Emittance Evolution in Hadron Colliders

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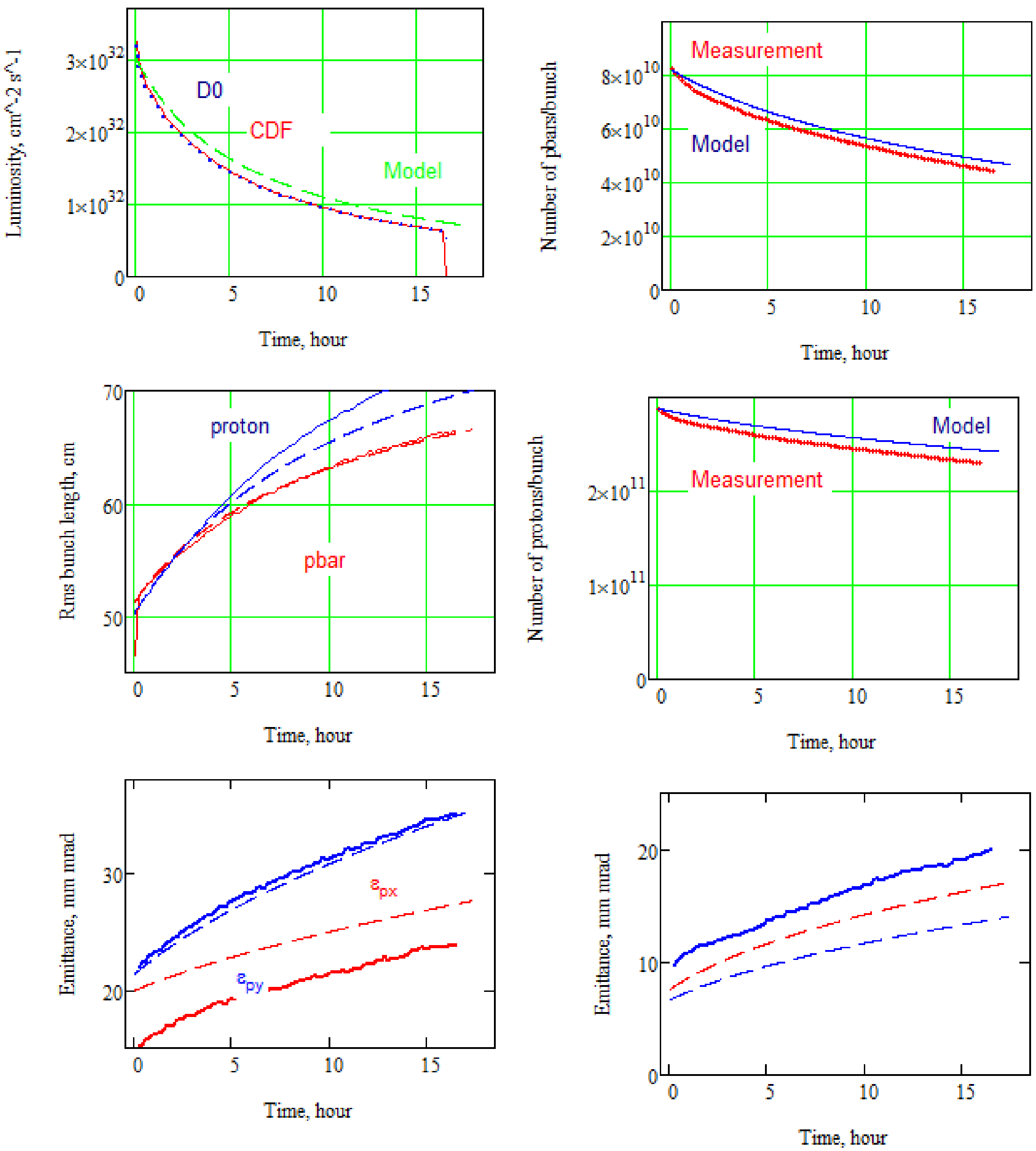
FCC Week 2015
March 22-27, 2015
Washington, DC

1. Luminosity Evolution Model

- The model was developed to describe Tevatron stores
- It accounts for major beam heating and particle loss mechanisms
  - Phenomena taken into account
    - Interaction with residual gas
      - Emittance growth due to multiple electromagnetic scattering
      - Particle loss due to single nuclear and electromagnetic interaction
    - Particle interaction in IPs (proportional to the luminosity)
      - Elastic + inelastic scattering
    - IBS
      - Multiple - momentum spread and emittance growth
      - Single – jump out of RF bucket
    - Bunch lengthening due to RF noise
      - Associated particle loss from the bucket
    - LHC and FCC specific
      - SR damping and diffusion
      - Emittance growth due to noise of transverse damper and e.-m. noise
      - Orbit length variation due to micro-seism
  - Phenomena ignored in the model: Beam-beam effects, Ring non-linearity, Diffusion amplification by coherent effects (typically small corrections)

Luminosity Evolution Model (Tevatron)



Store 7732

Effects taken into account

- IBS (L & ⊥)
- Gas scattering
- Loss due to luminosity
- RF noise

In. luminosity lifetime–5hour

Emittance lifetime:

Protons - ~20 hour

Pbars - ~7 hour

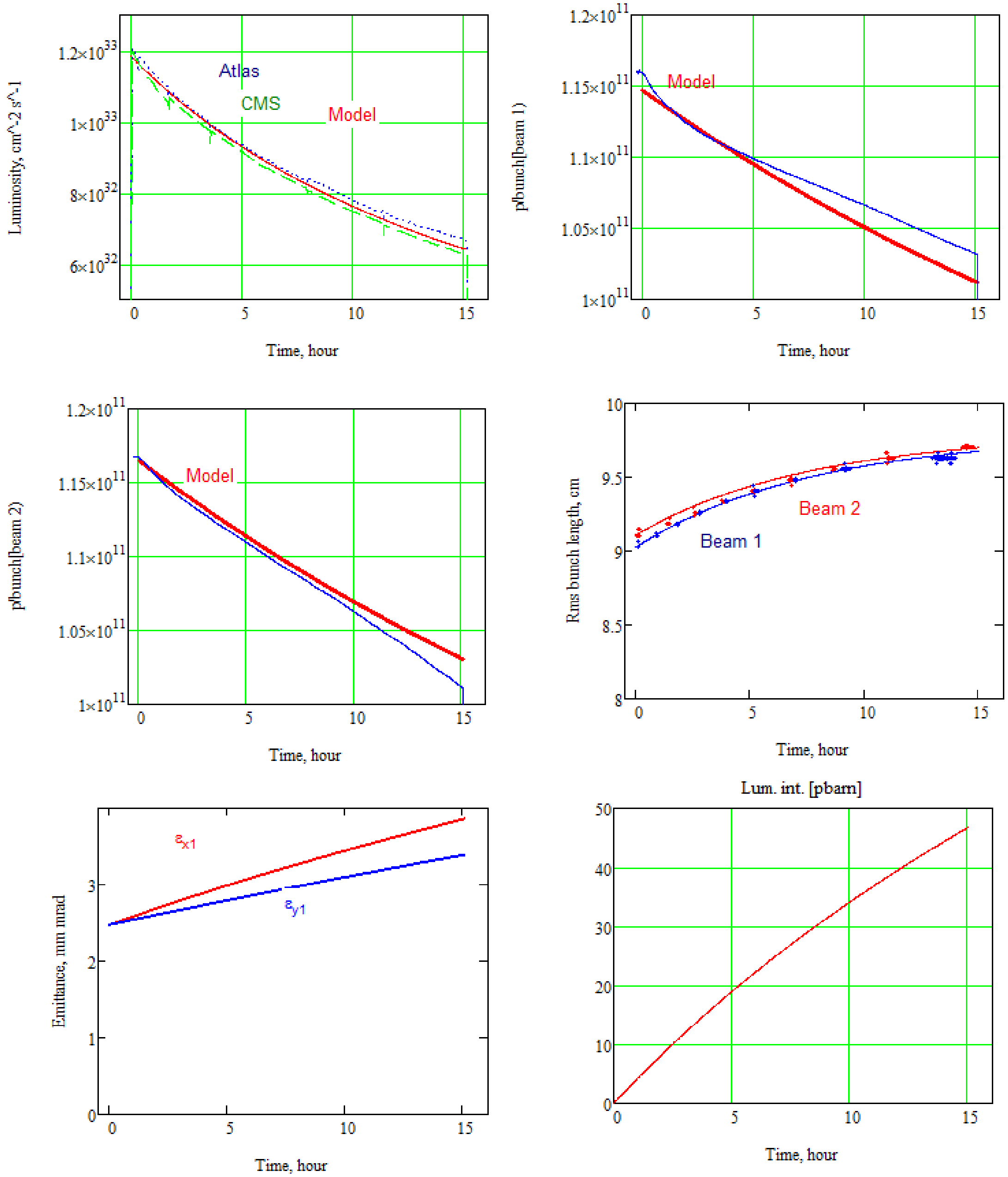
Intensity lifetime

Protons - ~61 hour

Pbars - ~19 hour

Beam-beam effects results in ~10% loss in lum. integral

Luminosity Evolution Model (LHC: 2*3.5 TeV)



Fill 1852

Effects taken into account

- IBS (L & ⊥ (64 hour))
- Emittance growth due to noise of transverse damper (44 hour)
- Gas scattering
- Loss due to luminosity

Luminosity lifetime – 18 hour

Emittance lifetime:

Protons - ~25 hour

Intensity lifetime

Protons - ~100 hour

Beam-beam effects result in ~10% loss in lum. integral

IBS

- Growth rate estimates

$$\tau_{\|}^{-1} = \frac{1}{\overline{\theta_{\|}^2}} \frac{d}{dt}\left(\overline{\theta_{\|}^2}\right) \equiv \frac{d}{dt}\left(\frac{\overline{p_{\|}^2}}{p}\right) \approx \frac{r_p^2 c N L_C Q^{1/2}}{8\gamma^{3/2}\varepsilon_n^{3/2} R_0^{1/2}\sigma_s \theta_{\|}^2}, \quad \beta_x \approx \beta_y \approx \beta \equiv \frac{R_0}{Q}$$

$$\tau_x^{-1} = \frac{1}{\varepsilon_x}\frac{d\varepsilon_x}{dt} \approx \frac{r_p^2 c N L_C R_0^{3/2}}{8\gamma^{1/2}\varepsilon_n^{5/2} Q^{5/2}\sigma_s}, \qquad \sigma_x \approx \sigma_y \approx \sqrt{\varepsilon\beta}$$

$$\tau_y^{-1} = \frac{1}{\varepsilon_y}\frac{d\varepsilon_y}{dt} \approx 0, \qquad \varepsilon_n = \gamma\varepsilon$$

- Increase of $\tau_{||}$ for LHC is mainly related with increase of γ and R_0: $\propto\gamma^2$
- Increase of τ_x for LHC is mainly related with increase of betatron tune, Q

	$\tau_{\|\|}$ [hour]	τ_x [hour]
Tevatron (prot)	12	15
LHC (3.5 TeV)	20	32
LHC (6.5 TeV)	50	80
FCC (50 TeV)	75	60

- IBS is a major mechanism of the emittance growth (luminosity loss) for Tevatron
- It plays comparatively small role for higher energy machines

IBS in Tevatron

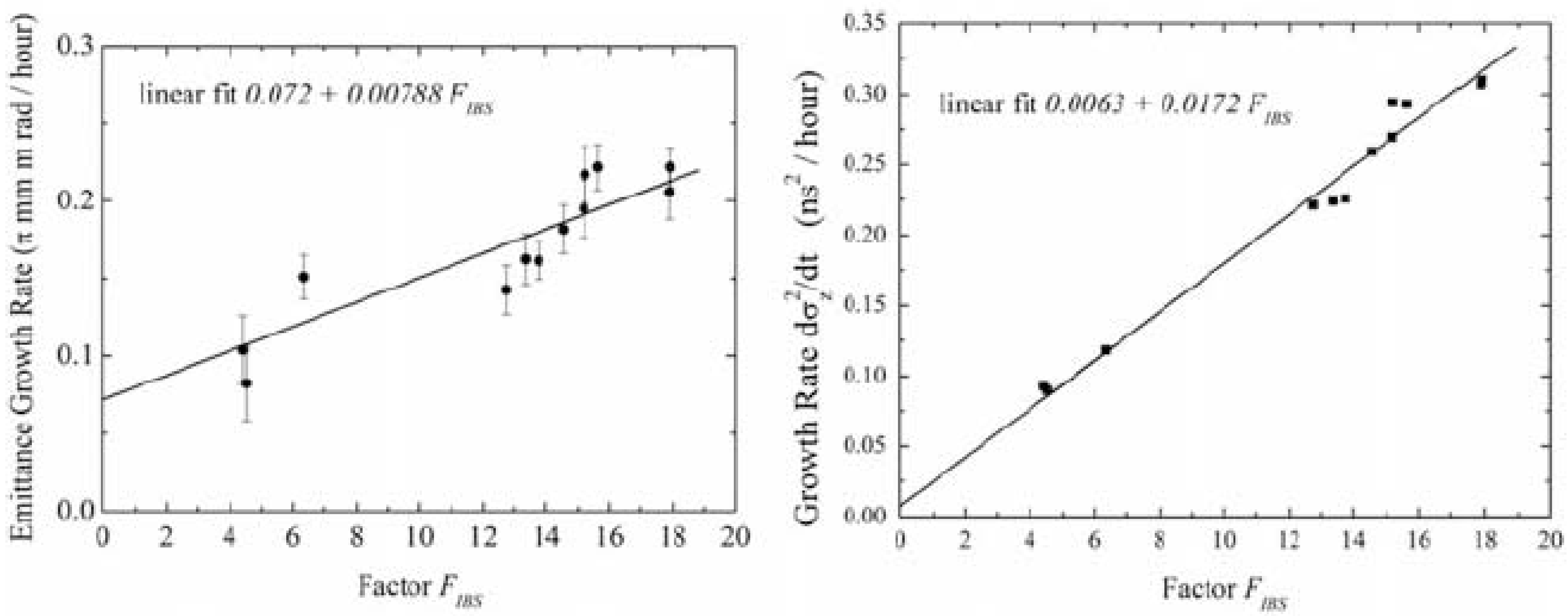


Fig. 6.12 Vertical emittance growth rates (rms, norm.) of proton bunches vs the IBS factor F_{IBS} (*left*); the rms bunch length growth rates vs the IBS factor F_{IBS} (*right*) [20]

$$F_{IBS} = \frac{N_p}{\varepsilon_\perp^{3/2}\sigma_s}$$

- $\perp$ emittance growth in Tevatron has a contribution additional to IBS
  - It significantly exceeds dε/dt driven by residual gas scattering
  - Present understanding – noise driven growth
    - Schottky monitor signal (~20 MHz) exceeds actual Schottky signal by about an order of magnitude
- At injection energy the "residual" emittance growth is dominated by multiple scattering on the residual gas ($d\varepsilon/dt_{gas} \propto 1/\gamma^2$; $d\varepsilon/dt_{noise} \sim const$)

Emittance Growth due to Transverse Noise

$$\frac{d\varepsilon}{dt}=\frac{16\pi^2\overline{\Delta\nu^2}}{g^2}\left(\left(\frac{d\varepsilon}{dt}\right)_0+\frac{f_0 g^2}{2\beta_{BPM}}\overline{x_{BPM}}^2\right)$$

$$\left(\frac{d\varepsilon_{x,y}}{dt}\right)_0=\beta_{x,y}\left(\frac{el}{Pc}\right)^2\frac{\omega_0^{\ 2}}{4\pi}\sum_{n=-\infty}^{\infty}S_{\delta B}\left((\nu-n)\omega_0\right)$$

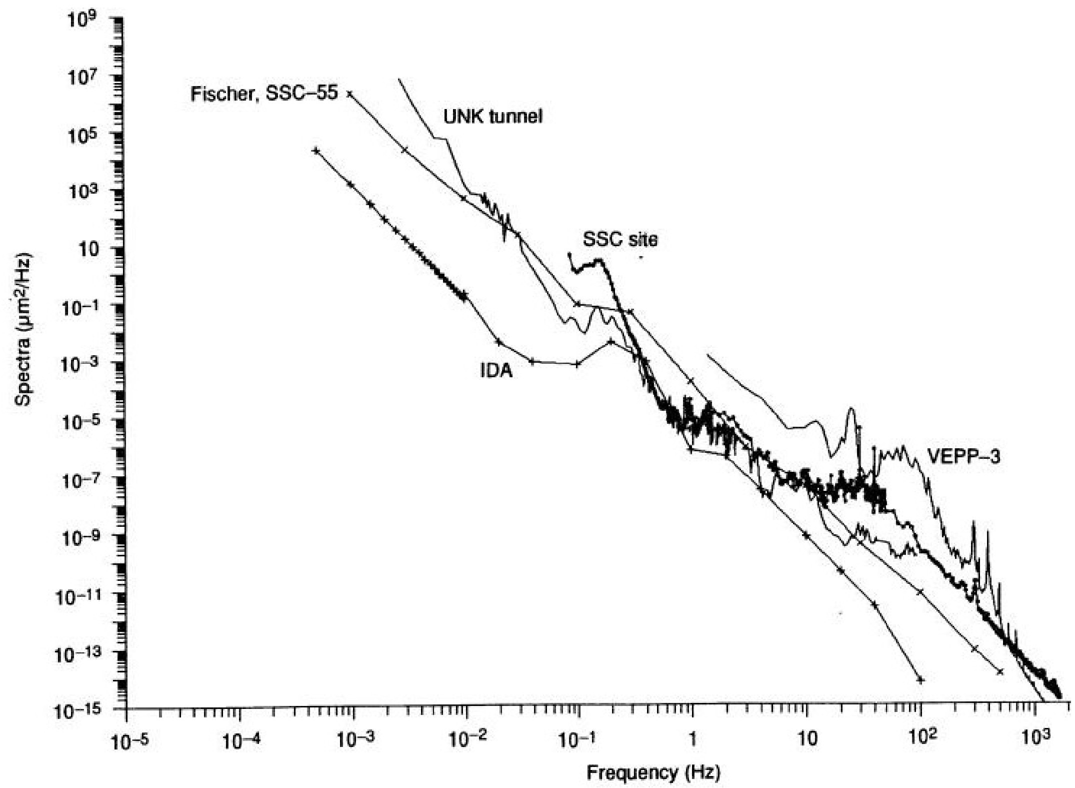
- The growth of feedback system gain, *g*, does not affect dε/dt
- For a collider the tune spread is dominated by beam-beam tune shift

$$\sqrt{\overline{\Delta\nu^2}}\simeq 0.2\xi_{tot}$$

- Observed emittance growth for the LHC fill 1852 corresponds to the effective noise of ~0.2 μm for 2 systems (H&V)
- Required noise in magnetic field of single dipole for the FCC (no damper, white noise):
  ΔB/B~1.5·10^{-9} for the 2 hour emittance growth time
  - Spectral density of ΔB/B fluctuations at ~1 kHz is unknown
    - Study is required
- Required BPM resolution: 0.5 μm for 2 hour emittance growth time
  - Close to what has been achieved for the LHC (SR helps)

FCC versus LHC

- Next hadron collider main features
  - ~1.5 times larger magnetic field & ~7 times larger energy
    ⇨ SR damping time: $1.5^2 \times 7 \approx 15$ times faster or ~1 hour
    ⇨ Revolution frequency $7/1.5 \approx 5$ times smaller
    ⇨ Spectral density of seismic noise $\propto 1/f^{3.5}$ => ~200 times larger
- Noise driven emittance growth is not a negligible problem for the LHC
  - It is suppressed by low noise transverse dampers (together with instabilities)
  - If not properly addressed the noise is going to be a major source of emittance growth



- If noise is too large the emittance growth cannot be suppressed to the required level
  ⇨ FRS requirements, mech. design & experimental studies

RF Noise

- At small amplitude the bunch lengthening due to RF phase noise is determined by its spectral density at synchrotron frequency,

$$\left.\frac{d\left(\sigma_\varphi{}^2\right)}{dt}\right|_{RF} = \pi\Omega_s{}^2 \sum_{n=-\infty}^{\infty} \mathrm{P}_\varphi\left(\Omega_s + nf_0\right) \quad ,$$

where the spectral density of RF phase noise is normalized as

$$\overline{\phi_{RF}{}^2} = \int_{-\infty}^{\infty} \mathrm{P}_\phi(\omega)d\omega$$

- For the white noise: $$\left.\frac{d\left(\sigma_\varphi{}^2\right)}{dt}\right|_{RF} = \frac{f_0}{2}\Omega_s{}^2\overline{\delta\phi^2}$$

- For Tevatron the measured $P_\phi(\omega)$ and bunch lengthening are in decent agreement:

$$P_{\phi f}\left(\Omega_s/2\pi\right) = 4\pi P_\phi\left(\Omega_s\right) \approx 6\cdot 10^{-12} \quad \mathrm{rad}^2/\mathrm{Hz} \quad \Leftrightarrow \quad \left.\frac{d\left(\sigma_\phi{}^2\right)}{dt}\right|_{RF} \approx 16 \quad \mathrm{mrad}/\sqrt{\mathrm{hour}}$$

- For the FCC the required turn-by-turn rms phase stability in the absence of damping (f_s=2.5 Hz): ~2 deg (not a problem, SR helps)
  - ♦ Corresponding requirements to path lengthening due to microseism at f_s is not a problem: δL < 0.5 cm (rms)

Summary

- Noise in the magnetic field of dipoles can drive unacceptably large emittance growth
  - Experimental measurements of noise spectral density in magnetic field of dipoles are required
  - Engineering, including civil construction, has to be aimed it reduction of mechanical vibrations and noise of power supplies at the betatron sidebands (frequency of the lowest one is ~1 kHz)
  - The liner should not have mechanical frequencies at betatron sidebands
- Requirements to the noise of transverse damper are similar to those obtained at the LHC
- Transverse noise of the damper and noise of magnetic field in dipoles have to be included in the luminosity evolution model
- Requirements to the noise of longitudinal damper are similar to those obtained at the LHC

Backup Slides

Residual Gas Scattering

- Beam life time

$$\tau_{scat}^{-1} = \frac{2\pi c r_p^{\,2}}{\gamma^2 \beta^3}\left(\sum_i n_i Z_i (Z_i+1)\right)\left(\frac{\overline{\beta_x}}{\varepsilon_{mx}}+\frac{\overline{\beta_y}}{\varepsilon_{my}}\right)+\sum_i n_i \sigma_i c\beta$$

  only second addend is important. It has weak dependence on energy
  - Typical lifetimes
    - Tevatron: 300 – 600 hours
    - LHC > 1000 hours (much better average vacuum)
    - FCC – should be close to Tevatron (SR will affect vacuum)
- Emittance growth time

$$\frac{1}{\varepsilon_{x,y}}\frac{d\varepsilon_{x,y}}{dt} = \frac{2\pi c r_p^{\,2}}{\gamma\beta^2 \varepsilon_{n_{x,y}}}\left(\sum_i n_i Z_i (Z_i+1) L_{C_i}\right)\overline{\beta_{x,y}}$$

  $\beta_{x,y} \propto \gamma$ => weak dependence on energy
  - Typical growth rate times
    - Tevatron: 300 – 600 hours
    - LHC >> 1000 hours (much better average vacuum)
    - FCC – should be close to Tevatron (SR will affect vacuum)

Bunch lengthening due to RF phase noise

$$\ddot{x}+\Omega_s{}^2\sin(x-\psi(t))=0 \quad \Rightarrow \quad \ddot{x}+\Omega_s{}^2\sin(x)=\Omega_s{}^2\cos(x)\psi(t)$$

Action - $I=\frac{1}{2\pi}\oint p\,dx$ Frequency - $\omega\equiv\omega(I)=2\pi\left(\oint\frac{dx}{p}\right)^{-1}$

Introduce the diffusion coefficient using the following form of diff. eq.

$$\frac{\partial f}{\partial t}=\frac{1}{2}\frac{\partial}{\partial I}\left(I\frac{D(I)}{\omega(I)}\frac{\partial f}{\partial I}\right)$$

where diffusion coefficient is

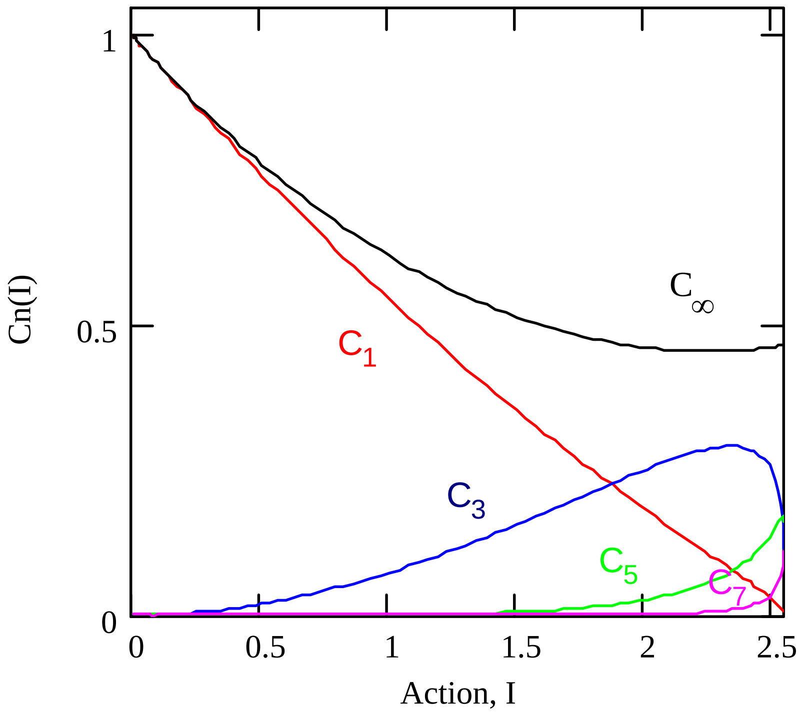
$$D(I)=\frac{\omega}{I}\frac{d}{dt}\overline{\delta I^2}=2\pi\Omega_s{}^2\sum_{n=0}^{\infty}C_n(I)P(n\omega(I)) \quad ,$$

and the spectral density is normalized a

$$\overline{\psi(t)^2}=\int_{-\infty}^{\infty}P(\omega)d\omega \quad .$$

For the white noise, $P(\omega)=P_0$, it yields

$D(I)=2\pi\Omega_s{}^2P_0C_\infty(I)$ where $C_\infty(I)=\sum_{n=0}^{\infty}C_n(I)$



For all even n, $C_n(I)=0$

- For the LHC the effect of RF noise is increased near bucket boundary due to bucket nonlinearity (spectral density goes to high f)

Direct measurement of RF noise performed by John Reid

- Microphonics - cavity mechanical resonances are at synchrotron frequency
  - ➢ Phase feedback suppresses microphonics by more than 20 Db
- Longitudinal damper is too noisy
  - ➢ Damper "white" noise hides mechanical resonances