5$ GeV and negligible for $q_T > 5$ GeV.

Conclusions

- NNLL+NNLO DY q_T-resummation [Bozzi,Catani,de Florian,G.F., Grazzini('11)].
- A public version of the DYqT code is available. Reduction of scale uncertainties from NLL to NNLL accuracy. The NNLL results consistent with the experimental data in a wide region of q_T.
- DYRES: NNLL q_T resummation with full kinematical dependence on the vector boson and on the final state leptons.
- Preliminary comparison with LHC data (implementing experimental cuts): good agreement between data and NNLL results without any model for Non Perturbative effects.
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