Lecture 8 - Circulating Beams and Imperfections

ACCELERATOR PHYSICS

MT 2014

E. J. N. Wilson

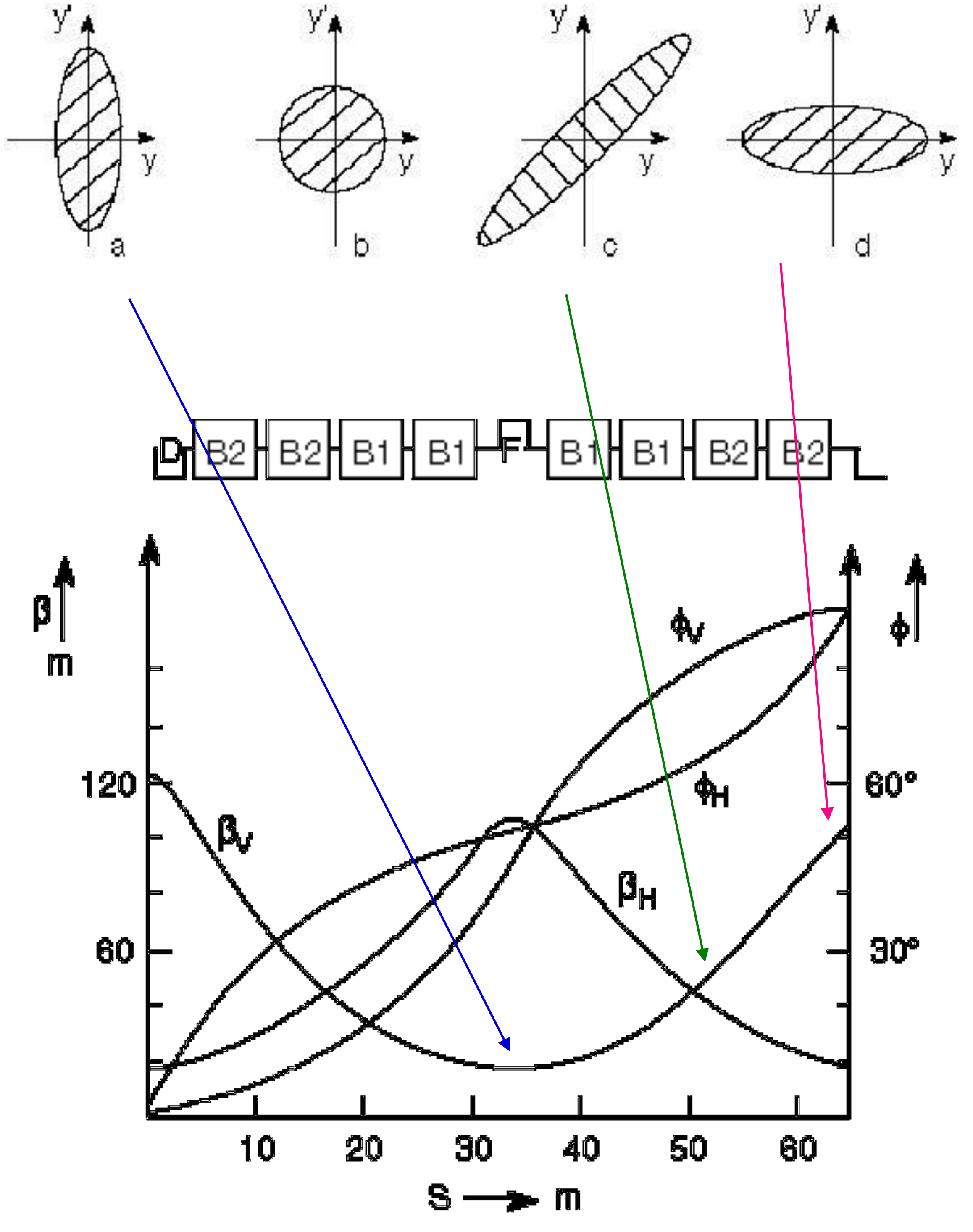
Previously - Longitudinal dynamics II

- **Transition - does an accelerated particle catch up - it has further to go**
- **Phase jump at transition**
- **Synchrotron motion**
- **Synchrotron motion (continued)**
- **Large amplitudes**
- **Buckets**
- **Buckets**
- **Adiabatic capture**
- **A chain of buckets**

Lecture 7 - Circulating Beams and Errors - Contents

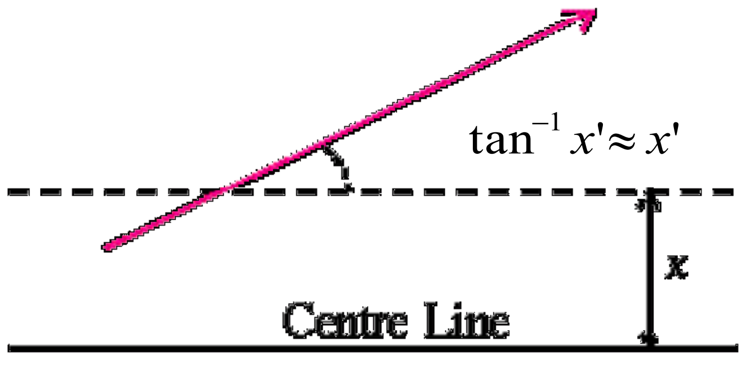
- Phase space at various points in a lattice
- Adiabatic damping
- Adiabatic damping of proton beam
- Liouville's theorem - Strict Version
- Emittance definitions
- Acceptance
- Making an orbit bump grow
- Circle diagram
- Closed orbit in the circle diagram
- Uncorrelated errors
- Sources of distortion
- FNAL measurement
- Diad bump
- Overlapping beam bumps
- Effect of quadrupole errors.
- Chromaticity
- Closed orbit in the circle diagram
- Gradient errors
- Working daigram

Betatron phase space at various points in a lattice

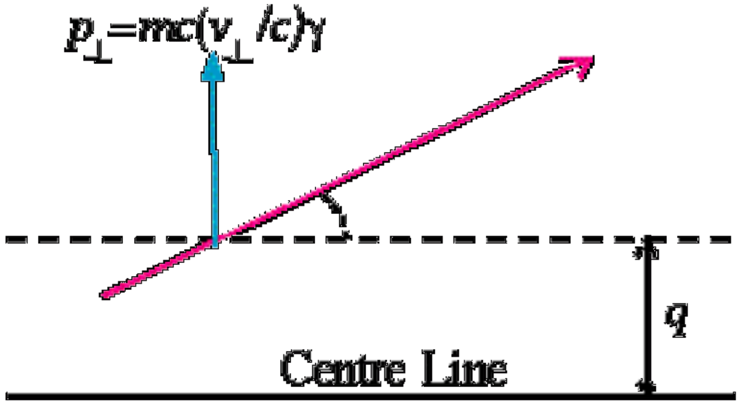


Adiabatic damping

- **Accelerator coordinates**



- **Canonical coordinates of Hamilton**

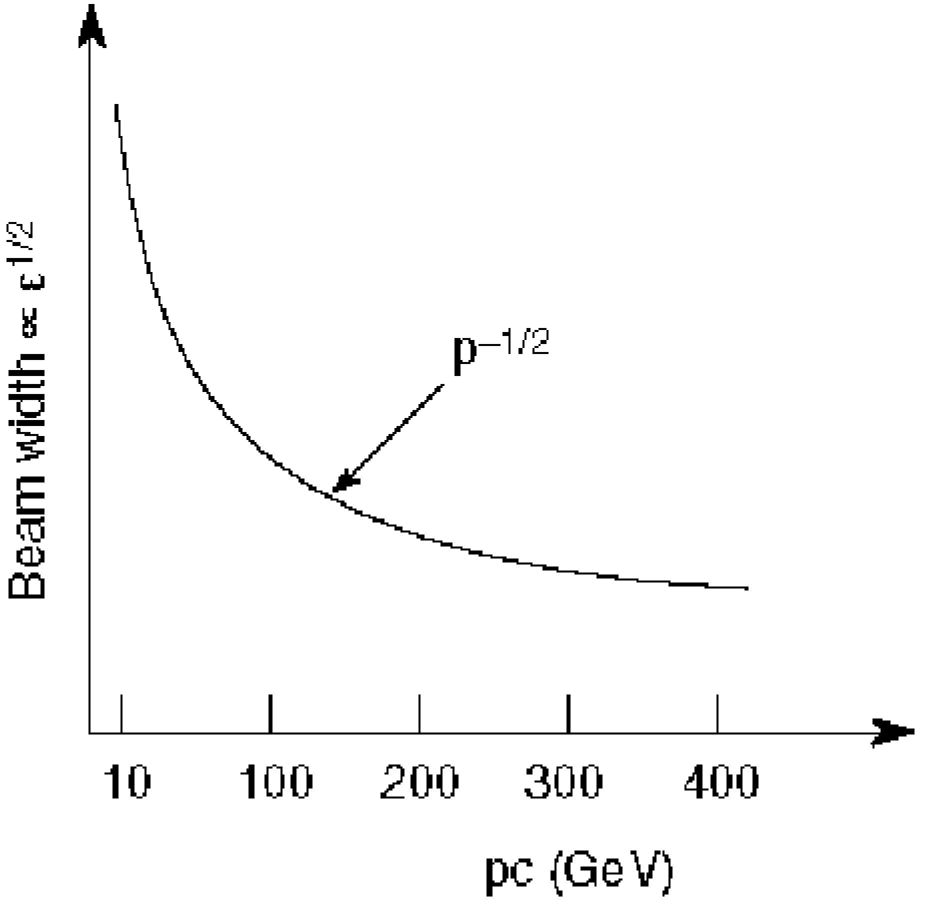


$$p = m\frac{dq}{dt}\gamma = m\frac{ds}{dt}\frac{dq}{ds}\gamma = mc(\beta\gamma)x'$$

$$\int p\,dq = mc(\beta\gamma)\int x'dx = p_0\int x'dx$$

$$\text{Emittance} = \pi\varepsilon = \int x'dx = \pi\varepsilon_{\text{normalised}}/(\beta\gamma) \propto 1/p_0$$

Adiabatic damping of proton beam

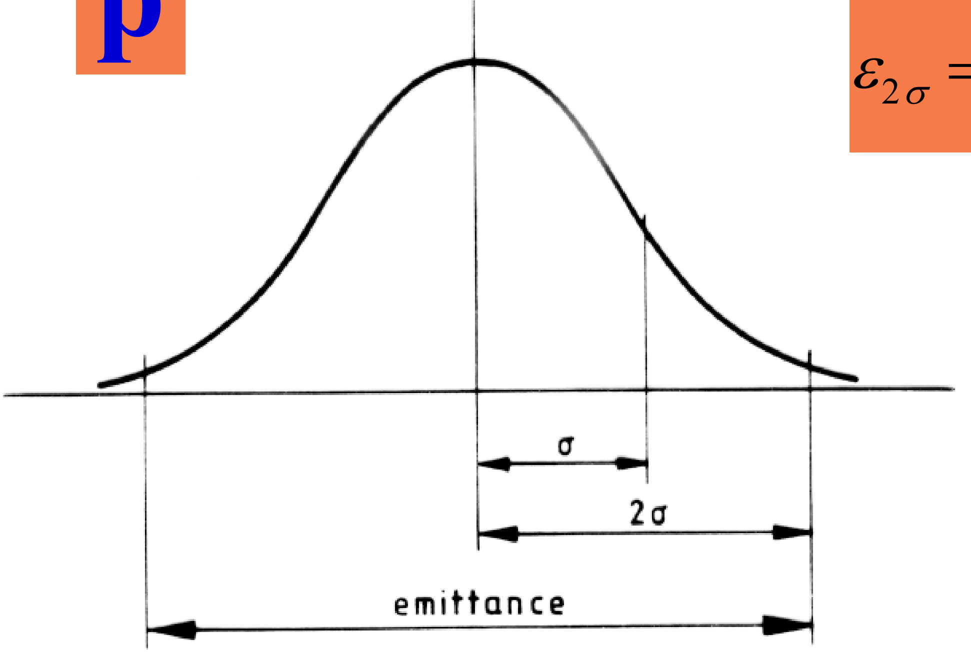


Emittance definitions

p

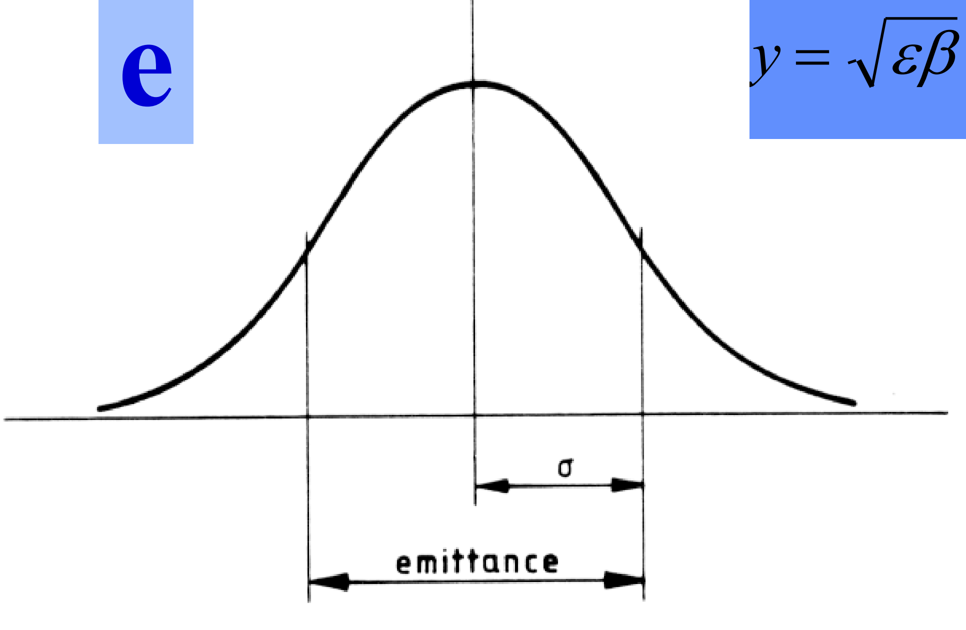
$$y = \sqrt{\varepsilon\beta} \qquad \therefore \varepsilon = \frac{y^2}{\beta}$$

$$\varepsilon_{2\sigma} = \frac{(2\sigma)^2}{\beta} = \frac{4\sigma^2}{\beta}$$

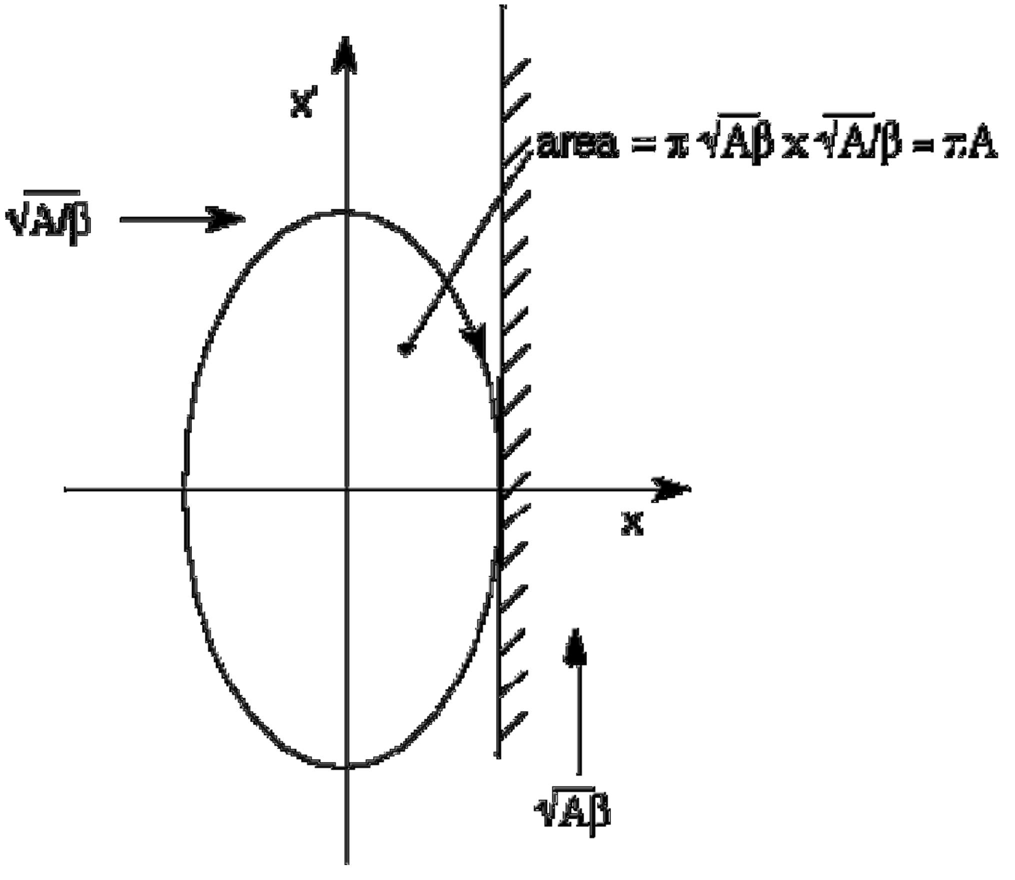


e

$$y = \sqrt{\varepsilon\beta} \qquad \varepsilon_{\sigma} = \frac{\sigma^2}{\beta}$$



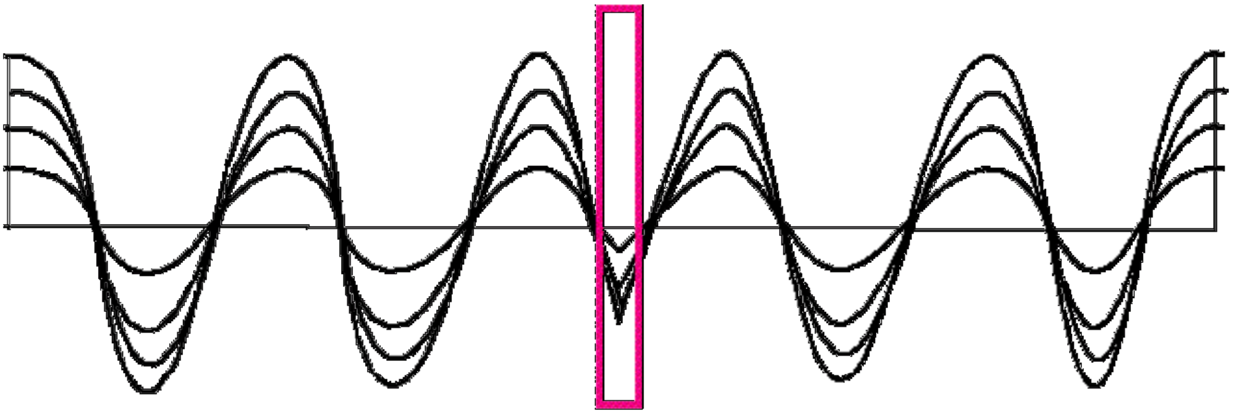
Acceptance



- **Largest particle grazing an obstacle defines acceptance.**
- **Acceptance is equivalent to emittance**

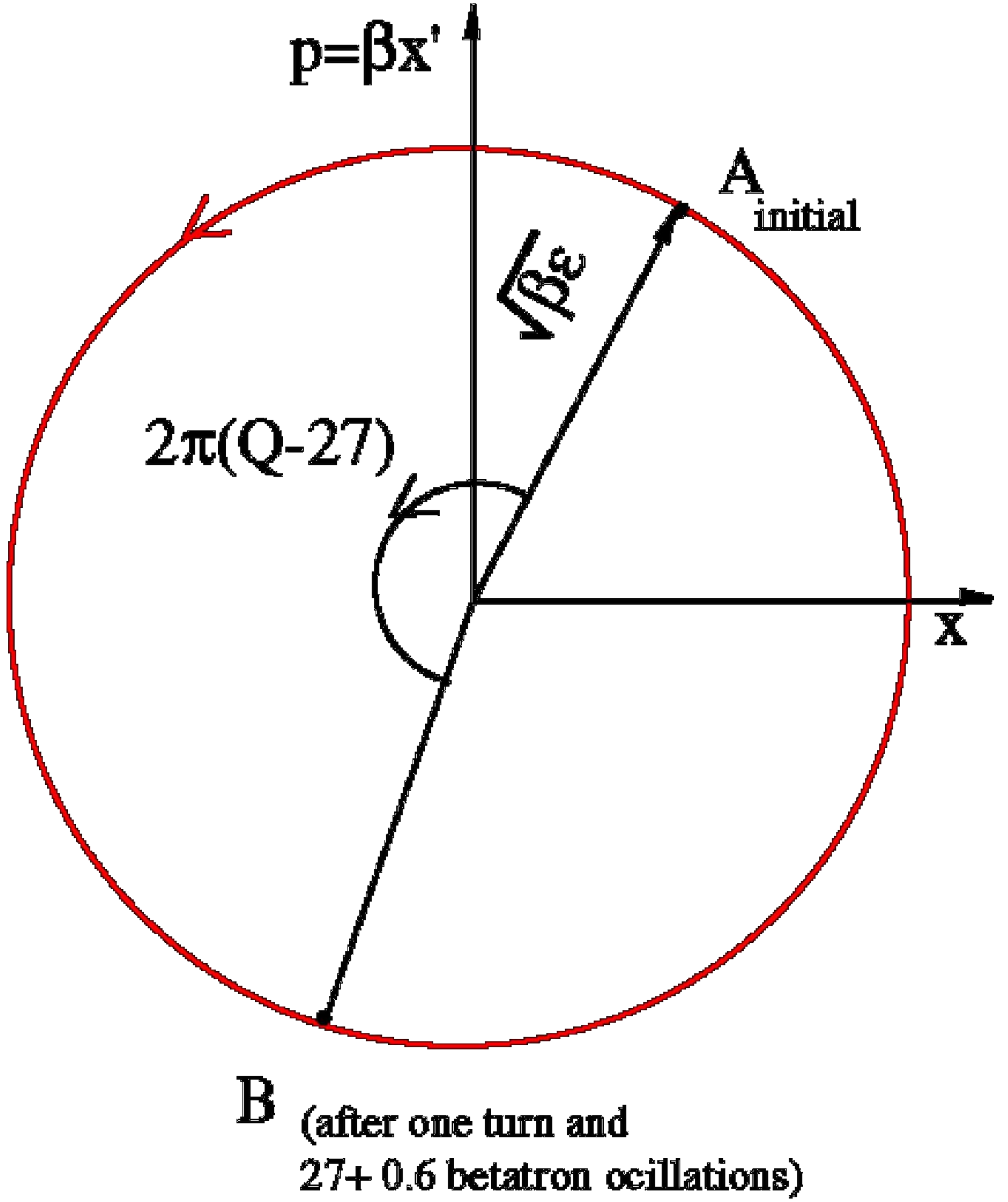
$$A = \frac{\hat{x}^2}{\beta}$$

Making an orbit bump grow



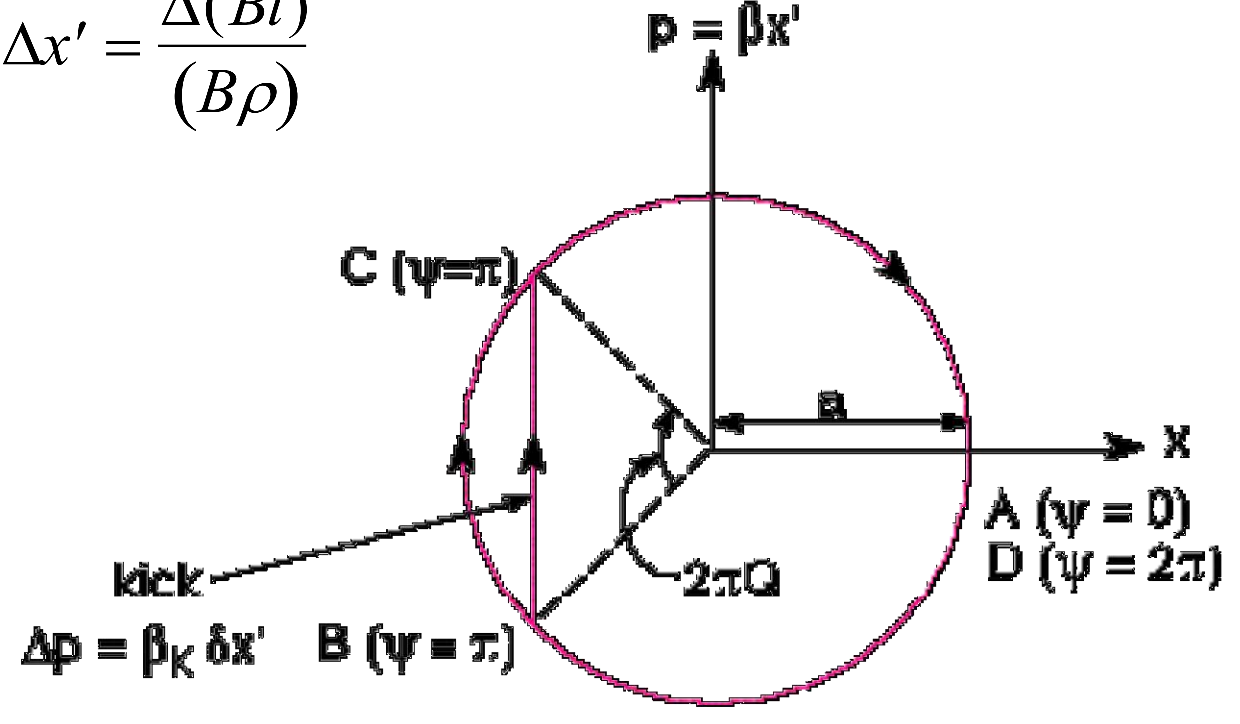
- As we slowly raise the current in a dipole:
- The zero-amplitude betatron particle follows a distorted orbit
- The distorted orbit is CLOSED
- It is still obeying Hill's Equation
- Except at the kink (dipole) it follows a betatron oscillation.
- Other particles with finite amplitudes oscillate about this new closed orbit

Circle diagram



Closed orbit in the circle diagram

$$\Delta x' = \frac{\Delta(Bl)}{(B\rho)}$$



Tracing a closed orbit for one turn in the circle diagram with a single kick. The path is ABCD.

$$\frac{\Delta p}{2} = \frac{\beta_k \delta x'}{2} = a \sin\left(\frac{2\pi Q}{2}\right)$$

$$a = \frac{\beta_k \delta x'}{2\sin \pi Q}$$ **elsewhere** $$\hat{x} = a\sqrt{\frac{\beta(s)}{\beta_k}} = \frac{\sqrt{\beta_k \beta(s)}}{2\sin \pi Q}\delta x'$$

Uncorrelated errors

- **A random distribution of dipole errors**

- **Take the r.m.s. average of** $\delta y_i' = \Delta(Bl)/(B\rho)$
- **Weighted according to the** β_k **values**
- **The expectation value of the amplitude is:**

$$\langle y(s) \rangle = \frac{\sqrt{\beta(s)}}{2\sqrt{2}\sin \pi Q}\sqrt{\sum_i \beta_i \delta y_i'^2}$$

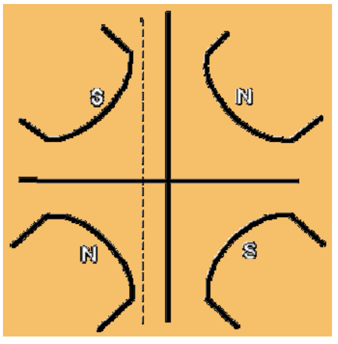
- **Kicks from the** ***N*** **magnets in the ring.**

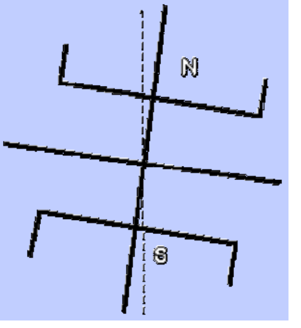
$$\approx \frac{\sqrt{\beta(s)\bar{\beta}}}{2\sqrt{2}\sin \pi Q} \sqrt{N} \frac{(\Delta B\ell)_{rms}}{B\rho}$$

- **The factor** $\sqrt{2}$ **takes into account the averaging over both sine and cosine phases**

- **A further factor 2 safety is applied to include 98% of all sample distributions.**

Sources of distortion





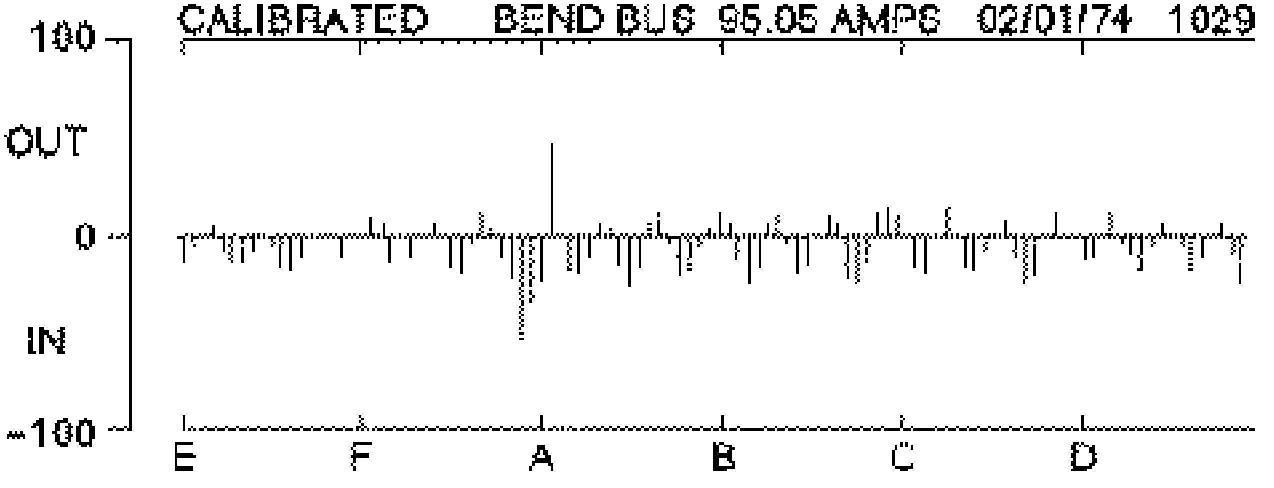
Δy **Δ**

Table 1

Sources of Closed Orbit Distortion

Type of element	Source of kick	r.m.s. value	$\langle \Delta B l / (B\rho) \rangle_{rms}$	plane
Gradient magnet	Displacement	$<\Delta y>$	$k_i l_i <\Delta y>$	x,z
Bending magnet (bending angle = θi)	Tilt	$<\Delta>$	$\theta_i <\Delta>$	z
Bending magnet	Field error	$<\Delta B/B>$	$\theta i <\Delta B/B>$	x
Straight sections (length = d_i)	Stray field	$<\Delta B_S>$	$d_i \langle \Delta B_s \rangle / (B\rho)_{inj}$	x,z

FNAL MEASUREMENT



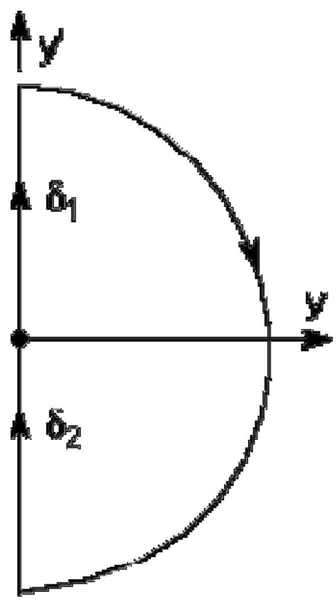
- Historic measurement from FNAL main ring
- Each bar is the position at a quadrupole
- +/- 100 is width of vacuum chamber
- Note mixture of 19th and 20th harmonic
- The Q value was 19.25

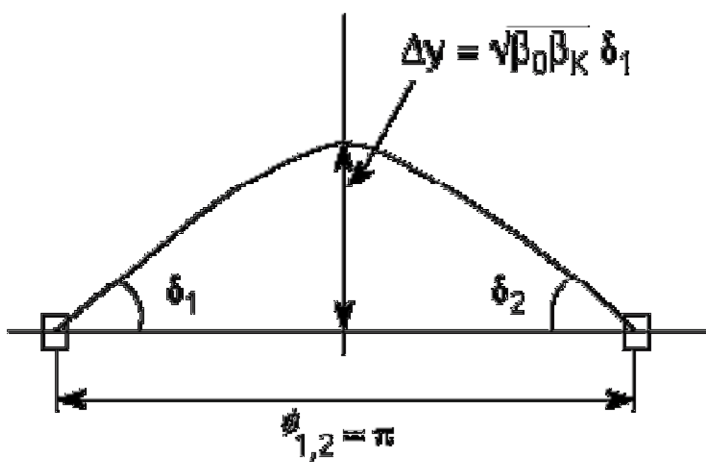
Diad bump

- Simplest bump is from two equal dipoles 180 degrees apart in betatron phase. Each gives:

$$\delta = \frac{\Delta(B\,l)}{B\rho}$$

- The trajectory is : $y(s) = \delta\sqrt{\beta(s)\,\beta_k}\,\sin(\phi - \phi_0)$

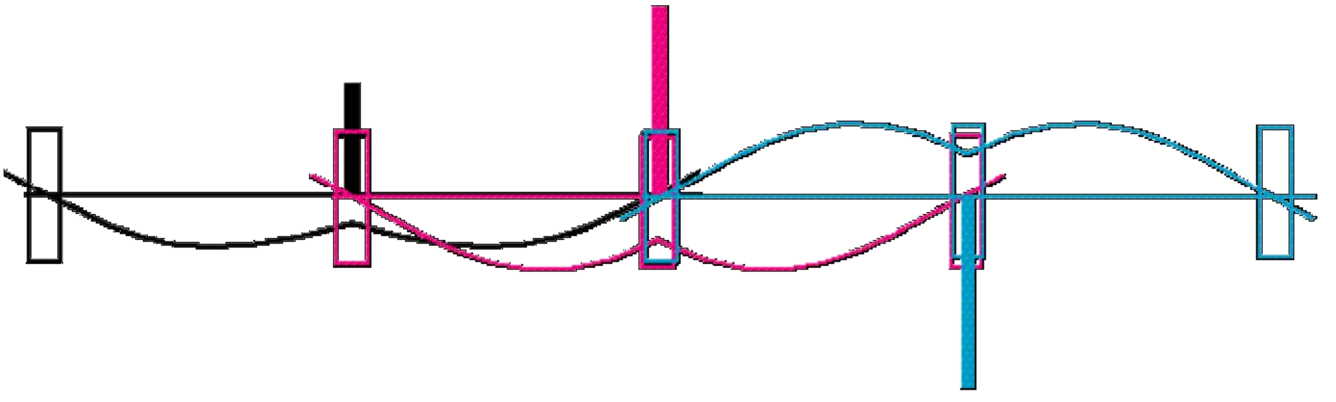




- The matrix is

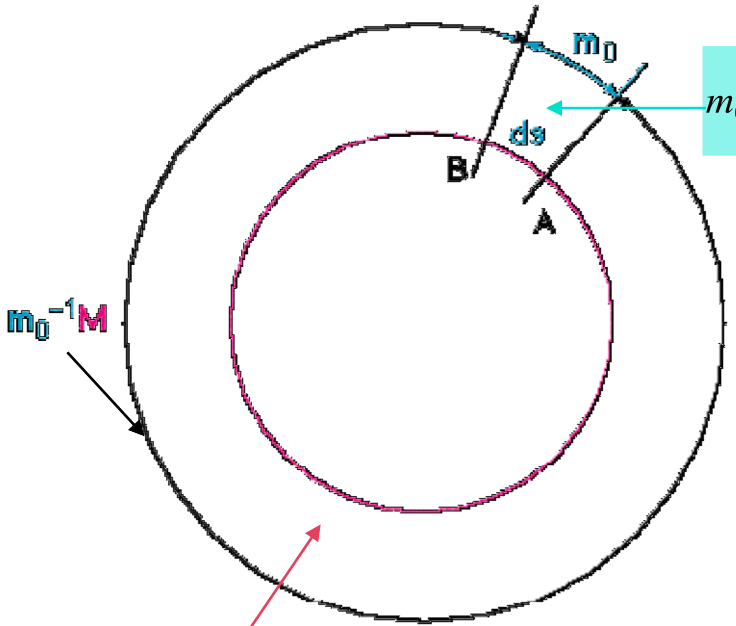
$$\begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} \left(\sqrt{\beta}/\sqrt{\beta_0}\right)\left(\cos\Delta\phi + \alpha_0 \sin\Delta\phi\right) & , \sqrt{\beta_0\beta}\,\sin\Delta\phi \\ \left(-1/\sqrt{\beta_0\beta}\right)\left\{\left(\alpha - \alpha_0\right)\cos\Delta\phi + \left(1 + \alpha\alpha_0\right)\sin\Delta\phi\right\}, & \left(\sqrt{\beta}/\sqrt{\beta_0}\right)\left(\cos\Delta\phi - \alpha\sin\Delta\phi\right) \end{pmatrix} \begin{pmatrix} y_0 \\ y'_0 \end{pmatrix}$$

Overlapping beam bumps



- Each colour shows a triad bump centred on a beam position measurement.
- A computer calculates the superposition of the currents in the dipoles and corrects the whole orbit simultaneously

Gradient errors



$$m_0 = \begin{pmatrix} 1 & 0 \\ -k_0(s)ds & 1 \end{pmatrix} ,$$

$$m = \begin{pmatrix} 1 & 0 \\ -\left[k_0(s) + \delta k(s)\right]ds & 1 \end{pmatrix} .$$

$$M_0(s) = \begin{pmatrix} \cos\varphi_0 + \alpha_0 \ \sin\varphi_0 \ , & \beta_0 \ \sin\varphi_0 \\ -\gamma_0 \ \sin\varphi_0 \ , & \cos\varphi_0 - \alpha_0 \ \sin\varphi_0 \end{pmatrix} .$$

$$M(s) = m m_0^{-1} M_0 \ .$$

$$m m_0^{-1} = \begin{pmatrix} 1 & 0 \\ -\delta k(s_1)ds & 1 \end{pmatrix} .$$

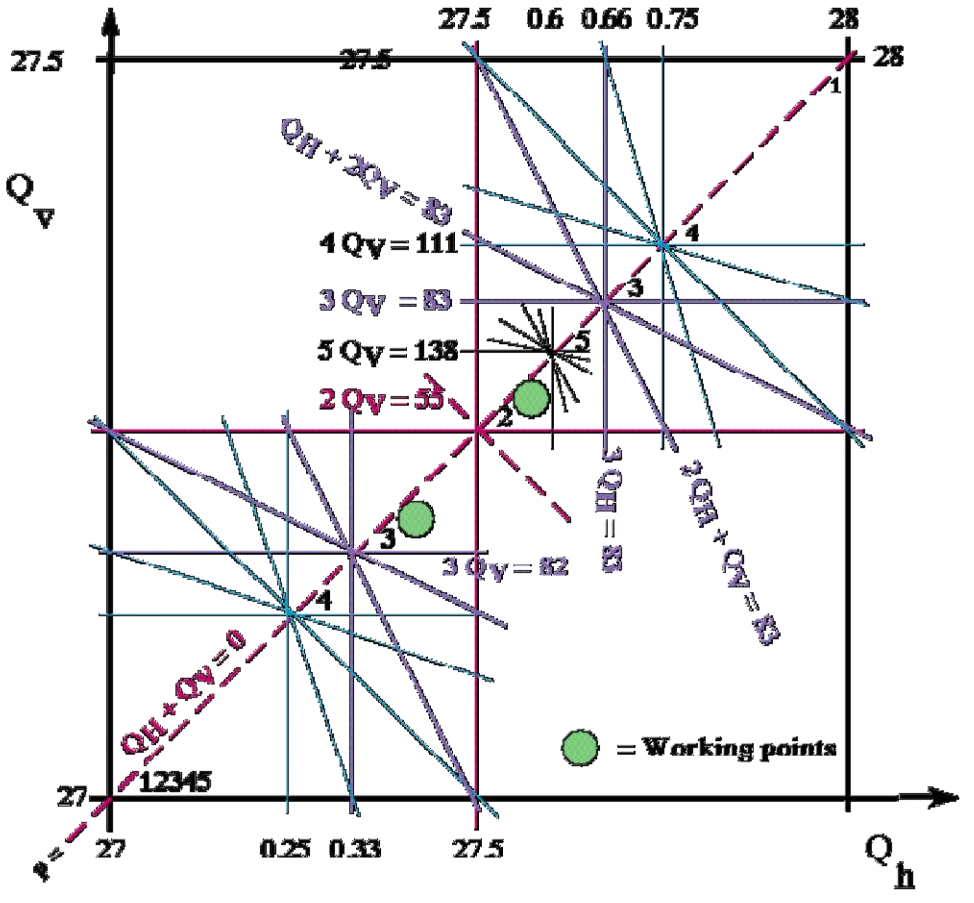
$$M = \begin{pmatrix} \cos\ \phi_0 + \alpha_0 \sin\ \phi_0 \ , & \beta_0 \sin\ \phi_0 \\ -\delta k(s)ds\left(\cos\ \phi_0 + \alpha_0 \sin\ \phi_0\right) - \gamma \sin\ \phi_0 \ , & -\delta k(s)ds\beta_0 \sin\ \phi_0 + \cos\ \phi_0 - \alpha_0 \sin\ \phi_0 \end{pmatrix} .$$

$$\Delta(\mathrm{Tr\ M})/\ 2 = \Delta(\cos\phi) = -\Delta\phi \sin\phi_0 = \frac{\sin\phi_0}{2}\beta_0(s)\delta k(s)ds$$

$$2\pi\Delta Q = \Delta\phi = \frac{\beta(s)\delta k(s)ds}{2} \ .$$

$$\boxed{\Delta Q = \frac{1}{4\pi}\int \beta(s)\delta k(s)\ \ ds \ .}$$

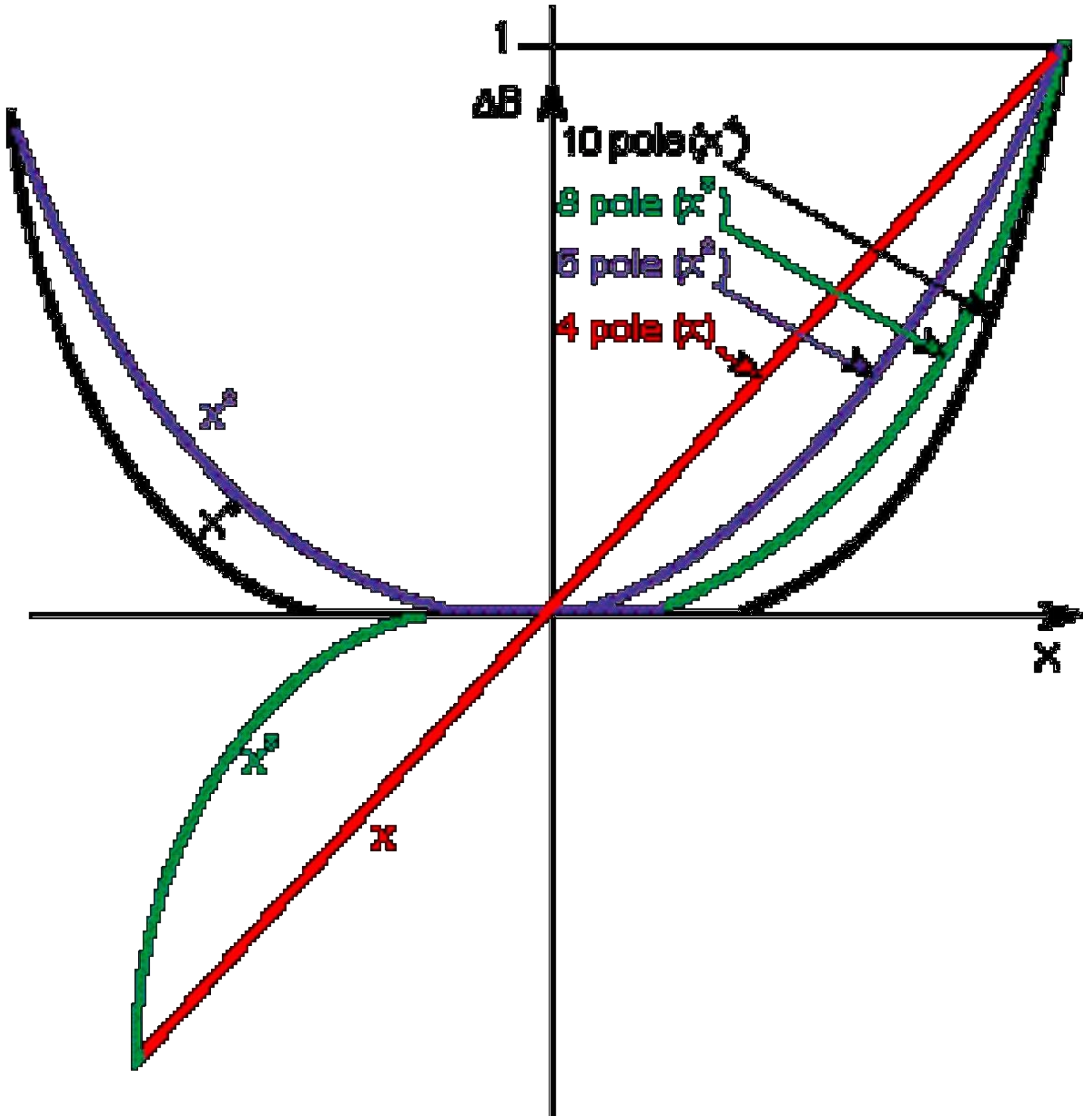
Q diagram



$$nQ = p \ ,$$

$$\boxed{1Q_H + mQ_V = p \ ,}$$

Multipole field shapes



Physics of Chromaticity

- The Q is determined by the lattice quadrupoles whose strength is:

$$k = \frac{1}{(B\rho)} \frac{dB_z}{dx} \quad \propto \frac{1}{p}$$

- Differentiating: $\quad \frac{\Delta k}{k} = -\frac{\Delta p}{p} \; .$
- Remember from gradient error analysis

$$\Delta Q = \frac{1}{4\pi} \int \beta(s)\delta k(s) \; ds \; .$$

- Giving by substitution

$$\Delta Q = \frac{1}{4\pi} \int \beta(s)\Delta k(s) ds = \left[\frac{-1}{4\pi} \int \beta(s) k(s) ds \right] \frac{\Delta p}{p} \; .$$

$$\Delta Q = Q' \frac{\Delta p}{p}$$

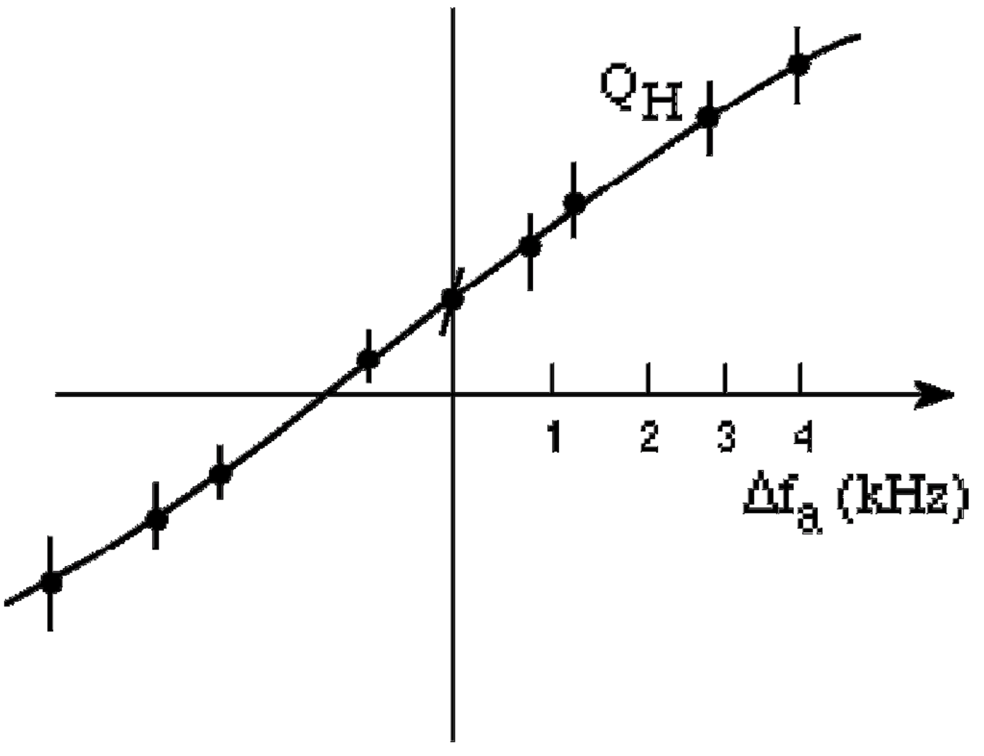
Q' **is the chromaticity**

- "Natural" chromaticity

$$Q' = -\frac{1}{4\pi} \oint \beta(s) k(s) ds \approx -1.3Q$$

N.B. Old books say $\xi = \frac{p}{Q} \frac{dQ}{dp} = \frac{Q'}{Q}$

Measurement of Chromaticity



- **We can steer the beam to a different mean radius and a different momntum by changing the rf frequency and measure Q**

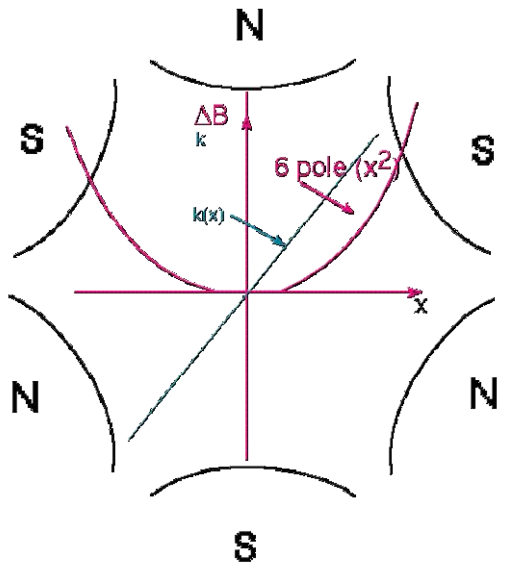
$$\Delta f_a = f_a \eta \frac{\Delta p}{p} \qquad \Delta r = D_{av} \frac{\Delta p}{p}$$

- **Since** $\Delta Q = Q' \frac{\Delta p}{p}$

- **Hence**

$$\therefore Q' = f_a \eta \frac{dQ}{df_a}$$

Correction of Chromaticity



- **Parabolic field of a 6 pole is really a gradient which rises linearly with x**
- **If x is the product of momentum error and dispersion**

$$\Delta k = \frac{B'' D}{(B\rho)} \frac{\Delta p}{p} \ .$$

- **The effect of all this extra focusing cancels chromaticity**

$$\Delta Q = \left[\frac{1}{4\pi} \int \frac{B''(s)\beta(s)D(s)\,ds}{(B\rho)} \right] \frac{dp}{p} \ .$$

- **Because gradient is opposite in v plane we must have two sets of opposite polarity at F and D quads where betas are different**

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- **Phase space at various points in a lattice**
- **Adiabatic damping**
- **Adiabatic damping of proton beam**
- **Liouville's theorem - Strict Version**
- **Emittance definitions**
- **Acceptance**
- **Making an orbit bump grow**
- **Circle diagram**
- **Closed orbit in the circle diagram**
- **Uncorrelated errors**
- **Sources of distortion**
- **FNAL measurement**
- **Diad bump**
- **Overlapping beam bumps**
- **Effect of quadrupole errors.**
- **Chromaticity**
- **Closed orbit in the circle diagram**
- **Gradient errors**
- **Working daigram**