Electron beam dynamics in storage rings

Synchrotron radiation
and its effect on electron dynamics

Lecture 1: Synchrotron radiation

Lecture 2: Undulators and Wigglers

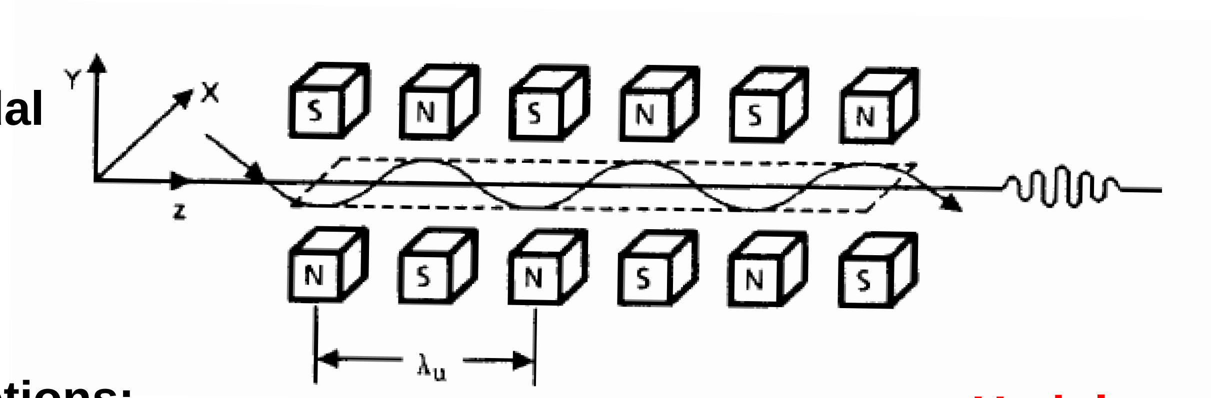
Lecture 3: Electron dynamics-I

Lecture 4: Electron dynamics-II

Contents

Undulators and wigglers

Periodic array of magnetic poles providing a sinusoidal magnetic field on axis:



$$B = (0, \quad B_0 \sin(k_u z), \quad 0,)$$

Solution of equation of motions:

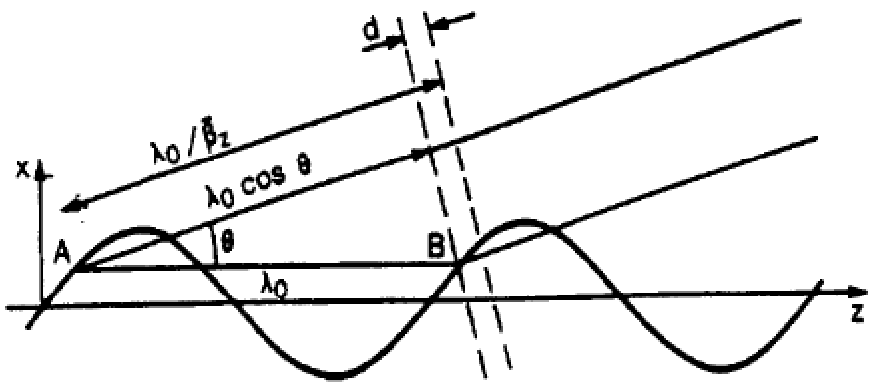
$$K = \frac{eB_0\lambda_u}{2\pi mc}$$

Undulator parameter

$$\bar{r}(t) = -\frac{\lambda_u K}{2\pi\gamma}\sin\omega_u t \cdot \hat{x} + \left(\bar{\beta}_z ct + \frac{\lambda_u K^2}{16\pi\gamma^2}\cos(2\omega_u t)\right)\cdot\hat{z}$$

$$\bar{\beta}_z = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

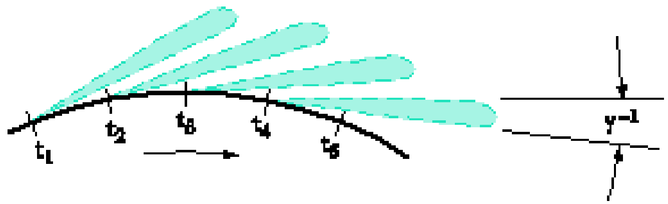
Constructive interference of radiation emitted at different poles



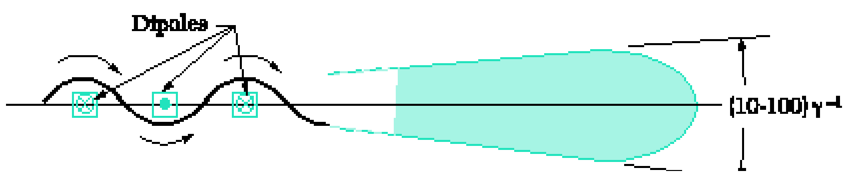
$$d = \frac{\lambda_u}{\bar{\beta}} - \lambda_u \cos\theta = n\lambda$$

$$\lambda_n = \frac{\lambda_u}{2\gamma^2 n}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

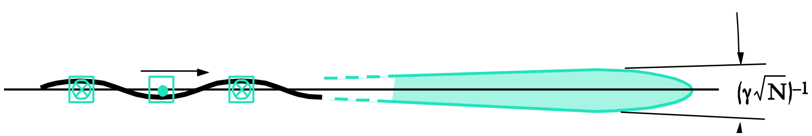
From the lecture on synchrotron radiation



bending magnet - a "sweeping searchlight"



wiggler - incoherent superposition K > 1
Max. angle of trajectory > 1/γ



undulator - coherent interference K < 1
Max. angle of trajectory < 1/γ

Continuous spectrum characterized by ε_c = critical energy

ε_c(keV) = 0.665 B(T)E^2(GeV)

***eg*: for B = 1.4T E = 3GeV ε_c = 8.4 keV**

(bending magnet fields are usually lower ~ 1 – 1.5T)

Quasi-monochromatic spectrum with peaks at lower energy than a wiggler

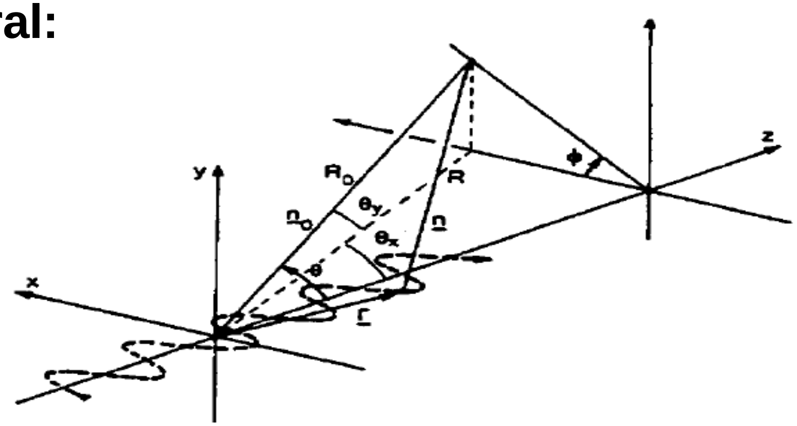
$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}\left(1+\frac{K^2}{2}\right) \approx \frac{\lambda_u}{n\gamma^2}$$

$$\varepsilon_n(eV) = 9.496\frac{nE[GeV]^2}{\lambda_u[m]\left(1+\frac{K^2}{2}\right)}$$

Radiation integral for a linear undulator (I)

The angular and frequency distribution of the energy emitted by a wiggler is computed again with the radiation integral:

$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2\omega^2}{4\pi\varepsilon_0 4\pi^2 c}\left|\int_{-\infty}^{\infty} \hat{n}\times(\hat{n}\times\overline{\beta})e^{i\omega(t-\hat{n}\cdot\bar{r}/c)}dt\right|^2$$



Using the periodicity of the trajectory we can split the radiation integral into a sum over N_u terms

$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2\omega^2}{4\pi\varepsilon_0 4\pi^2 c}\left|\int_{-\lambda_u/2\bar{\beta}c}^{\lambda_u/2\bar{\beta}c} \hat{n}\times(\hat{n}\times\overline{\beta})e^{i\omega(t-\hat{n}\cdot\bar{r}/c)}dt\right|^2 \left|1+e^{i\delta}+e^{i2\delta}+\ldots+e^{i(N_u-1)\delta}\right|^2$$

where

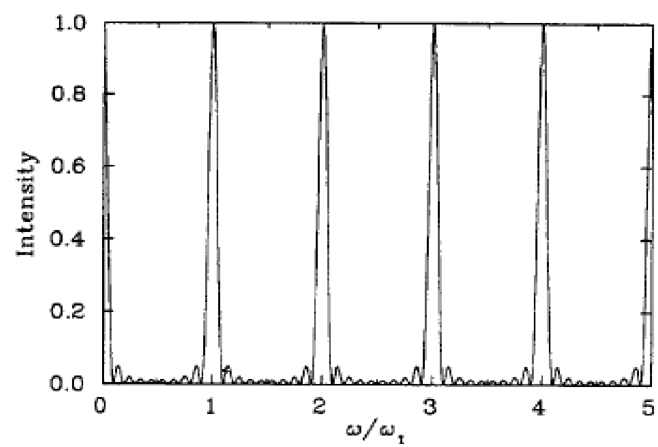
$$\delta = \frac{2\pi\omega}{\omega_{res}(\theta)} \qquad \omega_{res}(\theta) = \frac{2\pi c}{\lambda_{res}(\theta)} \qquad \lambda_{res}(\theta) = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)$$

Radiation integral for a linear undulator (II)

The radiation integral in an undulator or a wiggler can be written as

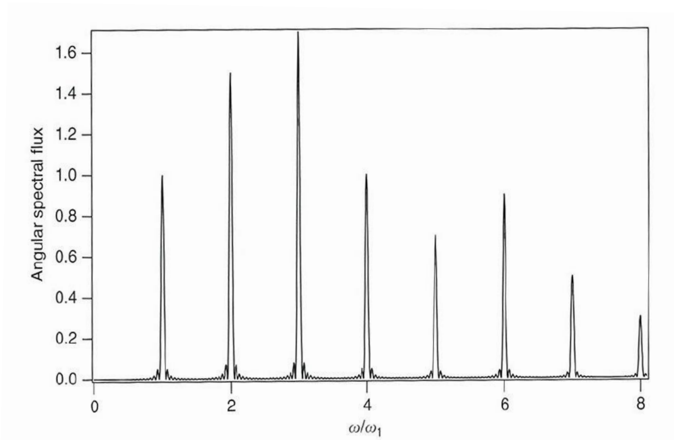
$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2\gamma^2N^2}{4\pi\varepsilon_0 c} L\left(N\frac{\Delta\omega}{\omega_{res}(\theta)}\right) F_n(K,\theta,\phi)$$

$\Delta\omega = \omega - n\,\omega_{res}(\theta)$



The sum on δ generates a series of sharp peaks in the frequency spectrum harmonics of the fundamental wavelength

$$L\left(N\frac{\Delta\omega}{\omega_{res}(\theta)}\right) = \frac{\sin^2(N\pi\Delta\omega/\omega_{res}(\theta))}{N^2\sin^2(\pi\Delta\omega/\omega_{res}(\theta))}$$



The integral over one undulator period generates a modulation term F_n which depends on the angles of observations and K

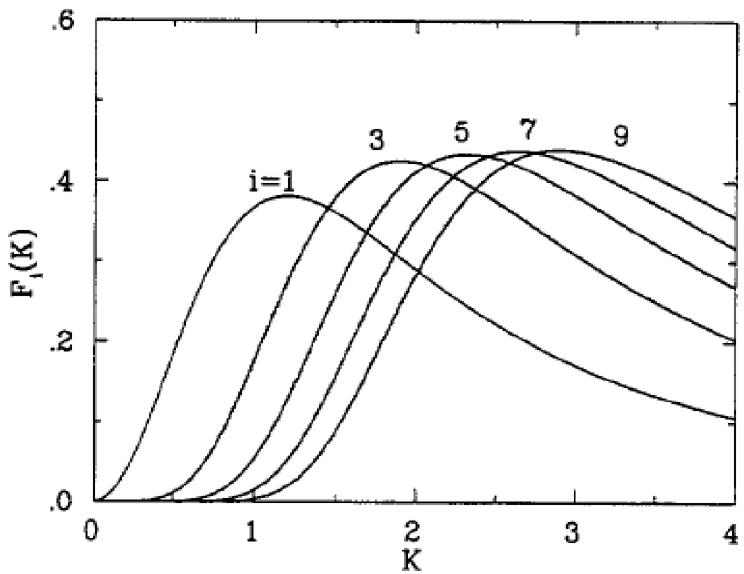
$$F_n(K,\theta,\phi) \propto \left| \int_{-\lambda_0/2\bar{\beta}c}^{\lambda_0/2\bar{\beta}c} \hat{n}\times(\hat{n}\times\bar{\beta})e^{i\omega(t-\hat{n}\cdot\bar{r}/c)}dt \right|^2$$

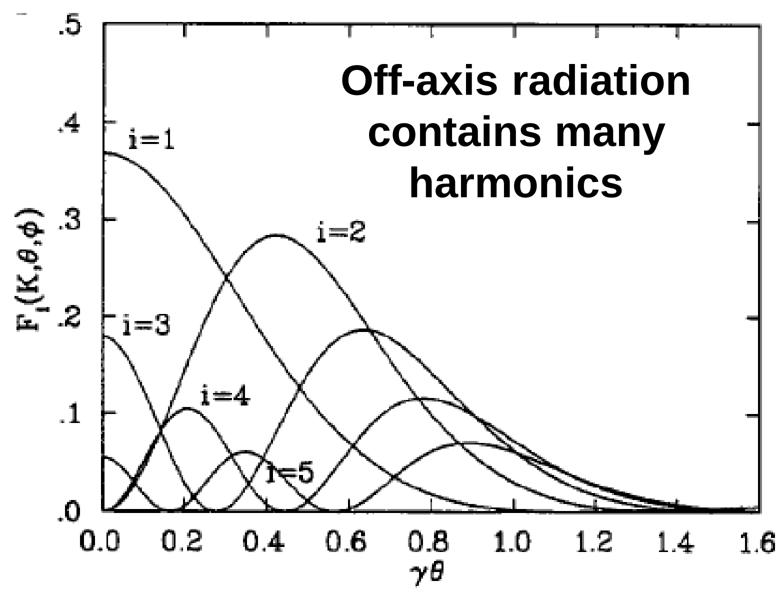
Radiation integral for a linear undulator (II)

e.g. on axis (θ = 0, φ = 0):

$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2\gamma^2N^2}{4\pi\varepsilon_0 c} L\left(N\frac{\Delta\omega}{\omega_{res}(0)}\right) F_n(K,0,0)$$

$$F_n(K,0,0) = \frac{n^2K^2}{(1+K^2/2)}\left[J_{\frac{n+1}{2}}(Z) - J_{\frac{n-1}{2}}(Z)\right]^2 \qquad Z = \frac{nK^2}{4(1+K^2/2)}$$



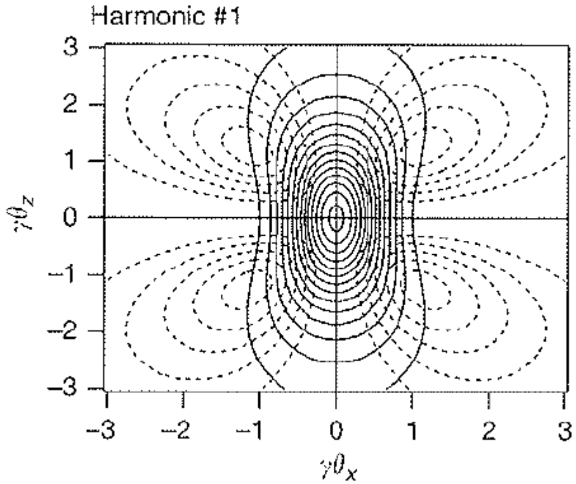


Only odd harmonic are radiated on-axis;

as K increases the higher harmonic becomes stronger

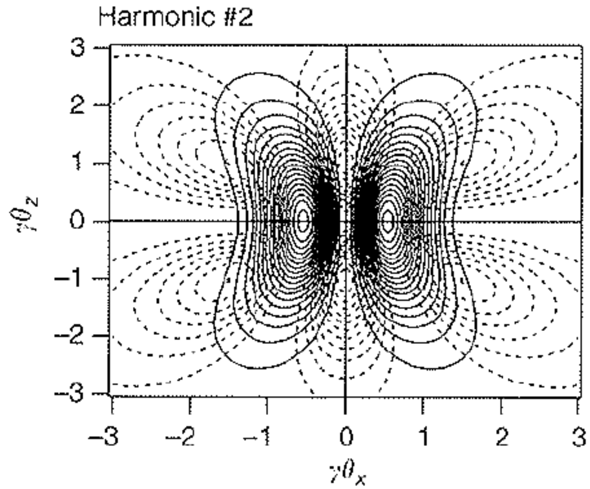
Angular patterns of the radiation emitted on harmonics – on axis

Angular spectral flux as a function of frequency for a linear undulator; linear polarisation solid, vertical polarisation dashed (K = 2)



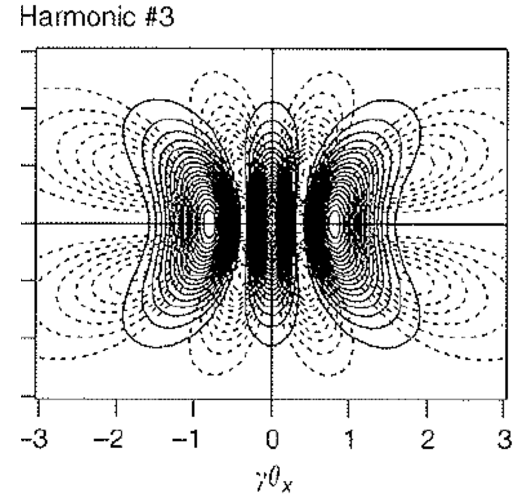
$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

Fundamental wavelength emitted by the undulator



$$\lambda_2 = \frac{\lambda_1}{2}$$

2nd harmonic, not emitted on-axis !

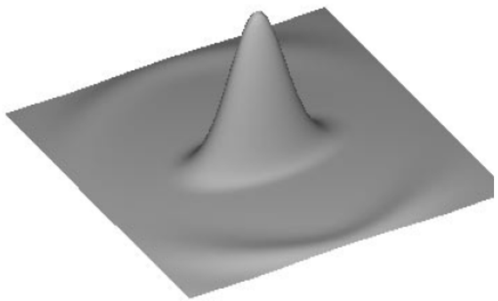


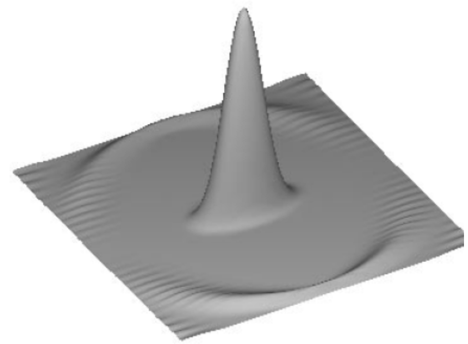
$$\lambda_2 = \frac{\lambda_1}{3}$$

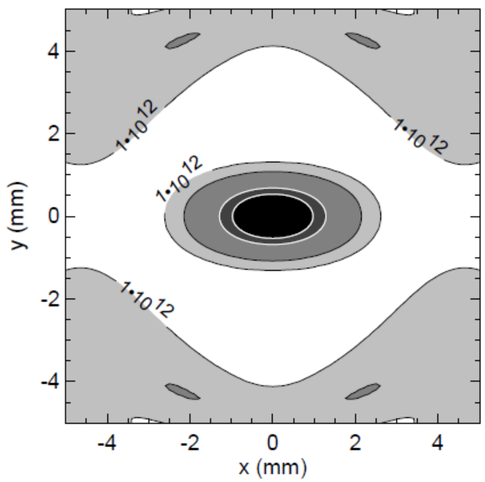
3rd harmonic, emitted on-axis !

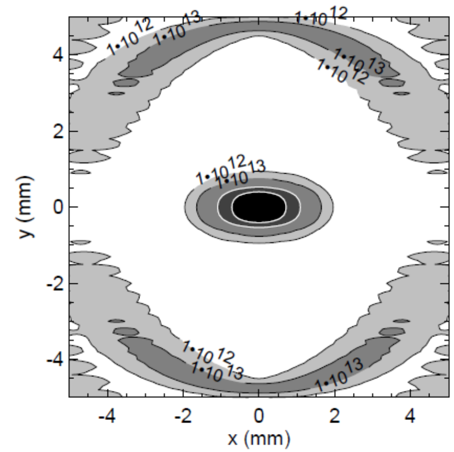
Angular patterns of the radiation emitted on harmonics

Angular spectral flux as a function of frequency for a linear undulator; linear polarisation (K = 2)







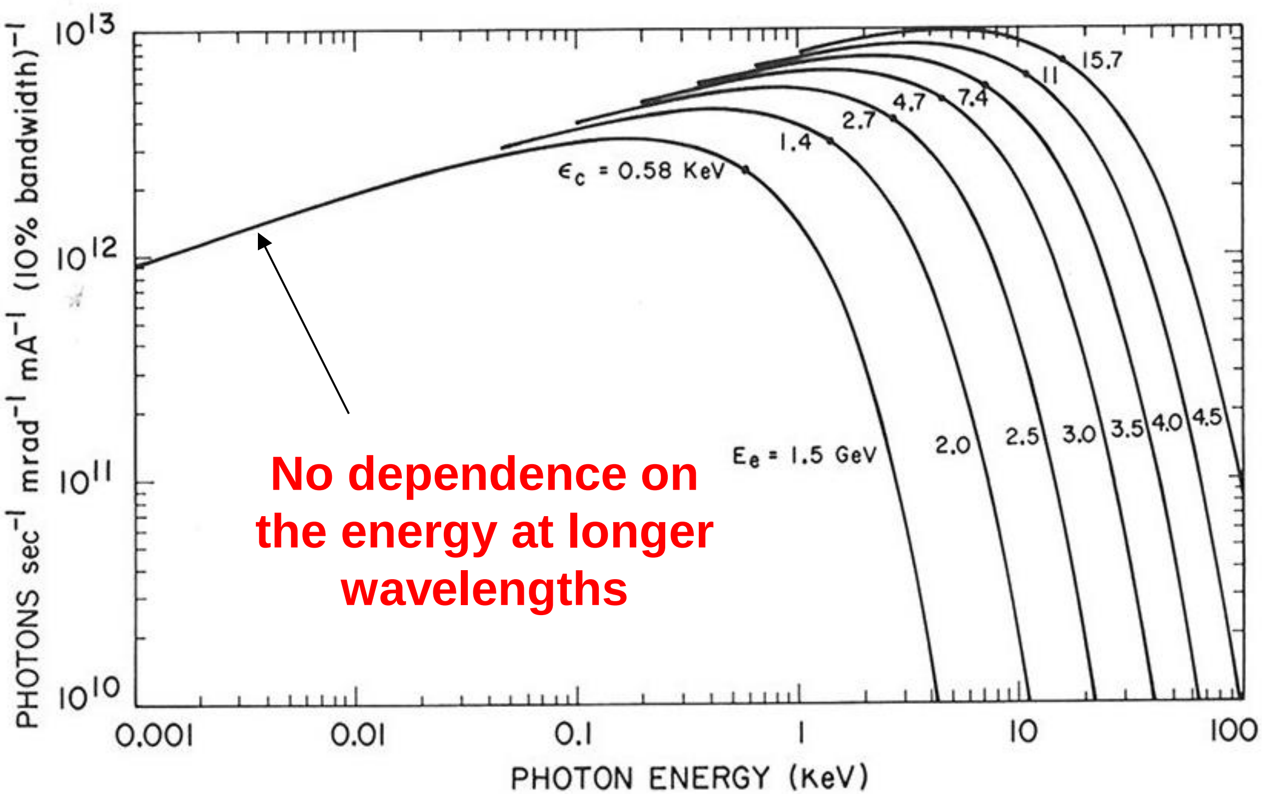


The fundamental frequency is also emitted off axis as part of the “second harmonic” according to the relation

$$\lambda_{res}(\theta) = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

Synchrotron radiation emission from a bending magnet

Dependence of the frequency distribution of the energy radiated via synchrotron emission on the electron beam energy



Critical frequency

$$\omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$$

Critical angle

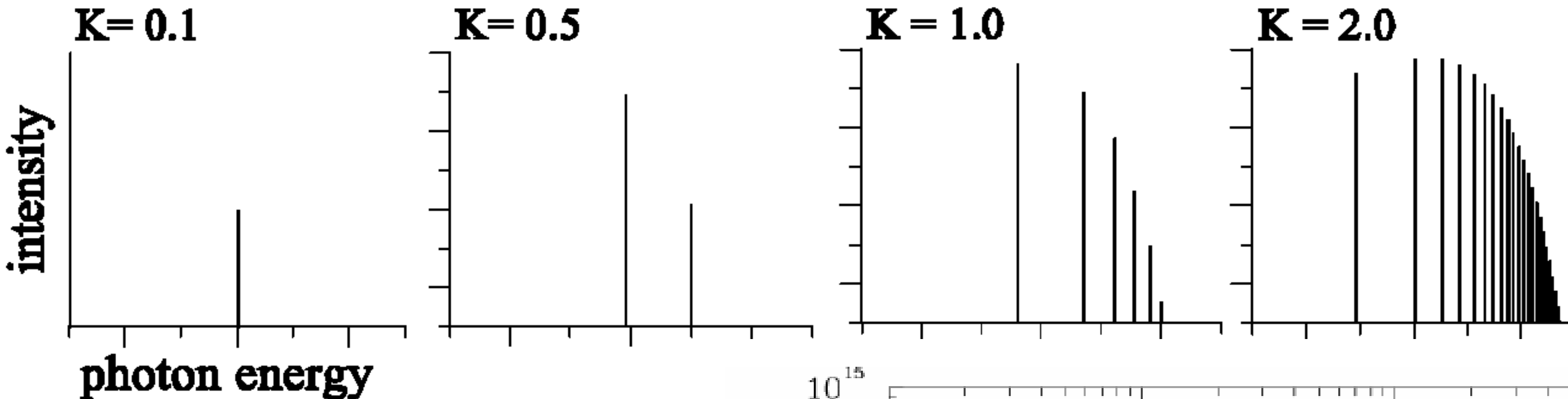
$$\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}$$

Critical energy

$$\varepsilon_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$

Undulators and wigglers (large K)

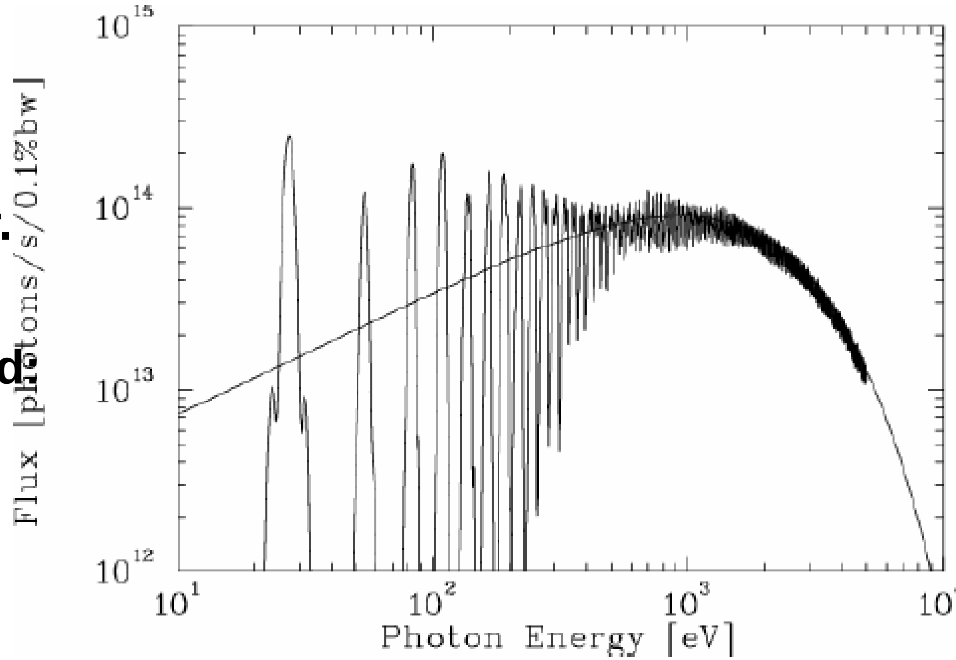
Radiated intensity emitted vs K



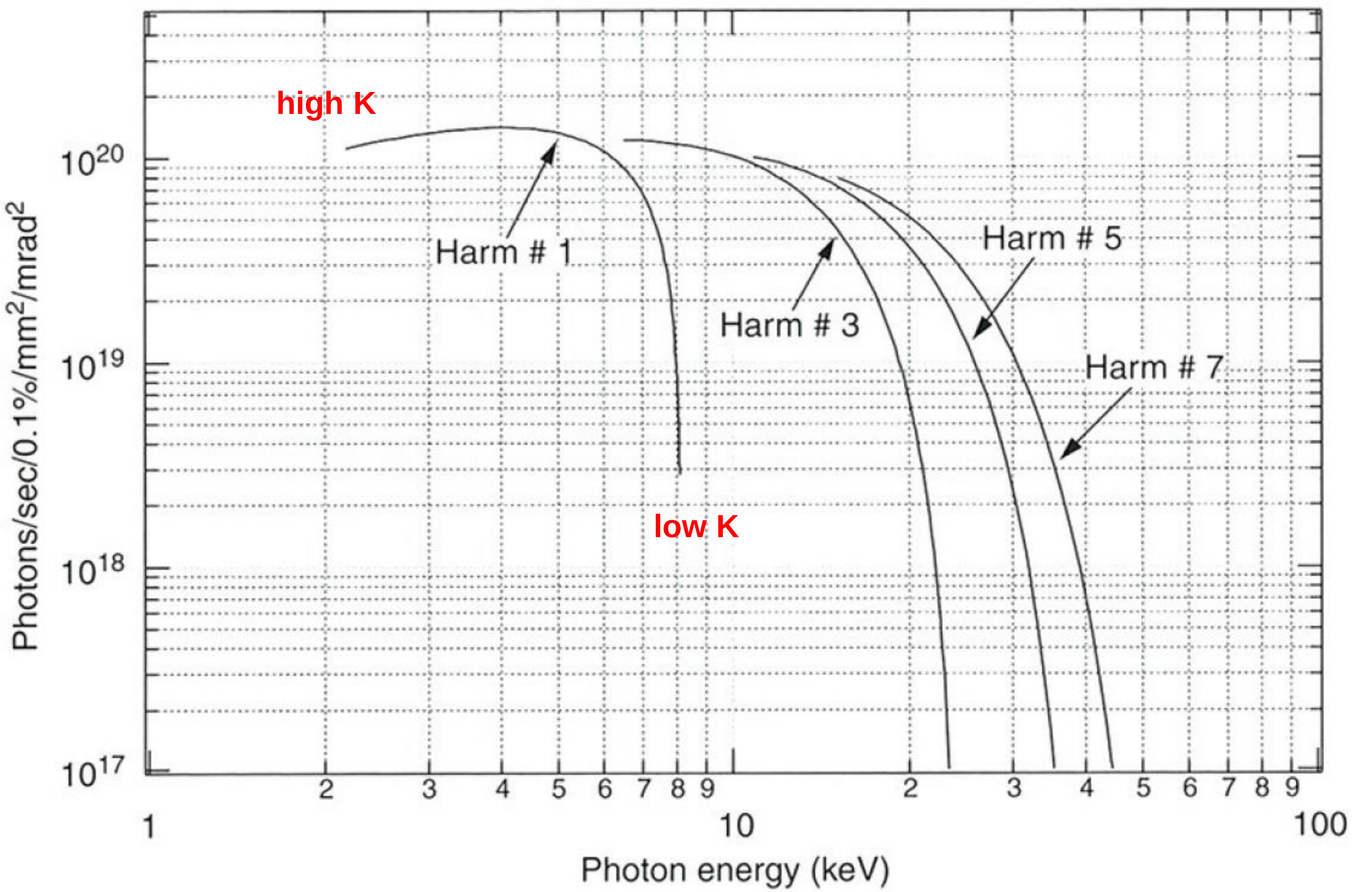
For large K the wiggler spectrum becomes similar to the bending magnet spectrum, $2N_u$ times larger.

Fixed $B_{0,}$ to reach the bending magnet critical wavelength we need:

K	1	2	10	20
n	1	5	383	3015



Undulator tuning curve (with K)



$$K = \frac{eB_0\lambda_u}{2\pi mc}$$

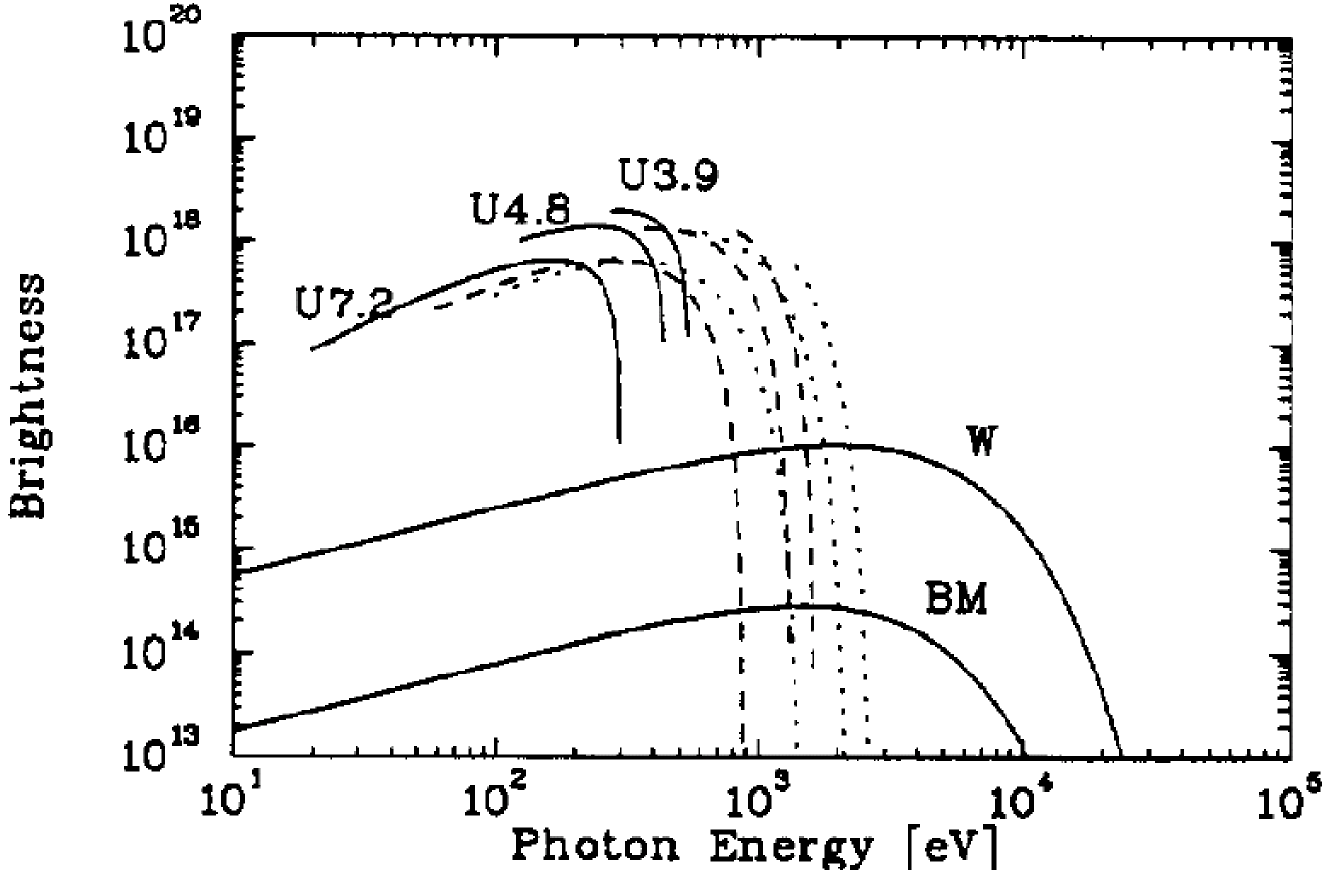
Undulator parameter

K decreases by opening the gap of the undulator (reducing B)

Brightness of a 5 m undulator 42 mm period with maximum K = 2.42 (ESRF)
Varying K one varies the wavelength emitted at various harmonics
(not all wavelengths of this graph are emitted at a single time)

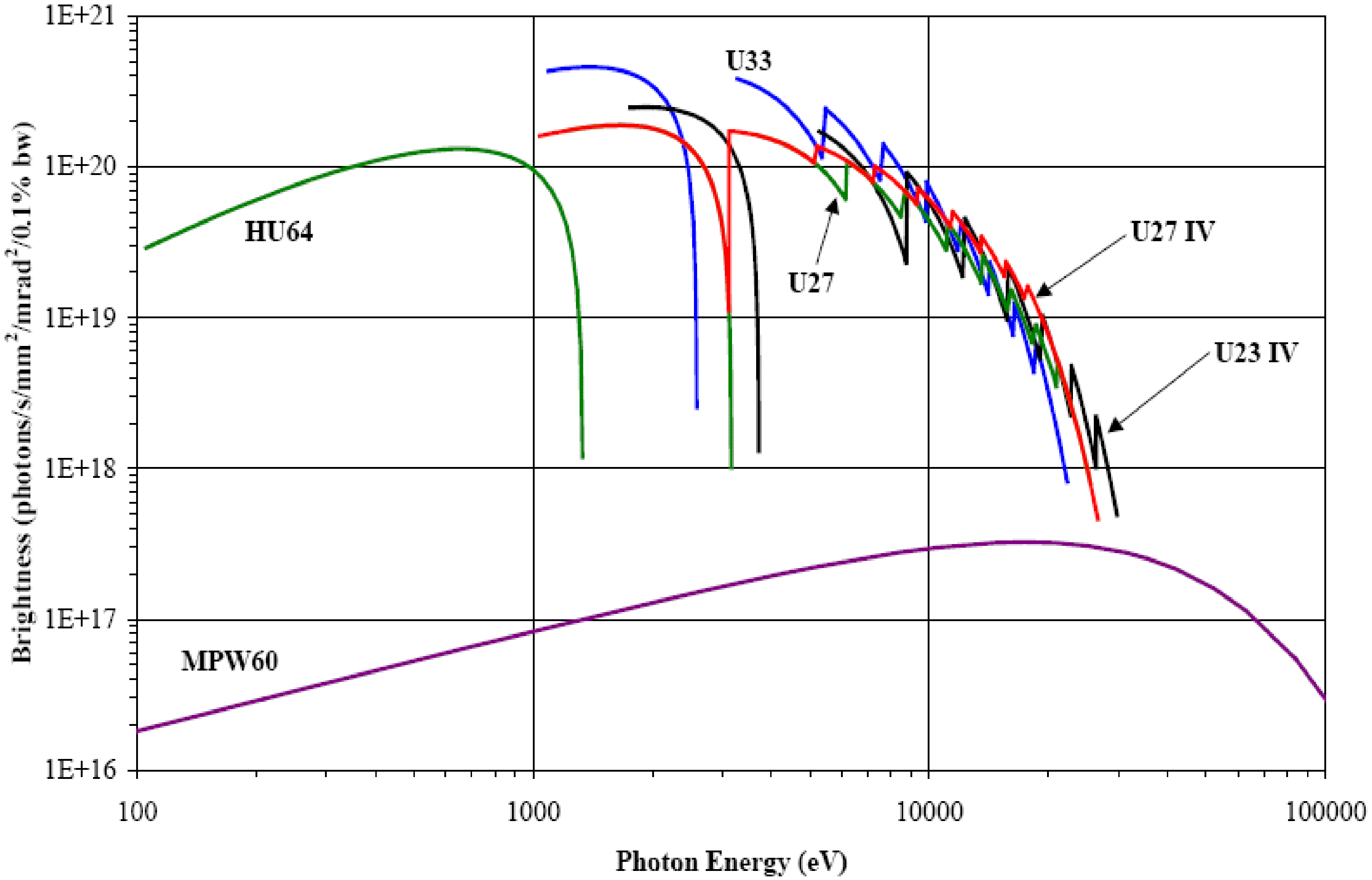
Spectral brightness of undulators of wiggler

Comparison of undulators for a 1.5 GeV ring for three harmonics (solid dashed and dotted) compared with a wiggler and a bending magnet (ALS)



Diamond undulators and wiggler

Spectral brightness for undulators and wigglers in state-of-the-art 3rd generation light sources



Summary of radiation characteristics of undulators or wiggler

Undulators have weaker field or shorter periods (K< 1)

Produce narrow band radiation and harmonics

Intensity is proportional to N_u^2

Wigglers have higher magnetic field (K >1)

Produce a broadband radiation

Intensity is proportional to N_u

Type of undulators and wigglers

Electromagnetic undulators: the field is generated by current carrying coils; they may have iron poles;

Permanent magnet undulators: the field is generated by permanent magnets Samarium Cobalt (SmCo; 1T) and Neodymium Iron Boron (NdFeB; 1.4T); they may have iron poles (hybrid undulators);

APPLE-II: permanent magnets arrays which can slide allowing the polarisation of the magnetic field to be changed from linear to circular

In-vacuum: permanent magnets arrays which are located in-vacuum and whose gap can be closed to very small values (< 5 mm gap!)

Superconducting wigglers: the field is generated by superconducting coils and can reach very high peak fields (several T, 3.5 T at Diamond)

Electromagnetic undulators (I)

HU64 at SOLEIL:

variable polarisation electromagnetic undulator

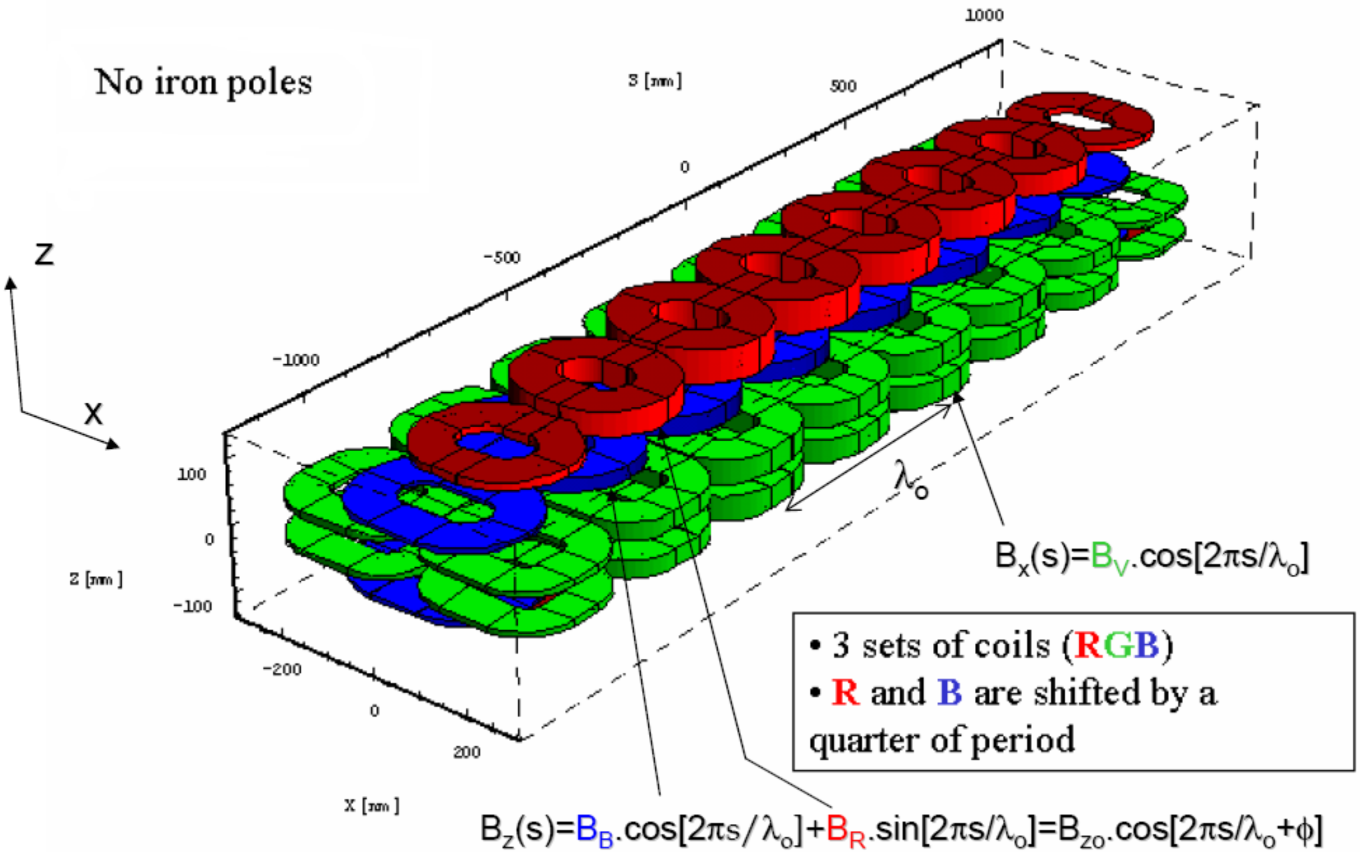
Period 64 mm

14 periods

Min gap 19 mm

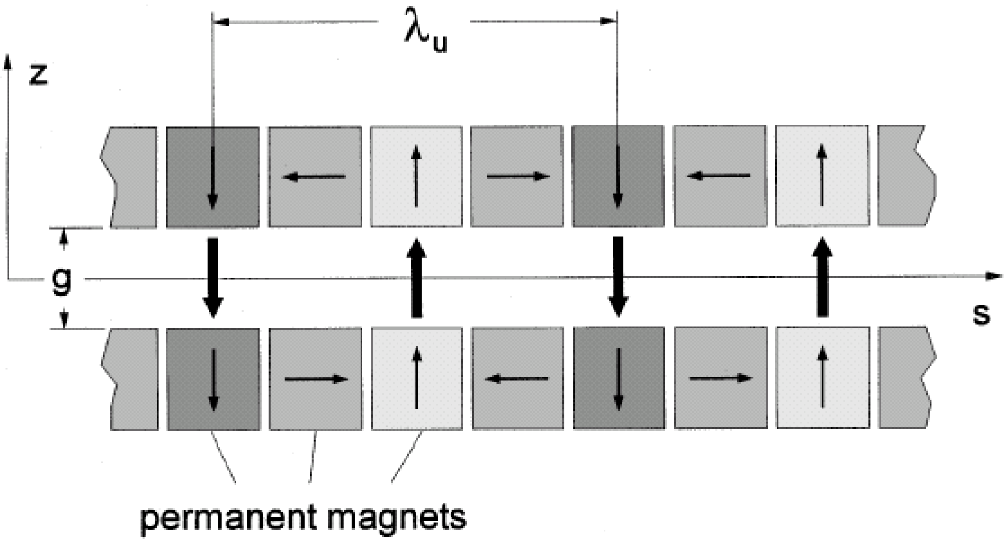
Photon energy < 40 eV (1 keV with EM undulators)

Electromagnetic undulators (II)

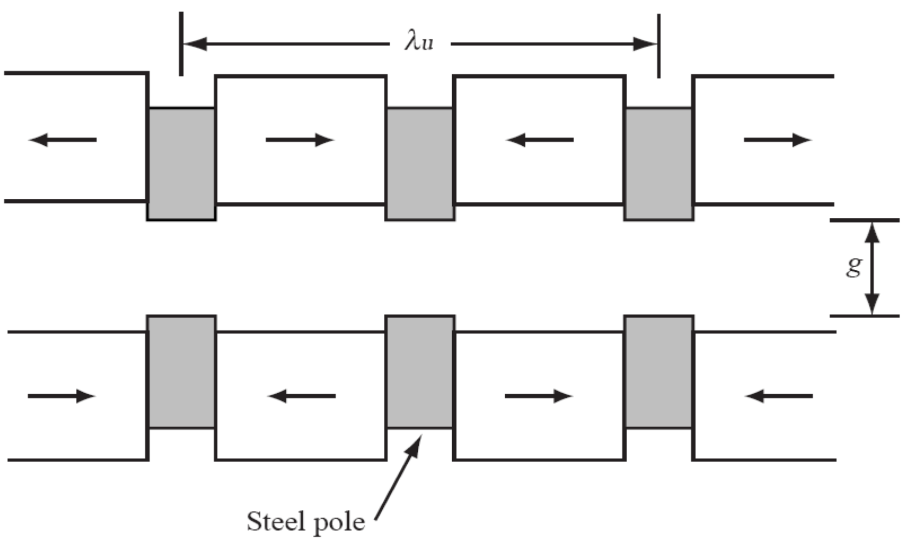


Depending on the way the coil power supplies are powered it can generate linear H, linear V or circular polarisations

Permanent magnet undulators



Halback configuration

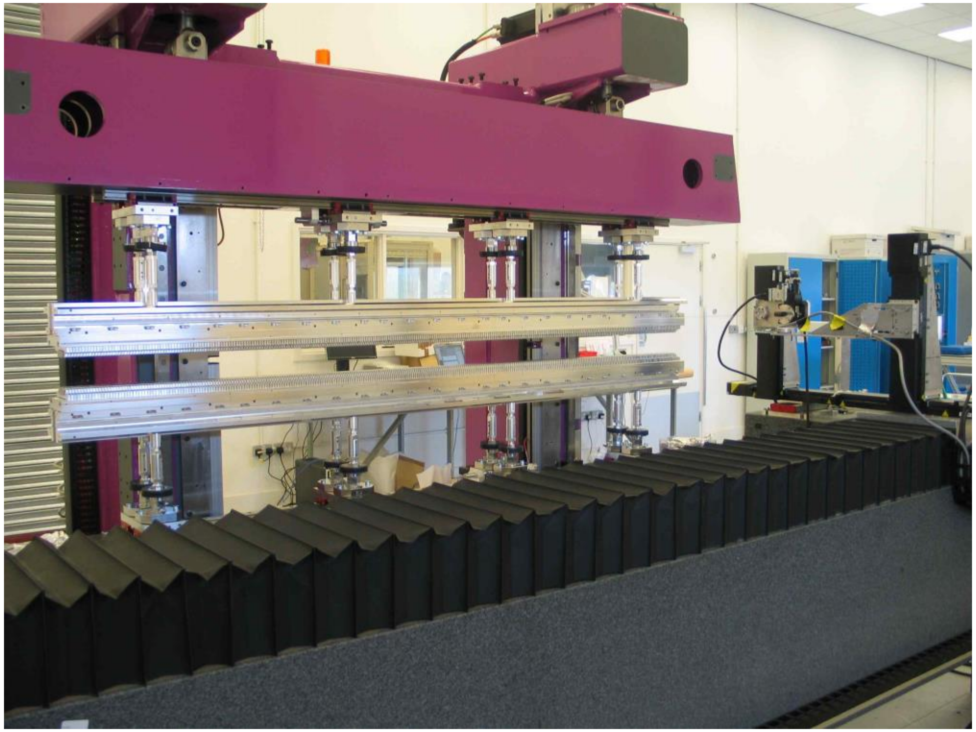


hybrid configuration with steel poles

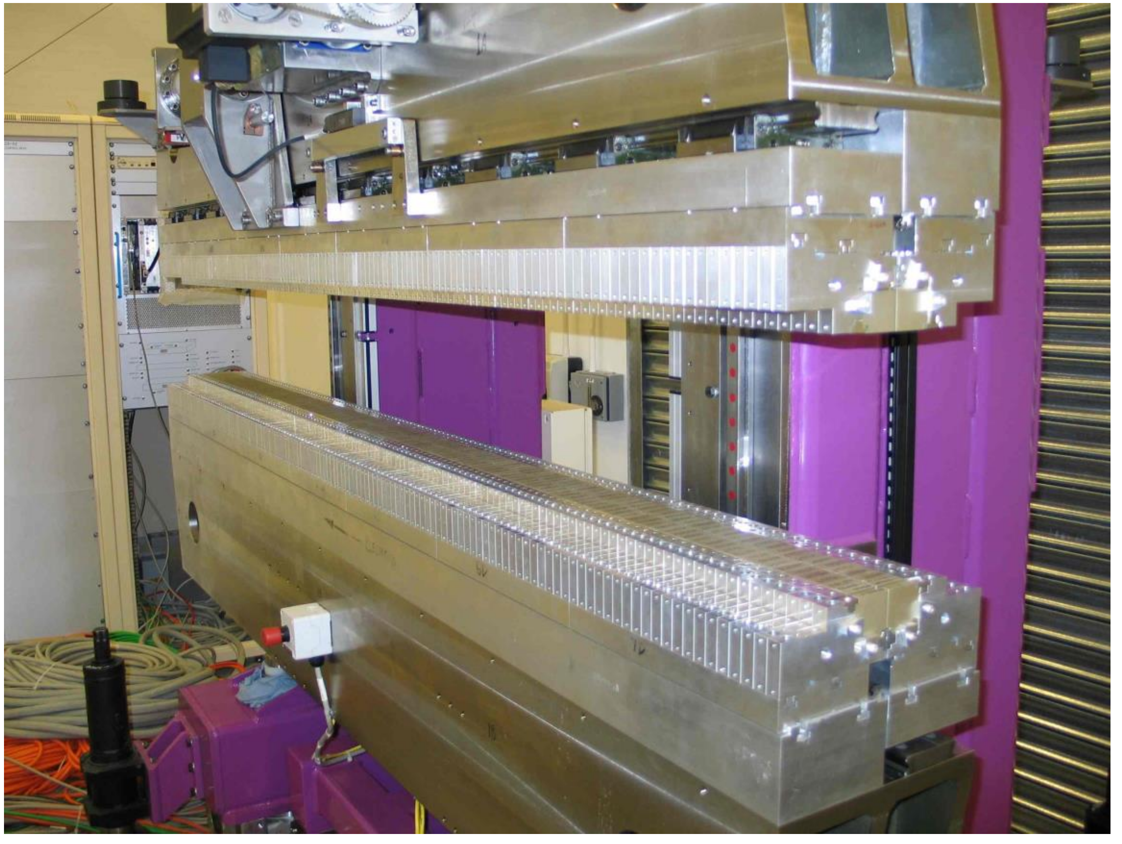
In-vacuum undulators

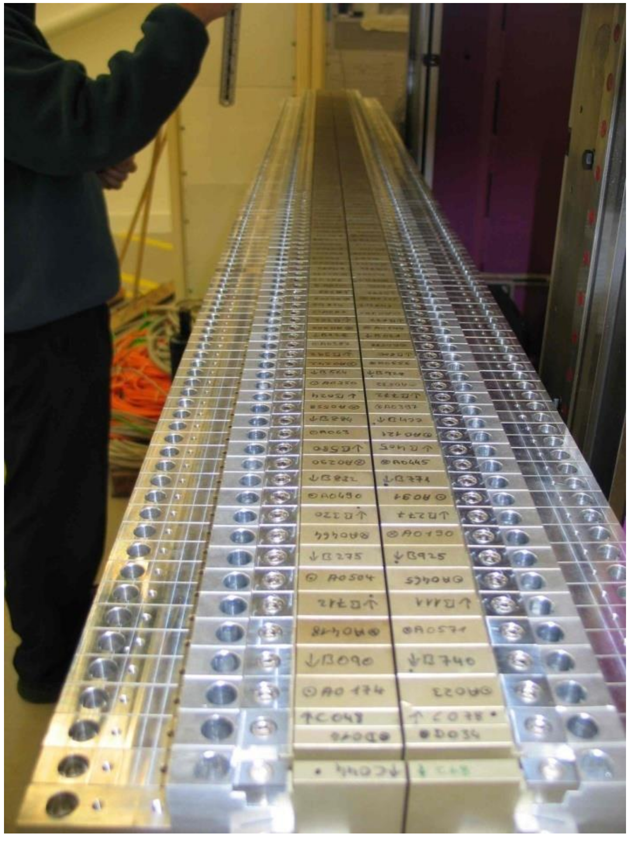
U27 at Diamond

27 mm, 73 periods 7 mm gap,
B = 0.79 T; K = 2



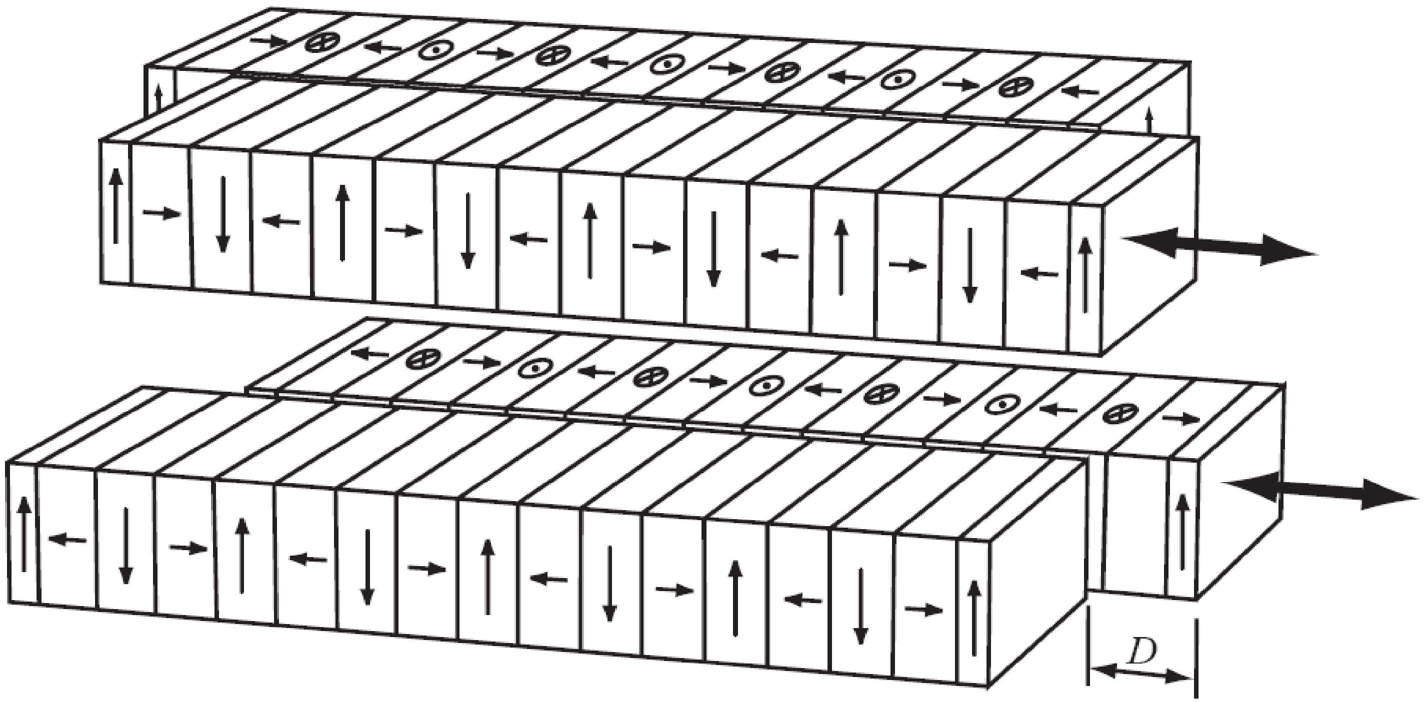
Apple-II type undulators (I)





HU64 at Diamond; 33 period of 64 mm; B = 0.96 T; gap 15 mm; Kmax = 5.3

Apple-II type undulators (II)



Four independent arrays of permanent magnets

Diagonally opposite arrays move longitudinal, all arrays move vertically

Sliding the arrays of magnetic pole it is possible to control the polarisation of the radiation emitted

Superconducting Wigglers

Superconducting wigglers are used when a high magnetic field is required

3 - 10 T

They need a cryogenic system to keep the coil superconductive

Nb_3Sn and NbTi wires

SCMPW60 at Diamond

3.5 T coils cooled at 4 K

24 period of 64 mm

gap 10 mm

Undulator K = 21

Equations of motion in an undulator

- **Assume electrons travelling on the s axis**
- **Paraxial approximation (small angular deflection)**
- **In case of a linear wiggler with $B_y \neq 0$** $\quad \overline{\mathrm{B}} = \mathrm{B}_0 \left(0, \quad \cos(\mathrm{k_u z}), \quad 0\right)$

$$\frac{dp_x}{dt} = -\frac{eB_0}{c} v_z \cos(k_u z)$$

$$\frac{dp_y}{dt} = 0$$

$$\frac{dp_z}{dt} = \frac{eB_0}{c} v_x \cos(k_u z)$$

The solution reads (γ = constant)

Max amplitude of oscillations $\propto$ K

$$\mathrm{v_x} = -\frac{\mathrm{cK}}{\gamma} \sin(\mathrm{k_u} \beta_{\mathrm{z0}} \mathrm{ct})$$

$$\mathrm{v_y} = 0$$

$$\mathrm{v}_z \approx c\left[1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right) + \frac{K^2}{4\gamma^2}\cos(2k_u z)\right]$$

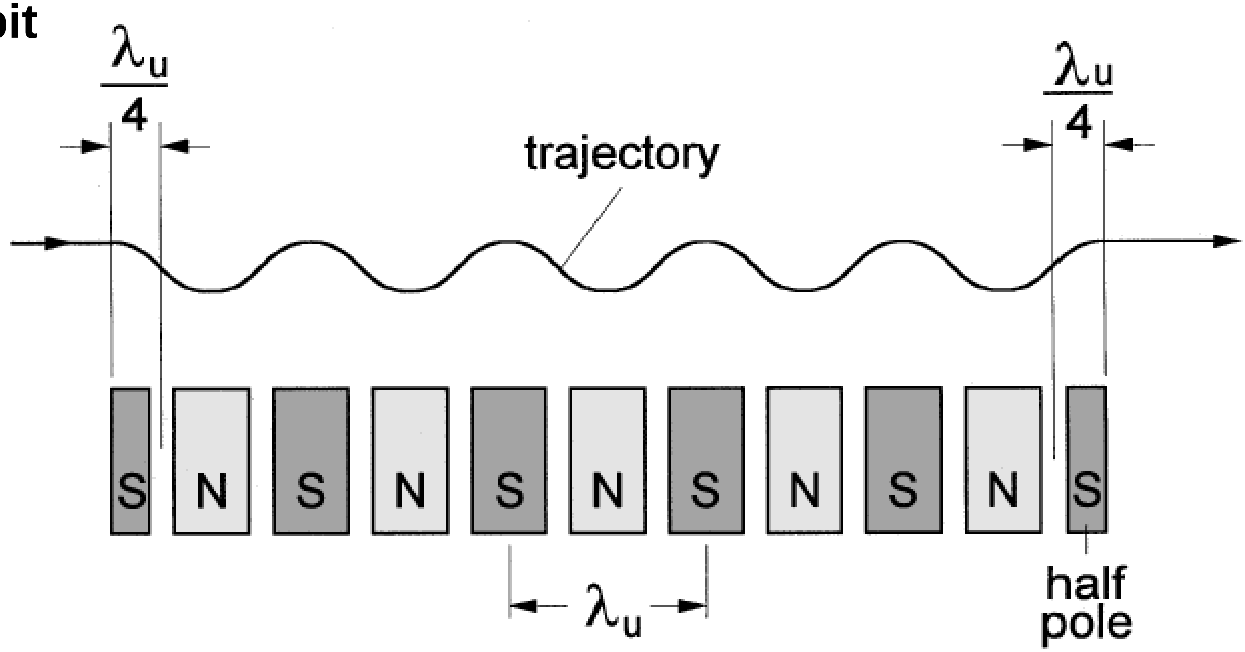
$$x = \frac{K}{k_u \beta_{z0} \gamma} \cos(k_u \beta_{z0} ct) + x_0$$

$$\mathrm{y} = \mathrm{y}_0$$

$$\mathrm{z} = \beta_{\mathrm{z0}} \mathrm{ct} + \frac{\mathrm{K}^2}{8\gamma^2 \mathrm{k_u} \beta_{\mathrm{z0}}} \sin(2\mathrm{k_u} \beta_{\mathrm{z0}} \mathrm{ct}) + \mathrm{z}_0$$

Closed Orbit errors induced by an undulator

The integral of the magnetic field seen in the nominal trajectory path must be zero, otherwise the undulator induces an overall angular kick or an overall offset to orbit



End poles and trim coils are used to ensure that

$$\int_0^L B_y ds = 0$$ **First field integral (angle)**

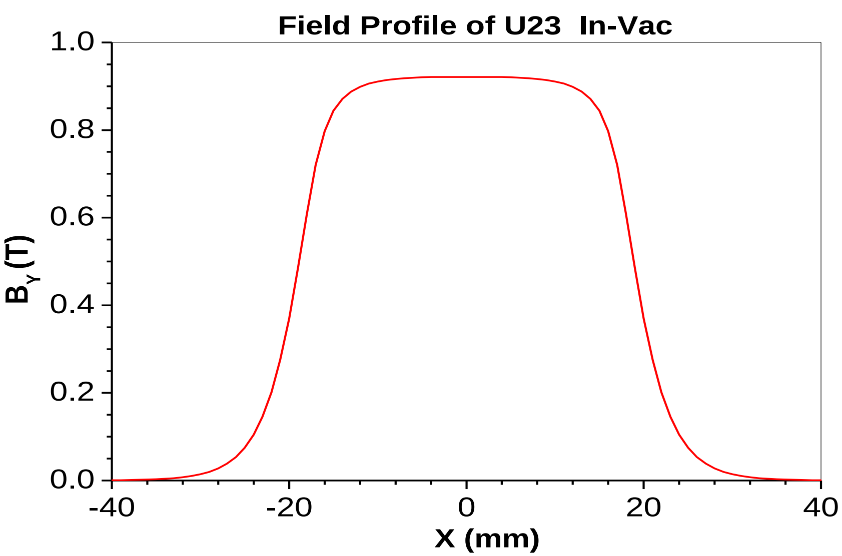
$$\int_0^L ds \int_0^s B_y ds' = 0$$ **second field integral (offset)**

Roll-off of transverse magnetic field

A more realistic analytical expression for the magnetic field of an undulator with a finite pole transverse width is given by:

$$B_x = 0$$
$$B_y = B_w \cosh(k_w y)\cos(k_w z) \qquad \overline{B} = B_0\left(0, \quad \cos(k_w z), \quad 0\right)$$
$$B_z = -B_w \sinh(k_w y)\sin(k_w z)$$

The magnetic fields in real structures exhibit an even more complicated transverse dependence.



Linear focussing and non-linear term appear in the equations of motion and have to be integrated numerically.

e.g. numerically computed field roll-off for an in-vacuum undulator (U23) at Diamond

Quadrupole effect of an undulator (I)

Analytical calculation of the motion can be still performed by keeping the lowest order in y in the expansion of the magnetic field around the nominal oscillatory trajectory

$$B_x = 0$$
$$B_y = B_w \cosh(k_w y)\cos(k_w z) \approx B_w \left[1 + \frac{(k_w y)^2}{2}\right]\cos(k_w z)$$
$$B_z = -B_w \sinh(k_w y)\sin(k_w z) \approx -B_w k_w y \cdot \sin(k_w z)$$

Expanding the generic trajectory (x, y) as

$$x = x_r + x_{ref} \qquad\qquad y = y_r + y_{ref}$$

and averaging over one undulator period we end up with

$$x_r'' = 0 \qquad\qquad y_r'' = -2\left(\frac{\pi K}{\lambda_u \gamma}\right)^2 y_r$$

An undulator behaves as a focussing quadrupole in the vertical plane and as a drift in the horizontal plane, to the lowest order in the deviation form the reference trajectory.

Quadrupole effect of an undulator (II)

The quadrupole strength is proportional to B^2 and $1/E^2$

$$K_y = 2\left(\frac{\pi K}{\lambda_u \gamma}\right)^2 = \frac{1}{2}\left(\frac{eB}{mc\gamma}\right)^2$$

Unlike a true vertically focussing quadrupole, an ideal undulator does not have a corresponding defocussing effect in the horizontal plane in first order; in the horizontal plane it may have a weak defocussing due to the finite width of the magnetic poles;

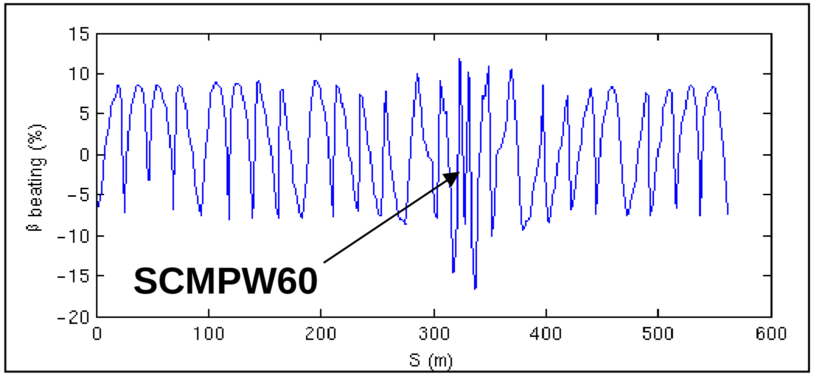
The quadrupole associated to the undulator generated a tune shift and a beta-beating

$$\Delta Q_y = \frac{K_y L_w}{2\pi}\beta_y \qquad \frac{\Delta\beta_y}{\beta_y} = \frac{-K_y L \beta_y}{2\sin(2\pi Q_y)}$$

e.g. Diamond Superconducting wiggler

$\Delta Q_y = 0.012$

$\Delta\beta/\beta \sim 10\%$



Effect of undulators and wigglers on beam dynamics and cures

Principal effects of undulators and wigglers on beam dynamics

Closed orbit distortion

Betatron Tune shift

Optics variation (β - beating)

Dynamic aperture reduction

Variation of damping times; Emittances; Energy spread

Remedies improving field qualities

Correction of the field integral + Trim coil for closed orbit distortion

Wide transverse gap (reduced roll-off) for linear optics

“Magic fingers” to decrease the multipole component of the wiggler

Remedies using beam optics methods

Feed forward tables for trim coil orbit corrections

Local correction of optics functions (alpha matching schemes, LOCO)

Non-linear beam dynamics optimisation with wiggler

Summary and additional bibliography

Undulators and Wigglers enhance synchrotron radiation

- **Undulators produce a narrow band series of harmonics**
- **Wigglers produce a broadband radiation**

Radiation can have linear or elliptical polarisation

Undulators and wiggler perturb the beam dynamics in the storage ring

- **Field quality must be excellent**
- **Effective correction schemes for orbit and linear optics are available**

R.P. Walker: CAS 98-04, pg. 129

A. Ropert: CAS 98-04, pg. 91

P. Elleaume in Undulators, Wigglers and their applications, pg. 69-107