Accelerator Physics Exercises

Answers to be handed in before lecture on 29 October 2014

1. In the "weak focusing" accelerators, the field gradient is defined as

$$k = -\frac{1}{(B_0\rho_0)}\frac{\partial B_z}{\partial x}$$

Show that the equation of motion of a particle in the horizontal degree of freedom is

$$\frac{1}{p_0}\frac{d}{ds}\left(p_0\frac{dx}{ds}\right)+\left(\frac{1}{\rho^2}-k\right)x=0$$

and in the vertical plane:

$$\frac{1}{p_0}\frac{d}{ds}\left(p_0\frac{dz}{ds}\right)+kz\ =0$$

Vertical oscillations are stable as long as $k >$ o, discuss the limit on k for horizontal stability.

2. A new 50 GeV (kinetic energy) proton synchrotron, the PS2 accelerator, is considered to replace the CERN PS. The new accelerator will sit in a new ring tunnel which has a mean radius of 215 m. and will receive an injected beam at 4 GeV (kinetic energy) from a new linear accelerator - the Superconducting Proton Linac (SPL). The 1.8 T magnetic field of the bending magnets is excited by a sine wave which oscillates between injection and top energy at a frequency of 0.3 Hz. Given that the mass of the proton is 0.9383 GeV:

a) What is the momentum at 4 GeV and at 50 GeV?

b) What is the (B rho) at 4 GeV and at 50 GeV?

c) What is the bending radius, rho?

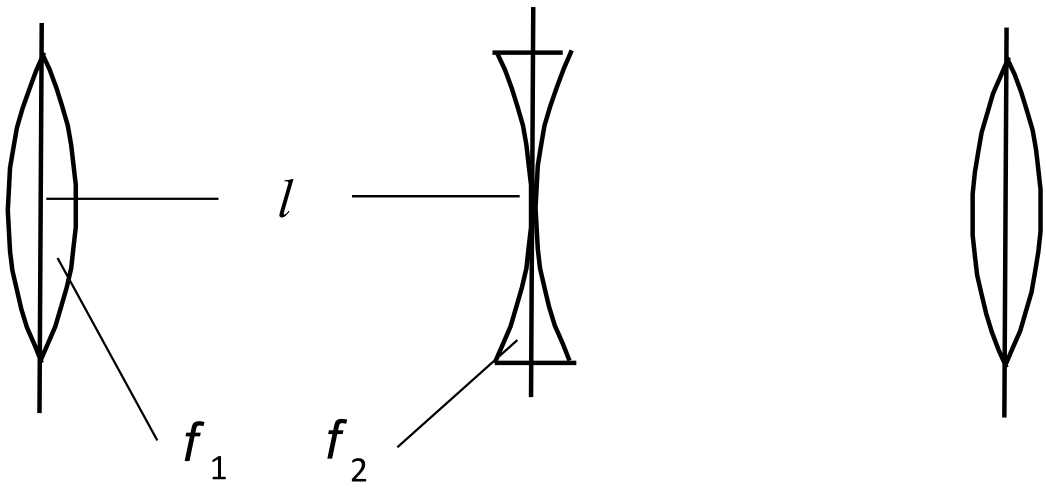
d) What is the fraction of the ring filled with dipole magnets?

3. (a) A quadrupole doublet (half of the FODO cell from the mid-point of the F to the mid point D) consists of two lenses of focal length f_1 and f_2 separated by a drift length of l m. Assume that the lenses are thin and show, by writing the three matrices for the lenses, that the product matrix is:

$$M = \begin{pmatrix} 1 - l/f_1 & \ell \\ -1/f^* & 1 - l/f_2 \end{pmatrix}$$

where:

$$\frac{1}{f^*} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{l}{f_1 f_2}$$



(b) A FODO cell (from mid-F, through D and to the midpoint of the next F) may be considered to be one such matrix with $f_1 = +2f$ and $f_2 = -2f$ followed (and multiplied) by another matrix with $f_1 = -2f$ and $f_2 = +2f$. Using the result of the last question, write down these two matrices and show by taking their product that the matrix for a FODO cell from mid-F to mid-F quadrupole

$$= \begin{pmatrix} 1 - l^2/2f^2 & 2l(1 + l/2f) \\ -l/2f^2(1 - l/2f) & 1 - l^2/2f^2 \end{pmatrix}$$

(c) The matrix for a FODO period must have the form:

$$M = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

Take the trace of this matrix and equate it to the result of (b) to obtain an expression for μ, the phase advance per period as a function of l and f

Now equate m_{12} to find β at the mid plane of the F quadrupole.

You are given the following data for the new PS2 ring:

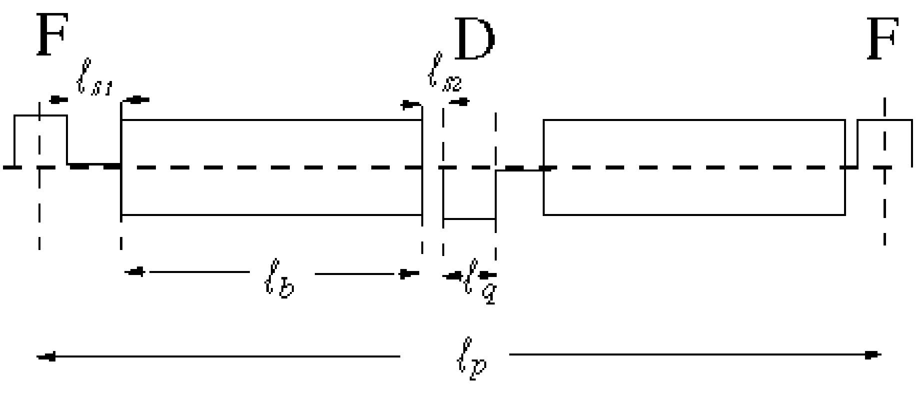
(B rho) to be calculated from Exercise 2b.

Quadrupole half-length (lq/2) = (1.49/2) m.

Quadrupole gradient = 17 T/m

Distance between quadrupole centres = 11.605 m.

Substitute the data to obtain the numerical value of μ and beta.



4) The emittance of a proton beam at injection in the CERN SPS is 2 mm mrad.

a) Calculate the half-width of the beam at an F quadrupole where β = 108 m.
b) What is the maximum value of the divergence in the beam if the β at a D quadrupole is 18 m. and α = 0?
c) What is the normalized emittance of this beam if the above data refer to a proton momentum of 10 GeV/c?
d) If this normalized emittance is accelerated to 400 GeV/c, what will be the half-width of the beam at an F quadrupole (β = 109 m.)?