



SAPIENZA  
UNIVERSITÀ DI ROMA

DIPARTIMENTO DI SCIENZE DI BASE  
E APPLICATE PER L'INGEGNERIA



## Impedance and collective effects (preliminary results)

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Acknowledgements:

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N. Monet, A. Mostacci, L. Palumbo, F. Zimmermann, M. Zobov

# Parameter list

parameter	Symbol	Value
Circumference	L	100 km
Beam energy	E	45.5 GeV
Beam current	$I_0$	1450 mA
Bunches/beam	$N_b$	16700
Bunch population	$N_p$	$1.8 \times 10^{11}$
Bunch length(*)	$\sigma_z$	2.3 mm
Energy spread(*)	$\sigma_\epsilon$	$3.7 \times 10^{-4}$
RF voltage	$V_{RF}$	2.5 GV
RF frequency	$f_{RF}$	400 MHz
Harmonic number	h	133600
Synchrotron tune	$Q_s$	0.46
Energy loss/turn	$U_0$	0.03 GeV
momentum compaction	$\alpha_c$	$18 \times 10^{-5}$

(\*) without beamstrahlung (no collision)

## Short range wakefield: Resistive wall

- To evaluate the RW impedance, we consider high conductivity such that the skin depth is much smaller than the wall thickness and

$$c\chi / b \ll \omega \ll c\chi^{-1/3} / b$$

with  $\chi = \frac{1}{Z_0 \sigma_c b}$

$$\chi^{1/3} b \ll z \ll b / \chi$$

$$10.6 \ll \omega \ll 5.7 \times 10^{12} \text{ rad/s}$$

$$Z_{\parallel}(\omega) = [1 - i \operatorname{sgn}(\omega)] \frac{L}{2\pi b} \sqrt{\frac{Z_0 |\omega|}{2c\sigma_c}}$$

$$Z_{\perp}(\omega) = [\operatorname{sgn}(\omega) - i] \frac{L}{2\pi b^3} \sqrt{\frac{2cZ_0}{|\omega| \sigma_c}}$$

$$52.5 \times 10^{-6} \ll z \ll 2.8 \times 10^7 \text{ m}$$

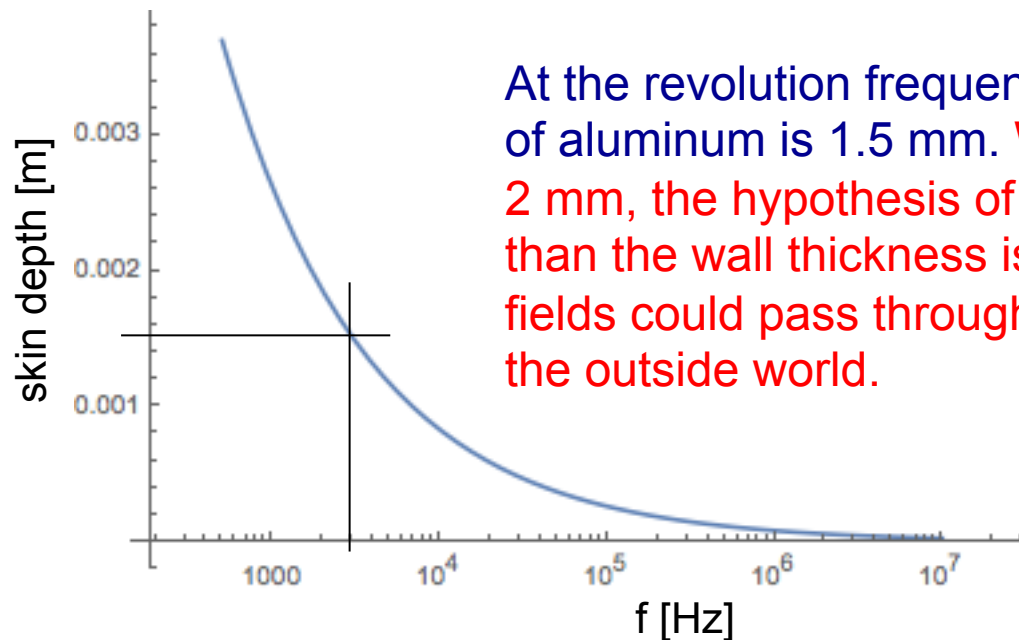
$$w_{\parallel}(z) = -\frac{Lc}{4\pi b} \sqrt{\frac{Z_0}{\pi\sigma_c}} \frac{1}{|z|^{3/2}}$$

$$w_{\perp}(z) = \frac{Lc}{\pi b^3} \sqrt{\frac{Z_0}{\pi\sigma_c}} \frac{1}{|z|^{1/2}}$$

## Short range wakefield: Resistive wall

- Vacuum pipe of aluminum

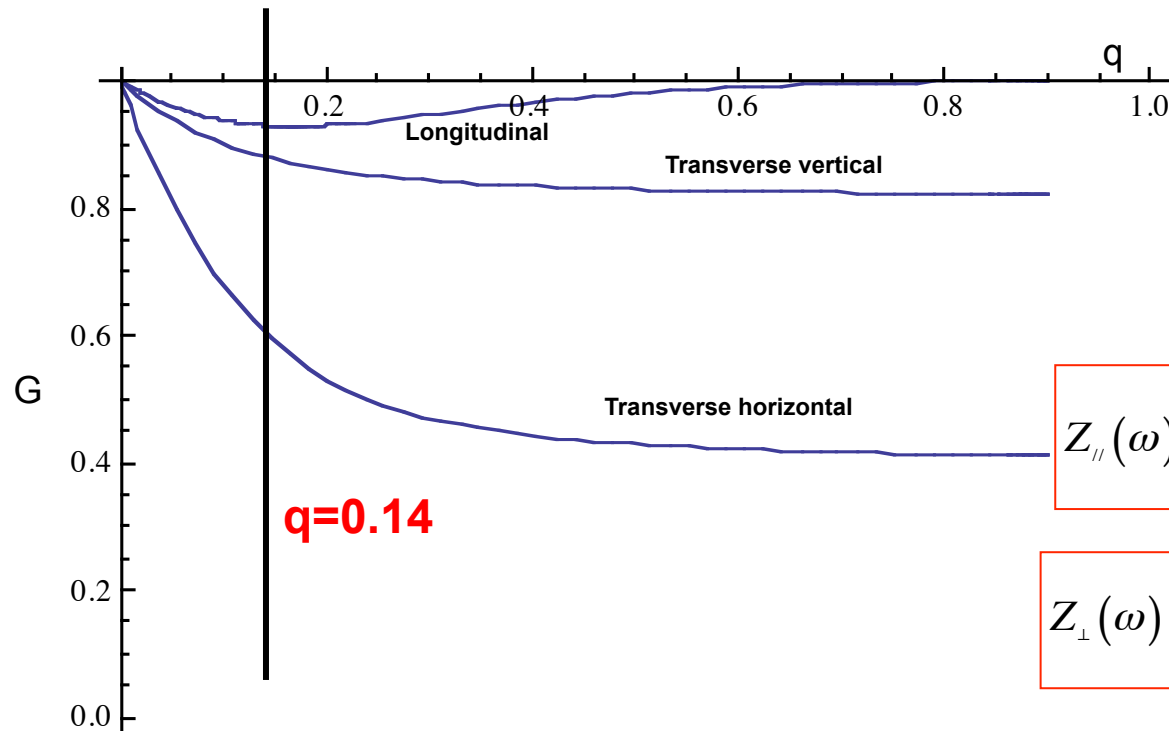
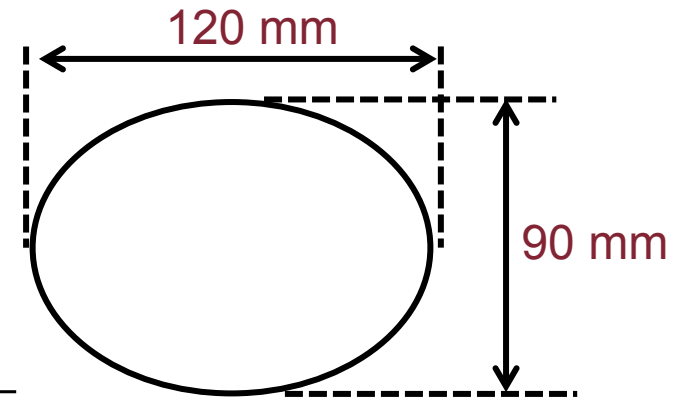
$$\sigma_c = 3.7 \times 10^7 \text{ S/m} \quad L = 10^5 \text{ m}$$



At the revolution frequency of 3 kHz, the skin depth of aluminum is 1.5 mm. With a vacuum chamber of 2 mm, the hypothesis of skin depth much smaller than the wall thickness is not fully satisfied and em fields could pass through the pipe interacting with the outside world.

# Short range wakefield: Resistive wall

- Elliptic chamber



Form factors  $G(u_0)$   
as a function of

$$q = \frac{a-b}{a+b} \quad q = e^{-2u_0}$$

$$Z_{||}(\omega) = [1 - i \operatorname{sgn}(\omega)] \frac{L}{2\pi b} \sqrt{\frac{Z_0 |\omega|}{2c\sigma_c}} G_{||}(u_0)$$

$$Z_{\perp}(\omega) = [\operatorname{sgn}(\omega) - i] \frac{L}{2\pi b^3} \sqrt{\frac{2Z_0 c}{|\omega| \sigma_c}} G_{\perp}(u_0)$$

## Short range longitudinal wakefield: Resistive wall

- The wake potential of a Gaussian bunch with 2.3 mm bunch length can be obtained by

$$W_{\parallel}(z) = \frac{cL}{8\sqrt{2\pi}b\sigma_z^{3/2}} \sqrt{\frac{Z_0}{\sigma_c}} F(z/\sigma_z) G_{\parallel}(u_0)$$

with

$$F(x) = |x|^{3/2} e^{-\frac{x^2}{4}} (I_{1/4} - I_{-3/4} \pm I_{-1/4} \mp I_{3/4})$$

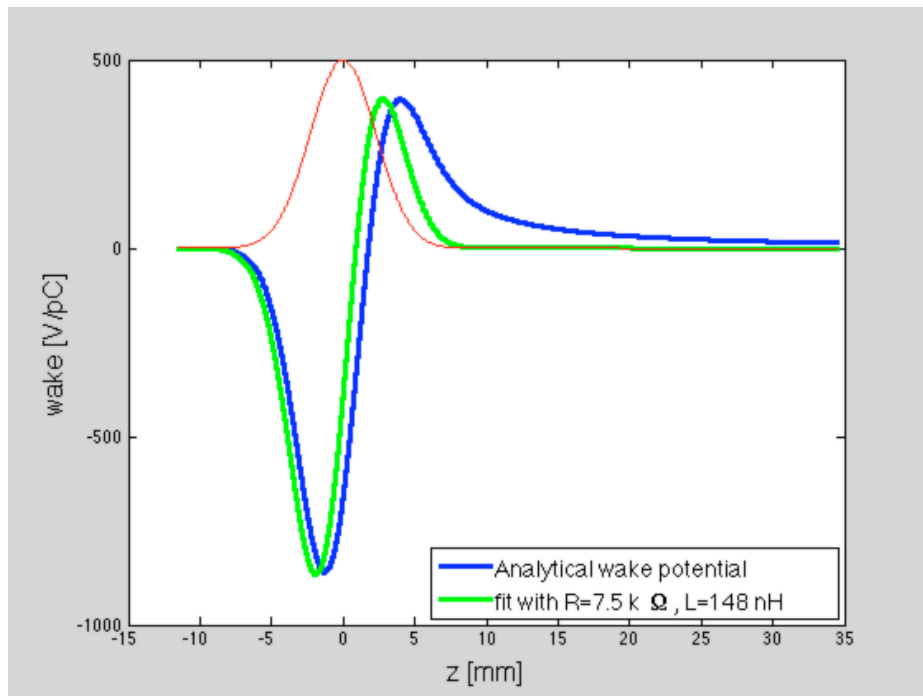
and  $I_n$  the modified Bessel function

fit with R-L impedance model

$$W(z) = -Rc\lambda(z) - Lc^2\lambda'(z)$$

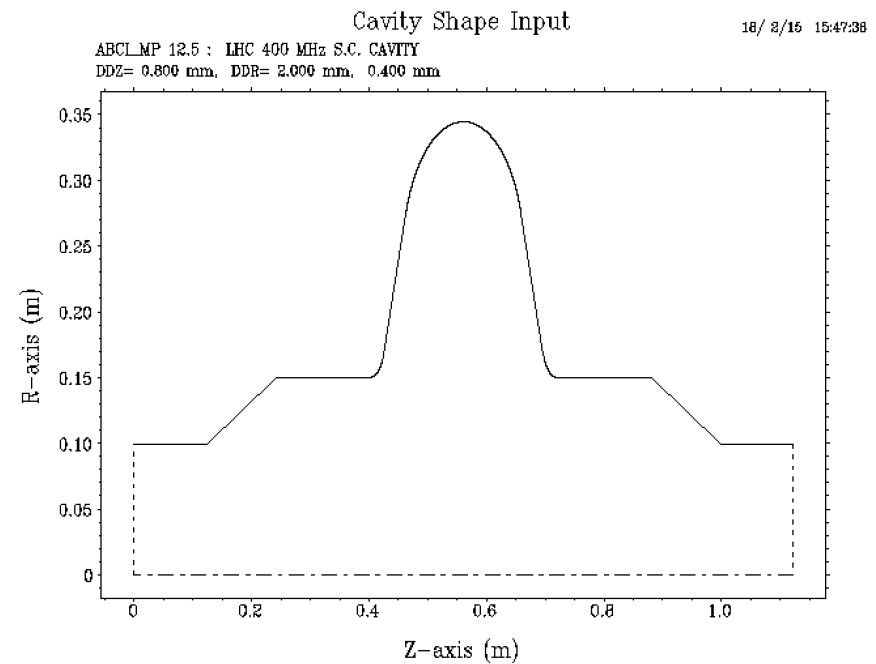
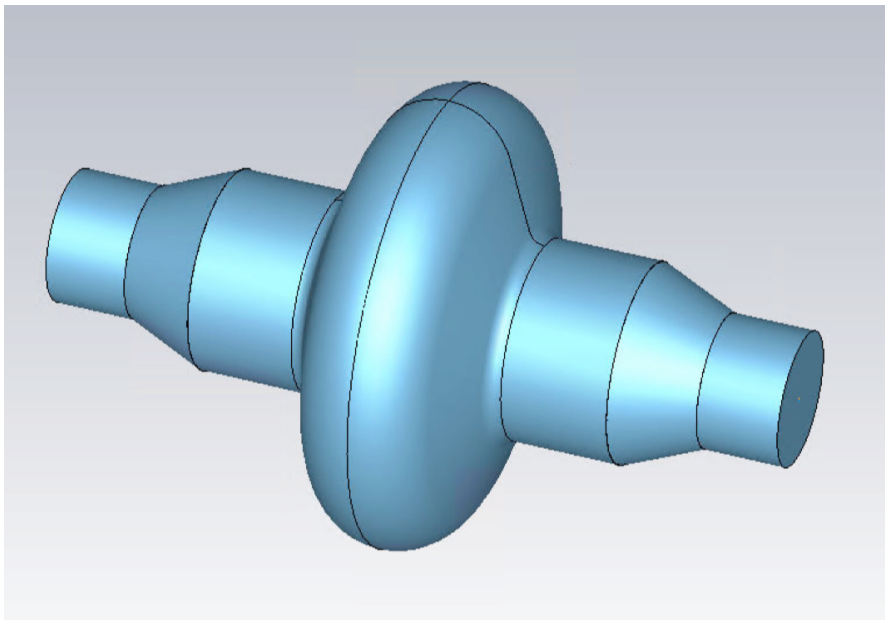
$R=7.5 \text{ k}\Omega$  and  $L=148 \text{ nH}$ .

The loss factor is 276 V/pC



## Short range wakefield: RF cavities

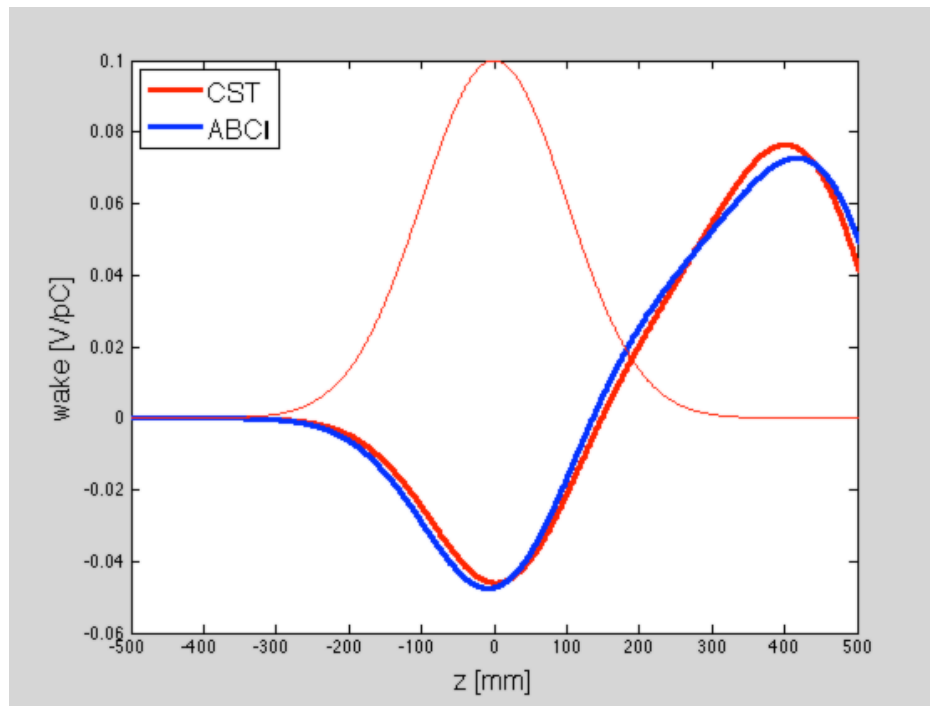
- 400 MHz cavities have been considered
- Both CST Microwave Studio and ABCI have been used to compare wakefields for a single cell



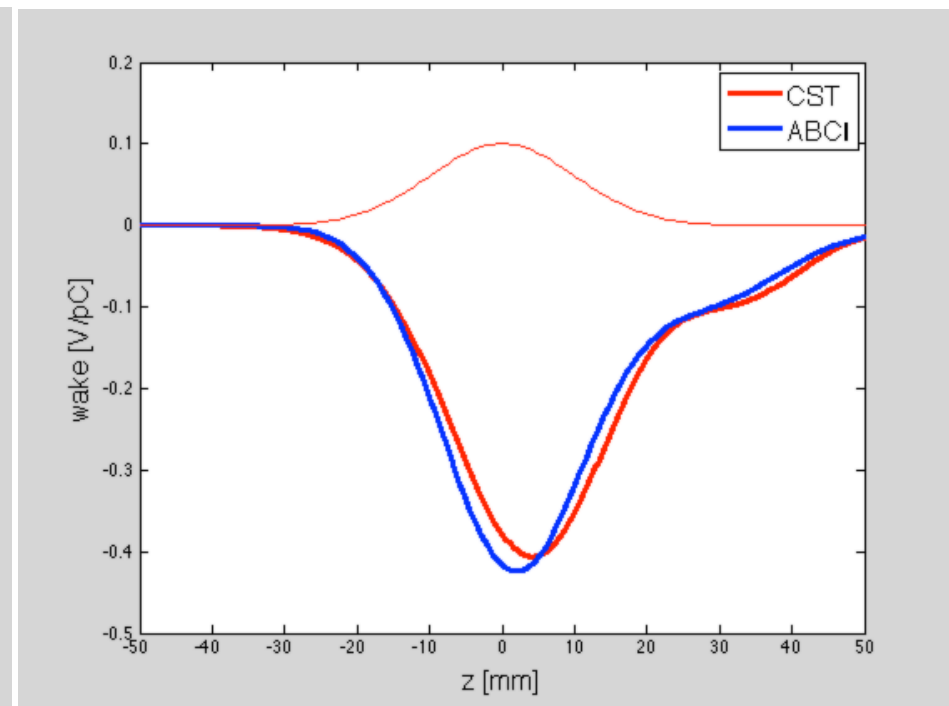
## Short range longitudinal wakefield: RF cavities

- Comparisons between ABCI and CST for different bunch lengths.

10 cm bunch length



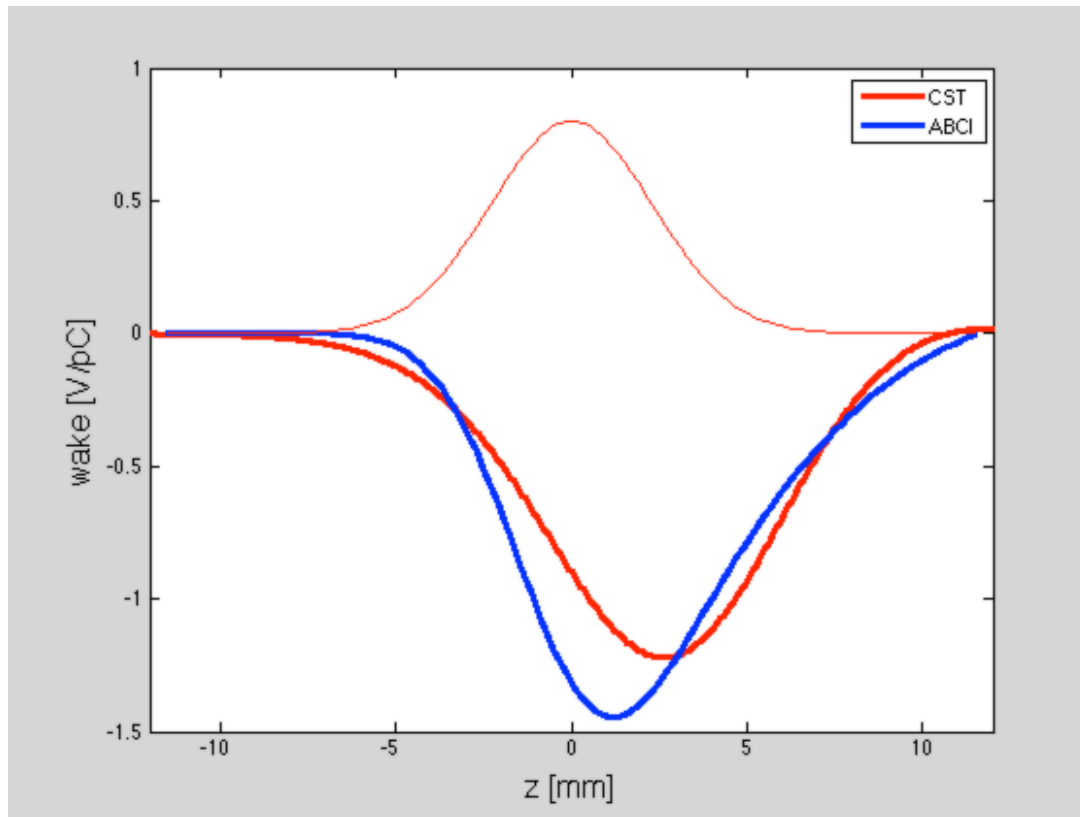
1 cm bunch length





## Short range longitudinal wakefield: RF cavities

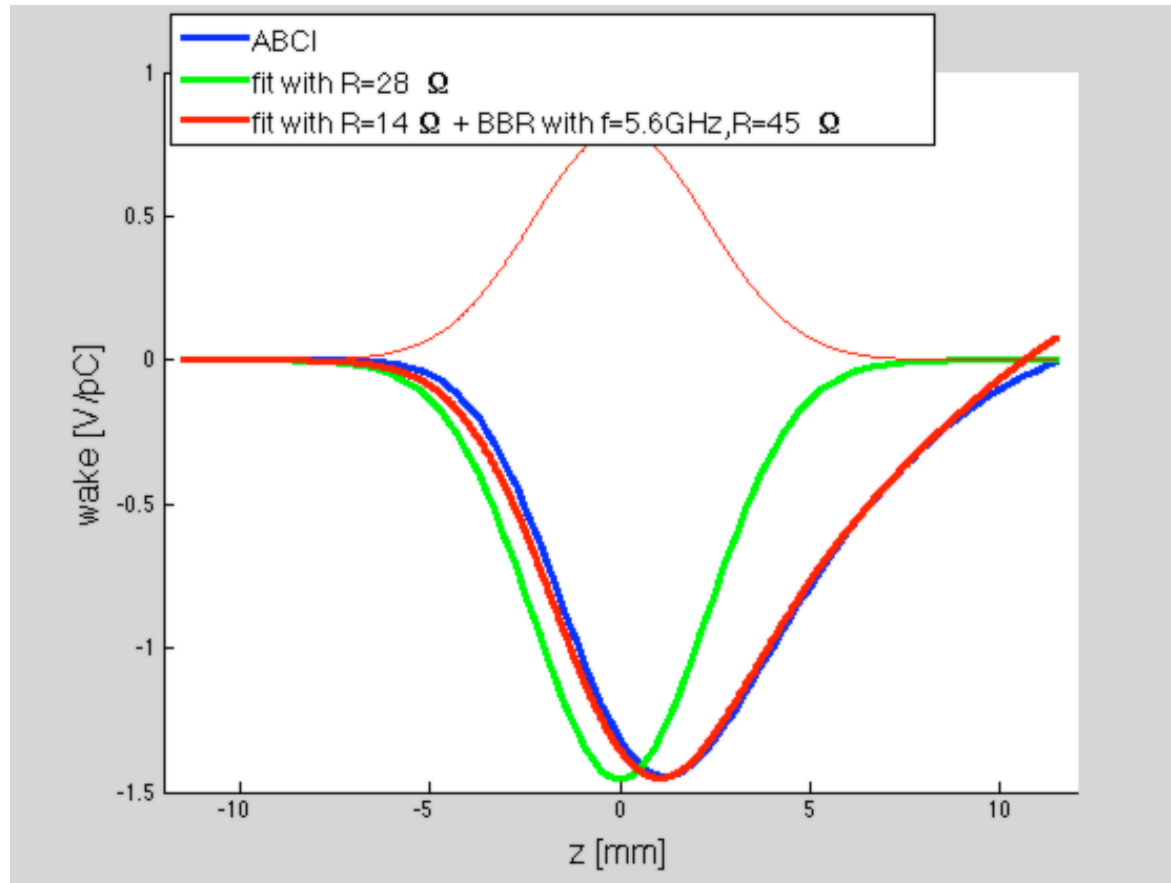
- For 2.3 mm bunch length ABCI and CST present some differences.



CST could have mesh problems because only a low number of lines per wavelength has been used. Nevertheless, in this example, 1.6 billion meshes have been used.

## Short range longitudinal wakefield: RF cavities

- Let us consider ABCI results ( $\sigma_z=2.3$  mm)



We still have a proportionality of the wake potential with the inverse of the bunch length

The loss factor for one cell is  **$K_{\text{loss}}=1.043$  V/pC**

The fit with the wake potential of the R-L impedance model is poor.

## Short range longitudinal wakefield: RF cavities

Rf Region	Frequency (MHz)	Per ring				
		Tubes 1 MW ea	Modules per tube	Modules 2 couplers ea	Voltage (MV)	Length (m)
1	400	13	8	104	1248	260
2	400	13	8	104	1248	260
3a	400	6	8	48	576	120
3b	400	6	8	48	576	120
4a	400	6	8	48	576	120
4b	400	6	8	48	576	120
<b>Total</b>		<b>50</b>		<b>400</b>	<b>4800</b>	<b>1000</b>
<b>Total for shared rf</b>		<b>100</b>		<b>800</b>	<b>9600</b>	<b>2000</b>

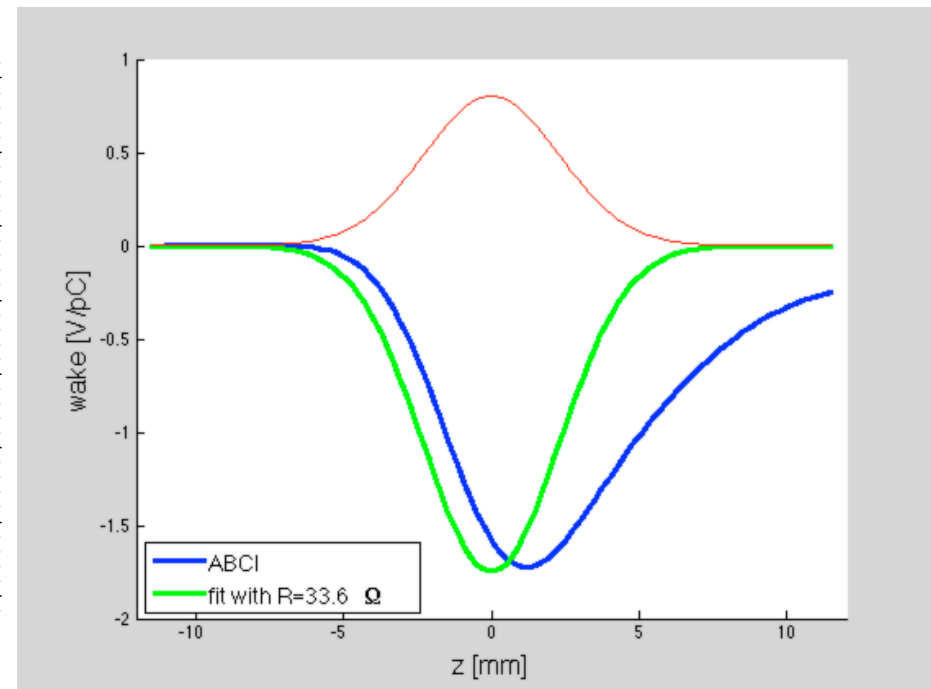
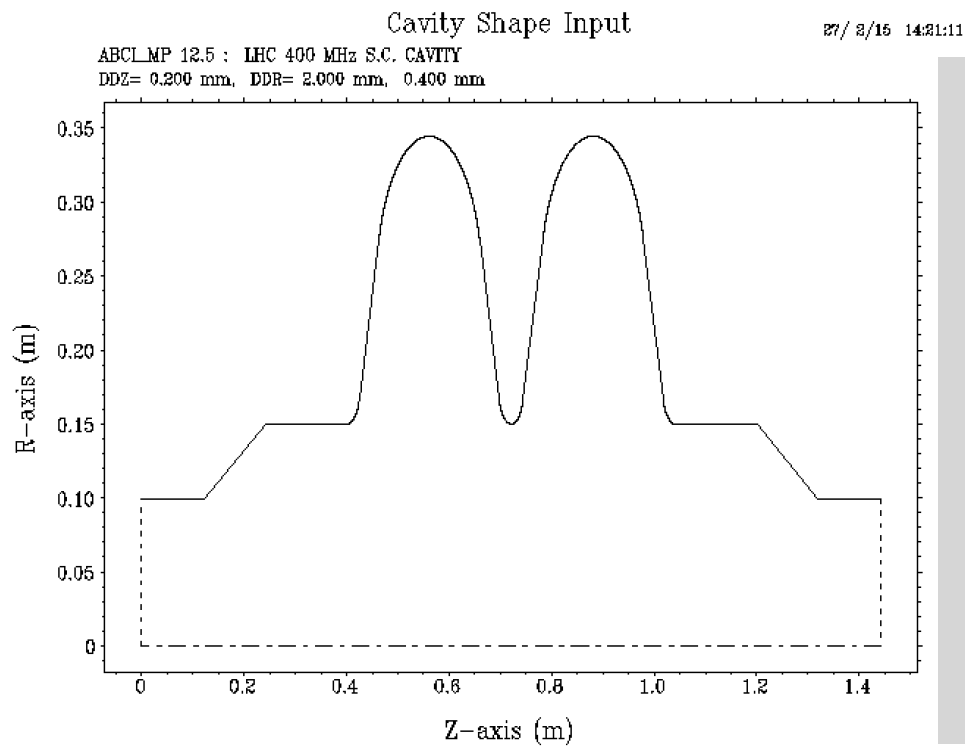
Courtesy of  
U. Wienands, SLAC  
Aspen Physics Center  
31-Jan-2015

- By considering 2 two-cell cavities per module, we get a total of 1600 cells, obtaining a total of 44.8 k $\Omega$
- For CEPC\* ring  $R=28.1$  k $\Omega$  (N. Wang, et al., 55th ICFA Advanced Beam Dynamics Workshop on High Luminosity Circular e+e- Colliders – Higgs Factory 2014)

\* Circular Electron Positron Collider

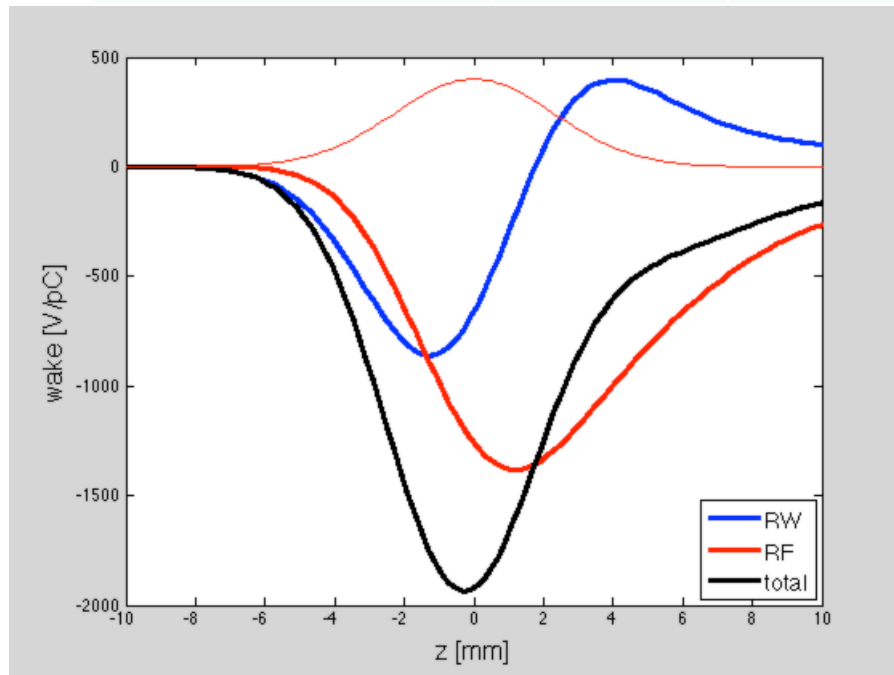
## Short range longitudinal wakefield: RF cavities

- However, if we take the short range wake of the two cell cavity, then  $R \approx 33.6 \Omega$ , the loss factor is 1.25 V/pC, and the total impedance becomes 26.9 k  $\Omega$  (800 cavities)



# Longitudinal Impedance Budget

Element	R [k $\Omega$ ]	L [nH]	$k_{\text{loss}}$ [V/pC]	Z/n  (m $\Omega$ )
Resistive wall (Al)	7.5	148	276	1.1*+2.8
RF cavities	26.9	-	1000	3.9*
total	34.3	148	1276	7.8
CEPC total	37.8	128.8	1205.1	4.4



\* at  $\omega = c / \sigma_z$

- The total loss factor, with a bunch charge of about 29 nC gives an energy lost per turn of about 0.037 GeV.
- The RF cavities contribute mainly to the real part of the impedance and to the total loss factor.
- The impedance budget is comparable to CEPC.

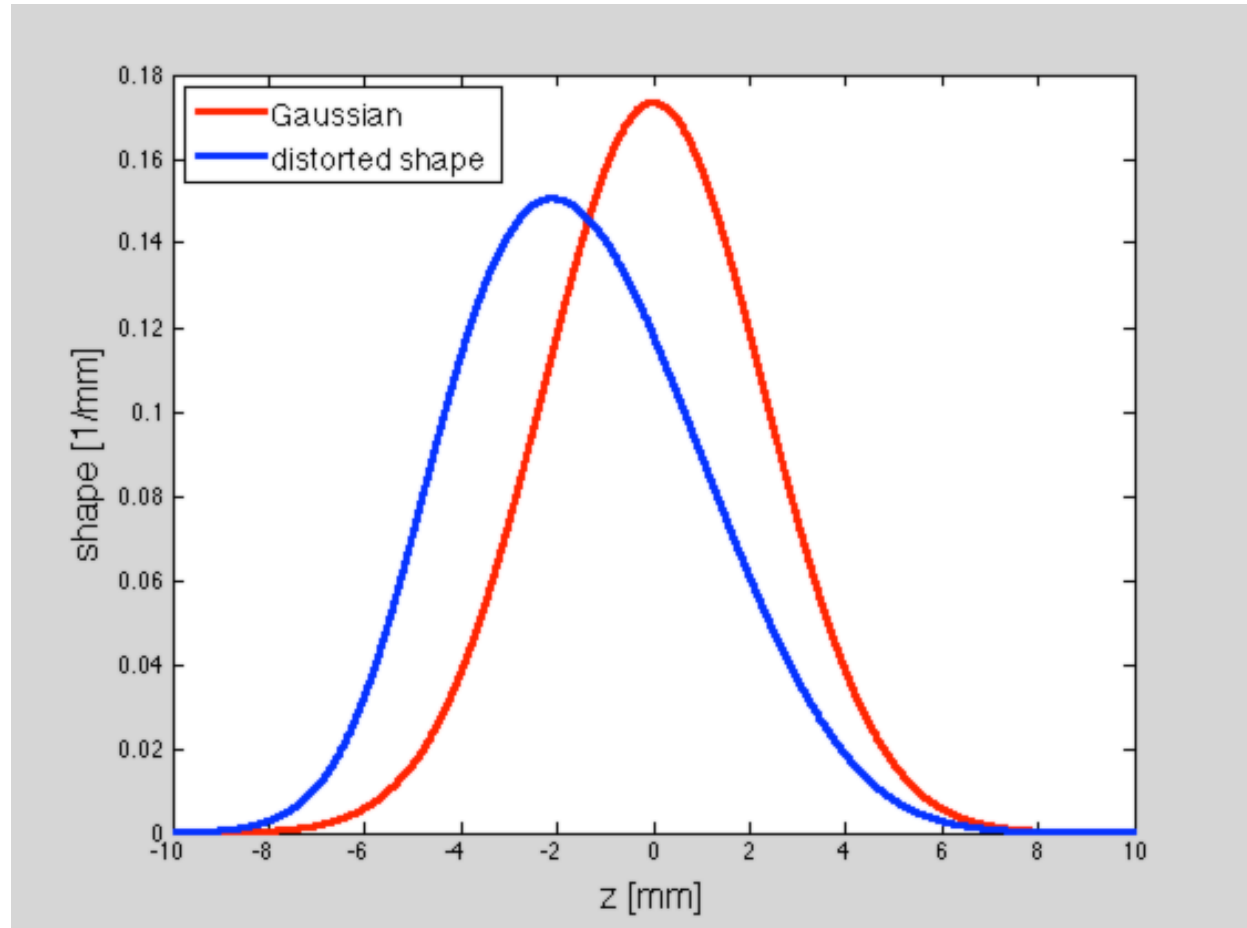
## Single bunch longitudinal effects

- Bunch lengthening is obtained in the potential well distortion regime by numerically solving the Haissinski equation with the R-L impedance model due to RF cavities and RW.

Initial bunch length: 2.30 mm

Final bunch length: 2.56 mm

Bunch is lengthened by about 10%



## Single bunch longitudinal effects

- Microwave instability threshold is estimated according to the Boussard or Keil-Schnell criterion

$$\left| \frac{Z_{\parallel}}{n} \right| = \frac{(2\pi)^{3/2} \alpha_c E \sigma_{\varepsilon}^2 \sigma_z}{ceN_p} \cong 5 \text{ m}\Omega$$

- This is a very small value ... further investigation is necessary<sup>(\*)</sup>.
- CEPC longitudinal impedance threshold is  $|Z/n|=25 \text{ m}\Omega$  (N. Wang, et al., 55th ICFA Advanced Beam Dynamics Workshop on High Luminosity Circular e+e- Colliders – Higgs Factory 2014)
- LEP:  $|Z/n|=30 \text{ m}\Omega$  (B. Zotter, EPAC'92, p.273)

(\*) With the new parameters,  $|Z/n|=13 \text{ m}\Omega$ , in line with LEP (three times shorter) and CEPS (twice)

## Single bunch longitudinal effects

- Alternative crab-waist scenario using the 175 GeV optics at the Z pole (45 GeV) with beamstrahlung

parameter	Symbol	Value
Bunch population	$N_p$	$0.5 \times 10^{11}$
Bunch length	$\sigma_z$	3.2 mm
Energy spread	$\sigma_\varepsilon$	$1.24 \times 10^{-3}$
momentum compaction	$\alpha_c$	$0.5 \times 10^{-5}$

$$\left| \frac{Z_{\parallel}}{n} \right| = \frac{(2\pi)^{3/2} \alpha_c E \sigma_\varepsilon^2 \sigma_z}{ceN_p} \cong 7.2 \text{ m}\Omega$$

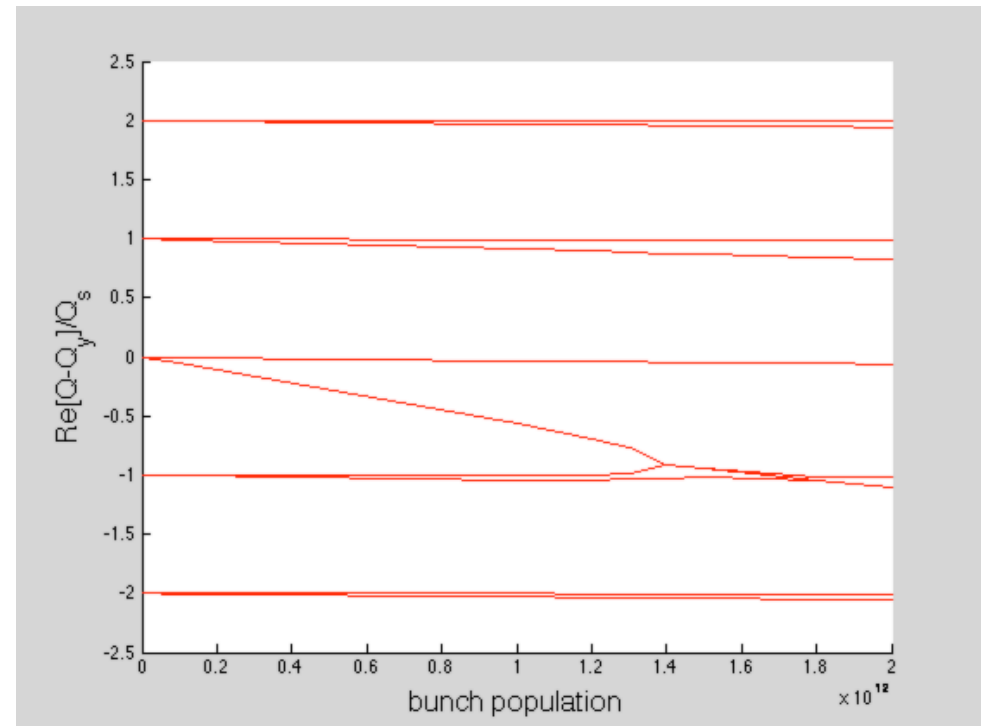
Reduced by a factor of 2 even if here we considered BS



## Transverse mode coupling instability

- The TMCI has been evaluated with the RW impedance by using the code DELPHI<sup>(\*)</sup> (Discrete Expansion over Laguerre Polynomials and Headtail modes for Instabilities computation). It is a semi-analytic Vlasov solver, which computes eigenfrequencies (tune shifts and growth rates) of azimuthal and radial modes for a given transverse impedance.

(\*) N. Mounet PhD Thesis at EPFL.



TMCI threshold (about  $1.3\text{-}1.4 \times 10^{12}$ ) seems to be higher than the nominal single bunch current ( $N_p = 1.8 \times 10^{11}$ ), but only RW impedance has been considered.

## Transverse mode coupling instability

- In order to include RF, we have considered the total longitudinal wake potential and fitted it with a wake potential of a Gaussian bunch with a BBR. Even if the fit is not very good, a rough estimate of the impedance can be done (RW+RF cavities). We get:

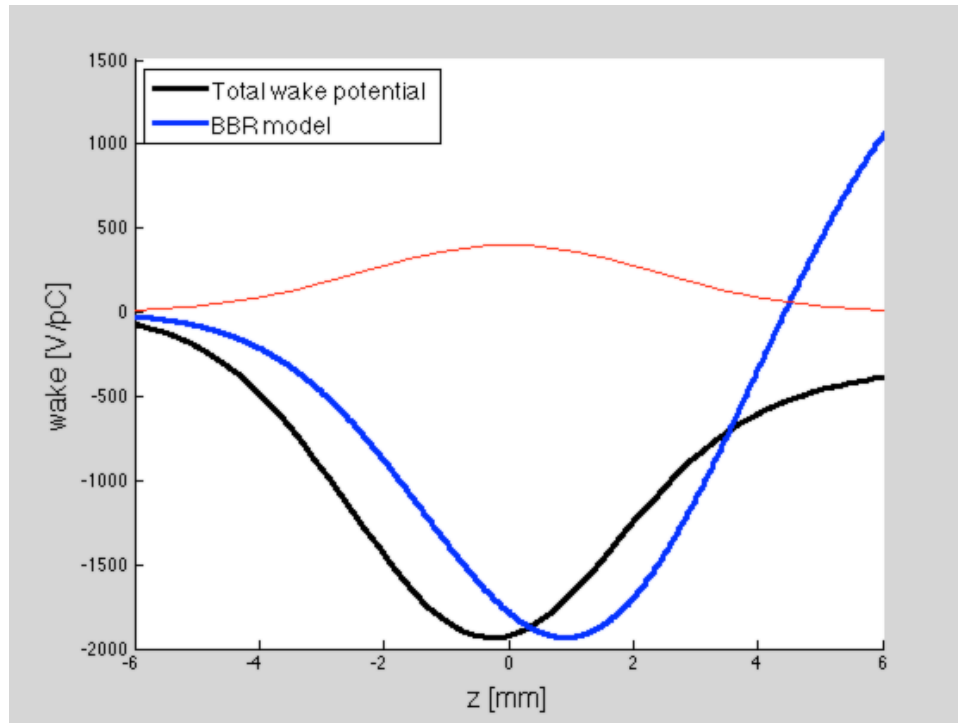
$$R_s = 77 \text{ k}\Omega, Q = 1, f_r = 15 \text{ GHz}$$

From the longitudinal impedance we have then obtained the transverse one by using the relation

$$Z_{\perp} = \frac{2c}{b^2 \omega} Z_{\parallel}$$

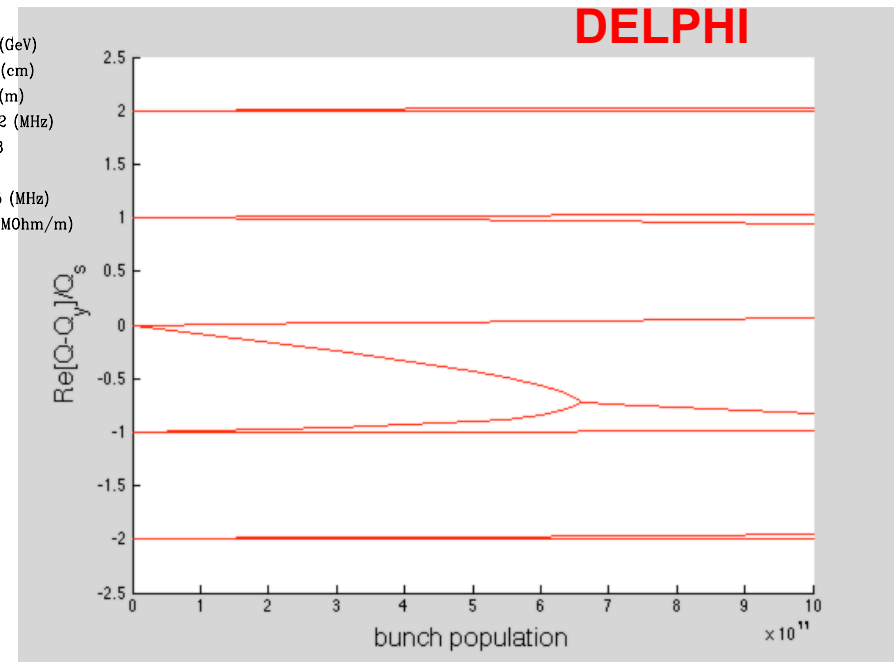
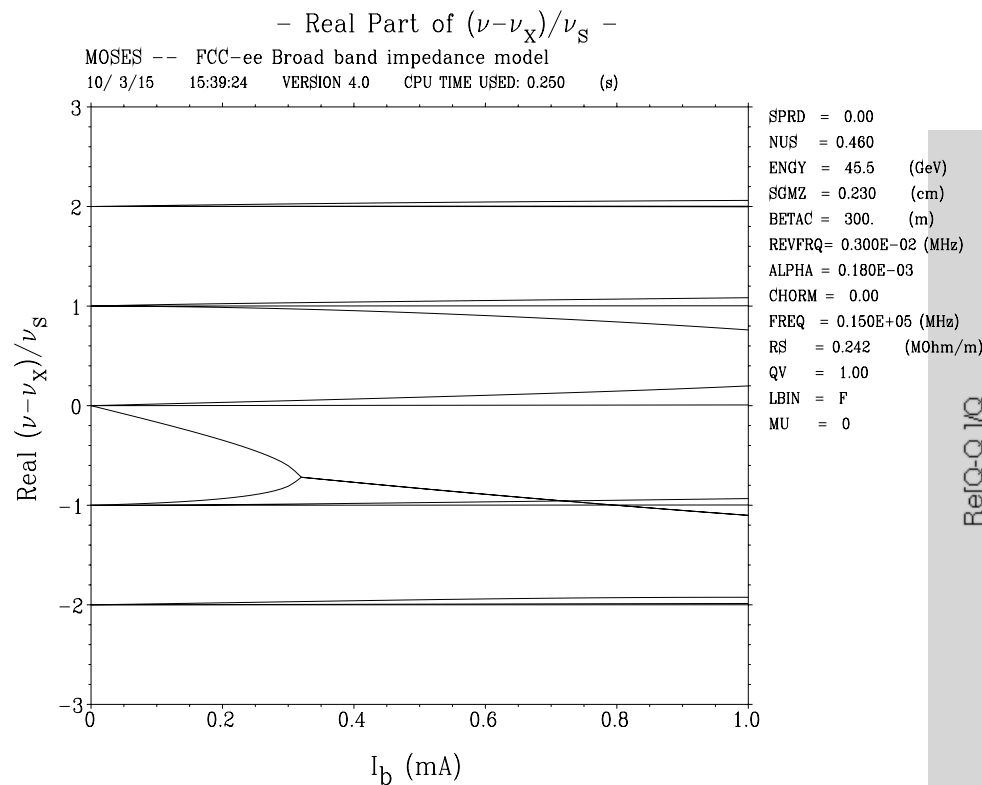
which, strictly speaking, is valid only for the RW impedance in circular beam pipe.

(see, e.g. A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, p. 81)



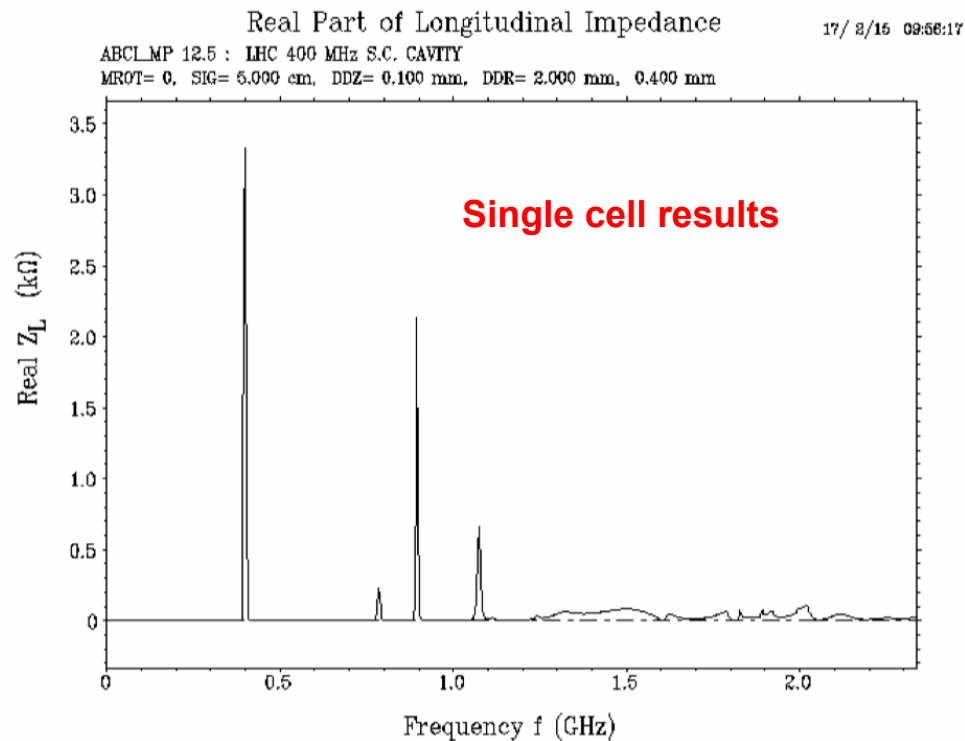
# Transverse mode coupling instability

- The transverse broad band resonator model has then been used with the code MOSES (M<sub>O</sub>de-coupling Single bunch instability in an Electron Storage ring) to evaluate the TMCI, giving a threshold of about 0.32 mA corresponding to  $N_p = 6.7 \times 10^{11}$



## Coupled bunch instability - longitudinal

- High quality resonant modes trapped, for example, in the RF cavities can induce coupled bunch instabilities.
- The cut-off frequency of the TM<sub>01</sub> mode for a circular pipe with radius of 10 cm (corresponding to the tubes attached to the RF cavities) is about 1.15 GHz.



- Below 1.1 GHz we can see, in addition to the fundamental mode at 400 MHz, other trapped HOMs.
- Considering the whole RF system, due to construction tolerances there is a spread in the resonant frequencies of HOMs which reduces the maximum shunt resistance.

## Coupled bunch instability - longitudinal

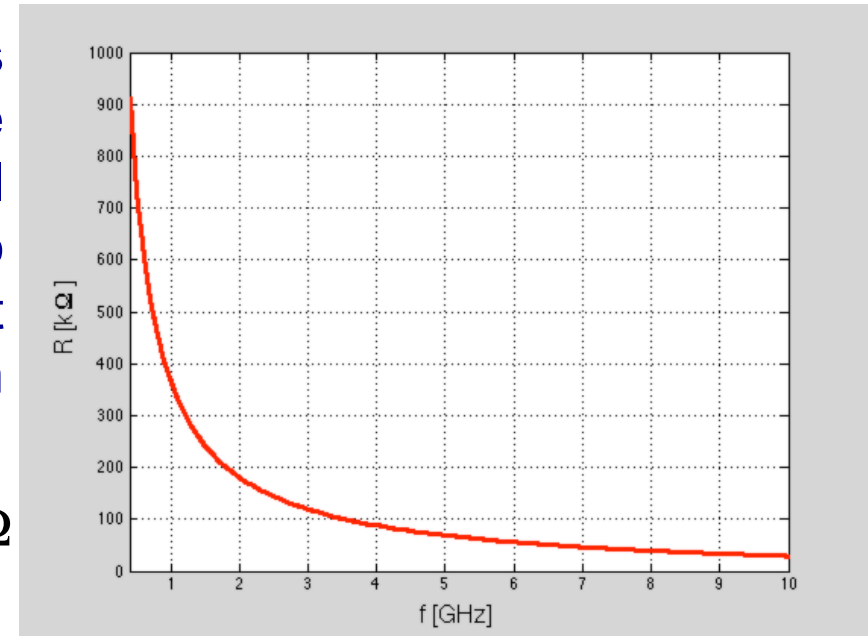
- In the worst case of resonant condition, the grow rate of the instability is

$$\alpha = \frac{\alpha_c I_0 f R_s}{2(E/e)Q_s} G(x) \quad G(x) = \frac{2}{x^2} e^{-x^2} I_1(x^2) \quad x = \frac{2\pi f}{c} \sigma_z$$

- Without any feedback, this grow rate can only be compensated by the natural damping rate (1320 turns) so that the maximum shunt resistance of a HOM is given

$$R_s = \frac{2(E/e)Q_s}{\alpha_c I_0 f \tau_z} G(x) \cong \frac{365}{f[\text{GHz}]} G(x) \text{ k}\Omega$$

longitudinal maximum shunt impedance



# RF and coupled bunch instabilities: 800 MHz system

- An alternative option to damping HOMs could be a design of a single mode cavity (no HOMs) – Example for 800 MHz

Courtesy of M. Zobov



Meeting 3: Harmonic RF System Review  
CERN, 3 November 2014

## Alternative Options for 800 MHz Harmonic Cavity



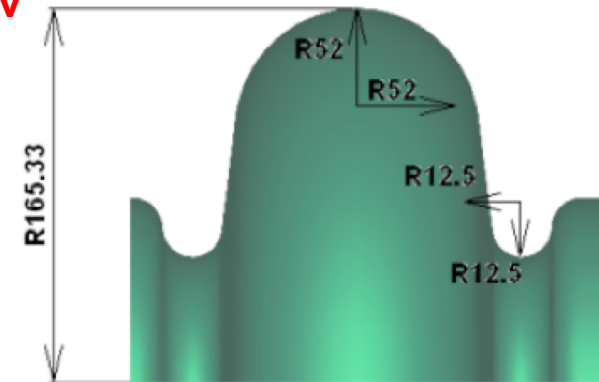
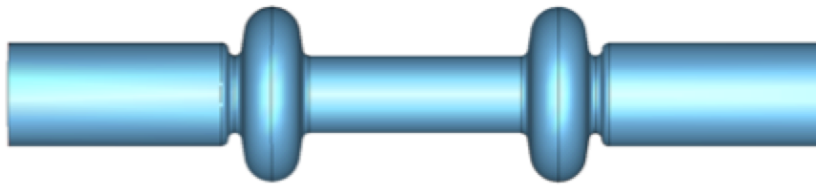
M. Zobov  
*LNF INF, Frascati, Italy*

Ya. Shashkov, N. Sobenin *MEPHI, Moscow, Russia*



# RF and coupled bunch instabilities: 800 MHz system

Courtesy of M. Zobov



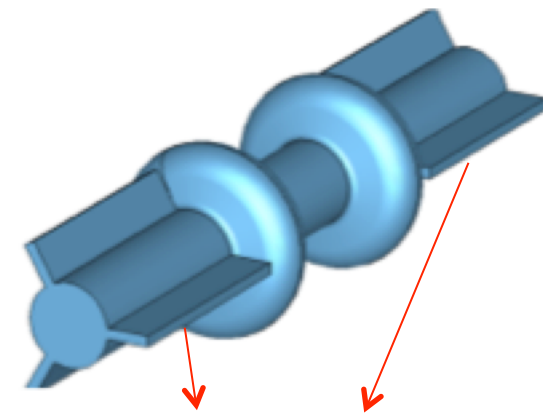
Geometry is perfectly azimuthally symmetric

There are no dangerous HOM

There is no need to use additional HOM couplers  
(8 couplers are required in the baseline version)

Cavities do not communicate with each other due to the small radius of the connecting pipe.

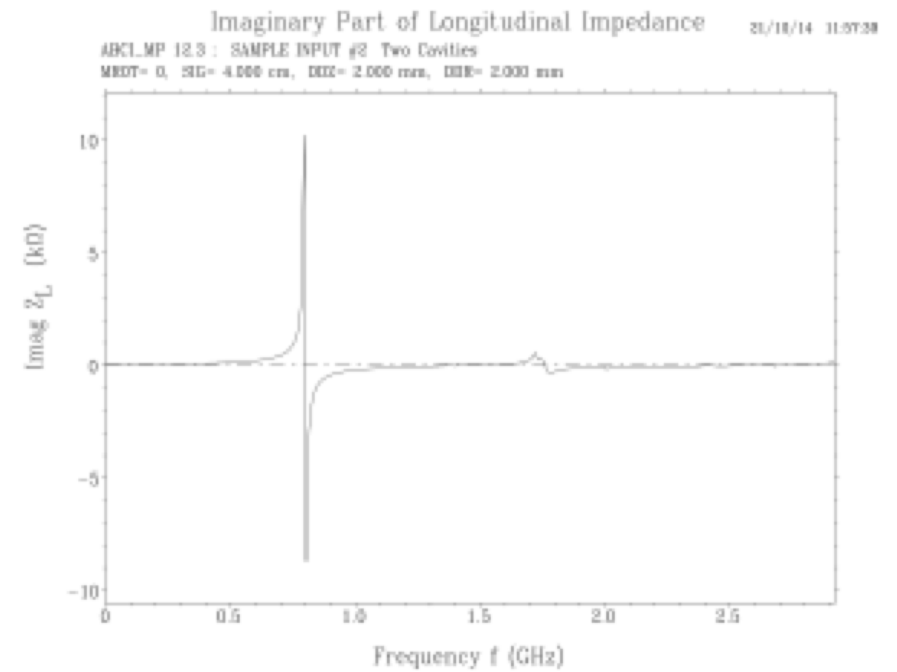
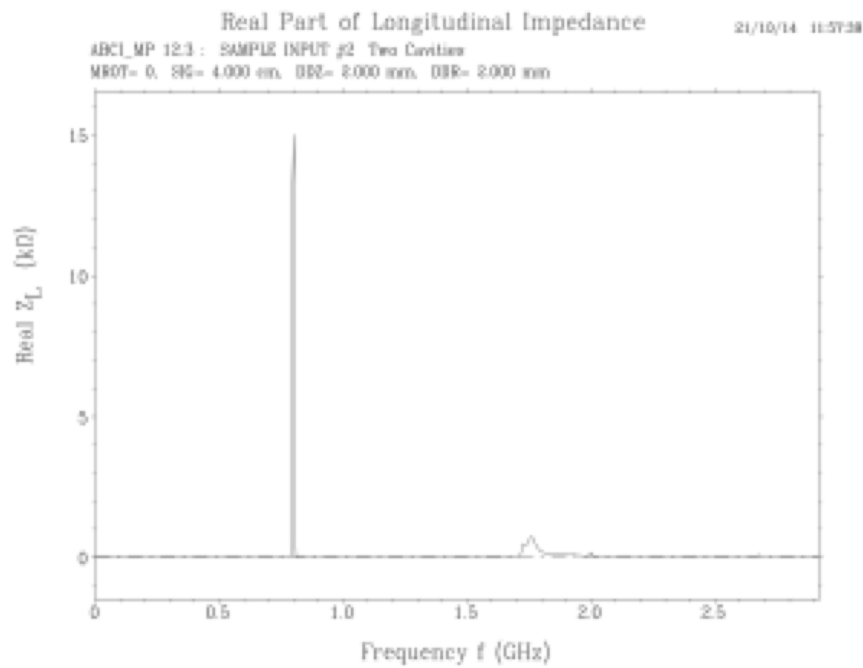
Main coupler can be placed on the beam pipe with a smaller radius



Waveguide dampers

# RF and coupled bunch instabilities: 800 MHz system

Courtesy of M. Zobov





## Coupled bunch instability – transverse RW

- Let us consider the azimuthal mode  $m=0$  of a Gaussian bunch coupling a single frequency line of the transverse RW impedance. The growth rate can be obtained with

$$\alpha = -\frac{cN_b I_b}{4\pi(E/e)Q_\beta} \operatorname{Re}\left[Z_\perp(\omega_q)\right] G_\perp\left(\frac{\sigma_z}{c}\omega'_q\right)$$

- where

$$\omega_q = \omega_0(qN_b + \mu + Q_\beta) \quad \omega'_q = \omega_q - \xi \frac{\omega_\beta}{\alpha_c} \quad G_\perp(x) = e^{-x^2} I_0(x^2) \cong 1$$

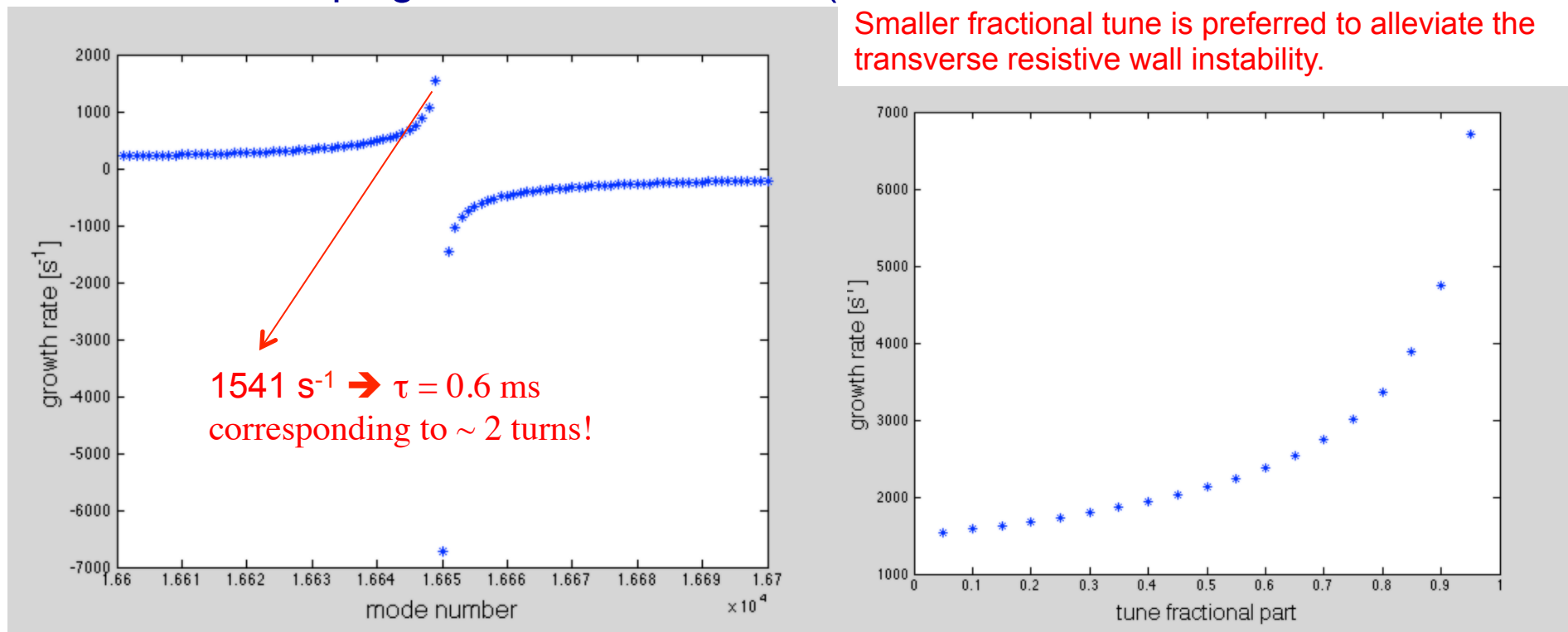
- The most dangerous instability occurs at the betatron line with the lowest negative frequency, and the growth rate is given by:

$$\alpha = \frac{cN_b I_b}{4\pi(E/e)Q_\beta} \frac{L}{2\pi b^3} \sqrt{\frac{LZ_0}{\pi|1-\nu_\beta|\sigma_c}} G_\perp\left(\frac{\sigma_z}{c}\omega'_q\right)$$

fractional part of the tune

## Coupled bunch instability – transverse RW

- The growth rate goes as  $1/Q_\beta$  and depends strongly on the fractional part of the tune  $\nu_\beta$ . By considering, for example,  $Q_\beta = 50.05$ , we get a rise time of about 0.6 ms, the natural damping time is about 0.88 s (2640 turns).



## Conclusions

- The evaluation of the impedance model and single beam effects of FCC-ee has just started.
- A first, rough, estimate of the contribution of RW and 400 MHz RF system has been done.
- Microwave instability seems critical and has to be studied more in detail, also with the help of simulation codes.
- The transverse coupled bunch instability due to RW seems very dangerous and cures (feedback) should be studied.
- To avoid RF HOMs and dangerous beam heating, single mode cavity should be evaluated.
- TMCI threshold at the moment is higher than the single bunch current by a factor of about 3-4.
- Several parameters are changing as the work is going on and the scenario may be modified, as well as the priorities.
- **There is a huge amount of work to do and any help is very much appreciated!**

