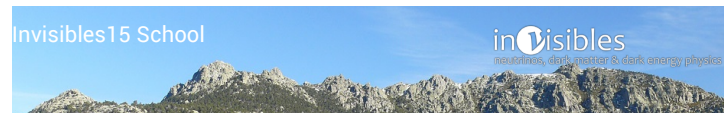


LHC Phenomenology

Gilad Perez

Weizmann Inst.

Invisibles School 2015



Outline (totally subjective)

Lecture I:

- Some motivation.
- Calculating LHC cross sections (σ).
- Parton distribution functions, parton luminosities.

Lecture II:

- Example, top-pair σ calculation.
- Kinematics & resonance search.

Outline

Lecture III:

- Resonance production vs. EFT production.
- Intro to jet phys.

Lecture IV:

- Jets cont'.
- Jet substructure phys., boosted massive jets. (if time permits)

Lecture I:

Some motivation (SM problems, naturalness);

How to calculate Xsections @ the LHC;

Parton distribution functions (PDFs) parton luminosities.

Link to notes: https://www.dropbox.com/s/znmb3xod9en41hi/LHC_Gilad_Perez_Lectures%20new.pdf?dl=0

Mathematica notebook+PDF files that are public, if you are interested in doing the ex.:

https://www.dropbox.com/s/xnr0449ehjndri1/Example_invisibles_LHC.nb?dl=0

<https://www.dropbox.com/s/q1mdtbt5qyoj229/Lall14.txt?dl=0>

<https://www.dropbox.com/s/7j6xelcg7k38m8r/Lall100.txt?dl=0>

Credit: my student, Yotam Soreq.

For advanced tools, see Fabio Maltoni's lectures.

Why the LHC? What are the problems of the Standard Model* (SM), before the LHC started?

| WW/unitarity, masses | fine tuning, naturalness | neutrino masses | flavor puzzle |
|-------------------------|-----------------------------|-----------------|--|
| | | dark matter | (strong CP) |
| | | baryogenesis | unification, charge quantisation |

* Let's set quantum gravity aside for simplicity ...

Why the LHC? What are the problems of the Standard Model* (SM), before the LHC started?

| data driven, clear scale | conceptual, vague scale | data driven, no clear reachable scale | conceptual |
|-----------------------------|-----------------------------|---|--|
| WW/unitarity, masses | fine tuning, naturalness | neutrino masses | flavor puzzle |
| | | dark matter | (strong CP) |
| | | baryogenesis | unification, charge quantisation |

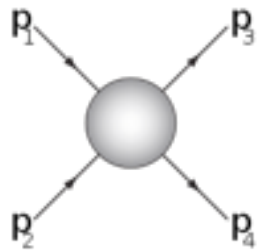
Why the LHC? (2 subjective reasons)

- Higgs & unitarity, suggests physics $< \text{TeV}$.
- Given the Higgs, the fine tuning problem requires new physics at a scale, generically, within the reach of the LHC.

[Fermion masses: another unitarity problem, relevant to LHC H-phys. (no time to discuss)]

The SM Higgsless Unitarity Problem

$$\mathcal{L}_{\text{eff}} = M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2$$



$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

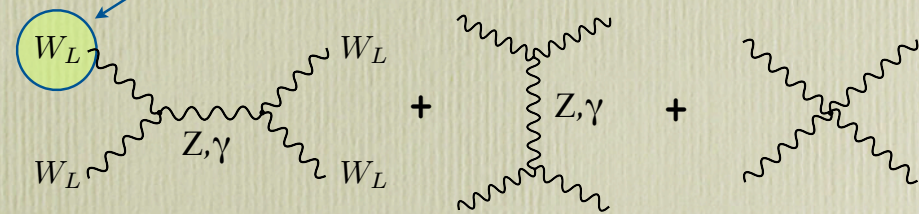
Mandelstam variables

Unitarity is restored by adding diagrams with intermediate Higgs in them as long as $m_h < 800$ GeV .

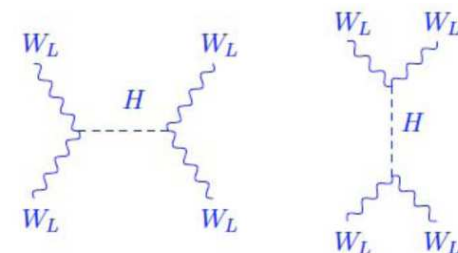
The amplitude for scattering of **longitudinal** W's and Z's grows with the energy and eventually violates the unitarity bound:

Ex: $A(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \frac{g_2^2}{4M_W^2} (s + t)$

each longitudinal polarization gives a factor E $\epsilon_L^\mu = \frac{p^\mu}{M_W} + O\left(\frac{E}{M_W}\right)$



Unitarity is violated at $\sqrt{s} \simeq \Lambda = 1.2$ TeV



The Higgs & the fine tuning/naturalness problem

't Hooft definition of technical naturalness:

a parameter is natural if when it's set to 0 there's an enhanced symmetry.

Additive renormalization (unnatural parameters): $d\lambda/d\ln\mu \propto \lambda g(\mu) + f(\mu)$

Multiplicative renormalization (natural parameters): $d\lambda/d\ln\mu \propto \lambda g(\mu)$

The Higgs mass parameter is subject to additive renormalisation.

Thus, it is sensitive to microscopic new physics dynamics.

Naturalness might give a hint: Higgs mass is additive, sensitive to microscopic scales. Within the SM it translates to UV sensitivity: $\frac{dm_H^2}{d\ln\mu} = \frac{3m_H^2}{8\pi^2} \left(2\lambda + y_t^2 - \frac{3g_2^2}{4} - \frac{3g_1^2}{20} \right)$.

See: Giudice (13)

Beyond the SM: any scale that couples to the Higgs (or even to tops, gauge ...) will induce a large shift to the Higgs mass, $\delta m_H^2 \approx \frac{\alpha}{4\pi} M^2$. Farina, Pappadopulo & Strumia (13)

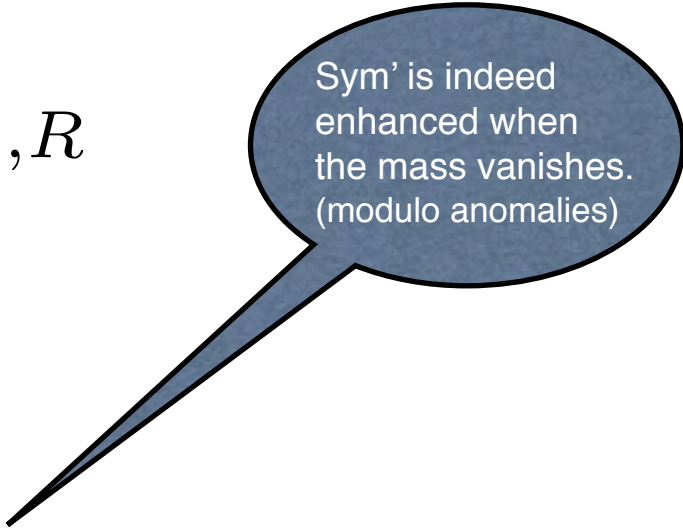
Tunning vs. fine tuning/naturalness problem

Flavor puzzle: the parameters' are small and hierarchical.

Is the flavor sector fine tuned? $m_u/m_t \sim 10^{-5}$.

Massless fermions theory: $\mathcal{L}_{\text{fermions}} \in \bar{\psi}_L \partial_\mu \gamma_\mu \psi_L + \bar{\psi}_R \partial_\mu \gamma_\mu \psi_R$

Two separate U(1)'s: $\psi_{L,R} \rightarrow e^{\theta_{L,R}} \psi_{L,R}$



Sym' is indeed enhanced when the mass vanishes. (modulo anomalies)

Mass term breaks it to a single U(1): $\bar{\psi}_L m \psi_R$

Only invariant under transformation with $\theta_L = \theta_R = \theta$

Flavor is natural, what's left for the LHC?

Flavor parameters are natural, subject to tuning & then radiatively stable, no UV sensitivity.

Within the SM the only exception is the Higgs mass. (& the QCD angle & the cosmological constant)



Motivates: study the Higgs & electroweak sym' breaking + naturalness.



Can be done at the LHC, a concrete task.

LHC physics

Why LHC?



Need more E!

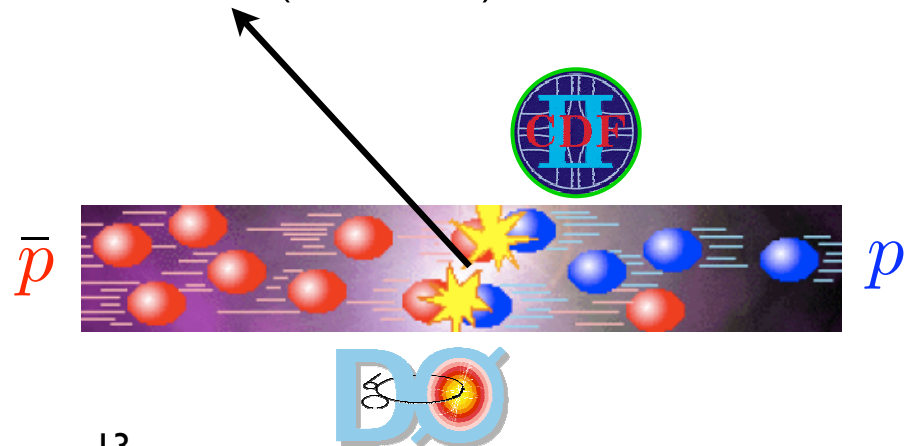
Sync' radiation,
problem for circular e-collider:

$$\frac{dW}{dt} \Big|_e \approx \left(\frac{e}{r}\right)^2 \left(\frac{E}{m_e}\right)^4 \sim 10^4 \text{ GeV s}^{-1} \Rightarrow \times 10^{12} e \sim \text{MWs radiation!}$$

10^{13} improvement when $e \Leftrightarrow$ proton



$E \sim 2 \text{ TeV}$ (2000 GeV)

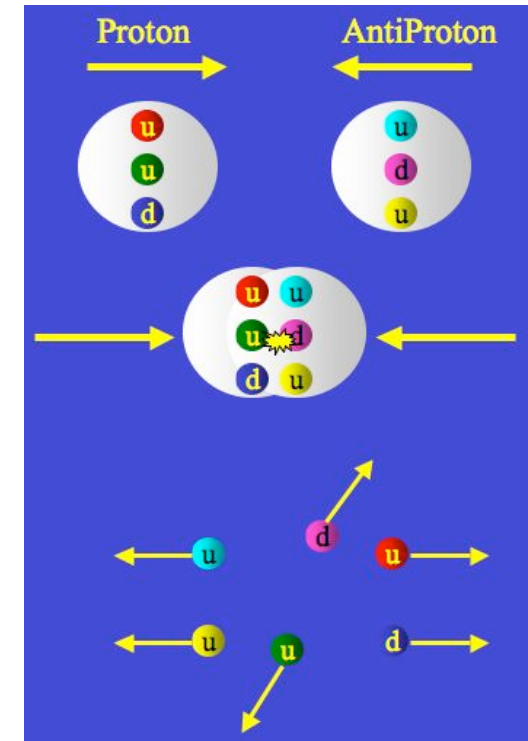
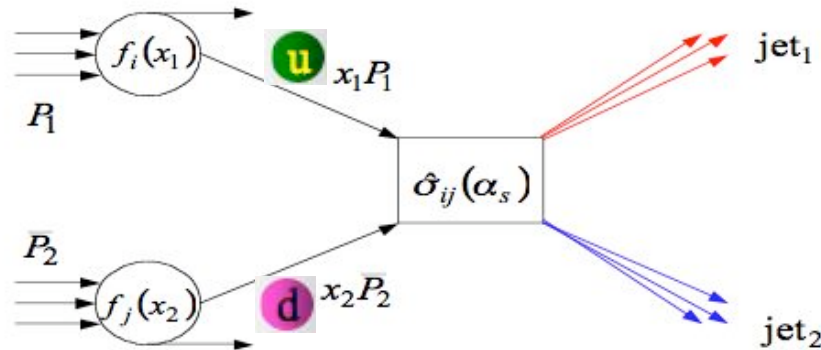


Nothing's free - QCD dust

- Expect $m_t = 130\text{-}200\text{ GeV}$, who needs 2TeV? Tevatron

- Proton anti-proton are composite:

- Typical E's much smaller: $E_{\text{event}}^2 = x_1 x_2 E_{p\bar{p}}^2$



- We don't know what is E_{CM} .
- We don't know which particles interacted.
- And ...

Calculating Xsections at the LHC: Parton Distribution Functions (PDFs)

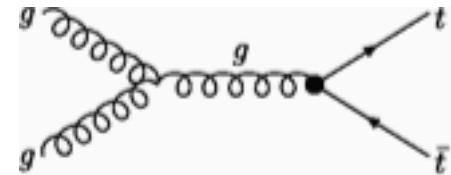
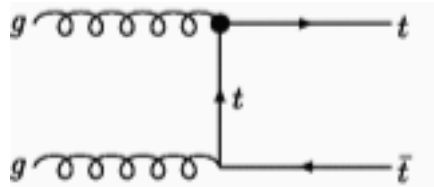
(assuming no p-rapidity or pt cuts)

$$\frac{d\sigma(pp \rightarrow f)}{d\hat{s}} = \sum_{ij} \hat{\sigma}_{ij}(\hat{s}) \int_0^1 \int_0^1 dx_i dx_j f_i(x_i) f_j(x_j) \delta(\hat{s} - x_i x_j s)$$

$\hat{\sigma}(\hat{s})$ Corresponds to the Born/hard/local/short distance Xsection that we would like to calculate/measure.

For instance $gg \rightarrow t\bar{t}$

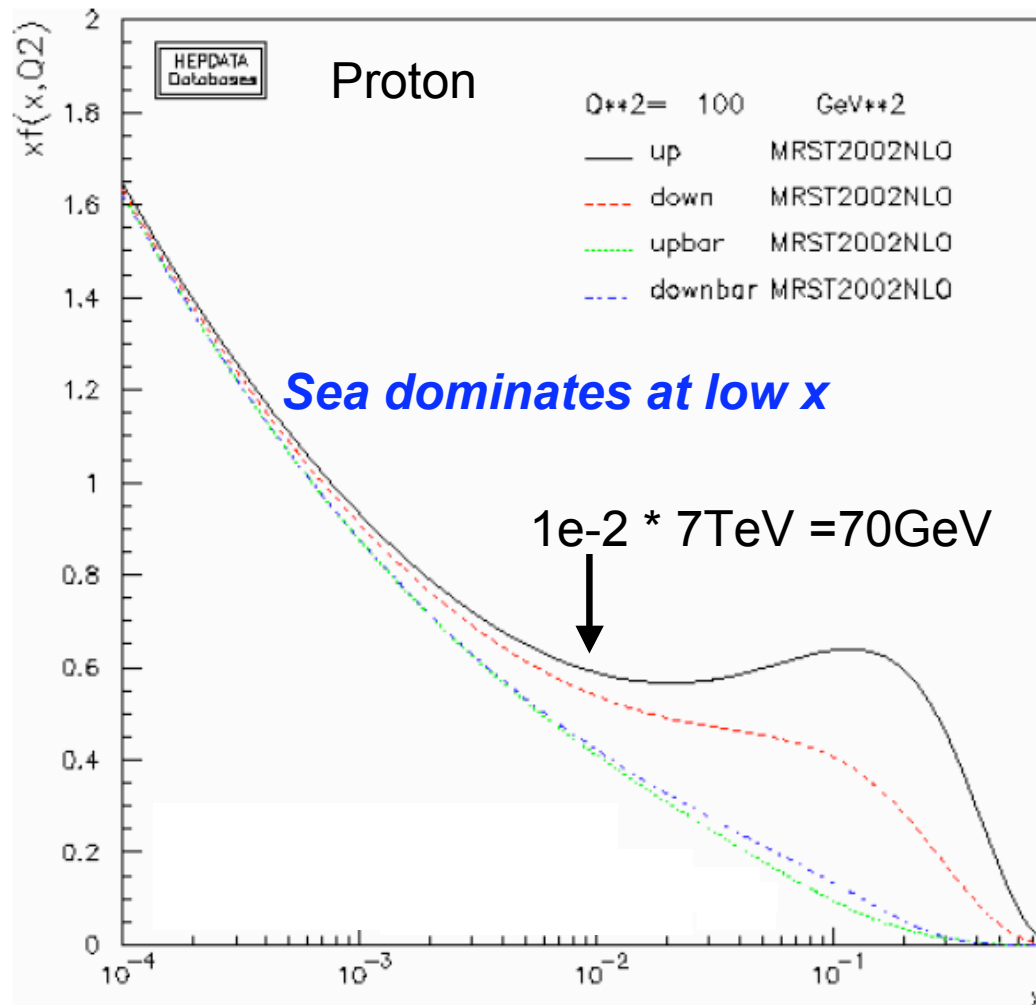
$$\hat{s} = (p_t + p_{\bar{t}})^2 = (p_g + p_{g'})^2$$



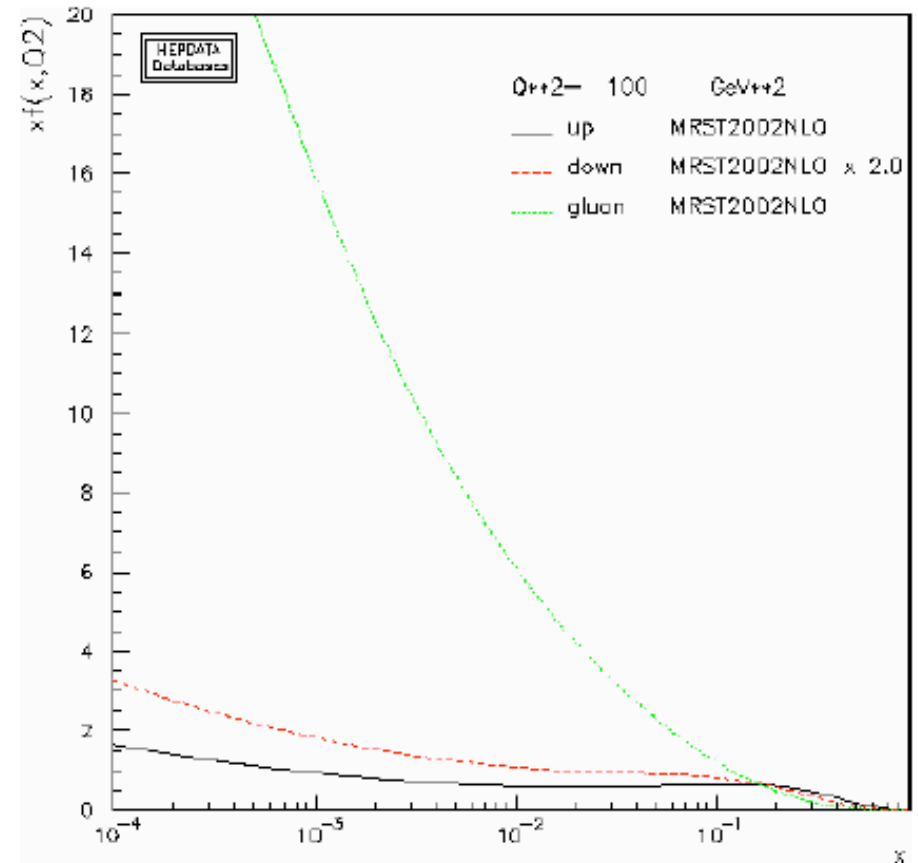
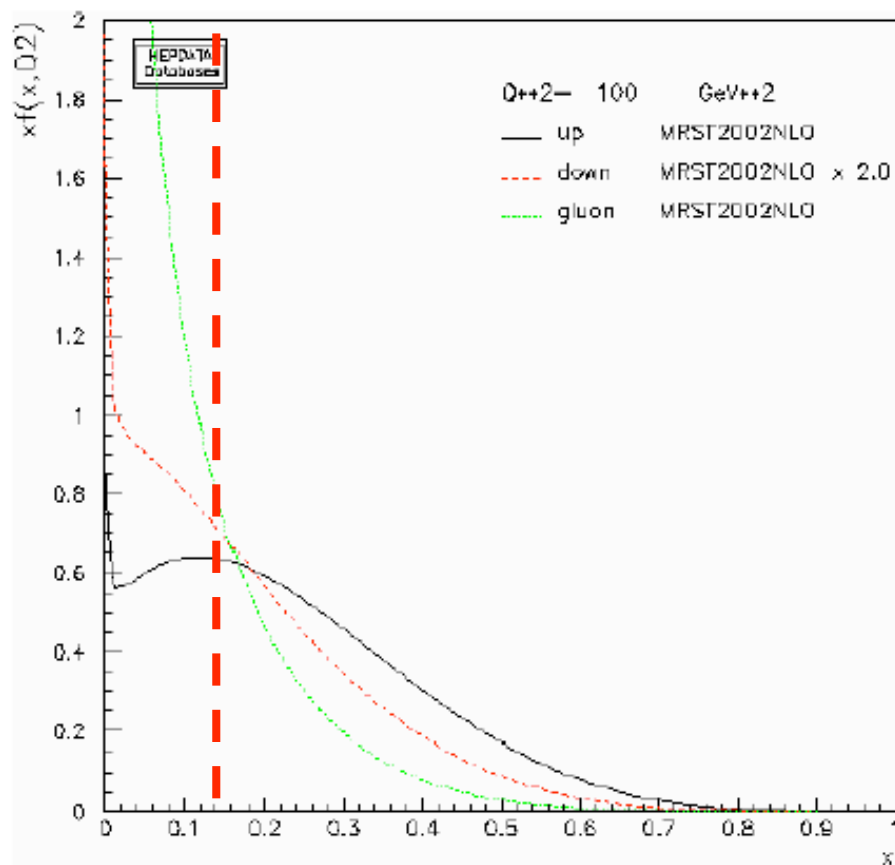
PDFs (What are they?)

PDFs are non-perturbative objects.

Probability of finding a constituent f with a longitudinal momentum fraction of $x \Rightarrow f_f(x)dx$



PDFs at the LHC



Gluons dominate at low x .

To set the scale, $x = 0.14$ at LHC is $0.14 * 7\text{TeV} = 1\text{TeV}$

\Rightarrow The LHC is a gluon collider !!!

Calculating Xsections at the LHC: Parton Distribution Functions (PDFs)

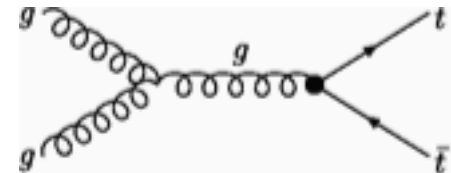
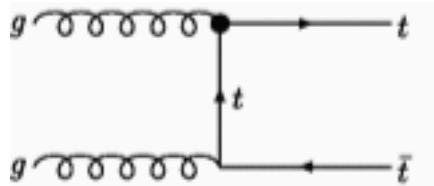
(assuming no p-rapidity or pt cuts)

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$\hat{\sigma}(\hat{s})$ Corresponds to the Born/hard/local/short distance Xsection that we would like to calculate/measure.

For instance $gg \rightarrow t\bar{t}$

$$\hat{s} = (p_t + p_{\bar{t}})^2 = (p_g + p_{g'})^2$$



Summary lecture I:

Some motivation (SM problems, naturalness);

How to calculate Xsections @ the LHC;

Parton distribution functions (PDFs) parton luminosities.

Mathematica notebook+PDF files that are public, if you are interested in doing the ex.:

https://www.dropbox.com/s/xnr0449ehjndri1/Example_invisibles_LHC.nb?dl=0

<https://www.dropbox.com/s/q1mdtbt5qyoj229/Lall14.txt?dl=0>

<https://www.dropbox.com/s/7j6xelcg7k38m8r/Lall100.txt?dl=0>

Homework:

1. is the electron mass a technical natural parameter? the up mass? neutrino Majorana masses? what happens if I will add to the SM a bare fermion mass? (say for the electron)
2. have a file with PDFs and parton luminosities where you can draw the above plots ...

Beginning of 2nd Lecture

- Parton Luminosities (cont').
- Example, top-pair Xsection calculation.
- Kinematics.

Physically only pairs of PDF are important

(assuming no p-rapidity or pt cuts)

$$\frac{d\sigma(pp \rightarrow f)}{d\hat{s}} = \sum_{ij} \hat{\sigma}_{ij}(\hat{s}) \int_0^1 \int_0^1 dx_i dx_j f_i(x_i) f_j(x_j) \delta(\hat{s} - x_i x_j s)$$

$$= \sum_{ij} \frac{\hat{\sigma}_{ij}(\hat{s})}{\hat{s}} \int_0^1 \int_0^1 dx_i dx_j f_i(x_i) f_j(x_j) \delta\left(1 - x_i x_j \frac{s}{\hat{s}}\right)$$

$$\tau = \frac{\hat{s}}{s}$$

$$\frac{d\sigma(pp \rightarrow f)}{d\tau} = \sum_{ij} \frac{\hat{\sigma}_{ij}(\hat{s})}{\tau} \int_0^1 \int_0^1 dx_i dx_j f_i(x_i) f_j(x_j) \delta\left(1 - \frac{x_i x_j}{\tau}\right)$$

$$\frac{d\sigma(pp \rightarrow f)}{d\tau} = \sum_{ij} \hat{\sigma}_{ij}(\hat{s}) \int_{\tau}^1 dx_i \frac{1}{x_i} f_i(x_i) f_j\left(\frac{\tau}{x_i}\right)$$

Parton-parton luminosities

$$\frac{dL_{ij}}{d\tau} = \frac{1}{1 + \delta_{ij}} \int_{\tau}^1 \frac{dx}{x} \left[f_i(x) f_j\left(\frac{\tau}{x}\right) + f_i\left(\frac{\tau}{x}\right) f_j(x) \right]$$

- Function of dimensionless quantity:
 - Scaling => independent of CM energy of proton proton collisions.
- However, $\hat{\sigma}_{ij}(\hat{s}) \equiv \hat{\sigma}_{ij}(\hat{E}^2)$ depends on E. The collider characteristics only help us understand the energy scale E^2 accessible given an S for proton-proton collisions.

Parton luminosity & cross section scaling

Let us use some simple rescaling to get some intuition for the behaviour:

$$\sigma(pp \rightarrow t\bar{t}) = \int_{\tau_{\min}}^1 d\tau \times \hat{\sigma}(\hat{s} = s\tau) \times \left. \frac{d\mathcal{L}}{d\tau} \right|_{\tau = \frac{\hat{s}}{s}}$$

$$= \int_{\tau_{\min}}^1 \frac{d\tau}{\tau} \times \left[\hat{s} \hat{\sigma}(\hat{s}) \right] \times \left. \frac{\tau d\mathcal{L}}{\hat{s} d\tau} \right|_{\tau = \frac{\hat{s}}{s}}$$

Why? Why?

Parton luminosity & cross section scaling

Let us use some simple rescaling to get some intuition for the behaviour:

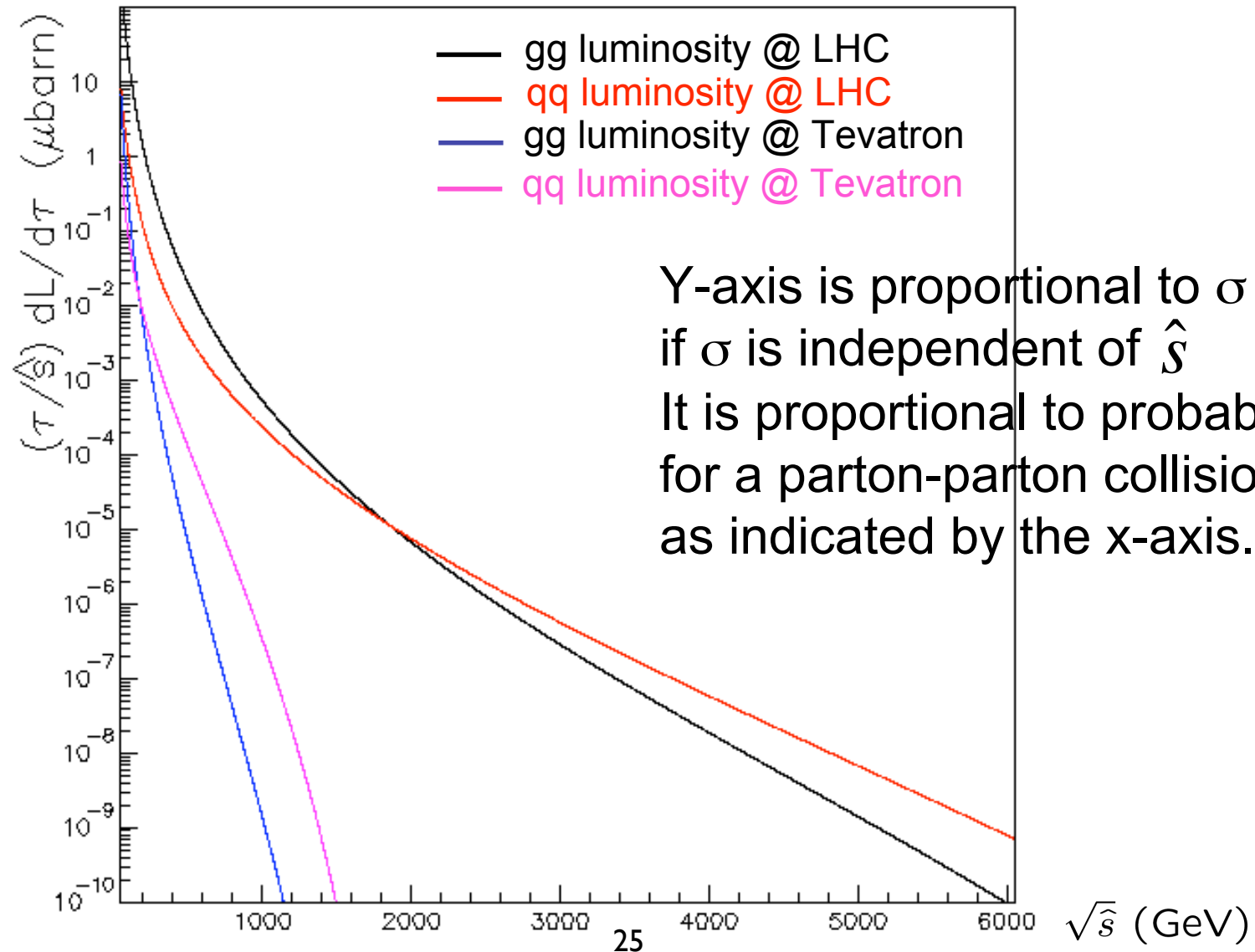
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$$= \int_{\tau_{\min}}^1 \frac{d\tau}{\tau} \times [\hat{s}\hat{\sigma}(\hat{s})] \times \left. \frac{\tau d\mathcal{L}}{\hat{s}d\tau} \right|_{\tau = \frac{\hat{s}}{s}}$$

order one
 τ tends to
be small

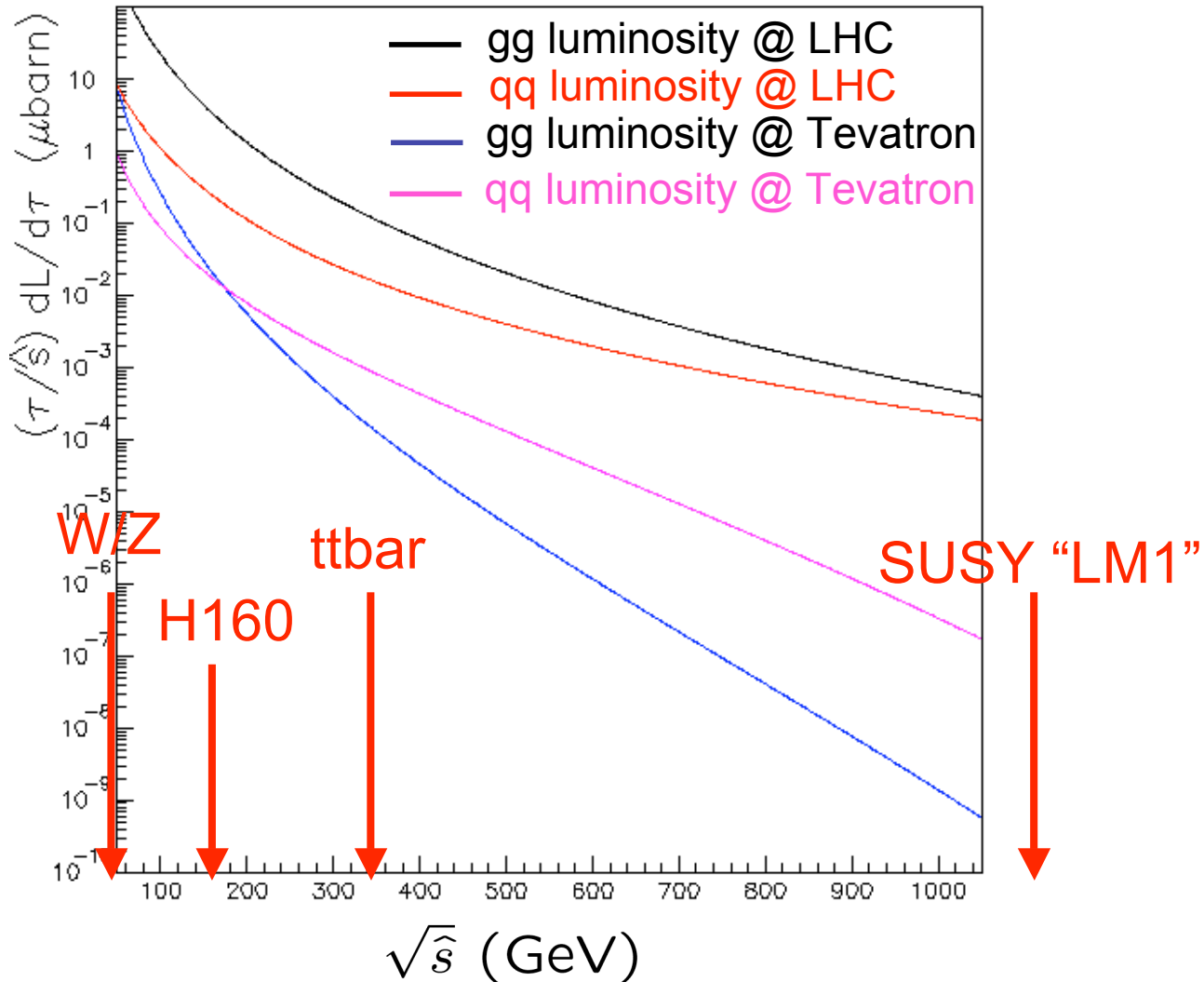
dimensionless:
naive dim' analysis
(NDA): $O(0.1-0.01)$

Luminosity functions, adding Xsection scale



Zooming-in on the < 1 TeV region

Protons are “empty”: $\text{GeV}^{-1} \sim 0.4 \text{ mb}$



Cross sections at 1.96TeV versus 14TeV Tevatron vs LHC

| | Cross section | | Ratio |
|------------------------------|---------------|--------|--------|
| $Z \rightarrow \mu\mu$ | 260pb | 1750pb | 6.7 |
| WW | 10pb | 100pb | 10 |
| $H_{160\text{GeV}}$ | 0.2pb | 25pb | 125 |
| $m\text{Sugra}_{\text{LM1}}$ | 0.0006pb | 50pb | 80,000 |

At $10^{32}\text{cm}^{-2}\text{s}^{-1}$ LHC might accumulate 10pb^{-1} in one day!

Collider Reach

Assuming similar scaling for background & signal => same number of events:

$$N_{\text{old}} = N_{\text{new}} \Leftrightarrow \frac{1}{\left(\frac{m_{\text{old}}^2}{s_{\text{old}}}\right)^{2,3}} \times \frac{1}{m_{\text{old}}^2} \times \mathcal{L}_{\text{old}} = \frac{1}{\left(\frac{m_{\text{new}}^2}{s_{\text{new}}}\right)^{2,3}} \times \frac{1}{m_{\text{new}}^2} \times \mathcal{L}_{\text{new}}$$



$$m_{\text{new}} \sim m_{\text{old}} \times \left(\frac{\mathcal{L}_{\text{new}}}{\mathcal{L}_{\text{old}}}\right)^{\frac{1}{6,8}} \times \left(\frac{\sqrt{s_{\text{new}}}}{\sqrt{s_{\text{old}}}}\right)^{\frac{2,3}{3,4}}$$

40% improvement, for the jump to 13/14 TeV for same Lumi and another 60% for 300 inv fb;
consequently, overall roughly increase of 2-2.5 in reach.

But, many searches will enter the boosted regime => qualitative change of physics!

Consider for example LHC top pair production

$$\sigma^{p(g)p(g) \rightarrow t\bar{t}} = \int_{\tau_{\min}}^1 d\tau \hat{\sigma}^{t\bar{t}}(\hat{s} = \tau s) \frac{d\mathcal{L}_{gg}}{d\tau} \quad \tau_{\min} = (2m_t/14 \text{ TeV})^2$$

$$\frac{d\mathcal{L}_{gg}}{d\tau} = \int_{\tau}^1 \frac{dx}{x} f_g(x) f_g(\tau/x)$$

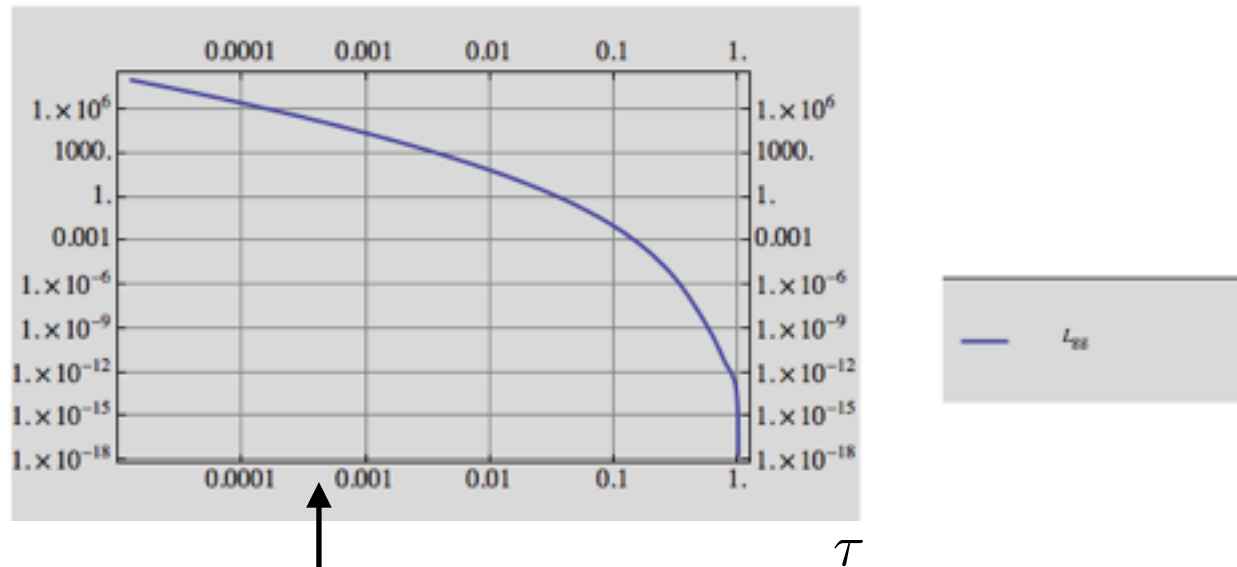
$$\beta = \sqrt{1 - 4m_t^2/\hat{s}}$$

$$\hat{\sigma}_{gg \rightarrow t\bar{t}} = \frac{\pi\alpha_s^2\beta}{48\hat{s}} \left(31\beta + \left(\frac{33}{\beta} - 18\beta + \beta^3 \right) \ln \left[\frac{1+\beta}{1-\beta} \right] - 59 \right)$$

The gluon luminosity function at LHC14

MSTW-PDF running factorisation scale as $Q^2 = \hat{s} = \tau s = \tau \times 14^2 \text{ TeV}^2$

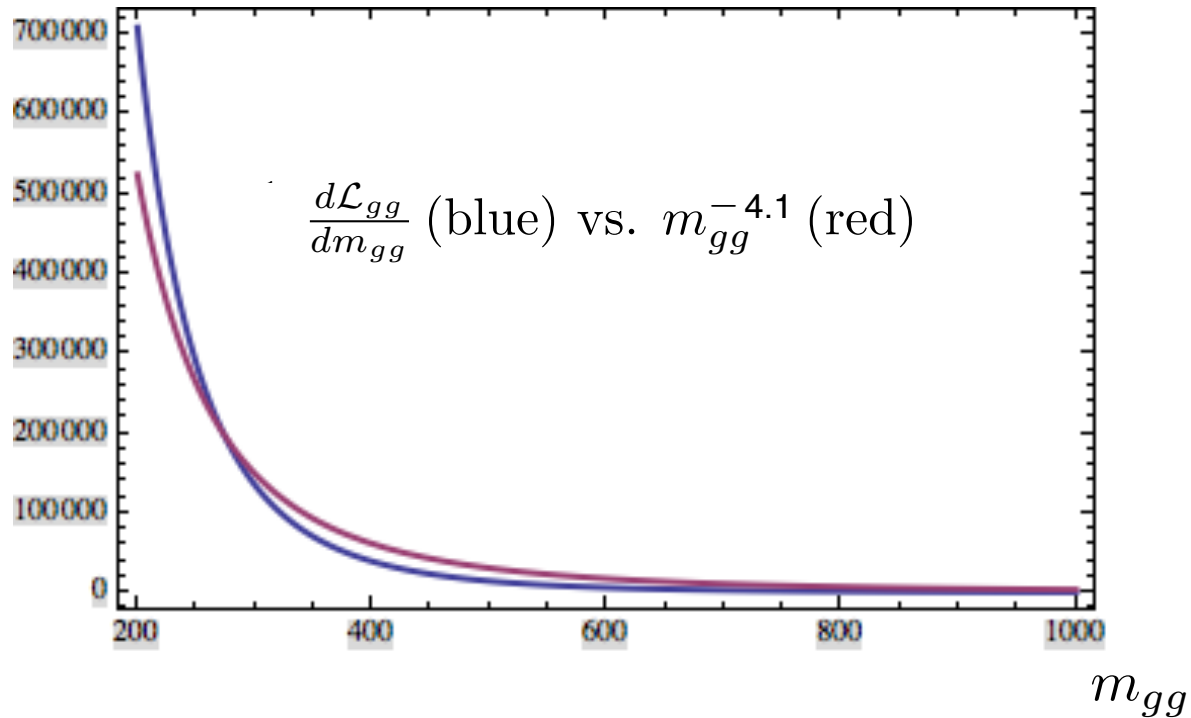
$$\frac{d\mathcal{L}_{gg}}{d\tau}$$



Typical τ for $t\bar{t}$ production at LHC14: $(2m_t/14 \text{ TeV})^2 \sim 6 \times 10^{-4}$.

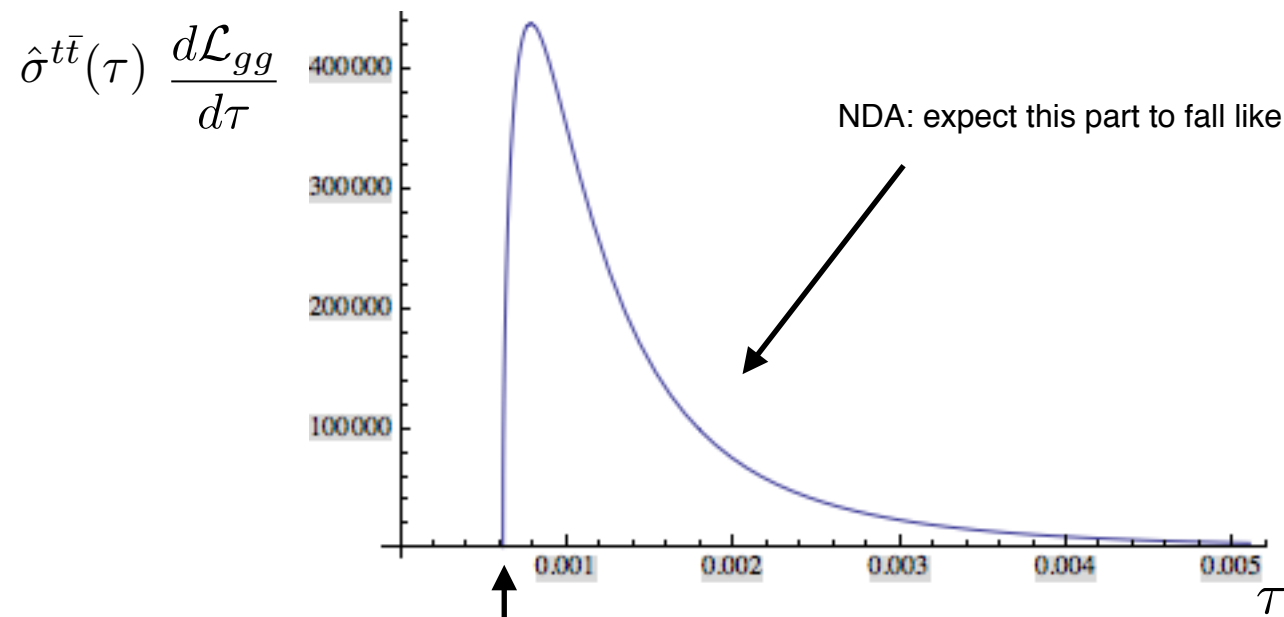
The luminosity functions are rapidly falling

MSTW-PDF running factorisation scale as $Q^2 = \hat{s} = \tau s = \tau \times 14^2 \text{ TeV}^2$



Generically, cross section falls even faster!

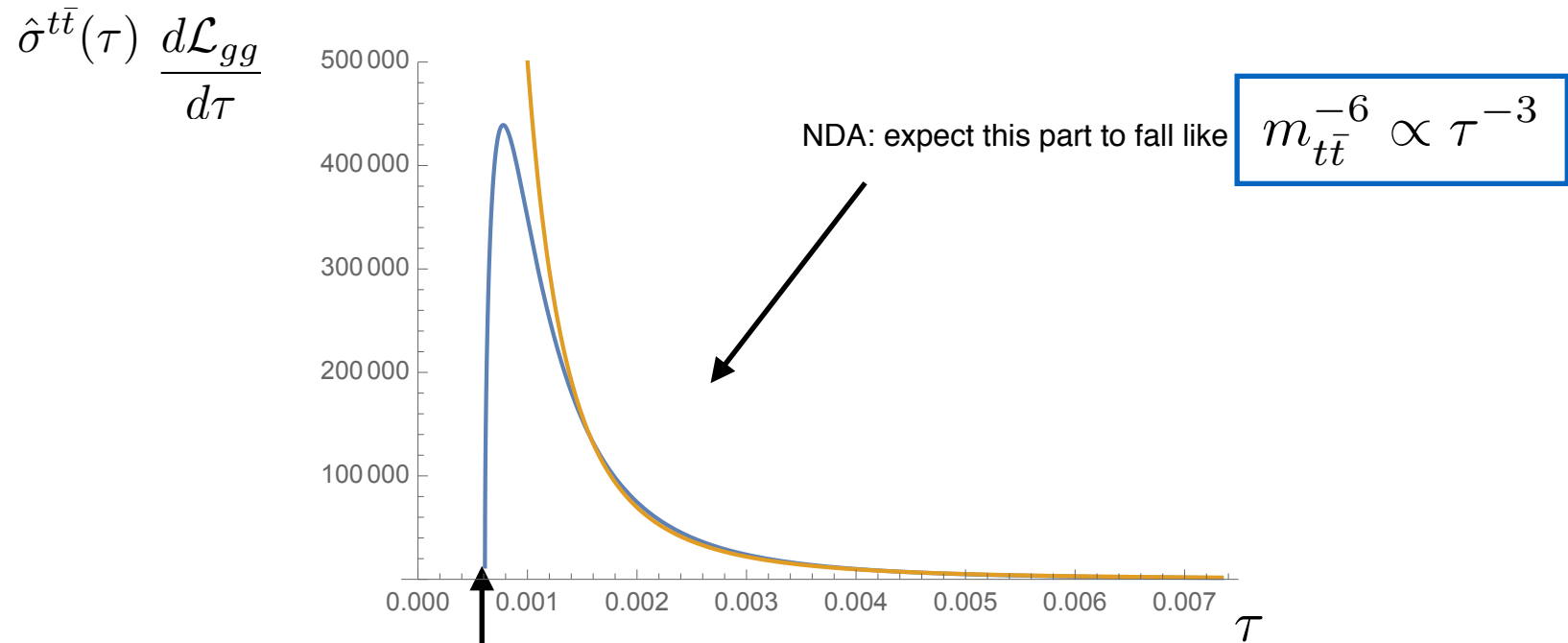
MSTW-PDF running factorisation scale as $Q^2 = \hat{s} = \tau s = \tau \times 14^2 \text{ TeV}^2$



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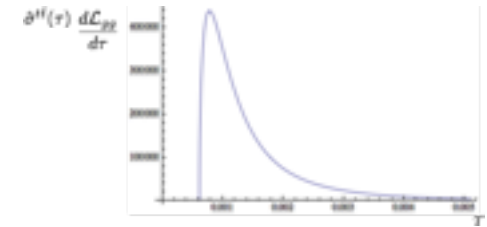
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Typical τ for $t\bar{t}$ production at LHC14: $(2m_t/14 \text{ TeV})^2 \sim 6 \times 10^{-4}$.

Back to estimating LHC cross section

What are the implications for this rapid fall?



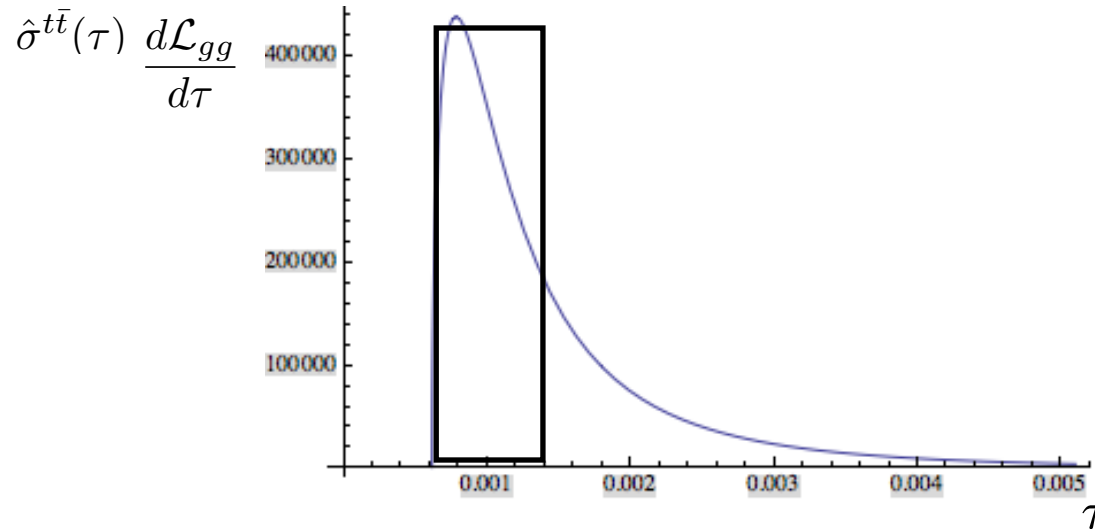
Massive particles ($h, W, Z, t, squarks, KK gluon \dots$) are produced near threshold.

Any dimensional cut (in the transverse direction), $m_{xx}, p_T, missing E_T, H_T$, implies that the signal and background distributions would peak right where the cut is located.

Maybe we can use this fact for a quick & rough estimation of the top pair Xsection?

Rough estimation for the LHC cross section step I:

Replacing the integral with differential



Let's replace the integral with differential:

$$\sigma^{p(g)p(g) \rightarrow t\bar{t}} = \int_{\tau_{\min}}^1 d\tau \hat{\sigma}^{t\bar{t}}(\hat{s}=\tau s) \frac{d\mathcal{L}_{gg}}{d\tau} \sim \Delta\tau \hat{\sigma}^{t\bar{t}}(\tau s) \frac{d\mathcal{L}_{gg}}{d\tau} \Big|_{\tau \rightarrow \frac{4}{3}\tau_{\min}}$$

$$\Delta\tau \sim \frac{4}{3}\tau_{\min}$$

Rough NDA estimation for the cross section step 1.1: Replacing the Born Xsection with its NDA value

NDA for 2->2 Xsection (far from threshold): $\hat{\sigma}(\hat{s}) \rightarrow \frac{1}{\hat{s}}$

$$\sigma^{p(g)p(g) \rightarrow t\bar{t}} = \int_{\tau_{\min}}^1 d\tau \hat{\sigma}^{t\bar{t}}(\tau s) \frac{d\mathcal{L}_{gg}}{d\tau}$$

$$\sim \Delta\tau \hat{\sigma}^{t\bar{t}}(\tau s) \frac{d\mathcal{L}_{gg}}{d\tau} \Big|_{\tau \rightarrow \frac{4}{3}\tau_{\min}}$$

$$\sim \Delta\tau \frac{\alpha_s^2}{\tau s} \frac{d\mathcal{L}_{gg}}{d\tau} \Big|_{\tau \rightarrow \frac{4}{3}\tau_{\min}}$$

And the results are:

Precise^{LO}: $\sigma^{p(g)p(g) \rightarrow t\bar{t}} = \int_{\tau_{\min}}^1 d\tau \cdot \hat{\sigma}^{t\bar{t}}(\tau s) \cdot \frac{d\mathcal{L}_{gg}}{d\tau} = 398.687 \text{ pb}$

Approx' luminosities: $\Delta\tau \hat{\sigma}^{t\bar{t}}(\tau s) \frac{d\mathcal{L}_{gg}}{d\tau} \Big|_{\tau \rightarrow \frac{4}{3}\tau_{\min}} = 354.212 \text{ pb}$

"NDA": $\Delta\tau \frac{\alpha_s^2}{\tau s} \frac{d\mathcal{L}_{gg}}{d\tau} \Big|_{\tau \rightarrow \frac{4}{3}\tau_{\min}} = 940.538 \text{ pb}$

my mathematica:

```
In[186]:= GeV2pb = 0.389 10^9 pb;
mt = 173.1;
βt[shat_] := Sqrt[1 - 4 mt^2/shat]
αs = 0.11;
σggtt[τ_] := (π αs^2 βt[τ s14])/
  48 τ s14 (31 βt[τ s14]^2 + (33/βt[τ s14] - 18 βt[τ s14] + βt[τ s14]^3) Log[(1 + βt[τ s14])/(1 - βt[τ s14])] - 59)
In[191]:= NIntegrate[dLdtaugg14Num[τp] σggtt[τp], {τp, (2 mt)^2/s14, 1}] GeV2pb
Out[191]= 398.687 pb
In[232]:= dLdtaugg14Num[4/3 (2 mt)^2/s14] σggtt[4/3 (2 mt)^2/s14] 4/3 (2 mt)^2/
s14 GeV2pb
Out[232]= 354.212 pb
In[233]:= dLdtaugg14Num[4/3 (2 mt)^2/s14] ( αs^2/(4/3 (2 mt)^2)) 4/3 (2 mt)^2/s14 GeV2pb
Out[233]= 940.538 pb
```

$t\bar{t}$ Xsection @ LHC14, compare with state of the art:

Precise^{LO}: $\sigma^{p(g)p(g)\rightarrow t\bar{t}} = \int_{\tau_{\min}}^1 d\tau \hat{\sigma}^{t\bar{t}}(\tau s) \frac{d\mathcal{L}_{gg}}{d\tau} = 398.687 \text{ pb}$

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"NDA": $\Delta\tau \frac{\alpha_s^2}{\tau s} \frac{d\mathcal{L}_{gg}}{d\tau} \Big|_{\tau \rightarrow \frac{4}{3}\tau_{\min}} = 940.538 \text{ pb}$

Theory: Xsection (Tevatron, LHC) now known to NNLO (+NNLL resum')

| Collider | σ_{tot} [pb] | scales [pb] | pdf [pb] |
|------------|----------------------------|------------------------------|------------------------------|
| Tevatron | 7.164 | +0.110(1.5%) -0.200(2.8%) | +0.169(2.4%) -0.122(1.7%) |
| LHC 7 TeV | 172.0 | +4.4(2.6%) -5.8(3.4%) | +4.7(2.7%) -4.8(2.8%) |
| LHC 8 TeV | 245.8 | +6.2(2.5%) -8.4(3.4%) | +6.2(2.5%) -6.4(2.6%) |
| LHC 14 TeV | 953.6 | +22.7(2.4%) -33.9(3.6%) | +16.2(1.7%) -17.8(1.9%) |

Bärnreuther, Czakon & Mitov; Czakon & Mitov x2 (12);
Czakon, Fiedler & Mitov (13).

Some kinematics

LHC, longitudinal vs. transverse

Relativistic invariant phase-space element:

$$d\tau = d^3p/E = dp_x dp_y dp_z/E$$

Define pp collision axis along *z-axis*:

From $p^\mu = (E, p_x, p_y, p_z)$ - which are invariant under boosts along *z*?

the two longitudinal components: E and p_z are NOT invariant the two transverse components: p_x and p_y (and dp_x, dp_y) ARE invariant

Need all variables invariant for boost along *z-axis*:

For convenience, define \mathbf{p}^μ with only 1 component not Lorentz invariant Choose p_T, m, ϕ as the "transverse" (invariant) coordinates

where $p_T \equiv p \sin(\theta)$ and ϕ is the azimuthal angle

As 4th coordinate define "rapidity": $y = 1/2 \ln [(E+p_z)/(E-p_z)]$

Rapidity

Form a boost of velocity β along z axis

- $p_z \Rightarrow \gamma(p_z + \beta E)$

- $E \Rightarrow \gamma(E + \beta p_z)$

- Transform rapidity \Rightarrow

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \Rightarrow \frac{1}{2} \ln \frac{\gamma(E + \beta p_z) + \gamma(p_z + \beta E)}{\gamma(E + \beta p_z) - \gamma(p_z + \beta E)}$$
$$= \frac{1}{2} \ln \frac{(E + p_z)(1 + \beta)}{(E - p_z)(1 - \beta)} = y + \ln \gamma(1 + \beta)$$

$$y \Rightarrow y + y_b$$

Boosts along the beam axis change y by a constant, y_b :

- $(p_T, y, \phi, m) \Rightarrow (p_T, y + y_b, \phi, m)$ with $y \Rightarrow y + y_b$, $y_b \equiv \ln \gamma(1 + \beta)$
rapidity is simply additive

Measure

Boosts along the beam axis change y by a constant, y_b :

$y \rightarrow y+y_b \Rightarrow$ rapidity is simply additive.

Can change coordinate from:

$dx_1 dx_2$ to $dy d\tau$, with identity Jacobian.

LHC: $q_1 = 1/2\sqrt{s} (x_1, 0, 0, x_1)$ $q_2 = 1/2\sqrt{s} (x_2, 0, 0, -x_2)$

Rapidity of system q_1+q_2 is: $y = 1/2 \ln[(E+p_z)/(E-p_z)] = 1/2 \ln(x_1/x_2)$

"Pseudo" and "Real" rapidity

The relation between y , β and θ can be seen using $p_z = p \cos\theta$ and $p = \beta E$:

$$y = \frac{1}{2} \cdot \ln \frac{(E+p_z)}{(E-p_z)} = \frac{1}{2} \cdot \ln \frac{(1+\beta \cos\theta)}{(1-\beta \cos\theta)}$$

This expression can almost associate the position in the detector (θ) with the rapidity y , apart from the β terms.

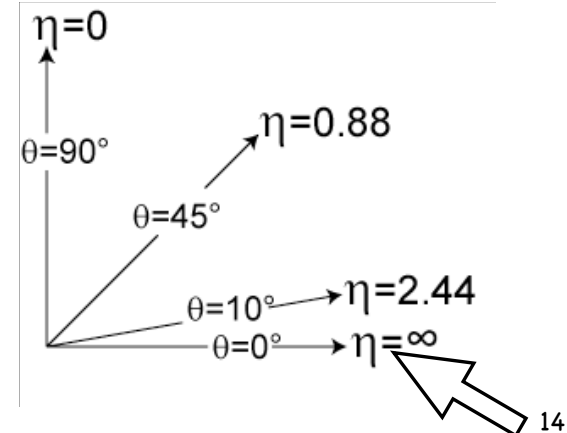
However, at the LHC (and Tevatron, HERA), $\approx 90\%$ of the particles in the detector are pions with $\beta \approx 1$. Therefore we can introduce the "pseudorapidity" defined as $\eta = y(\theta)$ for $\beta=1$:

$$\eta = \frac{1}{2} \cdot \ln \frac{(1+\cos\theta)}{(1-\cos\theta)} = \ln \frac{\cos(\theta/2)}{\sin(\theta/2)} = -\ln \left(\tan \frac{\theta}{2} \right)$$

$\cos^2\theta/2 = \frac{1}{2} \cdot (1+\cos\theta)$
 $\sin^2\theta/2 = \frac{1}{2} \cdot (1-\cos\theta)$

The pseudorapidity η is a good approximation of the true relativistic rapidity y when a particle is "relativistic".

It is a handy variable to approximate the rapidity y if the mass and the momentum of a particle are not known.



Summary lecture II:

How to calculate Xsections @ the LHC;

Parton luminosities;

Some kinematics

Homework:

1. How much gain in mass-reach will be achieved moving from 300/fb to HL 3000/fb?
2. Repeat for a 100TeV machine. What searches would benefit more from a HL upgrade?
3. How many tops were produced at the Tevatron? What was the dominant production mechanism?
4. Top-partners (appears in Little/Composite Higgs models), are heavy vector-like quarks; what is the bound on their masses such that, so far, < 10 events have been produced at the LHC run I?

Lecture III:

(Higgs) Resonance production @ LHC;

The EFT region;

Intro to Jets

Resonance based searches

Resonance based searches

Because of the large QCD uncertainties, it is much easier to search for bumps over continuous distribution, then to look for small depletions ...

Consider a particle H with a width and mass:

$$\Gamma_H \text{ and } m_H.$$

Resonances distribution described via Bright-Wigner

formula

$$\frac{1}{\pi} \frac{\hat{s}\Gamma_H/M_H}{(\hat{s} - M_H^2)^2 + (\hat{s}\Gamma_H/M_H)^2}$$

Resonance based searches

Let us suppose that the particle is narrow:

$$\Gamma_H \ll m_H.$$

(in many cases also, the LHC-exp' resolution is poor ...)



$$\frac{1}{\pi} \frac{\hat{s}\Gamma_H/M_H}{(\hat{s} - M_H^2)^2 + (\hat{s}\Gamma_H/M_H)^2} \longrightarrow \delta(\hat{s} - M_H^2)$$

$$\hat{\sigma}_{\text{LO}}(gg \rightarrow H) = \frac{\pi^2}{8M_H} \Gamma_{\text{LO}}(H \rightarrow gg) \delta(\hat{s} - M_H^2)$$

Resonance based estimation & scaling

$$\begin{aligned}\sigma(pp \rightarrow H) &\approx \int d\tau \frac{d\mathcal{L}}{d\tau} \hat{\sigma}(H \rightarrow gg) \approx \int d\tau \frac{d\mathcal{L}}{d\tau} \frac{\pi^2}{8M_H} \Gamma(H \rightarrow gg) \delta(\hat{s} - M_H^2) \\ &= \int d\tau \frac{d\mathcal{L}}{d\tau} \frac{\pi^2}{8M_H s} \Gamma(H \rightarrow gg) \delta(\tau - M_H^2/s) = \left. \frac{d\mathcal{L}}{d\tau} \right|_{\tau = \frac{M_H^2}{s}} \times \frac{\pi^2 \Gamma(H \rightarrow gg)}{8M_H s}\end{aligned}$$

The difference from the non-resonance scaling:
1/mass as opposed to 1/mass².

Final results are similar.

For bounds => background dominated =>
scaling unchanged.

$$N_{\text{old}} = N_{\text{new}} \Leftrightarrow \frac{1}{\left(\frac{m_{\text{old}}^2}{s_{\text{old}}}\right)^{2,3}} \times \frac{1}{m_{\text{old}}^2} \times \mathcal{L}_{\text{old}} = \frac{1}{\left(\frac{m_{\text{new}}^2}{s_{\text{new}}}\right)^{2,3}} \times \frac{1}{m_{\text{new}}^2} \times \mathcal{L}_{\text{new}}$$



$$m_{\text{new}} \sim m_{\text{old}} \times \left(\frac{\mathcal{L}_{\text{new}}}{\mathcal{L}_{\text{old}}}\right)^{\frac{1}{6,8}} \times \left(\frac{\sqrt{s_{\text{new}}}}{\sqrt{s_{\text{old}}}}\right)^{\frac{2,3}{3,4}}$$

Resonance based estimation, the Higgs

$$\begin{aligned}\sigma(pp \rightarrow H) &\approx \int d\tau \frac{d\mathcal{L}}{d\tau} \hat{\sigma}(H \rightarrow gg) \approx \int d\tau \frac{d\mathcal{L}}{d\tau} \frac{\pi^2}{8M_H} \Gamma(H \rightarrow gg) \delta(\hat{s} - M_H^2) \\ &= \int d\tau \frac{d\mathcal{L}}{d\tau} \frac{\pi^2}{8M_H s} \Gamma(H \rightarrow gg) \delta(\tau - M_H^2/s) = \left. \frac{d\mathcal{L}}{d\tau} \right|_{\tau = \frac{M_H^2}{s}} \times \frac{\pi^2 \Gamma(H \rightarrow gg)}{8M_H s}\end{aligned}$$

The example is Higgs. It is super narrow

its width is roughly 4 MeV. ($\Gamma_H/M_H \sim 10^{-5}$)

Why is the Higgs so narrow? calculate its width?
assume that the bottom's yield 50% of it for simplicity;

with:
$$\Gamma_{\text{scalar}} = \sum_i g_{fi}^2 m_H^2 / 8\pi$$

Higgs on-shell cross section (0th order)

$$\sigma(pp \rightarrow H) = \left. \frac{d\mathcal{L}}{d\tau} \right|_{\tau = \frac{M_H^2}{s}} \times \frac{\pi^2 \Gamma(H \rightarrow gg)}{8M_H s}$$

$$4\text{MeV} \quad 9\%$$

$$\Gamma_{h \rightarrow gg} = \Gamma_h \times BR(h \rightarrow gg) \sim 0.3 \text{ MeV}$$

Ex.: calculate the above for 14 and 100 TeV.

(I got $\sim 30\text{pb}$ using my code, correct answer is 50pb , large NLO/kfactor correction)

Higgs on-shell cross section, EFT+NDA

$$\Gamma(h \rightarrow gg) = \frac{G_F \alpha_s^2 m_h^3}{36\sqrt{2}\pi^3} \left| \sum_q \kappa_q A_{1/2}^H(\tau_q) \right|^2 .$$

$$\sum_q \kappa_q A_{1/2}^H(\tau_q) \approx 1.38\kappa_t - (0.044 - 0.048i)\kappa_b \approx (4/3)\kappa_t$$

$$\frac{\alpha_s^2 m_h^3}{72\pi^3 v}$$

We can indeed check that this form $\sim 9\%$ of the Higgs decays:

$$\mathcal{M}_{h \rightarrow gg} = \frac{\alpha_s}{4\pi v^2} f_{\text{LP}}^g H^\dagger H G^{\mu\nu a} G_{\mu\nu}^a$$

The amplitude scales as $1/v$, therefore the rate scales as $1/v^2$,

in order to get the right dimension for the rate (mass dim.)

we compensate by m_h^3 , such that $\Gamma \propto m_h^3/v^2$.

Resonance vs. EFT @ hadronic collisions

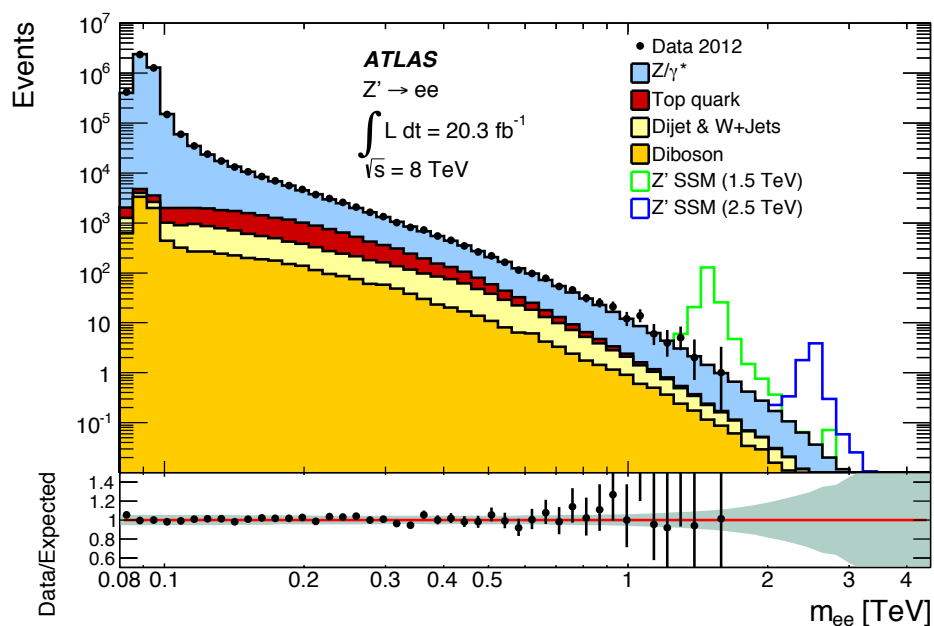
Resonance vs. EFT @ hadronic collisions

Often heavy & narrow resonances tends to “broaden” because of competition with off-shell production that are strongly supported by the rapidly falling PDFs.

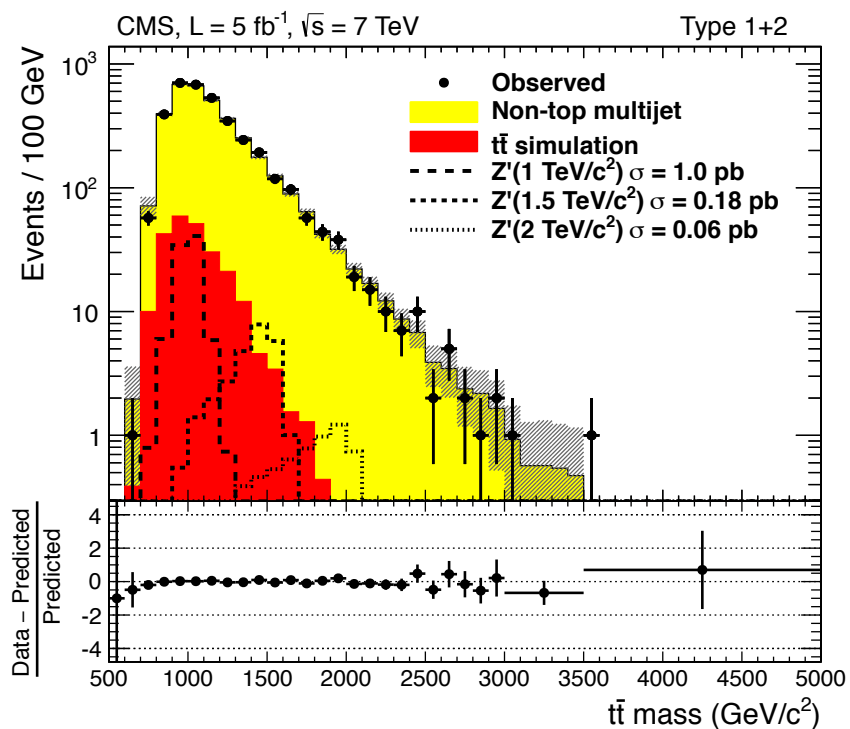
eventually, it is not useful anymore to search for them but to look at their virtual contributions.

Resonance vs. EFT @ hadronic collisions

Let us take as an example a narrow Z'



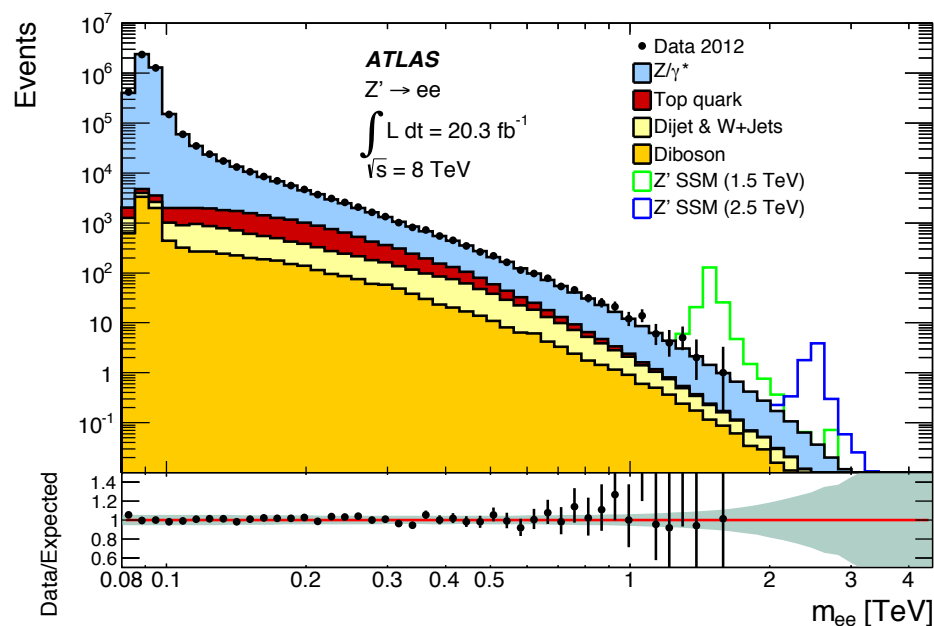
top-pair narrow resonance



Search for high-mass dilepton resonances with the ATLAS detector, 1405.4123v2 .

Resonance vs. EFT @ hadronic collisions

Let us take as an example a narrow Z' .



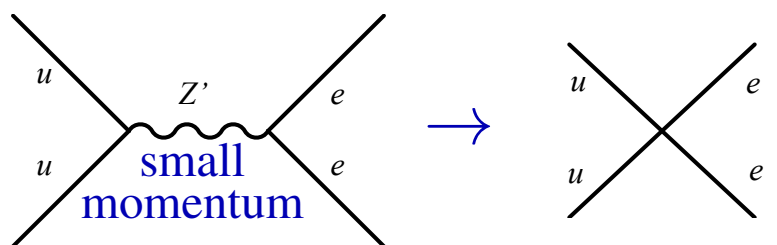
For dielectron masses above 200 GeV,
 the mass resolution is below 2% over the entire η range.

| Model | Width [%] |
|-------|--------------|
| Z' | 3.0 |

Search for high-mass dilepton resonances with the ATLAS detector, 1405.4123v2 .

Resonance vs. EFT @ hadronic collisions

EFT lectures (Kaplan):

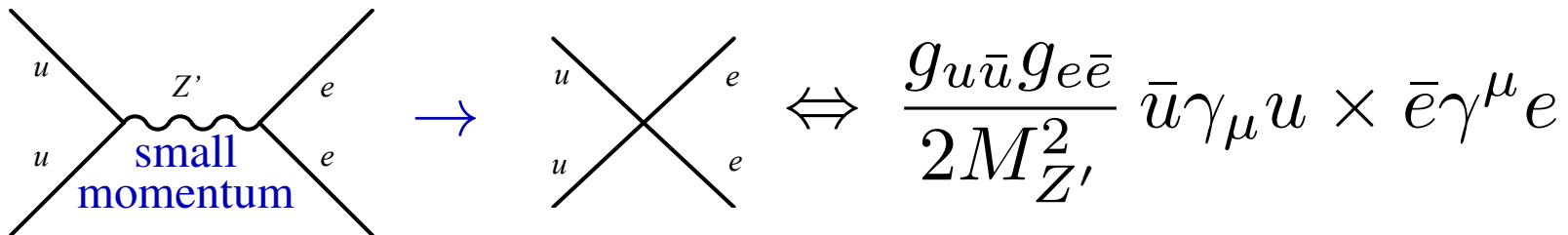


The diagram shows a transition from a resonance to an EFT operator. On the left, a Feynman diagram for a resonance process: two incoming u quark lines and two outgoing e electron lines are connected by a wavy line representing a Z' boson. The text "small momentum" is written in blue below the Z' line. A blue arrow points to the right, where a second Feynman diagram shows the same four external lines but without the Z' boson, representing the EFT operator. To the right of this diagram is an equivalence symbol \Leftrightarrow followed by the mathematical expression for the operator: $\frac{g_{u\bar{u}}g_{e\bar{e}}}{2M_{Z'}^2} \bar{u}\gamma_\mu u \times \bar{e}\gamma^\mu e$.

Neglecting interference, NDA, how should the cross section go like?

Resonance vs. EFT @ hadronic collisions

EFT lectures (Kaplan):



Non interfering, NDA:
$$\hat{\sigma}_{LO} \approx |g_{u\bar{u}} g_{e\bar{e}}|^2 \frac{\hat{E}^2}{4M_{Z'}^4}$$

EFT contributions rising with center of mass energy²!

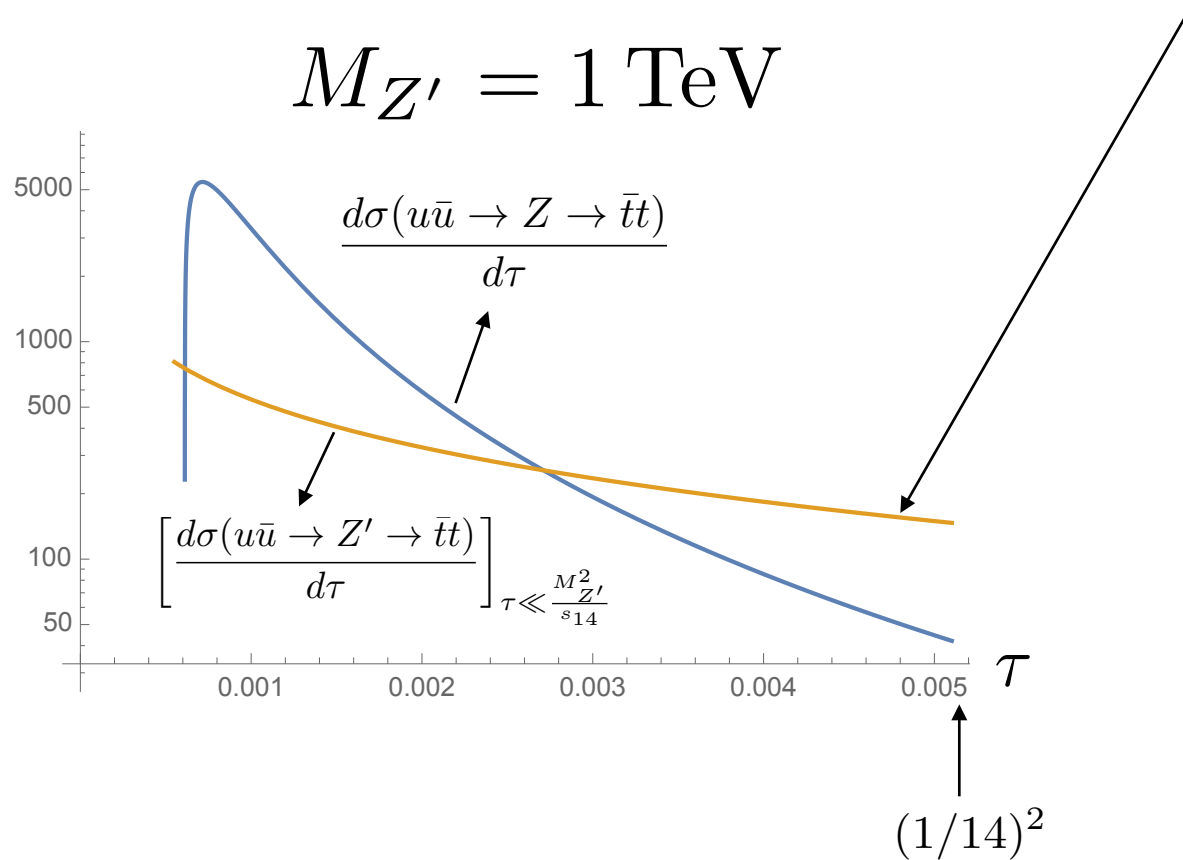
What is the corresponding scaling in the interference case?

Resonance vs. EFT @ hadronic collisions

$$\hat{\sigma}_{LO} \approx |g_{u\bar{u}}g_{e\bar{e}}|^2 \frac{\hat{E}^2}{4M_{Z'}^4}$$

EFT contributions rising with center of mass energy² !

$$M_{Z'} = 1 \text{ TeV}$$

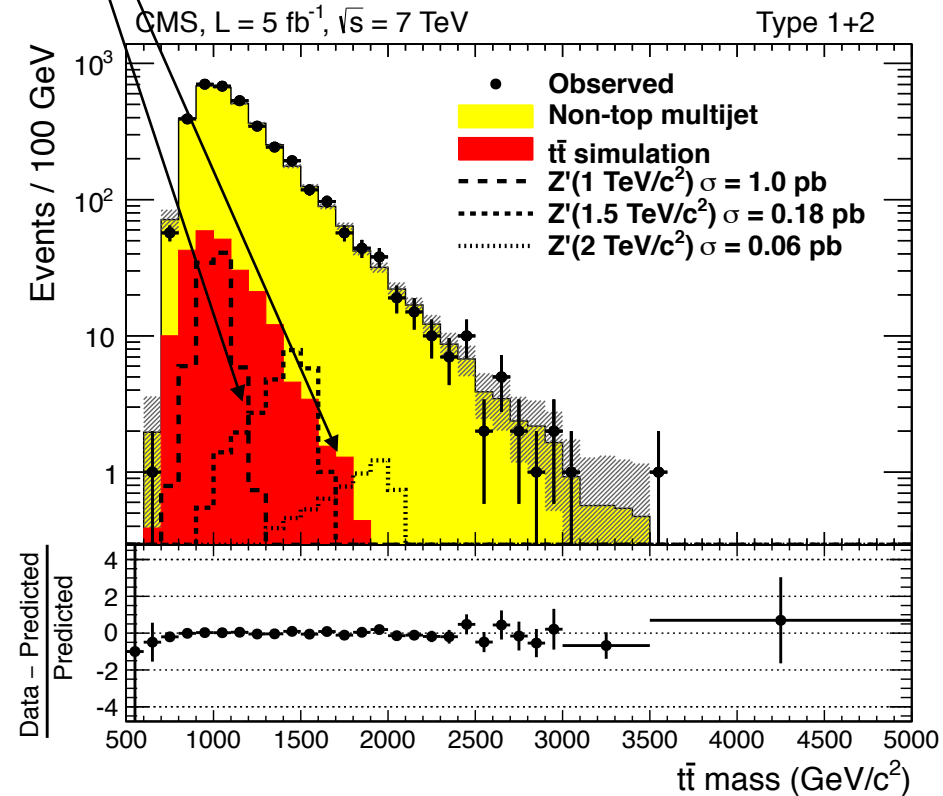
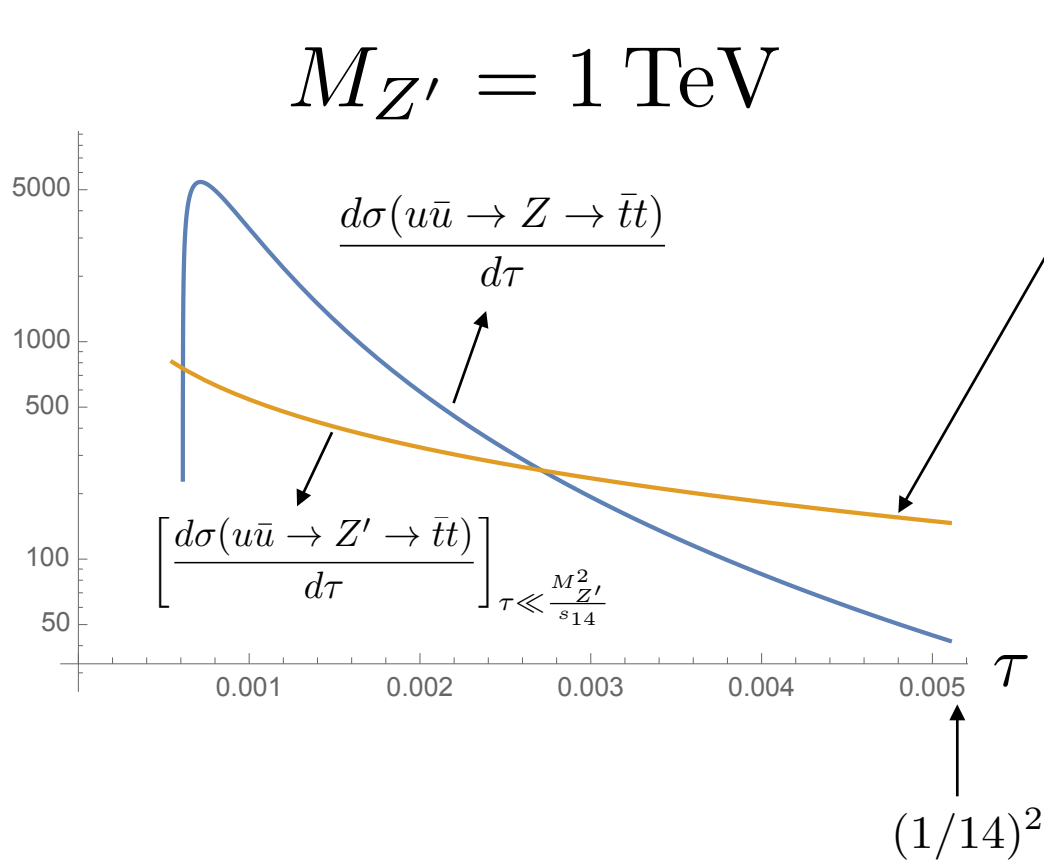


Resonance vs. EFT @ hadronic collisions

$$\hat{\sigma}_{LO} \approx |g_{u\bar{u}}g_{e\bar{e}}|^2 \frac{\hat{E}^2}{4M_{Z'}^4}$$

EFT, rising with COM energy², leads to IR-resonance broadening.

$$M_{Z'} = 1 \text{ TeV}$$

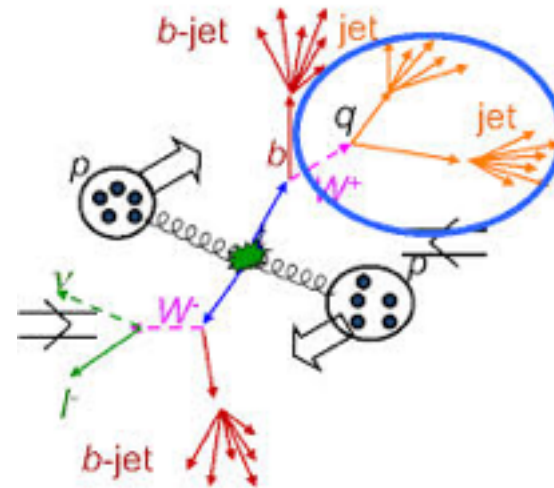
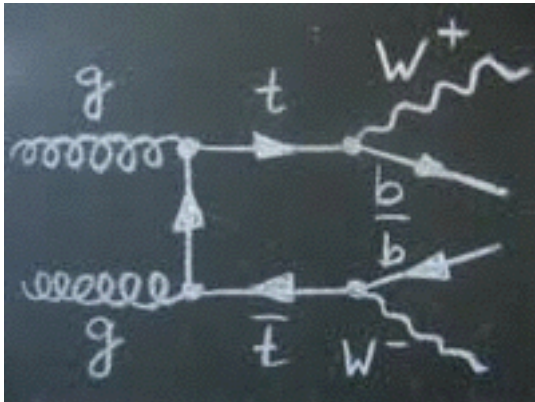


Few words about jets

Tops and jets

Tops decay almost instantly.

Thus at the LHC we identify tops via their decay products:



Unfortunately, isolated gluons/quarks are not gauge invariant objects, they are not observables, in real events we “see” jets.

But what are jets??

Intuitive definition: spray of particles moving in the same direction.

More precise: Objects that describe differential energy flow that are sensitive to microscopic (perturbative) dynamics & insensitive to long distance (non-perturbative) physics.

However, before going differentially, begin λ inclusive case.

Lecture III summary:

(Higgs) Resonance production @ LHC;

The EFT region;

Intro to Jets (ratio of had'/lepton in lepton collider, at NLO)

Homework:

Why is the Higgs narrow?

Calculate the Higgs width from the decay to bottoms, then using the amplitude given, verify that the gluon final state BR is ~9%.

Using the PDF calculate the Higgs production Xsec' using the narrow width approx.

What is the corresponding (to EFT w 4fermions) scaling in the interference case?

Show that: $s(1 - x_1) = m_{2g}$

Lecture IV:

Jets, cont';

Definitions, Sterman-Weinberg, Jade;

The k_t variety;

Boosted-massive-jets, jet substructure

Intro': $e^+e^- \rightarrow \text{quarks}$

$$R = \frac{\sigma(ee \rightarrow \text{hadrons})}{\sigma(ee \rightarrow \mu\mu)}$$

Far below the Z pole: $R = N_c \sum_q Q_q^2$

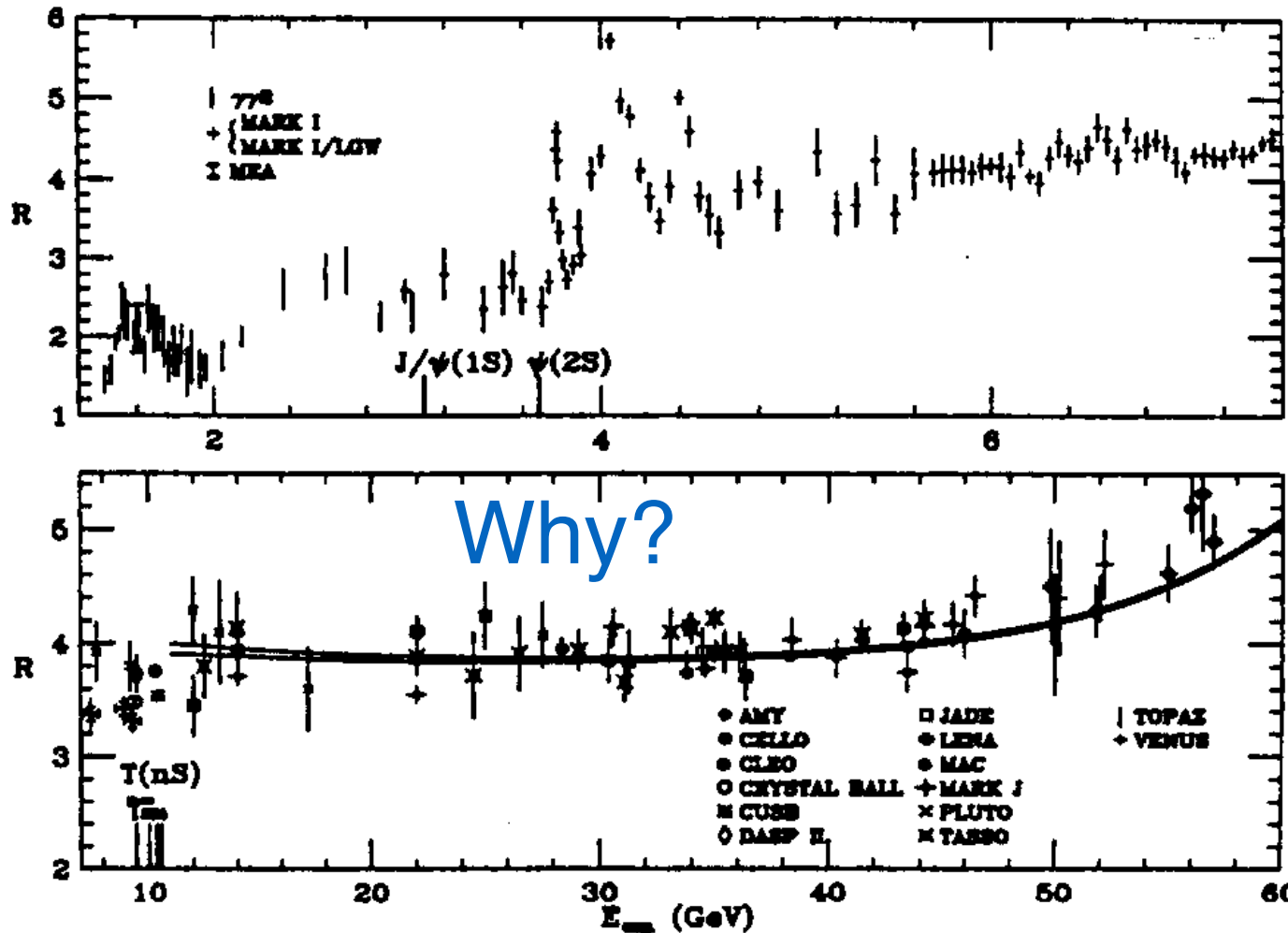
On the Z pole, the corresponding quantity is the ratio of the partial decay widths of the Z to hadrons and to muon pairs:

$$R_Z = \frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow \mu^+\mu^-)} = \frac{\sum_q \Gamma(Z \rightarrow q\bar{q})}{\Gamma(Z \rightarrow \mu^+\mu^-)} = \frac{3 \sum_q (a_q^2 + v_q^2)}{a_\mu^2 + v_\mu^2}.$$

Intro': $e^+e^- \rightarrow quarks$

For the 3 light quarks: $R = 3 \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 \right] = 2$

Adding $c, c + b$ yield $R = 10/3, 11/3$

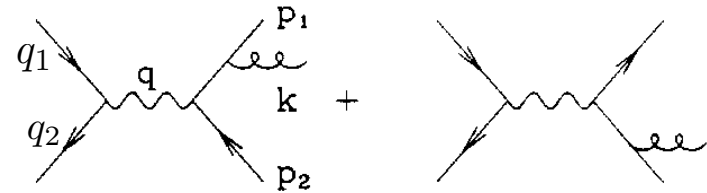


Results seem always higher!

Intro': $e^+e^- \rightarrow quarks @ NLO$

Contribution from higher orders ...

$$e^+ + e^- \rightarrow q + \bar{q} + g$$



$$x_{1,2} = 2E_{q,\bar{q}}/\sqrt{s}$$

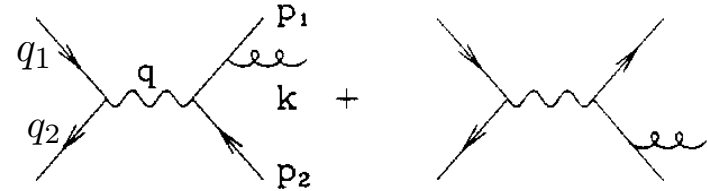
Question: are the x 's Lorentz invariant?

$$\text{Show that: } s(1 - x_1) = m_{2g}^2$$

Intro': $e^+e^- \rightarrow quarks @ NLO$

Contribution from higher orders ...

$$e^+ + e^- \rightarrow q + \bar{q} + g$$



$$\sigma^{q\bar{q}g} = N_c \sigma_0 \frac{C_F \alpha_s}{2\pi} \sum_q Q_q^2 \int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

where the integration region is for:

$$0 \leq x_{1,2} \leq 1, \quad x_1 + x_2 > 1$$

$$\sigma_0 = \frac{4\pi\alpha^2}{3s} Q_f^2$$

$$C_F = 4/3$$

$$x_{1,2} = 2E_{q,\bar{q}}/\sqrt{s}$$

Intro': $e^+e^- \rightarrow quarks @ \text{NLO}$

$$\sigma^{q\bar{q}g} = N_c \sigma_0 \frac{C_F \alpha_s}{2\pi} \sum_q Q_q^2 \int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

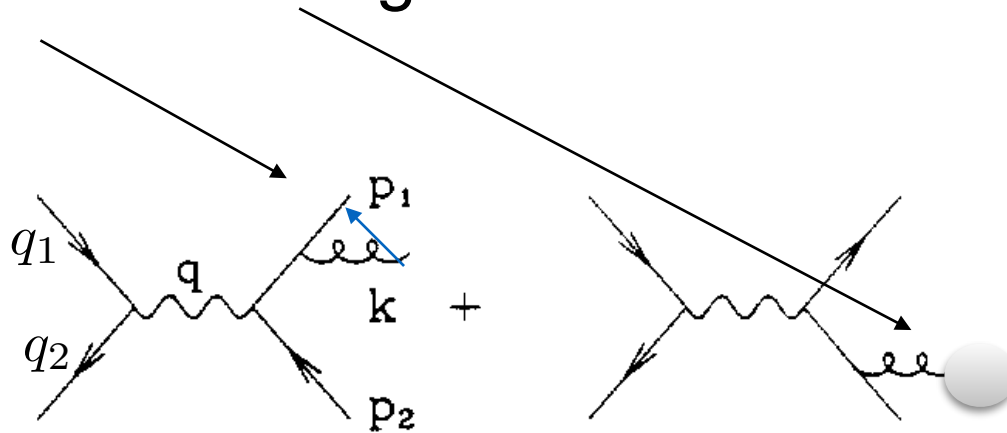
Integrals are divergent at $x_i = 1$, what is special about it?

$$1 - x_1 = x_2 \frac{E_g}{\sqrt{s}} (1 - \cos \theta_{2g})$$

The gluon is either soft, $E_g \rightarrow 0$;
or collinear $\theta_{2g} \rightarrow 0$.

$e^+e^- \rightarrow quarks$: Soft & collinear singularities of QCD

Both collinear and soft “gluon-states” are indistinguishable ...



These singularities are not physical due to the IR hadronic scale of QCD. However, the corresponding IR dynamics cannot be described in perturbation theory.

$e^+ e^- \rightarrow quarks$: regularization of the total Xsection

The above singularities can be regularised, say by Dim.

Reg.:

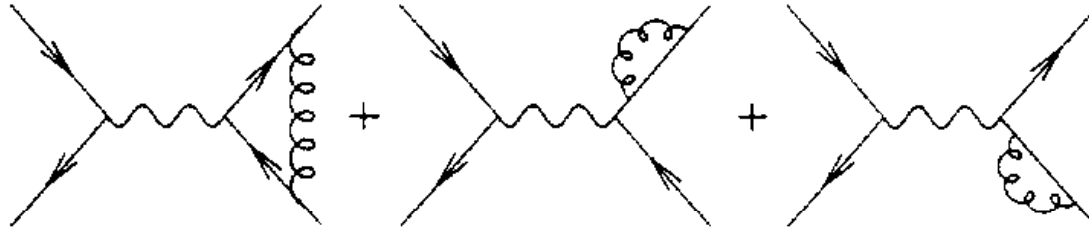
$$\sigma^{q\bar{q}g}(\epsilon) = \sigma_0 3 \sum_q Q_q^2 H(\epsilon) \int dx_1 dx_2 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2 - \epsilon(2 - x_1 - x_2)}{(1 - x_1)^{1+\epsilon}(1 - x_2)^{1+\epsilon}}$$

$$\text{with } \epsilon = \frac{1}{2}(4 - d), \text{ and } H(\epsilon) = \frac{3(1 - \epsilon)^2}{(3 - 2\epsilon)\Gamma(2 - 2\epsilon)} = 1 + O(\epsilon).$$

$$\sigma^{q\bar{q}g} \simeq N_c \sigma_0 \frac{C_F \alpha_s}{2\pi} \sum Q_q^2 \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} \right)$$

But we still have a divergent answer for the cross section
what is missing?

$e^+e^- \rightarrow quarks$: adding the virtual contributions



Virtual contributions can be computed in a similar fashion, again using Dim. Reg. to regularise the IR-divergencies:

$$\sigma^{q\bar{q}(g)} \simeq N_c \sigma_0 \frac{C_F \alpha_s}{2\pi} \sum Q_q^2 \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 \right)$$



$$R^{\text{NLO}} = N_c \sum_q Q_q^2 \left(1 + \frac{\alpha_s}{\pi} \right)$$

This 5% increase leads to much better agreement with data.

So what?

Jets

The previous success, regarding the total rate, didn't tell us anything about the distribution of energy flow & how to linked it with the partonic Xsec':

$$\text{LO} - \frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2 Q_f^2}{2s} (1 + \cos^2\theta)??$$

$$\text{NLO} - \frac{1}{\sigma} \frac{d^2\sigma}{dx_1 dx_2} = C_F \frac{\alpha_S}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}??$$

We expect the fragmented hadrons to roughly follow the parton direction, as seen in data from the 50s in cosmic ray & then latter on consistently in many exp'.

Jets

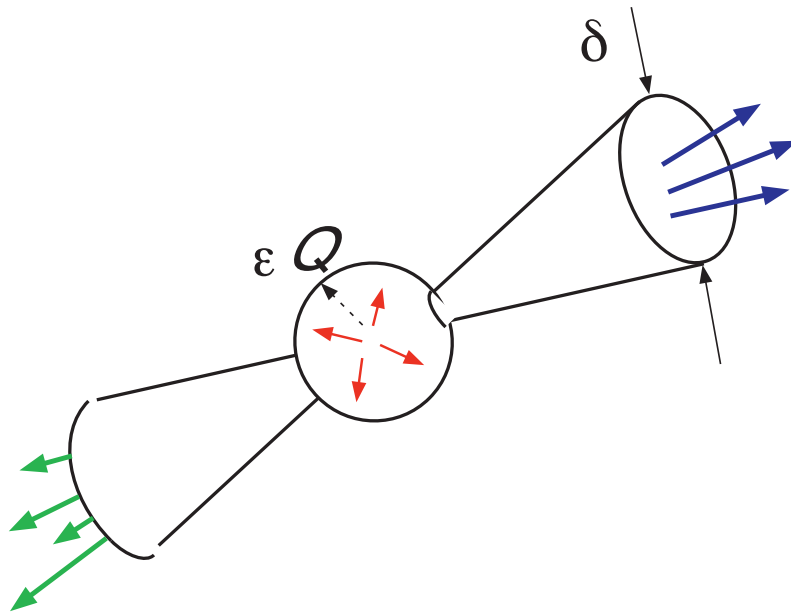
Then the soft/collinear gluons events would still have energy flow of 2 outgoing partons - “2 jets” topology.

On the other hand a well separated Xtra gluon emission is suppressed & look like an Xtra energy flow source - “3 jets”

Cone Jets, IRC safety (Sterman-Weinberg, 77)

Need to find a definition of these object, calculable in perturbation theory & yield finite rates (IRC_{collinear} safe).

Sterman-Weinberg a final state is classified as a 2-jet-like if -
All but a fraction ϵ of the total energy is contained in a pair of cones of half-angle δ .



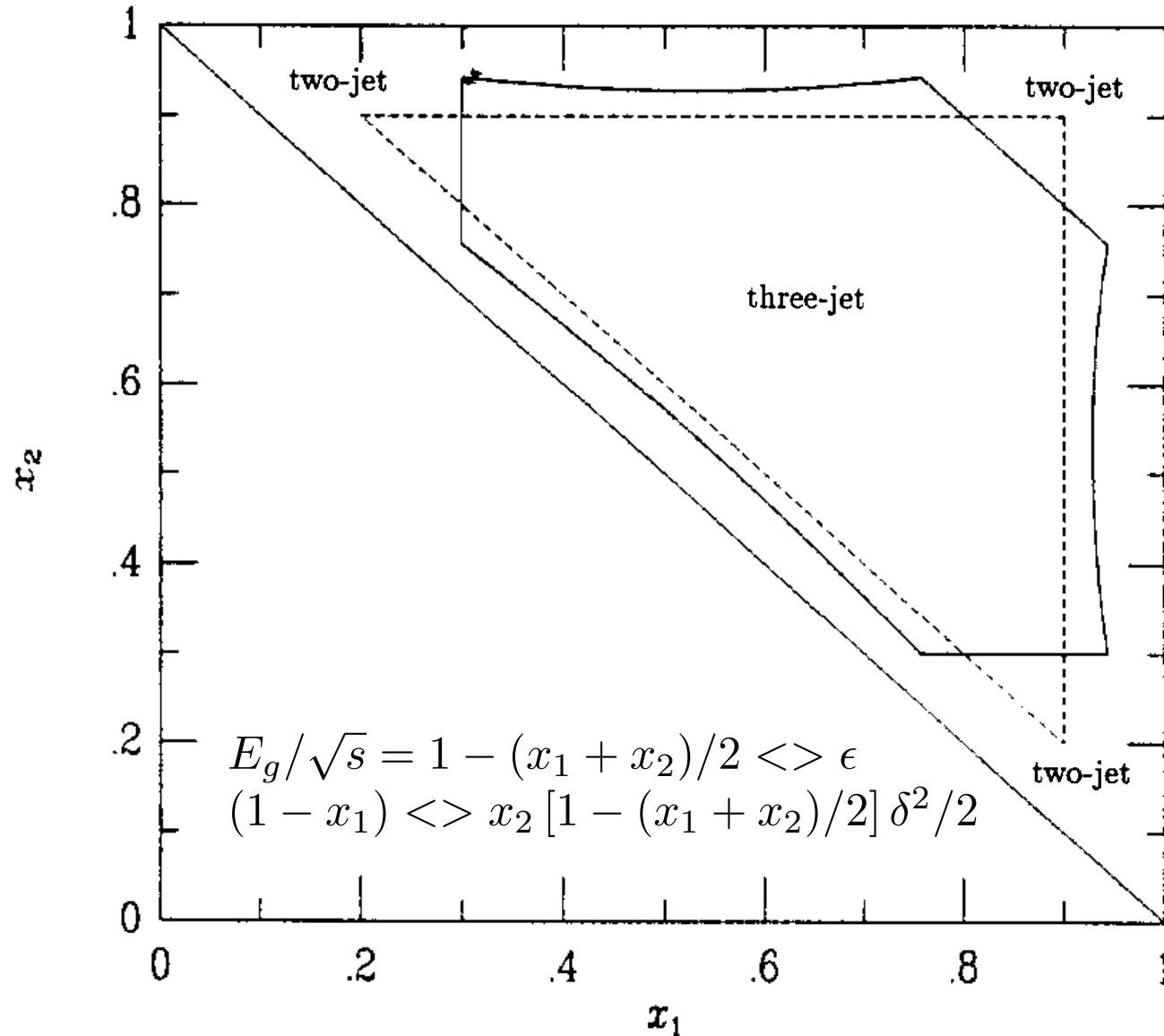
Cone Jets, IRC safety (Sterman-Weinberg, 77)

2-jet cross section: int. matrix elements over phase-space given by ϵ & δ .

Lowest order \Rightarrow leading order picture.

At $O(\alpha_s)$, 2-jet X_{sec} ' is obtained by appropriate integration.

Cone Jets, IRC safety



Boundaries between the two- and three-jet regions in the (x_1, x_2) plane for (a) Sterman-Weinberg jets with $(\epsilon, \delta) = (0.3, 30^\circ)$ (solid lines), and (b) JADE algorithm jets with $y = 0.1$ (dashed lines). 79

2-jet vs 3-jet Xsections

At this order: $\sigma = \sigma_2 + \sigma_3$,

Let's define $f_{2,3} = \sigma_{2,3}/\sigma$,

$$f_2 \simeq 1 - 8C_F \frac{\alpha_s}{2\pi} \left[\ln \delta(\ln 2\epsilon - 1) + \frac{3}{4} + \frac{\pi^2}{12} - \frac{7}{12} \right]$$

$$f_3 = 1 - f_2$$

These are IRC safe, observables as well as derivatives, such as angular dist' etc ...

So what are jets?

When $\epsilon, \delta \ll 1$ $O(\alpha_s) \Rightarrow$ log enhanced.

Residues of the singularities, improved when resummed.
(usefulness limited)

Number of jets is not a physical parameter!

Intuitive partons & jets link holds only at LO.

Higher order in pert. th. $\Rightarrow \geq 4$ jets.

Cones in hadron colliders

Sterman-Weinberg cones give inefficient ‘tiling’ of the phase-space 4π solid angle.

Similarly for hadronic machine one needs to use different E threshold and not COM.

And, also non trivial to implement in practice, “where to place the cone?” And, “how to deal with overlaps?”. Thus, alternatives were constructed.

One needs to find way to cluster partons (energy) in an IR safe manner.

Also practical issues: seeds and overlaps ...

Sequential recombination jet algorithms

Jade (Jade Collab' 88)

$$\min (p_i + p_j)^2 = \min 2E_i E_j (1 - \cos \theta_{ij}) > y s, \quad i, j = q, \bar{q}, g,$$

$$0 < x_1, x_2 < 1 - y, \quad x_1 + x_2 > 1 + y.$$

$$f_3 = C_F \frac{\alpha_S}{2\pi} \left[(3 - 6y) \log \left(\frac{y}{1 - 2y} \right) + 2 \log^2 \left(\frac{y}{1 - y} \right) + \frac{5}{2} - 6y - \frac{9}{2} y^2 + 4 \operatorname{Li}_2 \left(\frac{y}{1 - y} \right) - \frac{\pi^2}{3} \right],$$

$$f_2 = 1 - f_3,$$

where Li_2 is the dilogarithm function,

$$\operatorname{Li}_2(x) = - \int_0^x dy \frac{\log y}{1 - y}.$$

Sequential recombination jet algorithms

Jade: (Jade Collab' 88)

$$\min(m_{ij}^2) = 2E_i E_j (1 - \cos \theta_{ij}) > y \times s$$

$$0 \geq x_{1,2} < 1 - y, \quad x_1 + x_2 = 1 + y$$

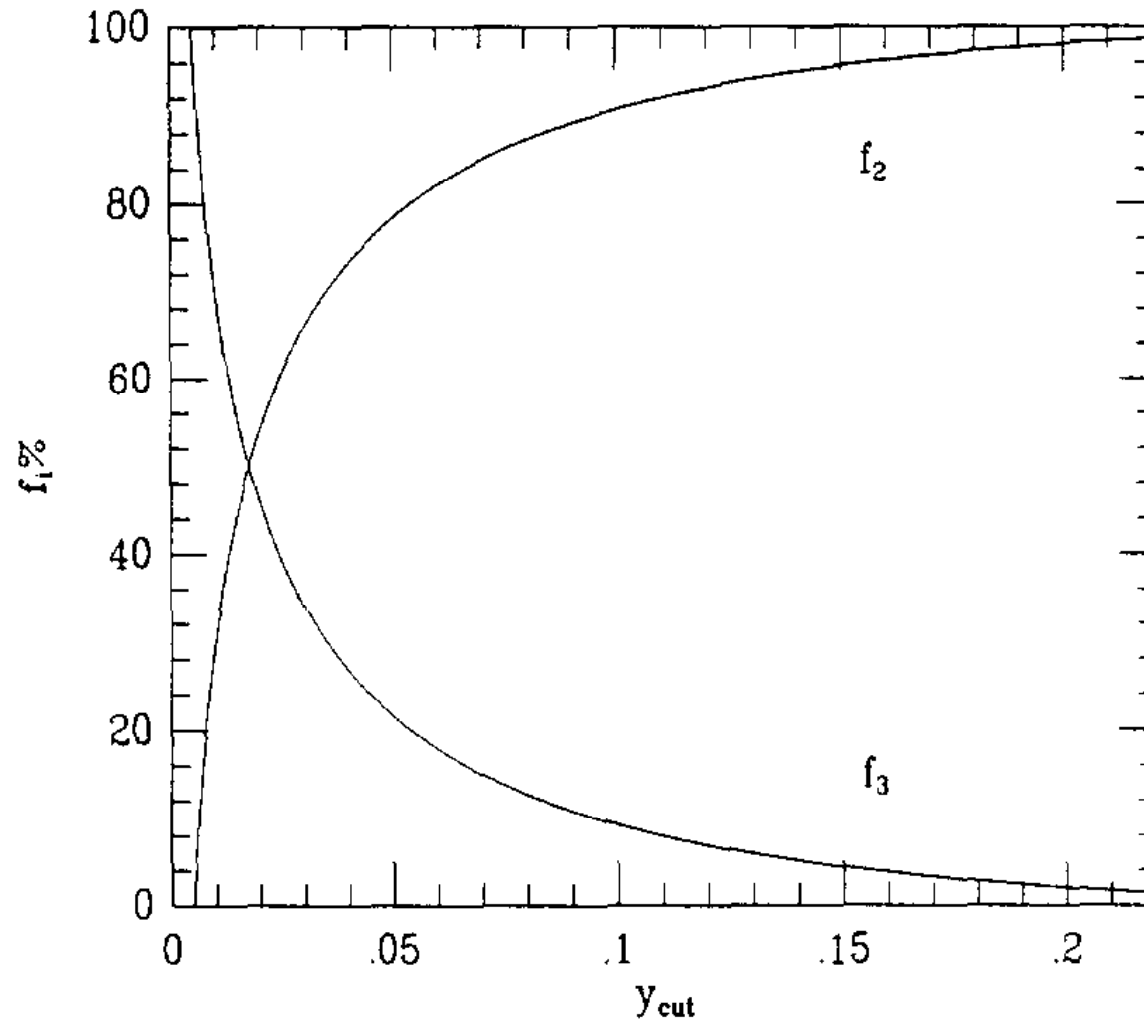
$$f_3 = C_F \frac{\alpha_S}{2\pi} \left[(3 - 6y) \log \left(\frac{y}{1 - 2y} \right) + 2 \log^2 \left(\frac{y}{1 - y} \right) + \frac{5}{2} - 6y - \frac{9}{2}y^2 + 4 \operatorname{Li}_2 \left(\frac{y}{1 - y} \right) - \frac{\pi^2}{3} \right],$$

$$f_2 = 1 - f_3,$$

where Li_2 is the dilogarithm function,

$$\operatorname{Li}_2(x) = - \int_0^x dy \frac{\log y}{1 - y}.$$

Jade



The values of f_3 and f_2

The above valid for $y < 1/3$, the Fig. shows the two and three jet ratios. Soft and collinear singularities again reappear as large logarithms in the limit where y is small.

The k_t algorithm in e^+e^-

The e^+e^- , k_t algorithm is similar to the JADE algorithm except as concerns the distance measure, which is

$$y_{ij} = \frac{2 \min(E_i^2, E_j^2) (1 - \cos \theta_{ij})}{Q^2} .$$

Q^2 is the square of total $E \sim s$.

In the collinear limit, $\theta_{ij} \ll 1$, numerator $\sim (\min(E_i, E_j) \theta_{ij})^2 \Rightarrow$

the squared transverse momentum of i relative to j , hence the name k_t .

The k_t algorithm in e^+e^-

k_t -measure: $y_{ij} \Leftrightarrow$ inverse splitting probability
for parton k to go into i and j , when i or j is

soft and collinear, $\frac{dP_{k \rightarrow ij}}{dE_i d\theta_{ij}} \sim \frac{\alpha_s}{\min(E_i, E_j)\theta_{ij}}$

Maltoni's talk.

The k_t algorithm with incoming hadrons

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2) \frac{\Delta R_{ij}^2}{R^2}, \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2,$$
$$d_{iB} = p_{ti}^2,$$

1. Work out all the d_{ij} and d_{iB} ac
2. Find the minimum of the d_{ij} and d_{iB} .
3. If it is a d_{ij} , recombine i and j into a single new particle and return to step 1.
4. Otherwise, if it is a d_{iB} , declare i to be a [final-state] jet, and remove it from the list of particles. Return to step 1.
5. Stop when no particles remain.

One can generalise the k_t :

$$d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta R_{ij}^2}{R^2}, \quad \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2,$$

$$d_{iB} = p_{ti}^{2p},$$

$p = -1, 0$ for anti- k_t and Cambridge/Aachen (C/A).

Intermediate summary

Jets (spikes of energy flow) in QCD at high energies are due to asymptotic freedom & its non-abelian nature.

Jet algorithms obtain finite (IRC safe) & perturbative differential description.

Distributions (jets numb. etc.) are prescription-dep., within an algorithm \Rightarrow short distance physics is transparent.

Allow us to make contact \w microscopical partonic calculation, with quarks/gluons final states.

Massive boosted jets

Jets substructure

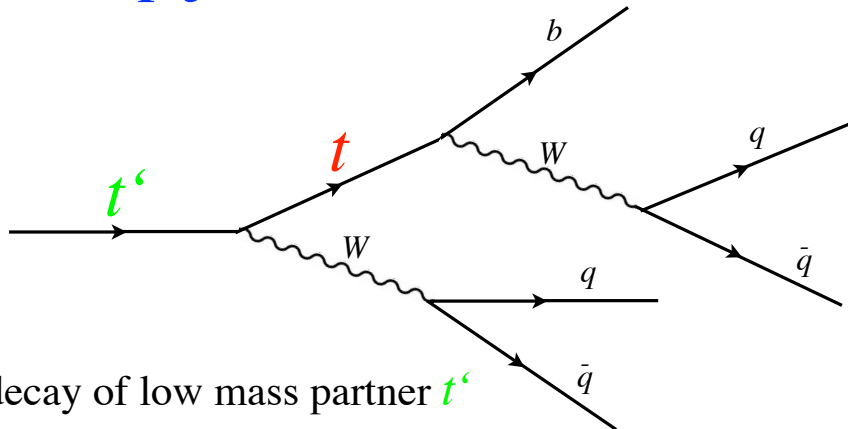
(Briefly ...)

Boosted tops EW bosons: $m_{t'} \gg m_t$

- ◆ The challenge of searching for heavy resonance top-partners:

As $m_{t'} \gg m_t$ outgoing tops are ultra-relativistic, their products collimate

\Rightarrow top jets.

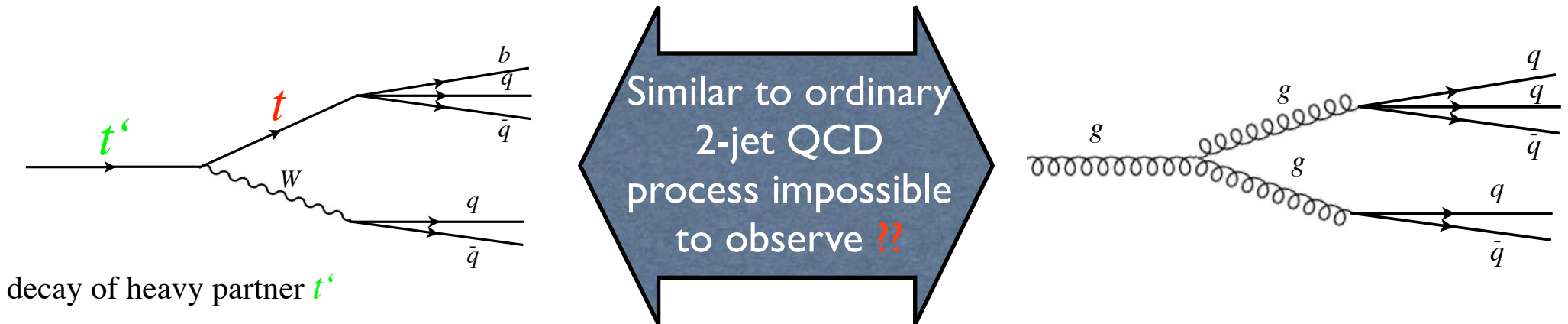


decay of low mass partner t'

Boosted tops EW bosons: $m_{t'} \gg m_t$

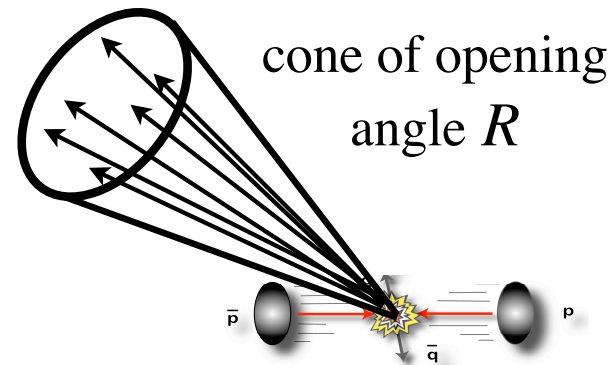
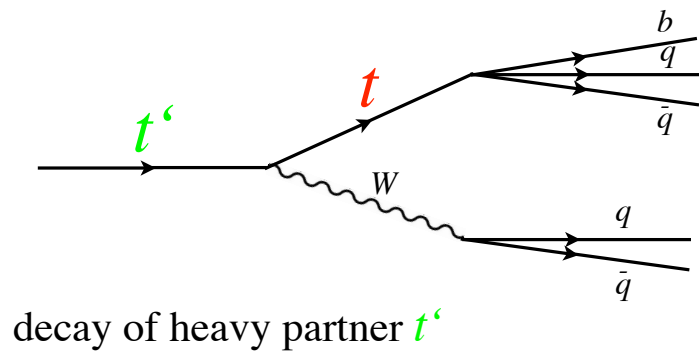
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How big is the opening angle?

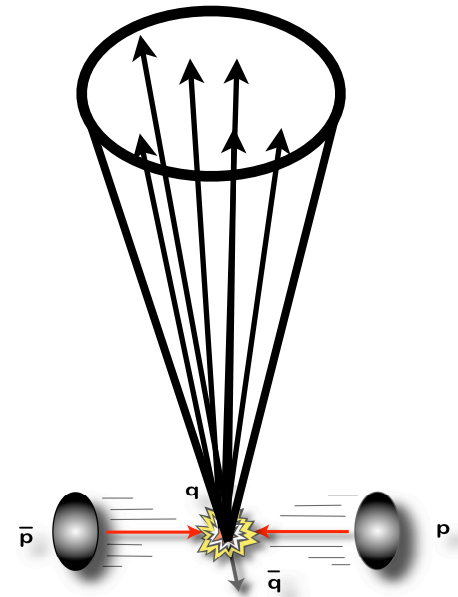
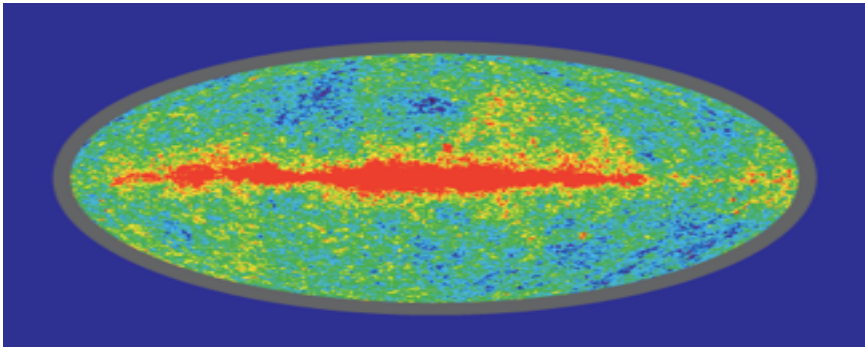
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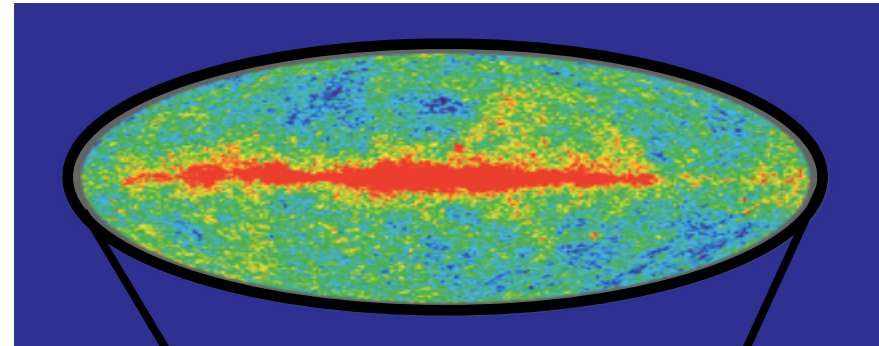
$$\delta R \sim \frac{2m_X}{E_{J_X}}$$

What is the opening angle of a 2 TeV top?

Understanding the inside of massive boosted jets



Jet substructure



- (i) Mass;
- (ii) Angularity (filtering) & planar flow;
- (iii) Beyond shapes, template function.

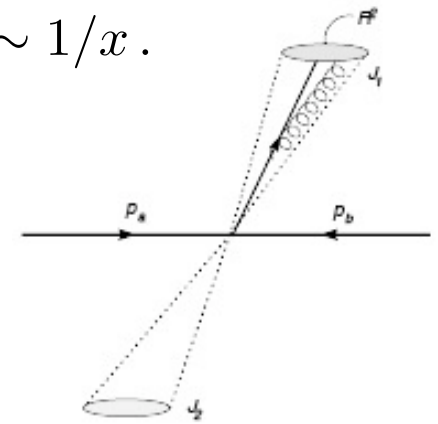


The Splitting Function (leading log, gluon emission)

In the limit where the emitted gluon is soft and collinear we find:

In QCD the probability for a parton j to emit a parton i with energy fraction x at angle θ is

$$d\sigma \propto \alpha_s P_{ij}(x) dx \frac{d\theta}{\theta} \quad P_{ij}(x) \text{ is the Altarelli-Parisi matrix} \quad P_{ij} \sim 1/x.$$



As discussed below, above limit seems (fortunately) to be valid for a search for massive boosted jets:

$$\Lambda_{\text{QCD}} \ll m_{\text{peak}} \ll m_J \ll P_T R, \quad R \ll 1$$

Large mass => perturbative control (asymptotic freedom)

- ◆ Use simple perturbation theory to define & compute set of jet-shape variables.

$$\alpha_s(m_J) \sim 1$$

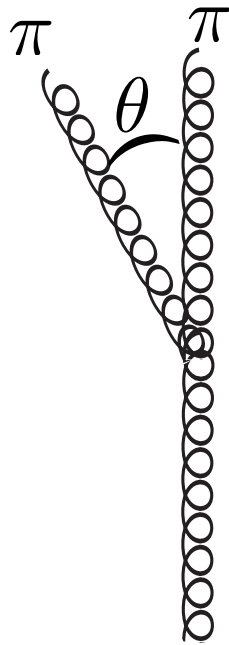


$$m_J \sim m_\pi \sim \Lambda_{\text{QCD}}$$

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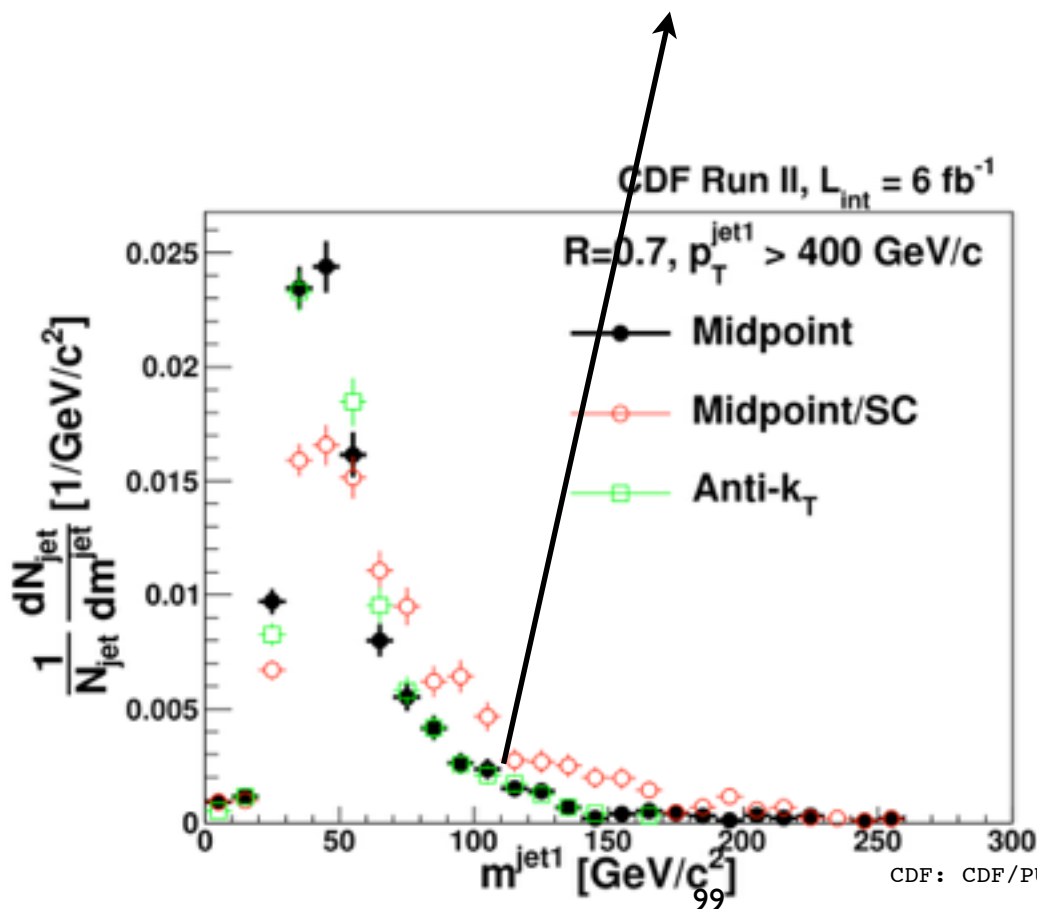


$$m_J \sim p_\pi \theta \gg \Lambda_{\text{QCD}}$$

The big picture: Energy flow of massive narrow jets, QCD first

◆ Interested in narrow, massive energetic

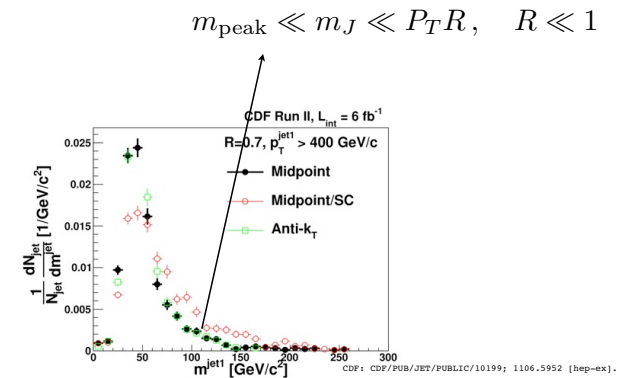
(boosted) jets: $m_{\text{peak}} \ll m_J \ll P_T R, \quad R \ll 1$



Jet substructure

Use splitting function to get some qualitative understanding:

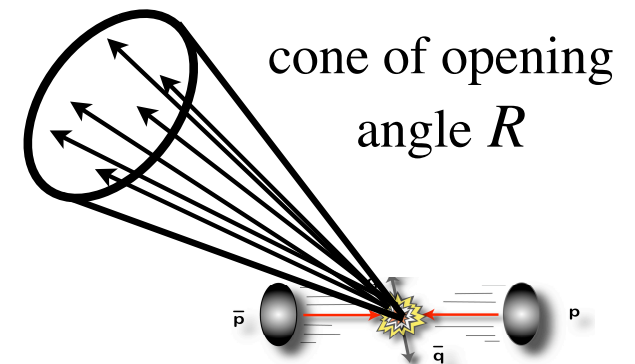
2-body partonic IR-safe approx' for jet substructure.



Since signal is EW mass boosted particles, obvious variable to distinguish between signal & QCD background is the jet mass.

Jet mass definition:

$$m_J^2 = \left(\sum_{i \in R} P_i \right)^2, \quad P_i^2 = 0$$



Jet mass from splitting function (leading log)

$$d\sigma \propto \alpha_s P_{ij}(x) dx \frac{d\theta}{\theta} \text{ with } P_{ij} \sim 1/x.$$

$$\text{Given } m_J^2 \approx x E_J^2 \theta^2 \Rightarrow \frac{d\sigma}{dm_J^2} \propto \alpha_s \frac{C_F}{m_J^2} \int_{\frac{m_J}{E_J}}^R \frac{d\theta}{\theta} \propto \alpha_s \frac{C_F}{m_J^2} \log \left(\frac{E^2 R^2}{m_J^2} \right)$$

$C_F = 4/3$ for quarks, $C_A = 3$ for gluons.

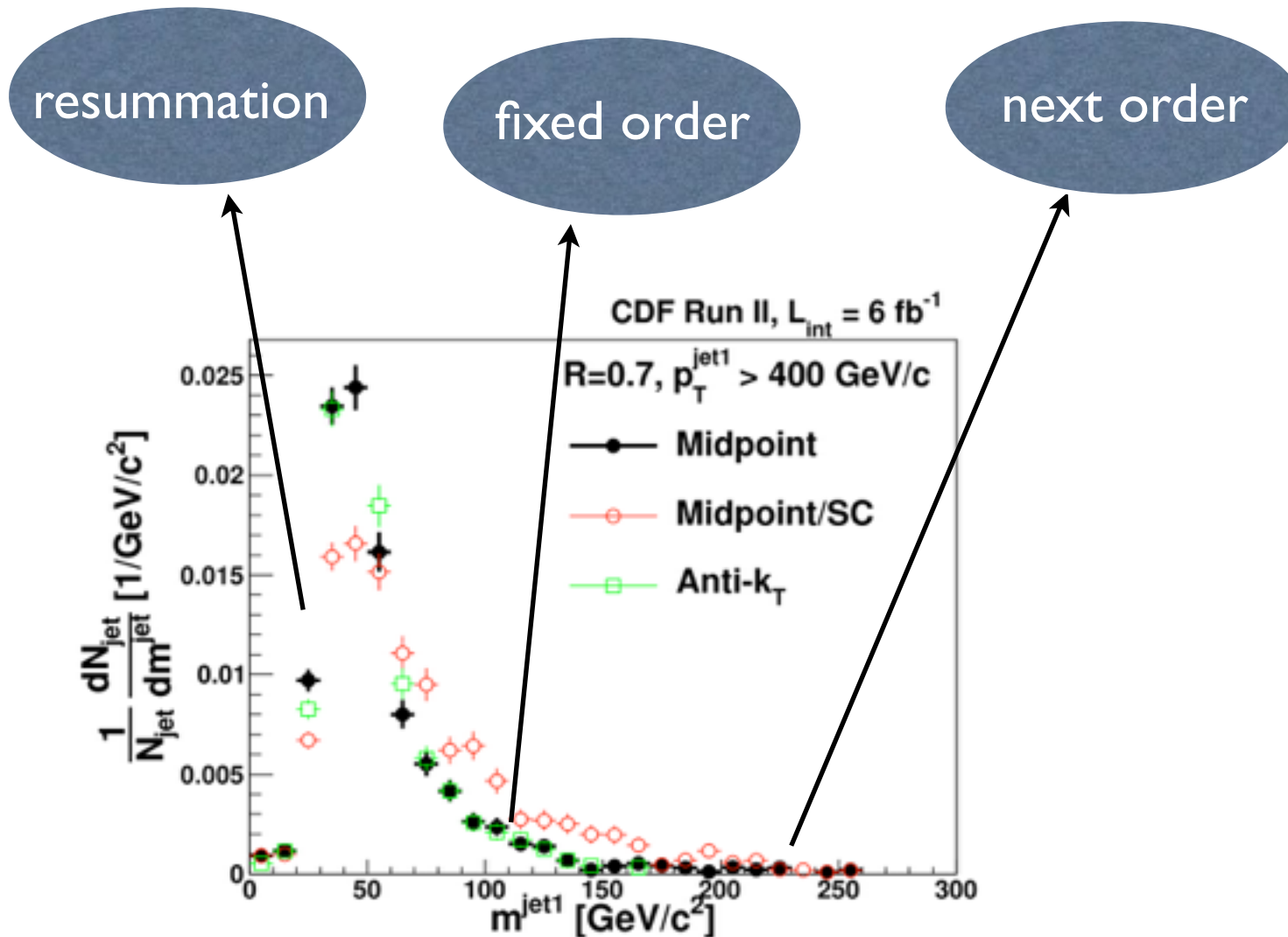
As long as $\alpha_s(m_J^2) \ll \alpha_s(m_J^2) \log \left(\frac{p_T^2 R^2}{m_J^2} \right) \ll 1$

We can use fix order perturbation theory.

Questions: what are the relevant mass range for this approx' for jet of $E \sim 1 \text{ TeV}$ & $R=0.4$?

What is the average jet mass for these parameters?

Summary QCD jet mass



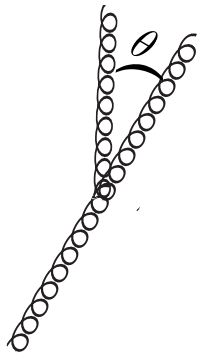
Questions: What is the shape of top jet mass distribution?

Jet substructure beyond mass

2-body partonic approximation actually tells us more:

Kinematics is trivial, for given mass & momenta: a single more variable, distribution extracted from splitting function.

angular distribution: $\frac{d^2\sigma}{dm_J^2 d\theta} \propto \frac{C_F}{m_J^2 \theta}$, and $\theta_{\min} = \frac{2m_J}{E_J}$



Questions: Show that the Higgs jet angular distribution is given by θ^{-3} , with the same min' angle.

Testing with real data



Alon, Duchovni, GP & Sinervo, for the CDF, 10199, 10234, 1106.5952 [hep-ex];

Boosted jets' angular distribution, angularity τ_{-2}

$$\frac{d\sigma}{d\theta} \rightarrow \frac{d\sigma}{d\tau_{-2}} \approx 1/\tau_{-2}, \quad \tau_{-2}^{\min} = \left(\frac{m_J}{2E_J} \right)^3 \quad (\tau_{-2} \sim \sum_{i \in J} E_i \theta_i^4)$$

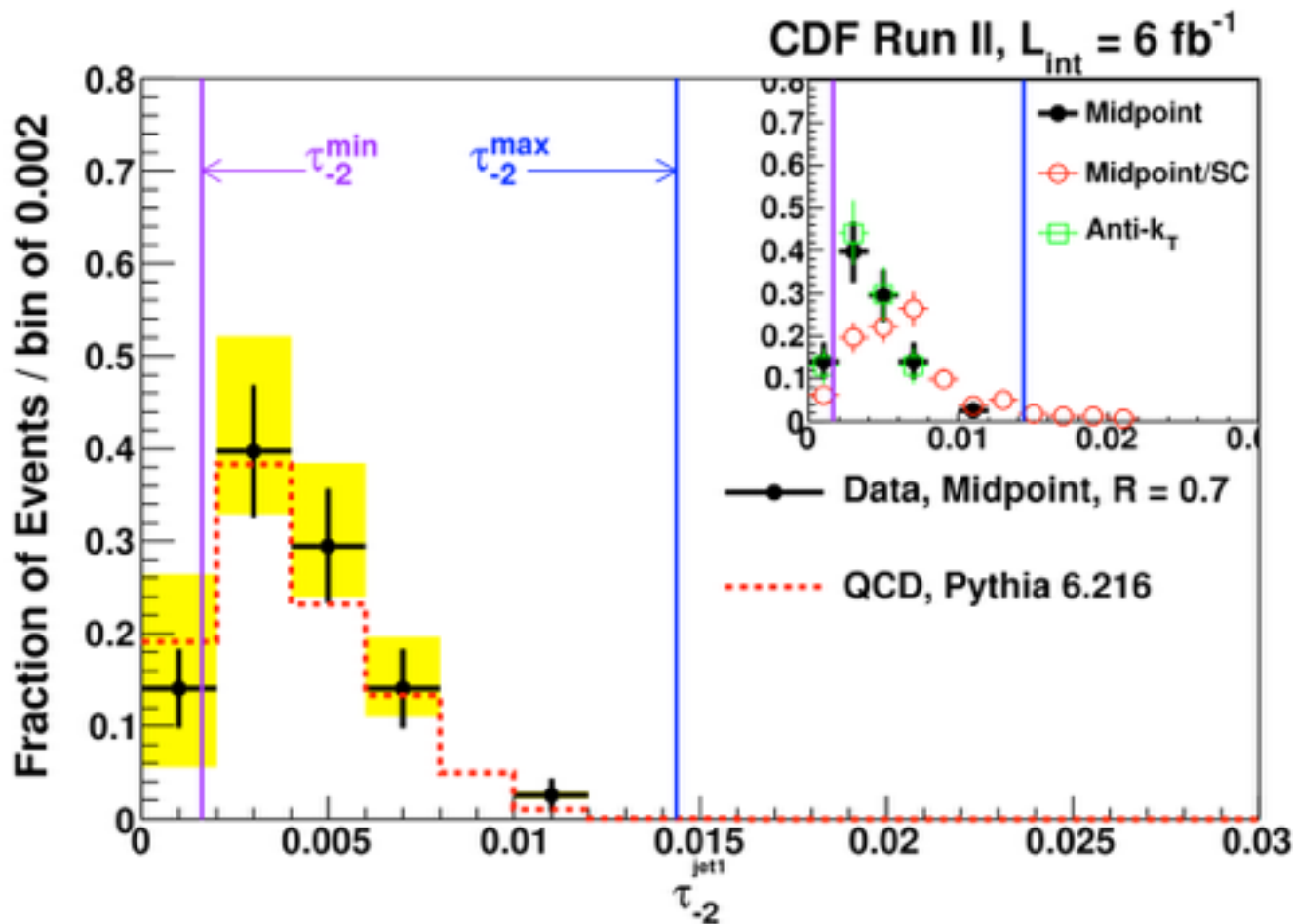
Almeida, Lee, GP, Sterman & Sung (10)

Questions: Derive the above angularity dist' (for large angles).

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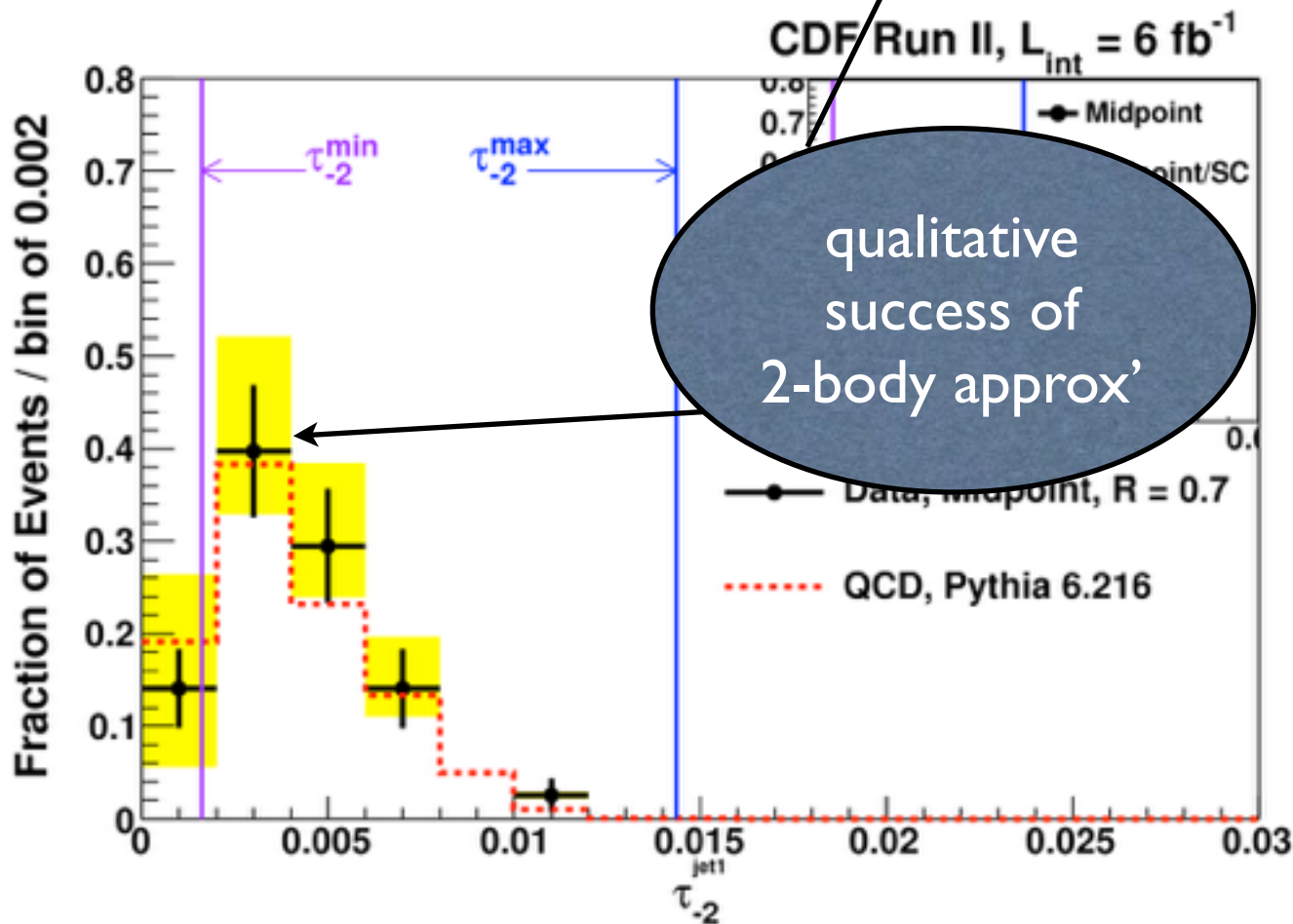


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Summary

LHC opens a new era: colliders energy $>$ electroweak (EW) scale.

Probing the mechanism of EW symmetry breaking.

New phenomena is kinematically allowed a shot of looking at new physics related to naturalness.

Calculation at the LHC are challenging due to nature of incoming composite particles.

Yet simple concepts as parton luminosities & understanding kinematics & jets allow for (semi-)quantitative control.