Scaling Behavior of Circular Colliders Dominated by Synchrotron Radiation

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2 Outline

- Radius x Power Scaling Law Invariant; Why “Bigger is Cheaper”
- Lattice Parameter Scaling Laws. Why “Bigger is Better”
- Staged Optimization and Costs
- Maximum β_y Phenomenology; Why β_y must not be too large
- Extra Material
- $L^* \times \mathcal{L}$ Luminosity x Detector Length Invariant Product
- Cost Implications of Doubling R
- Recommendations:
  - FCC-ee, CEPC, SPPC Cell Lengths are Way Too Short.
  - New FCC-pp 215 m Cell Length (and Lattice) is Great
  - “Luminosity” is a Dependent Variable, Not an Input Parameter
  - “Ground Up” Rather Than “Constrained Parameter” Design
- One Ring or Two?
  - For Higgs production one Ring is cheaper and better, but the luminosity is 5 or 10 times less for Z_0.
  - But it doesn’t matter. One ring or two can be decided later.
- Geographical Cost Optimization

- It would be wonderful for the world FCC collaboration if everyone used the same circumference—which would be 100 km.
- To achieve this we need only prove that doubling the CEPC circumference, while cutting the power per beam from 50 MW to 25 MW, is better (more luminosity), almost as cheap initial cost, and "greener" (lower power bill).
- Obviously "bigger is better" for the p,p collider (to maximize the beam energy for an achievable magnetic field).
- A Radius x RF-power invariant product scaling law shows that "bigger is **as good** as smaller" for e+e-, (because increasing R and decreasing P_{RF} proportionally leaves the luminosity constant.)
- But "bigger is also **better**" (because the ratio of dynamic aperture to beam size increases with increasing R.)

4 Radius x Power Scaling Law Invariant.

- Dominating everything is the synchrotron radiation formula

$$\boxed{\Delta E \propto \frac{E^4}{R},} \tag{1}$$

  relating energy loss per turn ΔE, beam energy E and bend radius R.
- 100 TeV (for example) is such a high energy that synchrotron radiation will “dominate” p,p design, just as it has always dominated e+e- design.
- For a given RF power P_{rf}, the maximum total number of stored particles is proportional to R^2—doubling the ring radius cuts in half the energy loss per turn and doubles the time interval over which the loss occurs. Expressed as a scaling law

$$\boxed{n_1 = \text{number of stored electrons per MW } \propto R^2.} \tag{2}$$

5 Proof of Radius x Power Scaling Law

$$\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}} \propto \frac{f}{N_b} \frac{(n_1 P_{\mathrm{rf}}[\mathrm{MW}])^2}{\sigma_x^* \sigma_y^*}. \tag{3}$$

- The dependencies on R are, $N_b \propto R$, $f \propto 1/R$, and $n_1 \propto R^2$. σ_x^* and σ_y^* are constant. Variations for which

  $$P_{\mathrm{rf}} \propto \frac{1}{R}. \tag{4}$$

  leave $\mathcal{L}^{\mathrm{RF}}_{\mathrm{pow}}$ invariant.
- This scaling law can be expressed in the form

  $$\boxed{\mathcal{L}(R, P_{\mathrm{rf}}) = f(RP_{\mathrm{rf}}),} \tag{5}$$

  The luminosity depends on R and P_{rf} only as a function $f(RP_{\mathrm{rf}})$ of their product.

6 Empirical $\mathcal{C}$ vs. E Scaling of Radiation-Dominated Circular Colliders

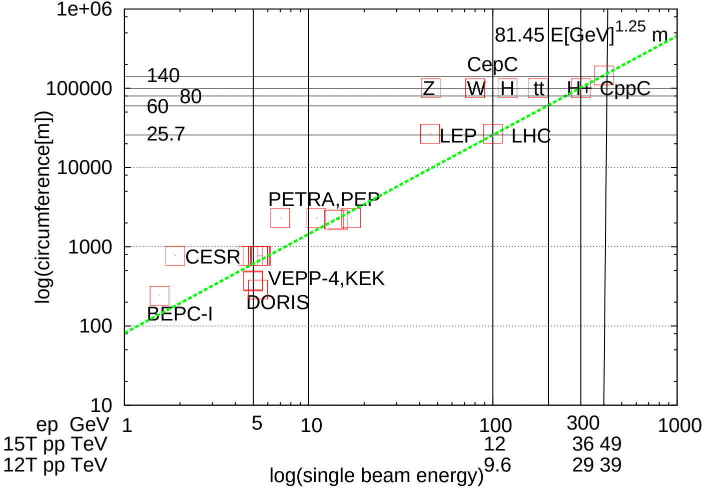

Figure: Dependence of circumference on beam energy, both for GeV-scale electron colliders, and for TeV-scale proton colliders of magnetic field 12T or 15T.

Parameter	Symbol	Proportionality	Scaling
phase advance per cell	μ		1
collider cell length	L_c		$R^{1/2}$
bend angle per cell	ϕ	$= L_c/R$	$R^{-1/2}$
quad strength $(1/f)$	q	$1/L_c$	$R^{-1/2}$
dispersion	D	ϕL_c	1
beta	β	L_c	$R^{1/2}$
tunes	Q_x, Q_y	R/β	$R^{1/2}$
Sands's "curly H"	$\mathcal{H}$	$= D^2/\beta$	$R^{-1/2}$
partition numbers	$J_x/J_y/J_\epsilon$	$= 1/1/2$	1
horizontal emittance	ϵ_x	$\mathcal{H}/(J_x R)$	$R^{-3/2}$
fract. momentum spread	σ_δ	$\sqrt{B}$	$R^{-1/2}$
arc beam width-betatron	$\sigma_{x,\beta}$	$\sqrt{\beta\epsilon_x}$	$R^{-1/2}$
-synchrotron	$\sigma_{x,synch.}$	$D\sigma_\delta$	$R^{-1/2}$
sextupole strength	S	q/D	$R^{-1/2}$
dynamic aperture	$x^{\rm max}$	q/S	1
relative dyn. aperture	$x^{\rm max}/\sigma_x$		$R^{1/2}$
pretzel amplitude	x_p	σ_x	$R^{-1/2}$

Table: *Constant dispersion* scaling is the result of choosing cell length $L \propto R^{1/2}$. The entry "1" in the last column of the shaded "dispersion" row, indicates that the dispersion is independent of R when the cell length L_c varies proportional to $\sqrt{R}$ with the phase advance per cell μ held constant.

8 Ring Parameters Scaled from LEP for 50 and 100 km Circumference

Parameter	Symbol	Value	Unit	Energy-scaled	Radius-	scaled
bend radius	R	**3026**	m	**3026** LEP	**5675**	**11350**
Beam Energy	E	**45.6**/91.5	GeV	**120**	**120**	**120**
Circumference	$\mathcal{C}$	26.66	km	26.66	50	100
Cell length	L_c		m	79	108	153
Momentum compaction	α_c	1.85e-4		1.85e-4	0.99e-4	0.49e-4
Tunes	Q_x	90.26		90.26	123.26	174.26
	Q_y	76.19		76.19	104.19	147.19
Partition numbers	$J_x/J_y/J_\epsilon$	1/1/2		1/1.6/1.4 !	1/1/2	1/1/2
Main bend field	B_0	0.05/0.101	T	0.1316	0.0702	0.0351
Energy loss per turn	U_0	0.134/2.05	GeV	6.49	3.46	1.73
Radial damping time	τ_x	0.06/0.005	s	0.0033	0.0061	0.0124
	τ_x/T_0	679/56	turns	37	69	139
Fractional energy spread	σ_δ	0.946e-3/1.72e-3		0.0025	0.0018	0.0013
Emittances (no BB), x	ϵ_x	22.5/30	nm	21.1	8.2	2.9
y	ϵ_y	0.29/0.26	nm	1.0	0.4	0.14
Max. arc beta functs	$\beta_x^{\max}$	125	m	125	171	242
Max. arc dispersion	$D^{\max}$	0.5	m	0.5	0.5	0.5
Beta functions at IP	β_x^*, β_y^*	2.0,0.05	m	1.25/0.04	N/Sc.	N/Sc.
Beam sizes at IP	σ_x^*, σ_y^*	211, 3.8	μm	178/11	N/Sc.	N/Sc.
Beam-beam parameters	ξ_x, ξ_y	0.037,0.042		0.06/0.083	N/Sc.	N/Sc.
Number of bunches	N_b	8		4	N/Sc.	N/Sc.
Luminosity	$\mathcal{L}$	2e31	$\mathrm{cm}^{-2}\mathrm{s}^{-1}$	1.0e32	N/Sc.	N/Sc.
Peak RF voltage	V_{RF}	380	MV	3500	N/Sc.	N/Sc.
Synchrotron tune	Q_s	0.085/0.107		0.15	N/Sc.	N/Sc.
Low curr. bunch length	σ_z	0.88	cm	$\frac{\alpha_c R \sigma_e}{Q_s E}$	N/Sc.	N/Sc.

Cell lengths are 47 m for CEPC, and 50 m for FCC-ee.

Staged Optimization

For best chance of initial approval and best eventual p,p performance, the cost of the first step has to be minimized and the tunnel circumference maximized. Surprisingly, these requirements are consistent. Consider optimization principles for three collider stages:

- **Stage I, e+e-:** Starting configuration. Minimize cost at “respectable” luminosity, e.g. 10^{34}. Constrain the number of rings to 1, and the number of IP’s to $N^* = 2$.
- **Stage II, e+e-:** Maximize luminosity/cost for production Higgs (etc.) running. Upgrade the luminosity by some combination of: $P_{\rm rf} \rightarrow 2P_{\rm rf}$ or $4P_{\rm rf}$, one ring → two rings, increasing N^* from 2 to 4, or decreasing β_y^*.
- **Stage III, pp:** Maximize the ultimate physics reach, i.e. center of mass energy, i.e. maximize tunnel circumference.

Exploiting $P_{\mathrm{rf}} \propto \mathcal{L}/R$, some estimated costs (in arbitrary cost units) and luminosities for Stages I (turn on) are given in the table.

	R km	P_{rf} MW	C_{tun} arb.	C_{acc} arb.	Phase-I cost arb.	$\mathcal{L}^I$ (Higgs) 10^{34}	$\mathcal{L}^I$ (Z_0) 10^{34}
1 ring	5	50	0.5	2.5	3.0	1.2	2.6
	10	25	1.0	2.87*	**3.87**	1.2	5.2
	10	50	1.0	3.58	4.58	2.3	10.4
2 rings	5	50	0.5	4.1†	4.6	1.2	21
	10	25	1.0	4.72	5.72	1.2	21
	10	50	1.0	5.89	6.89	2.3	42

Table: Estimated costs, one ring in the upper table, two in the lower. C_{tun} is the tunnel cost, C_{acc} is the cost of the rest of the accelerator complex. Costs have been extrapolated from CEPC pre-CDR proposal. *With one ring, changes $R \rightarrow 2R$ and $P \rightarrow P/2$ are estimated to increase the accelerator cost by a factor 1.15. †Changing from one ring to two rings with R and P held fixed is estimated to increase the cost by a factor 1.64.

The shaded row seems like the best deal.

11 Maximum β_y Phenomenology; Why β_y must not be too large

- To get higher luminosity requires reducing β_y^*.
- Reducing β_y^* increases $\beta_y^{\rm max}$, which invariably makes the collider more erratic, often unacceptably so. Sensitivity to beam-beam effects and other effects is greatly magnified by large β anywhere in the ring.
- There are inevitable unknown transverse element displacement errors $\Delta y_{\rm transverse}$.
- From the scaling laws derived earlier, to quantify the limitation imposed by a large $\beta_y^{\rm max}$ at one or a few points in the ring, one can introduce a transverse sensitivity length

$$\boxed{\text{transverse sensitivity length} = \frac{DL_C}{\beta^{\rm max}}.} \tag{6}$$

- The optical deviation caused by $\Delta y_{\rm transverse}$ will be negligible only in the limit

$$\Delta y_{\rm transverse} << \text{transverse sensitivity length}. \tag{7}$$

- The scale factor is phenomenological but, for empirical comparison purposes, it is to be taken to be independent of particle energy and type, electron or proton.

Maximum β_y Phenomenology Based on Transverse Orbit Sensitivity

- The inverse of the sensitivity length is a “figure of demerit, “FOD” $= \frac{\beta_Y^{\mathrm{max}}}{DL_c}$ that can be used to compare different rings, either proton or electron, independent of their beam energies.

β_y^* m	Ring		D m	L_c m	DL_c m^2	β_y^{max} m	$FOD = \frac{\beta_y^{\mathrm{max}}}{DL_c}$ 1/m
0.015	CESR	exp.	1.1	17	18.7	95	5.1
0.08	PETRA	exp.	0.32	14.4	4.6	225	49
	HERA	exp.	1.5	48	72	2025	28
0.05	LEP	exp.	0.8	79	63	441	7.0
0.007	KEKB	exp.	0.5	20	10	290	29
	LHC	exp.	1.6	79	126	4500	36
0.01	CepC$_1$	des.	0.31	47	14.6	1225	84
0.01	CepC$_2$	des.	1.03	153	158	1225	8.8
0.001	CEPC	des.	0.31	47	14.6	6000	410
0.001	FCC-ee	des.	0.10	50	5.0	9025	1805

- Empirically determined upper limit rule on $FOD_{\mathrm{trans.sens.}}$.

$$FOD_{\mathrm{trans.sens.}} < 40. \tag{8}$$

- CEPC exceeds this limit by a factor of 10, FCC-ee by a factor of 50. This is partly due to their way too short cell lengths.

13 Extra Material

- Note that kicker-free, septum free, vertical injection was invented by RT at Beijing in April 2014, and described in paper SAT4A3, “Lattice Optimization for Top-Off Injection” at the 55th ICFA Advanced Beam Dynamics Workshop on High Luminosity Circular e+e- Colliders, in the WG 6 Injection working group for HF2014 October 11.
- Vertical injection, on-axis horizontally, requires no septum.

$L^* \times \mathcal{L}$ Luminosity x Detector Length Invariant Product

L^* is the half-length of the drift space into which the detector must fit;

$$\mathcal{L} \overset{\text{e.g.}}{=} \frac{4 \times 10^{31}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{m}}{\beta_y^*} \tag{9}$$

or, using the relation between beta function β_Y* at the IP, and maximum beta function nearby, β_y^{max},

$$\mathcal{L} \overset{\text{e.g.}}{=} 1.6 \times 10^{31}\mathrm{cm}^{-2}\mathrm{s}^{-1}\mathrm{m} \times \frac{\beta_y^{\mathrm{max}}}{L^{*2}}. \tag{10}$$

- *The constant of proportionality in these equation is not determined by the scaling formula. They have been chosen to match a preliminary CEPC luminosity estimate.*
- For local chromaticity I.P. design (Yunhai Cai), lengths are scalable, quad strengths scale as 1/length, beta functions scale as length.
- An upper limit on β_y^{max} was set previously. As a result,

$$L^* \times \mathcal{L} \quad \text{product (upper limit) is fixed.} \tag{11}$$

15 Notes on Transverse Sensitivity Comparison Table

- The upper rows contain experimentally measured values, the lower rows contain design values.
- CepC_1 copies the L_c and D values from the CEPC pre-CDR report. CepC_2 obtains them from my tables. The IR designs are assumed identical, but with the β_y values shown.
- In theory nothing in the table depends directly on β_y^*. But, indirectly, large β_y^{max} values are correlated with small β_y^* values.

Justification for $FOD_{\mathrm{trans.sens.}}$ Comparison of Electron and Proton Rings

- Compared in this way the transverse tolerances of KEKB and LHC are close in value, even though, as storage rings, they could scarcely be more disimilar; KEKB is a “small” electron collider, LHC is a large proton collider.
- The near agreement between a modern electron ring KEKB and a modern proton beam LHC, lends some confidence in this sensitivity measure for comparing them.
- When β_y^{max} is large, it is always because β_y^* is small.
- But the value of β_y^*, in itself, does not influence the dynamic aperture. Nevertheless β_y^* values are given in the table.
- The pessimistic behavior of LEP can be blamed on the absence of top-off injection, which led to the tortuous ramping and beta squeeze operations. This limited the β_y^* to be not less than 5 cm.

Cost Implications of Doubling R

- My formulas suggest that leaving the bore unchanged (from LEP) is sensible for a first iteration.
- By reducing RF power, long term power costs have been reduced proportionally and three important start-up costs have also been reduced: installed RF, installed power, and installed cooling.
- But increasing R has increased other costs, of which the most serious is the vacuum chamber cost which will be more or less proportional to R.
- One might reflexively accept that doubling R will double the ring cost. But this is certainly not true.

Holding Magnet Cost Down

- According to CEPC cost estimates, the collider magnet cost will be a quite small fraction of the total cost. Here are some reasons for this, even with doubled ring radius.
- The optimized cell lengths L_c will be more than twice that of LEP. Only half as many magnets suggests “cheaper”.
- Iron electromagnet costs are sometimes expressed as dollars/energy where “energy” is the magnetic energy.
- From this (completely misleading) point of view, the magnet cost **falls** proportional to R because $B \propto 1/R$ and we are holding the transverse magnet area fixed.

More Magnet Considerations

- Immediate protest. The LEP magnets were already shorter than the cell length so the Higgs factory magnets will have more or less the same length and same cost.
- Some say "the costs are all in the magnet ends". Others say, "the cost is all in transporting and installation". Others: "the cost is all in the pedestals".
- All good points, **but they cannot be accepted.** To hold down magnet costs the magnets can be built *in situ*, or at least close enough for their installation to be an integral part of their construction.
- This is the only way to prevent the magnet cost from scaling proportional to the tunnel circumference, or worse.
- Built underground, the magnets can be almost arbitrarily long.

Why Top-Off Injection Permits Inexpensive Collider Magnets

- With top-off injection these magnets do not have to ramp up in field.
- As a result they have no eddy currents and therefore do not need to be laminated.
- Regrettably the same is not true for the injector magnet, which will be more challenging, and may be more expensive, than the collider magnet.

One Ring or Two? Limitation on Number of Bunches

- With one ring, the maximum number of bunches is limited to approximately ≤ 200.
- For $N_b > 200$ the luminosity $\mathcal{L}$ has to be de-rated accordingly; $\mathcal{L} \rightarrow \mathcal{L}_{\mathrm{actual}} = \mathcal{L} \times N_b/200$. This correction has been applied in Table 3 (showed earlier).
- When the optimal number of bunches is less than (roughly) 200, single ring operation is satisfactory, and hence favored.
- When the optimal number of bunches is much greater than 200, for example at the Z_0 energy, two rings are better.
- Note though, that the Z_0 single ring luminosities are still very healthy. In fact, with $\beta_y^*{=}10\,\mathrm{mm}$, which is a more conservative estimate than most others in this paper and in other FCC reports, the Z_0 single ring penalty is substantially less.

E GeV	β_y^* m	$\xi_{\rm sat}$	$\mathcal{L}_{\rm actual}$ 10^{34}	$N_{\rm actual}$	$P_{\rm rf}$ MW/beam
46	0.002	0.094	0.161	200	25
80	0.002	0.1	0.176	200	25
100	0.002	0.1	0.182	200	25
120	0.002	0.1	0.188	200	25
175	0.002	0.12	0.200	200	25
46	0.005	0.094	1.165	200	25
80	0.005	0.1	1.282	200	25
100	0.005	0.1	1.334	200	25
120	0.005	0.1	1.145	166	25
175	0.005	0.12	0.369	50	25
46	0.010	0.094	5.247	200	25
80	0.010	0.1	1.932	66.5	25
100	0.010	0.1	0.989	32.7	25
120	0.010	0.1	0.573	18.3	25
175	0.010	0.12	0.185	5.5	25

Table: Luminosites achievable with a single ring with number of bunches N_b limited to 200, 100 km circumference and 25 MW/beam RF power. The luminosity entries in (earlier) Table 2 were obtained from this table.

A Circular Collider is Not a Linear Collider

- “Final focus” (like “funeral”) is a place where electrons in a linear collider go to die.
- The “advantage” a circular collider has over a linear collider is that every particle has millions of chances to collide with a particle in the other beam.
- The term “intersection region” or “IR” is appropriate for a section of a storage ring in which the particles survive.
- Applying the term “final focus” to the IR of a circular collider is a crime against language.
- This is not just pedantry. It is the source of the common mistake of assuming the linear collider final focus optics can simply be inserted into a storage ring.
- The “disadvantage” of a circular collider is that a particle has to survive millions of passages through the other beam. This makes the storage ring IR optics far more difficult.

name	E GeV	ϵ_x nm	β_y^* mm	ϵ_y pm	$\xi_{\rm sat}$	$N_{\rm tot}$	σ_y μm	σ_x μm	u_c^* GeV	$n^*_{\gamma,1}$	$\mathcal{L}^{\rm RF}$ 10^{34}	$\mathcal{L}^{\rm bs}_{\rm trans}$ 10^{34}	$\mathcal{L}^{\rm bb}$ 10^{34}	N_b	β_x^* m	$P_{\rm rf}$ MW
Z	46	0.916	2	61.1	0.094	7.3e+14	0.35	5.24	0.000	1.97	52.5	96.8	52.513	33795	0.03	50
W	80	0.323	2	21.6	0.101	7.6e+13	0.208	3.12	0.001	2.06	9.66	16.2	9.661	5696	0.03	50
LEP	100	0.215	2	14.3	0.101	3.1e+13	0.169	2.54	0.002	2.10	4.95	8	4.947	2814	0.03	50
H	120	0.153	2	10.2	0.102	1.5e+13	0.143	2.15	0.003	2.13	2.86	4.48	2.863	1581	0.03	50
tt	175	0.077	2	5.12	0.118	3.3e+12	0.101	1.52	0.006	2.19	0.923	1.35	0.923	478	0.03	50
Z	46	16.5	5	1100	0.094	7.3e+14	2.35	35.21	0.001	2.12	21	33.2	21.005	1872	0.075	50
W	80	5.88	5	392	0.101	7.6e+13	1.4	20.99	0.003	2.22	3.86	5.52	3.864	313	0.075	50
LEP	100	3.91	5	261	0.101	3.1e+13	1.14	17.12	0.005	2.26	1.98	2.71	1.979	154	0.075	50
H	120	2.80	5	187	0.102	1.5e+13	0.966	14.50	0.007	2.30	1.15	1.52	1.145	86	0.075	50
tt	175	1.41	5	94	0.118	3.3e+12	0.686	10.28	0.016	2.38	0.369	0.455	0.369	26	0.075	50
Z	46	149	10	9900	0.094	7.3e+14	9.95	149.28	0.002	2.24	10.5	14.7	10.503	208	0.15	50
W	80	53.1	10	3540	0.101	7.6e+13	5.95	89.26	0.007	2.36	1.93	2.42	1.932	34	0.15	50
LEP	100	35.4	10	2360	0.101	3.1e+13	4.86	72.88	0.011	2.41	0.989	1.19	0.989	17	0.15	50
H	120	25.4	10	1700	0.102	1.5e+13	4.12	61.78	0.016	2.45	0.573	0.663	0.573	9.5	0.15	50
tt	175	12.9	10	857	0.118	3.3e+12	2.93	43.92	0.035	2.54	0.185	0.198	0.185	2.9	0.15	50

Table: Luminosity influencing parameters and luminosities with unlimited number of bunches N_b, assuming 50 km circumference ring and 50MW per beam RF power.

E GeV	β_y^* m	$\xi_{\rm sat}$	$\mathcal{L}_{\rm actual}$ 10^{34}	$N_{\rm b,actual}$	$P_{\rm rf}$ MW
46	0.002	0.094	0.174	112	50
80	0.002	0.1	0.190	112	50
100	0.002	0.1	0.197	112	50
120	0.002	0.1	0.203	112	50
175	0.002	0.12	0.216	112	50
46	0.005	0.094	1.256	112	50
80	0.005	0.1	1.380	112	50
100	0.005	0.1	1.434	112	50
120	0.005	0.1	1.145	86.6	50
175	0.005	0.12	0.369	26.1	50
46	0.010	0.094	5.644	112.0	50
80	0.010	0.1	1.932	34.7	50
100	0.010	0.1	0.989	17.1	50
120	0.010	0.1	0.573	9.5	50
175	0.010	0.12	0.185	2.9	50

Table: Luminosity influencing parameters and luminosities with the number of bunches limited to $N_b = 112$, assuming 50 km circumference ring and 50MW per beam RF power.

name	E GeV	ϵ_x nm	β_y^* mm	ϵ_y pm	$\xi_{\rm sat}$	$N_{\rm tot}$ 10^{12}	σ_y μm	σ_x μm	u_c^* GeV	$n_{\gamma,1}^*$	$\mathcal{L}^{\rm RF}$ 10^{34}	$\mathcal{L}_{\rm trans}^{\rm bs}$ 10^{34}	$\mathcal{L}^{\rm bb}$ 10^{34}	N_b	β_x^* m	$P_{\rm rf}$ MW
Z	46	0.949	2	63.3	0.094	1500	0.356	5.34	0.000	2.01	52.5	103	52.5	65243	0.03	25
W	80	0.336	2	22.4	0.101	150	0.212	3.17	0.001	2.10	9.66	17.2	9.6	10980	0.03	25
LEP	100	0.223	2	14.9	0.101	62	0.172	2.59	0.002	2.13	4.95	8.46	4.94	5421	0.03	25
H	120	0.159	2	10.6	0.102	30	0.146	2.19	0.003	2.17	2.86	4.74	2.86	3044	0.03	25
tt	175	0.078	2	5.33	0.118	6.6	0.103	1.55	0.006	2.24	0.923	1.43	0.92	920	0.03	25
Z	46	17.2	5	1140	0.094	1500	2.39	35.89	0.001	2.16	21	35.1	21.	3605	0.075	25
W	80	6.11	5	408	0.101	150	1.43	21.42	0.003	2.26	3.86	5.83	3.86	602	0.075	25
LEP	100	4.07	5	271	0.101	62	1.16	17.47	0.005	2.31	1.98	2.86	1.97	296	0.075	25
H	120	2.92	5	195	0.102	30	0.987	14.80	0.008	2.35	1.15	1.6	1.14	166	0.075	25
tt	175	1.47	5	98.1	0.118	6.6	0.7	10.51	0.017	2.43	0.369	0.479	0.37	49	0.075	25
Z	46	155	10	10300	0.094	1500	10.2	152.3	0.002	2.29	10.5	15.5	10.5	400	0.15	25
W	80	55.4	10	3690	0.101	150	6.08	91.17	0.007	2.41	1.93	2.55	1.93	66	0.15	25
LEP	100	37.0	10	2470	0.101	62	4.97	74.48	0.011	2.46	0.989	1.25	0.99	32	0.15	25
H	120	26.6	10	1770	0.102	30	4.21	63.15	0.016	2.50	0.573	0.696	0.57	18.3	0.15	25
tt	175	13.5	10	898	0.118	6.6	3.0	44.94	0.036	2.60	0.185	0.207	0.19	5.5	0.15	25

Table: The major factors influencing luminosity, assuming 100 km circumference and 25 MW/beam RF power. The predicted luminosity is the smallest of the three luminosities, $\mathcal{L}^{\rm RF}$, $\mathcal{L}_{\rm trans}^{\rm bs}$, and $\mathcal{L}^{\rm bb}$. All entries in this table apply to either one ring or two rings, except where the number of bunches N_b is too great for a single ring.

Cost Optimization

Treating the cost of the 2 detectors as fixed, and letting C be the cost exclusive of detectors, the cost can be expressed the sum of a term proportional to size and a term proportional to power;

$$C = C_R + C_P \equiv c_R R + c_P P_{\mathrm{rf}} \tag{12}$$

where c_R and c_P are unit cost coefficients. The radius x power scaling law gives

$$P_{\mathrm{rf}} = \frac{\mathcal{L}}{k_1 R}. \tag{13}$$

Minimizing C at fixed $\mathcal{L}$ leads to

$$\boxed{R_{\mathrm{opt}} = \sqrt{\frac{1}{k_1}\,\frac{c_P}{c_R}\,\mathcal{L}}.} \tag{14}$$

Conventional thinking has it that c_P is universal world wide but, at the moment, c_R is thought to be somewhat cheaper in China than elsewhere. If so, the optimal radius should be somewhat greater in China than elsewhere.