NEUTRINOS

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(YITP Stony Brook & ICREA U. Barcelona)

Invisibles15 School, June 18th, 2015

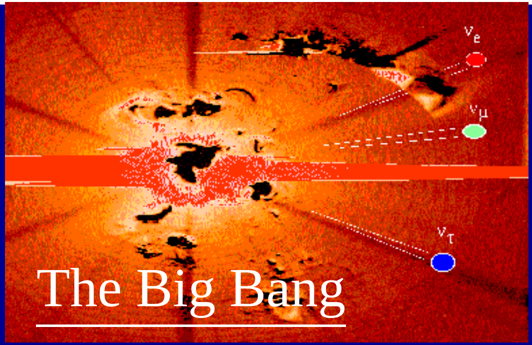
Global fit to neutrino oscillation data
http://www.nu-fit.org

invisibles
neutrinos, dark matter & dark energy physics

SEVENTH FRAMEWORK PROGRAMME

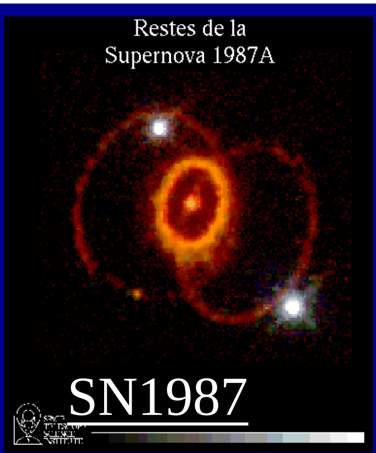
MARIE CURIE ACTIONS

Sources of ν's



$\rho_\nu = 330/\text{cm}^3$

p_ν =0.0004 eV



$E_\nu \sim$ MeV

The Sun

ν_e

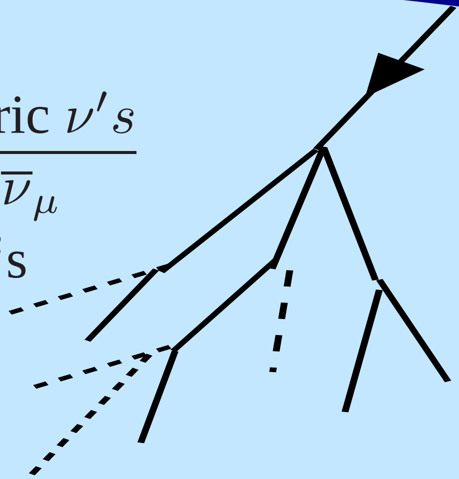
$\Phi_\nu^{Earth} = 6 \times 10^{10} \nu/\text{cm}^2\text{s}$

$E_\nu \sim$ 0.1–20 MeV

Atmospheric ν's

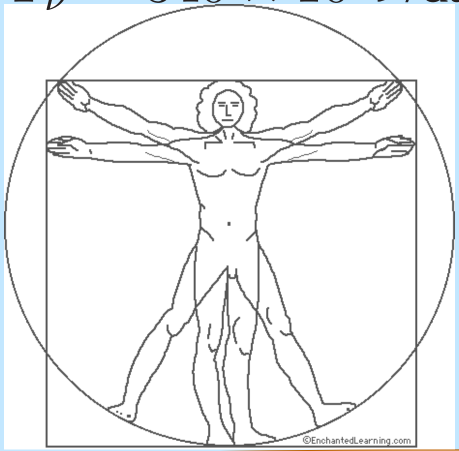
$\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu$

$\Phi_\nu \sim 1\nu/\text{cm}^2\text{s}$



Human Body

$\Phi_\nu = 340 \times 10^6 \nu/\text{day}$



Nuclear Reactors

$E_\nu \sim$ few MeV

$\overline{\nu}_e$

Earth's radioactivity

$\Phi_\nu \sim 6 \times 10^6 \nu/\text{cm}^2\text{s}$

Accelerators

$E_\nu \simeq$ 0.3–30 GeV

Fermilab Discovering the Nature of Nature
CERN
KEK

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NEUTRINOS

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Evidence of ν Masses

Determination of Lepton Flavour Parameters

Implications

ν in the SM

The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

$(1,2)_{-\frac{1}{2}}$	$(3,2)_{\frac{1}{6}}$	$(1,1)_{-1}$	$(3,1)_{\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} u^i \\ d^i \end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} c^i \\ s^i \end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	$\begin{pmatrix} t^i \\ b^i \end{pmatrix}_L$	τ_R	t^i_R	b^i_R

There is no ν_R

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There is no ν_R

$\Downarrow$

Accidental global symmetry: $B \times L_e \times L_\mu \times L_\tau$

$\Downarrow$

ν *strictly* massless

- By 2015 we have observed with high (or good) precision:
  - ∗ Atmospheric ν_μ & $\bar{\nu}_\mu$ disappear most likely to ν_τ (**SK**,MINOS, ICECUBE)
  - ∗ Accelerator ν_μ & $\bar{\nu}_\mu$ disappear at $L \sim 250[700]$ Km (K2K, bf T2K, MINOS)
  - ∗ Some accelerator ν_μ appear as ν_e at $L \sim 700$ Km (**T2K**, MINOS)
  - ∗ Solar ν_e convert to ν_μ/ν_τ (Cl, Ga, **SK, SNO, Borexino**)
  - ∗ Reactor $\overline{\nu_e}$ disappear at $L \sim 200$ Km (**KamLAND**)
  - ∗ Reactor $\overline{\nu_e}$ disappear at $L \sim 1$ Km (D-Chooz, **Daya Bay, Reno**)

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All this implies that L_α are violated

and There is Physics Beyond SM

The New Minimal Standard Model

- Minimal extension to introduce L_α violation $\Rightarrow$ give Mass to the Neutrino:

ν Mass Terms: Dirac Mass

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- One introduces ν_R which can couple to the lepton doublet by Yukawa interaction

$$\mathcal{L}_Y^{(\nu)} = -\lambda^\nu_{ij} \overline{\nu_{Ri}} L_{Lj} \tilde{\phi}^\dagger + \text{h.c.} \qquad (\tilde{\phi} = i\tau_2 \phi^*)$$

- Under spontaneous symmetry-breaking $\mathcal{L}_Y^{(\nu)} \Rightarrow \mathcal{L}_{\text{mass}}^{(\text{Dirac})}$

$$\mathcal{L}_{\text{mass}}^{(\text{Dirac})} = -\overline{\nu_R} M_D^\nu \nu_L + \text{h.c.} \equiv -\frac{1}{2}(\overline{\nu}_R M_D^\nu \nu_L + \overline{(\nu_L)^c} {M_D^\nu}^T (\nu_R)^c) + \text{h.c.} \equiv -\sum_k m_k \overline{\nu}_k^D \nu_k^D$$

$M_D^\nu = \frac{1}{\sqrt{2}} \lambda^\nu \, v$ =Dirac mass for neutrinos

${V_R^\nu}^\dagger M_D V^\nu = \text{diag}(m_1, m_2, m_3)$

$\Rightarrow$ The eigenstates of M_D^ν are Dirac particles (same as quarks and charged leptons)

$$\nu^D = {V^\nu}^\dagger \nu_L + {V_R^\nu}^\dagger \nu_R$$

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$$\nu^D = V^{\nu\,\dagger} \nu_L + V_R^{\nu\,\dagger} \nu_R$$

$\Rightarrow$ Total Lepton number is conserved *by construction* (not *accidentally*):

$$U(1)_L\, \nu = e^{i\alpha}\, \nu \quad \text{and} \quad U(1)_L\, \overline{\nu} = e^{-i\alpha}\, \overline{\nu}$$

$$U(1)_L\, \nu^C = e^{-i\alpha}\, \nu^C \quad \text{and} \quad U(1)_L\, \overline{\nu^C} = e^{i\alpha}\, \overline{\nu^C}$$

ν Mass Terms: Majorana Mass

• One does not introduce ν_R but uses that the field $(\nu_L)^c$ is right-handed, so that one can write a Lorentz-invariant mass term

$$\mathcal{L}^{(\mathrm{Maj})}_{\mathrm{mass}} = -\frac{1}{2}\overline{\nu^c_L} M^\nu_M \nu_L + \mathrm{h.c.} \equiv -\frac{1}{2}\sum_k m_k \overline{\nu}^M_i \nu^M_i$$

M^ν_M =Majorana mass for ν's is *symmetric* $\quad V^{\nu\,T} M_M V^\nu = \mathrm{diag}(m_1, m_2, m_3)$

$\Rightarrow$ The eigenstates of M^ν_M are Majorana particles

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$\Rightarrow$ But $SU(2)_L$ gauge inv is broken $\Rightarrow \mathcal{L}^{(\text{Maj})}_{\text{mass}}$ not possible at tree-level in the SM

• Moreover under any $U(1)$ symmetry with $U(1)\,\nu = e^{i\alpha}\,\nu$

$$U(1)\,\nu^c = e^{-i\alpha}\,\nu^c \quad \text{and} \quad U(1)\,\overline{\nu} = e^{-i\alpha}\,\overline{\nu} \quad \text{so} \quad U(1)\,\overline{\nu^c} = e^{i\alpha}\,\overline{\nu^c}$$

$\Rightarrow \mathcal{L}^{(\text{Maj})}_{\text{mass}}$ breaks $U(1)$ (so it can only appear for particles without electric charge)

$\Rightarrow$ Breaks Total Lepton Number $\Rightarrow \mathcal{L}^{(\text{Maj})}_{\text{mass}}$ not generated at any loop level in the SM

• in SM $B - L$ is non anomalous $\Rightarrow \mathcal{L}^{(\text{Maj})}_{\text{mass}}$ not generated non-perturbatively in SM

The New Minimal Standard Model

- Minimal extension to introduce L_α violation $\Rightarrow$ give Mass to the Neutrino:

  $*$ Introduce ν_R AND impose L conservation $\Rightarrow$ Dirac $\nu \neq \nu^c$:

$$\mathcal{L} = \mathcal{L}_{SM} - M_\nu \overline{\nu_L} \nu_R + h.c.$$

  $*$ NOT impose L conservation $\Rightarrow$ Majorana $\nu = \nu^c$

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2} M_\nu \overline{\nu_L} \nu_L^C + h.c.$$

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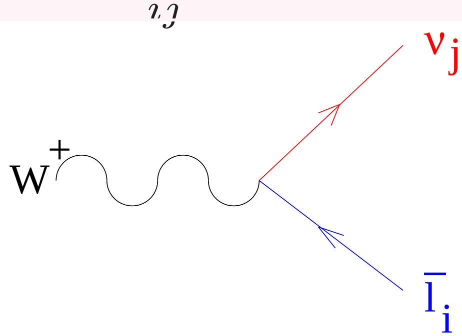
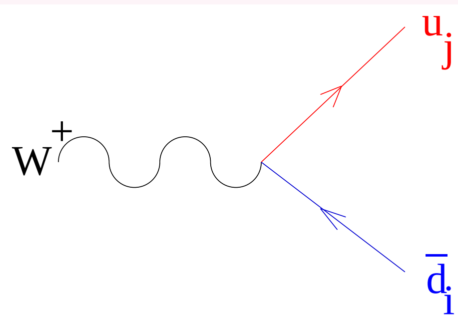
- The charged current interactions of leptons are not diagonal (same as quarks)

$$\frac{g}{\sqrt{2}} W_\mu^+ \sum_{ij} \left(U_{\rm LEP}^{ij}\, \overline{\ell^i}\, \gamma^\mu\, L\, \nu^j \;+\; U_{\rm CKM}^{ij}\, \overline{U^i}\, \gamma^\mu\, L\, D^j \right) \;+\; h.c.$$





Lepton Mixing

- Charged current and mass for 3 charged leptons ℓ_i and N neutrinos ν_j in weak basis

$$\mathcal{L}_{CC}+\mathcal{L}_M = -\frac{g}{\sqrt{2}}\sum_{i=1}^{3}\overline{\ell^W_{L,i}}\gamma^\mu \nu^W_i W^+_\mu - \sum_{i,j=1}^{3}\overline{\ell^W_{L,i}} M_{\ell\,ij}\,\ell^W_{R,j} - \frac{1}{2}\sum_{i,j=1}^{N}\overline{\nu^{c\,W}_i} M_{\nu\,ij}\,\nu^W_j + \text{h.c.}$$

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- Changing to mass basis by rotations

$$\ell^W_{L,i} = V^\ell_{Lij}\ell_{L,j} \qquad \ell^W_{R,i} = V^\ell_{Rij}\ell_{R,j} \qquad \nu_i^W = V^\nu_{ij}\nu_j$$

$$V_L^{\ell\,\dagger} M_\ell V_R^\ell = \text{diag}(m_e, m_\mu, m_\tau) \qquad V^{\nu\,T} M_\nu V^\nu = \text{diag}(m_1^2, m_2^2, m_3^2, \dots, m_N^2)$$

$V^\ell_{L,R} \equiv$ Unitary 3×3 matrices

$V^\nu \equiv$ Unitary $N\times N$ matrix.

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- The charged current in the mass basis

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\overline{\ell^i_L}\,\gamma^\mu\, U_{\text{LEP}}^{ij}\,\nu_j\, W_\mu^+$$

$U_{\text{LEP}} \equiv 3\times N$ matrix

$$U_{\text{LEP}}^{ij} = \sum_{k=1}^{3} P^\ell_{ii} {V_L^{\ell\,\dagger}}^{ik}\, V^{\nu\,kj} P^\nu_{jj}$$

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- P^{ℓ}_{ii} phase absorbed in l_i P^{ν}_{kk} phase absorbed in ν_i (only if ν_i is Dirac)
- $U_{\rm LEP} U_{\rm LEP}^{\dagger} = I_{3\times 3}$ but in general $U_{\rm LEP}^{\dagger} U_{\rm LEP} \neq I_{N\times N}$

  $\Rightarrow$ for $N = 3 + s$: $3s + 3$ angulos and $2s + 1$ $(3s + 3)$ phases for Dirac (Majorana)

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  $\Rightarrow$ for $N = 3 + s$: $3s + 3$ angulos and $2s + 1$ $(3s + 3)$ phases for Dirac (Majorana)
- For example for 3 Dirac ν's : 3 Mixing angles + 1 Dirac Phase

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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- For 3 Majorana ν's : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

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Effects of ν Mass: Flavour Transitions

- Flavour ($\equiv$ Interaction) basis (production and detection): ν_e, ν_μ and ν_τ
- Mass basis (free propagation in space-time): ν_1, ν_2 and ν_3 $\ldots$

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$\Rightarrow$ Flavour is not conserved during propagation

$\Rightarrow$ ν can be detected with different (or same) flavour than produced

- The probability $P_{\alpha\beta}$ of producing neutrino with flavour α and detecting with flavour β has to depend on:
  - Misalignment between interaction and propagation states ($\equiv U$)
  - Difference between propagation eigenvalues
  - Propagation distance

Vacuum Mass Oscillations

- If neutrinos have mass, a weak eigenstate $|\nu_\alpha\rangle$ produced in $l_\alpha + N \to \nu_\alpha + N'$ is a linear combination of the mass eigenstates ($|\nu_i\rangle$)

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U_{\alpha i}\, |\nu_i\rangle$$

U is the leptonic mixing matrix.

- After a distance L (or time t) it evolves

$$|\nu(t)\rangle = \sum_{i=1}^{n} U_{\alpha i} |\nu_i(t)\rangle$$

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- it can be detected with flavour β with probability

$$P_{\alpha\beta} = |\langle\nu_\beta(t)|\nu_\alpha(0)\rangle|^2 = |\sum_{i=1}^{n} U_{\alpha i} U^*_{\beta i} \langle\nu_i(t)|\nu_i(0)\rangle|^2$$

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- The probability

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- Under the approximations:

  $(\mathbf{1})$ $|\nu\rangle$ is a *plane wave* $\Rightarrow$ $|\nu_i(t)\rangle = \mathrm{e}^{\,-i\,E_i t}|\nu_i(0)\rangle$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i} \mathrm{Im}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin\left(\Delta_{ij}\right)$$

with $\Delta_{ij} = (E_i - E_j)t$

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  $(\mathbf{2})$ *relativistic* ν

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

Vacuum Oscillations

- The probability

$$P_{\alpha\beta} = |\langle \nu_\beta(t)|\nu_\alpha(0)\rangle|^2 = |\sum_{i=1}^{n} U_{\alpha i} U^*_{\beta i} \langle \nu_i(t)|\nu_i(0)\rangle|^2$$

- We call E_i the neutrino energy and m_i the neutrino mass
- Under the approximations:

(**1**) $|\nu\rangle$ is a *plane wave* $\Rightarrow$ $|\nu_i(t)\rangle = \mathrm{e}^{-i\,E_i t}|\nu_i(0)\rangle$

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i} \mathrm{Im}[U^\star_{\alpha i} U_{\beta i} U_{\alpha j} U^\star_{\beta j}] \sin\left(\Delta_{ij}\right)$$

with $\Delta_{ij} = (E_i - E_j)t$

(**2**) *relativistic* ν

$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

(**3**) Lowest order in mass $p_i \simeq p_j = p \simeq E$

$$\frac{\Delta_{ij}}{2} = 1.27 \frac{m_i^2 - m_j^2}{\mathrm{eV}^2} \frac{L/E}{\mathrm{Km/GeV}}$$

Vacuum Oscillations

- The oscillation probability:

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$$\frac{\Delta_{ij}}{2} = \frac{(E_i - E_j)L}{2} = 1.27 \frac{(m_i^2 - m_j^2)}{\mathrm{eV}^2} \frac{L/E}{\mathrm{Km/GeV}}$$

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$\to$ conserves CP

Vacuum Oscillations

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– The last piece $2\sum_{j\neq i}\mathrm{Im}[U^\star_{\alpha i}U_{\beta i}U_{\alpha j}U^\star_{\beta j}]\sin\left(\Delta_{ij}\right)$ opposite sign for $\overline{\nu}$

$\longrightarrow$ violates CP

Vacuum Oscillations

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$\longrightarrow$ violates CP

- $P_{\alpha\beta}$ depends on Theoretical Parameters
  - $\Delta m_{ij}^2 = m_i^2 - m_j^2$ The mass differences
  - $U_{\alpha j}$ The mixing angles (and Dirac phases)

and on Two *Experimental* Parameters:
  - E The neutrino energy
  - L Distance ν source to detector

Vacuum Oscillations

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$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{j\neq i}^{n} \mathrm{Re}[U^\star_{\alpha i}U_{\beta i}U_{\alpha j}U^\star_{\beta j}]\sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{j\neq i}\mathrm{Im}[U^\star_{\alpha i}U_{\beta i}U_{\alpha j}U^\star_{\beta j}]\sin\left(\Delta_{ij}\right)$$

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  - $\Delta m^2_{ij} = m_i^2 - m_j^2$ The mass differences
  - $U_{\alpha j}$ The mixing angles (and Dirac phases)

and on Two *Experimental* Parameters:
  - E The neutrino energy
  - L Distance ν source to detector

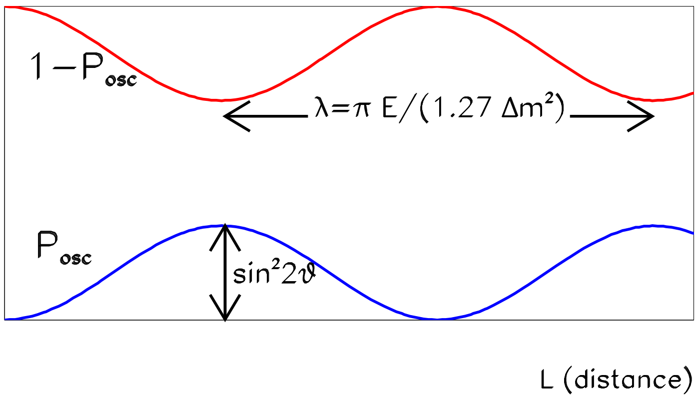
- No information on mass scale nor Majorana phases

2-ν Oscillations

- For 2-ν: $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

$$P_{osc} = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right) \quad \text{Appear}$$

$$P_{\alpha\alpha} = 1 - P_{osc} \quad \text{Disappear}$$



- For 2ν oscillation Prob in vaccum same for θ and $\frac{\pi}{2}$-θ

ν Interactions

- SM Weak Interactions $\Rightarrow \sigma^{\nu p} \sim 10^{-38} \mathrm{cm}^2 \frac{E_\nu}{\mathrm{GeV}}$
- Take atmospheric $\nu' s$: $\Phi_\nu^{\mathrm{ATM}} = 1\,\nu\,\mathrm{per\,cm}^2\,\mathrm{per\,sec}$ and $\langle E_\nu \rangle = 1$ GeV
- How many interact? In a human body:

$$N_{\mathrm{int}} = \Phi_\nu \times \sigma^{\nu p} \times N_{\mathrm{prot}}^{\mathrm{human}} \times T_{\mathrm{life}}^{\mathrm{human}} \quad (M \times T \equiv \mathrm{Exposure})$$

$$\left.\begin{array}{l} N_{\mathrm{protons}}^{\mathrm{human}} = \frac{M^{\mathrm{human}}}{gr} \times N_A = 80\mathrm{kg} \times N_A \sim 5 \times 10^{28}\mathrm{protons} \\ T^{\mathrm{human}} = 80\,\mathrm{years} = 2 \times 10^9\,\mathrm{sec} \end{array}\right\} \begin{array}{l} \mathrm{Exposure}_{\mathrm{human}} \\ \sim \mathrm{Ton} \times \mathrm{year} \end{array}$$

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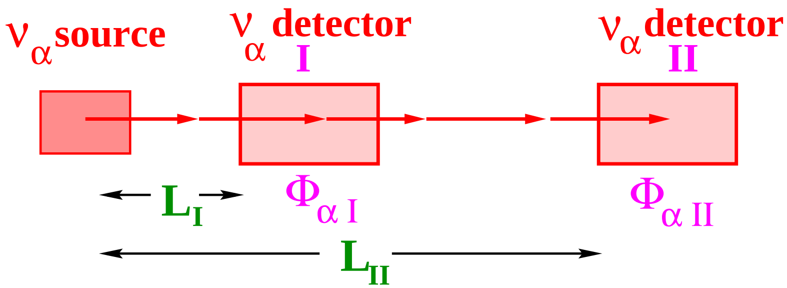
$$N_{\mathrm{int}} = (5 \times 10^{28})\,(2 \times 10^9) \times 10^{-38} \sim 1\,\mathrm{interaction\,per\,lifetime}$$

$\Rightarrow$ Need huge detectors with Exposure $\sim$ KTon $\times$ year

ν Oscillations: Experimental Probes

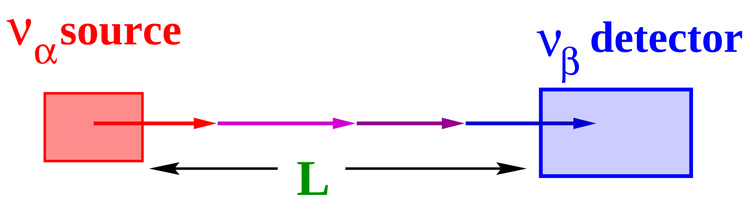
- Generically there are two types of experiments to search for ν oscillations :

Disappearance Experiment



Compares $\Phi_{\alpha\, I}$ and $\Phi_{\alpha\, II}$ to look for loss

Appearance Experiment



Searches for β diff α

Concha Gonzalez-Garcia

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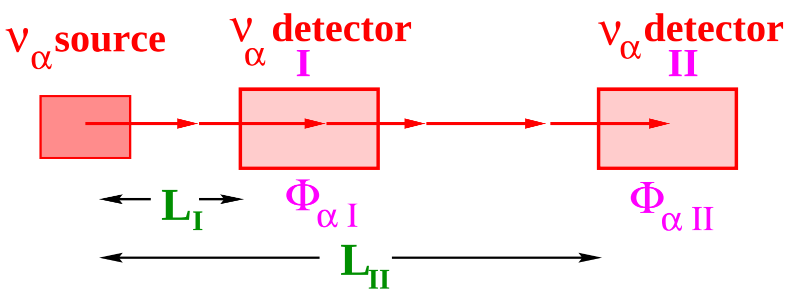

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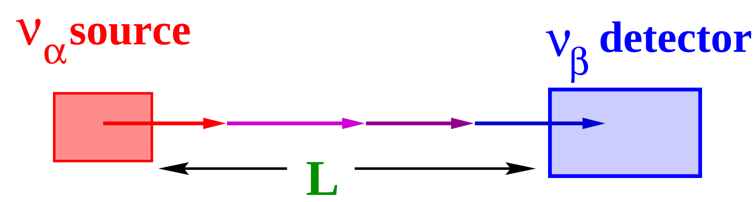
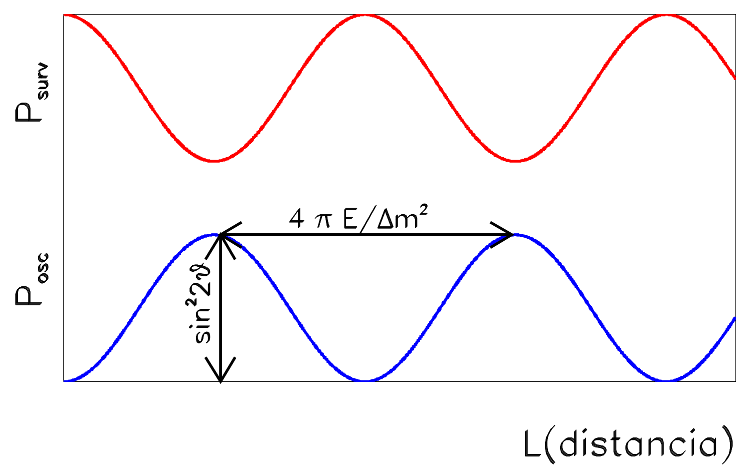


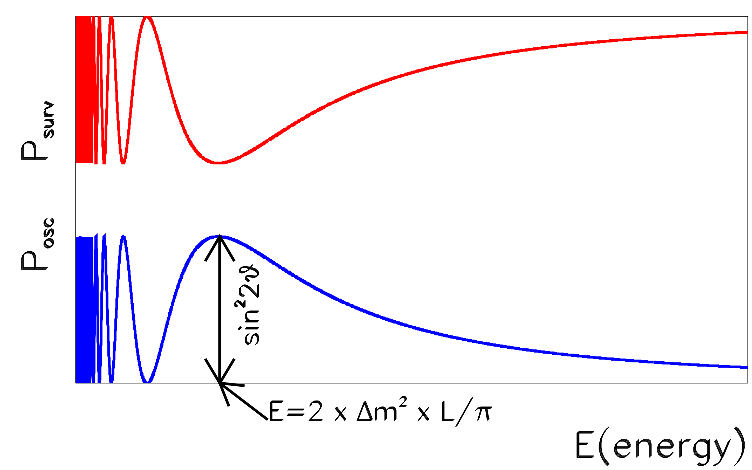
Searches for β diff α

- To detect oscillations we can study the neutrino flavour

as function of the Distance to the source



As function of the neutrino Energy



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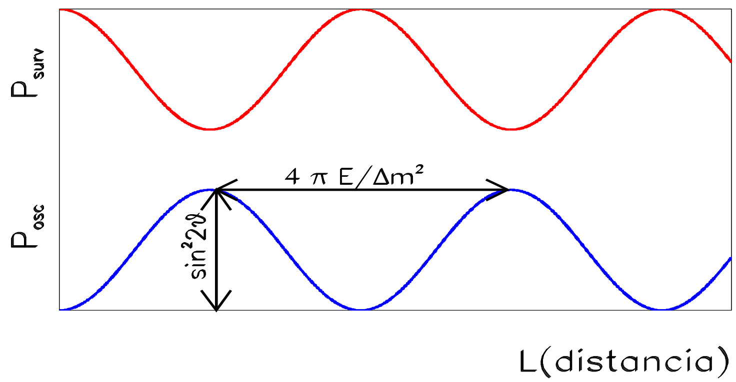

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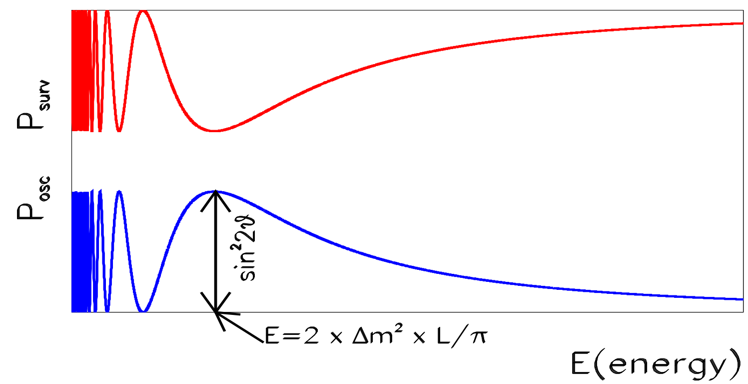
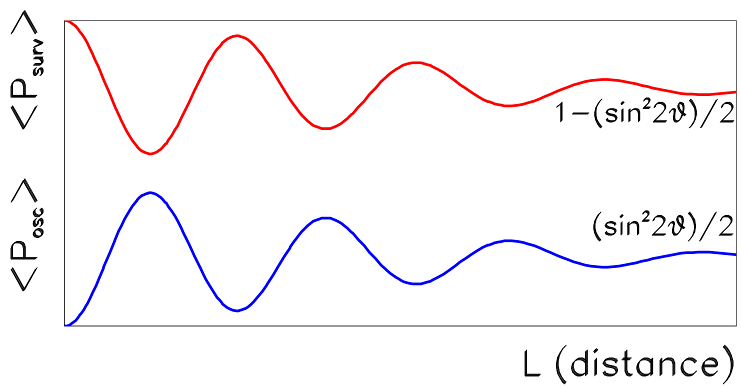
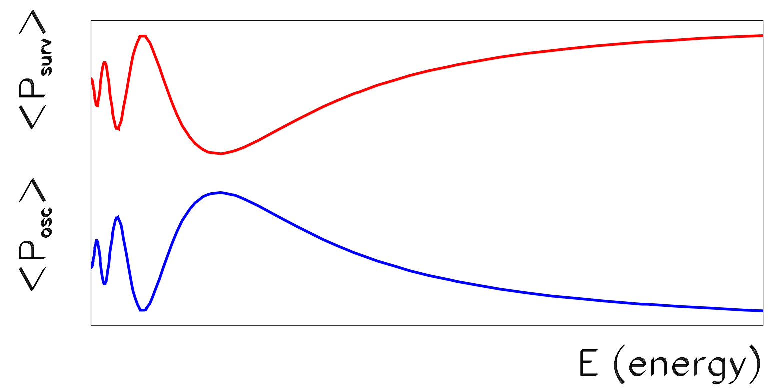


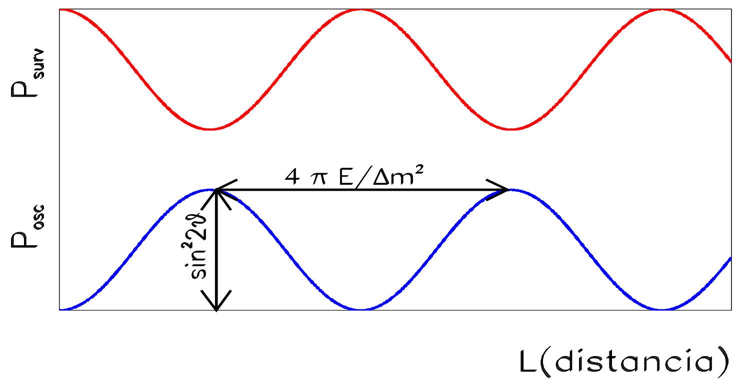
- In real experiments $\Rightarrow$ $\langle P_{\alpha\beta}\rangle = \int dE_\nu \frac{d\Phi}{dE_\nu}\sigma_{CC}(E_\nu)P_{\alpha\beta}(E_\nu)$

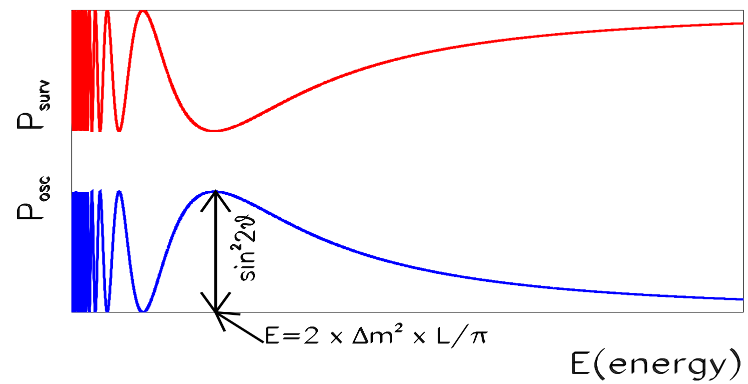




as function of the Distance to the source

As function of the neutrino Energy





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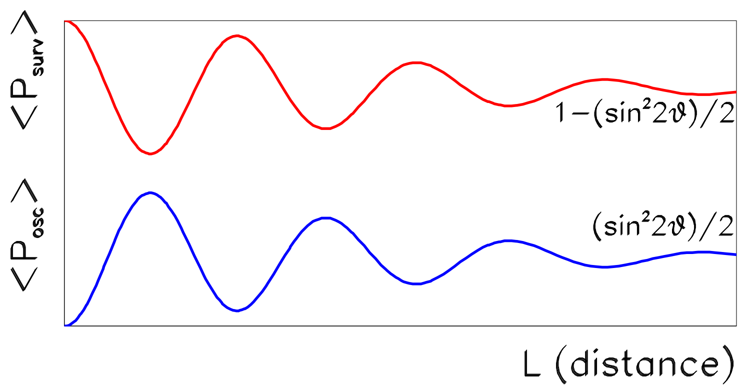
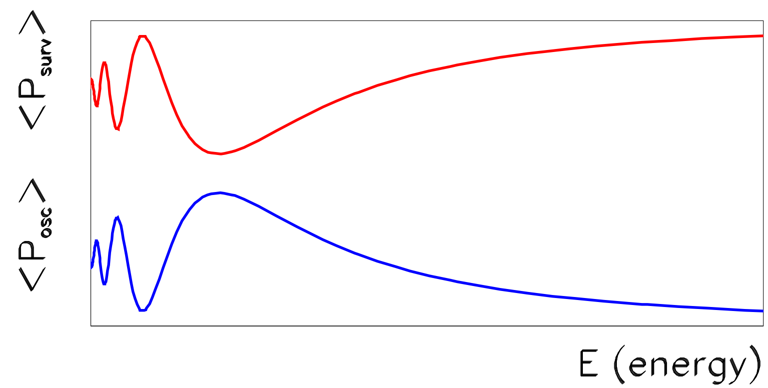




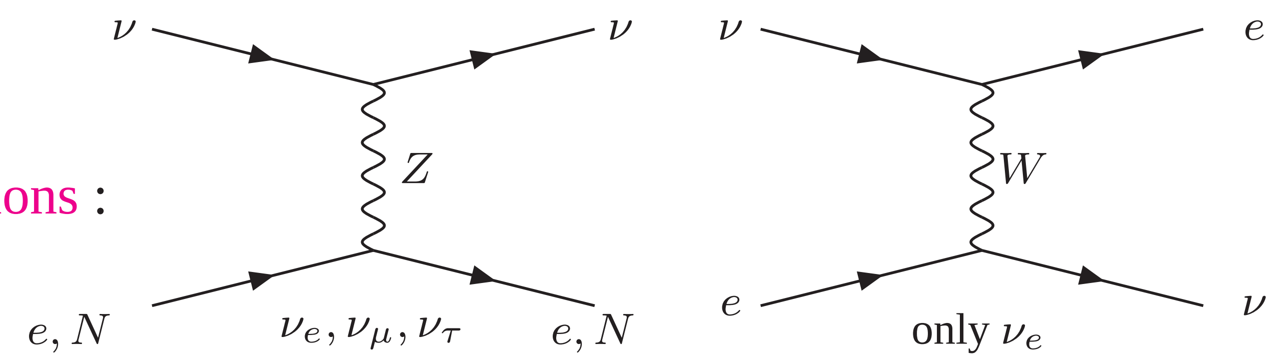
- Maximal sensitivity for $\Delta m^2 \sim E/L$

  – $\Delta m^2 \ll E/L \;\Rightarrow \langle \sin^2(1.27 \Delta m^2 L/E) \rangle \simeq 0 \rightarrow \langle P_{osc} \rangle \simeq 0$

  – $\Delta m^2 \gg E/L \;\Rightarrow \langle \sin^2(1.27 \Delta m^2 L/E) \rangle \simeq \frac{1}{2} \rightarrow \langle P_{osc} \rangle \simeq \frac{1}{2} \sin^2(2\theta)$

Matter Effects

- If ν cross matter regions (Sun, Earth...) it interacts *coherently*
  - But Different flavours have different interactions :



$\Rightarrow$ Effective potential in ν evolution : $V_e \neq V_{\mu,\tau} \Rightarrow \Delta V^{\nu} = -\Delta V^{\bar{\nu}} = \sqrt{2} G_F N_e$

$\Rightarrow$ *Modification of mixing angle and oscillation wavelength* $\equiv$ MSW effect

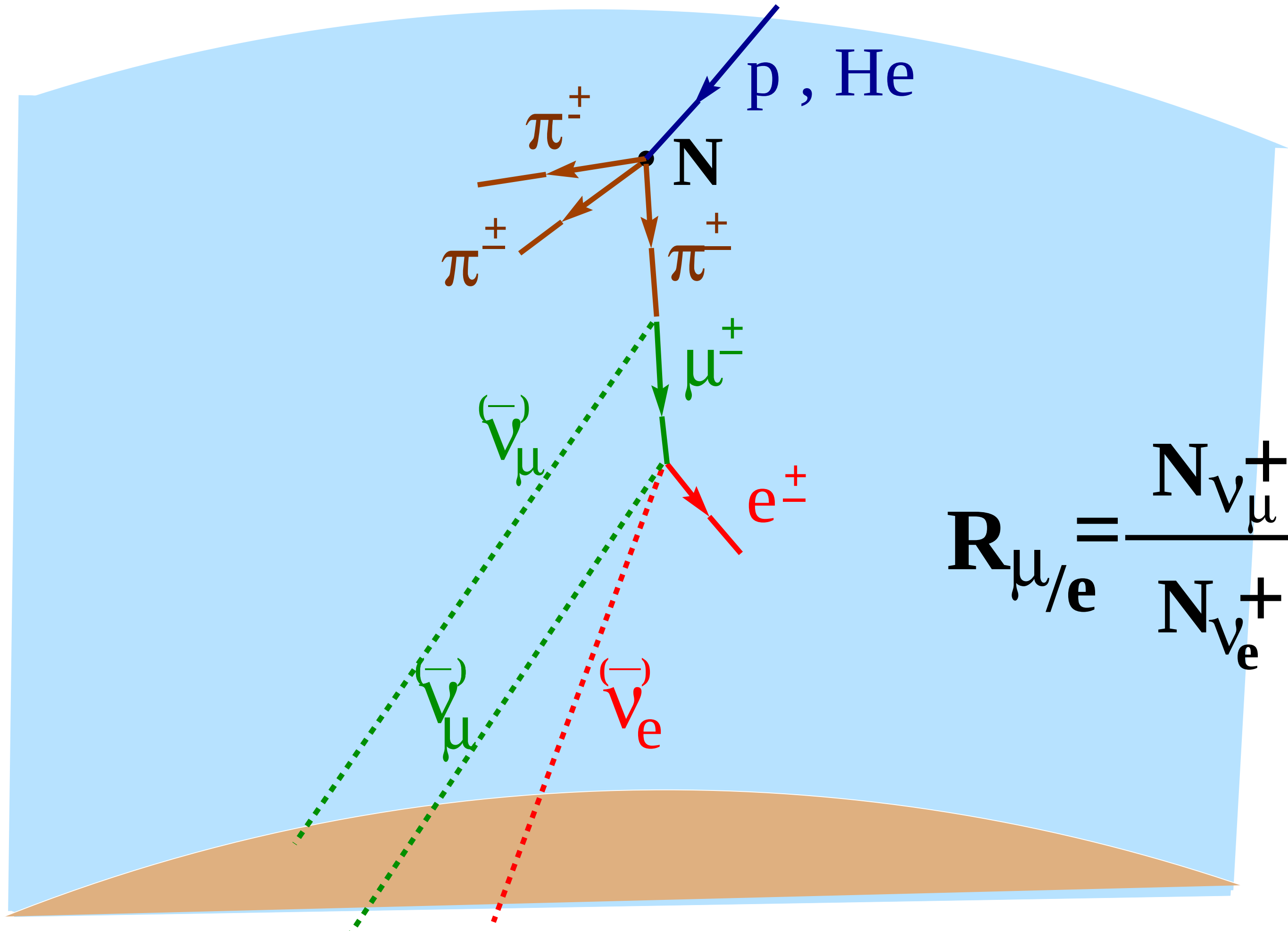
- The mixing angle in matter

$$\sin(2\theta_m) = \frac{\Delta m^2 \sin(2\theta)}{\sqrt{(\Delta m^2 \cos(2\theta) - 2E\Delta V)^2 + (\Delta m^2 \sin(2\theta))^2}}$$

- For solar neutrinos in adiabatic regime $P(\nu_e \to \nu_e) = \frac{1}{2}\left[1 + \cos(2\theta_m)\cos(2\theta)\right]$

Atmospheric Neutrinos

Atmospheric $\nu_{e,\mu}$ are produced by the interaction of cosmic rays (p, He . . .) with the atmosphere



$$R_{\mu/e} = \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} \sim 2$$

Atmospheric Neutrinos

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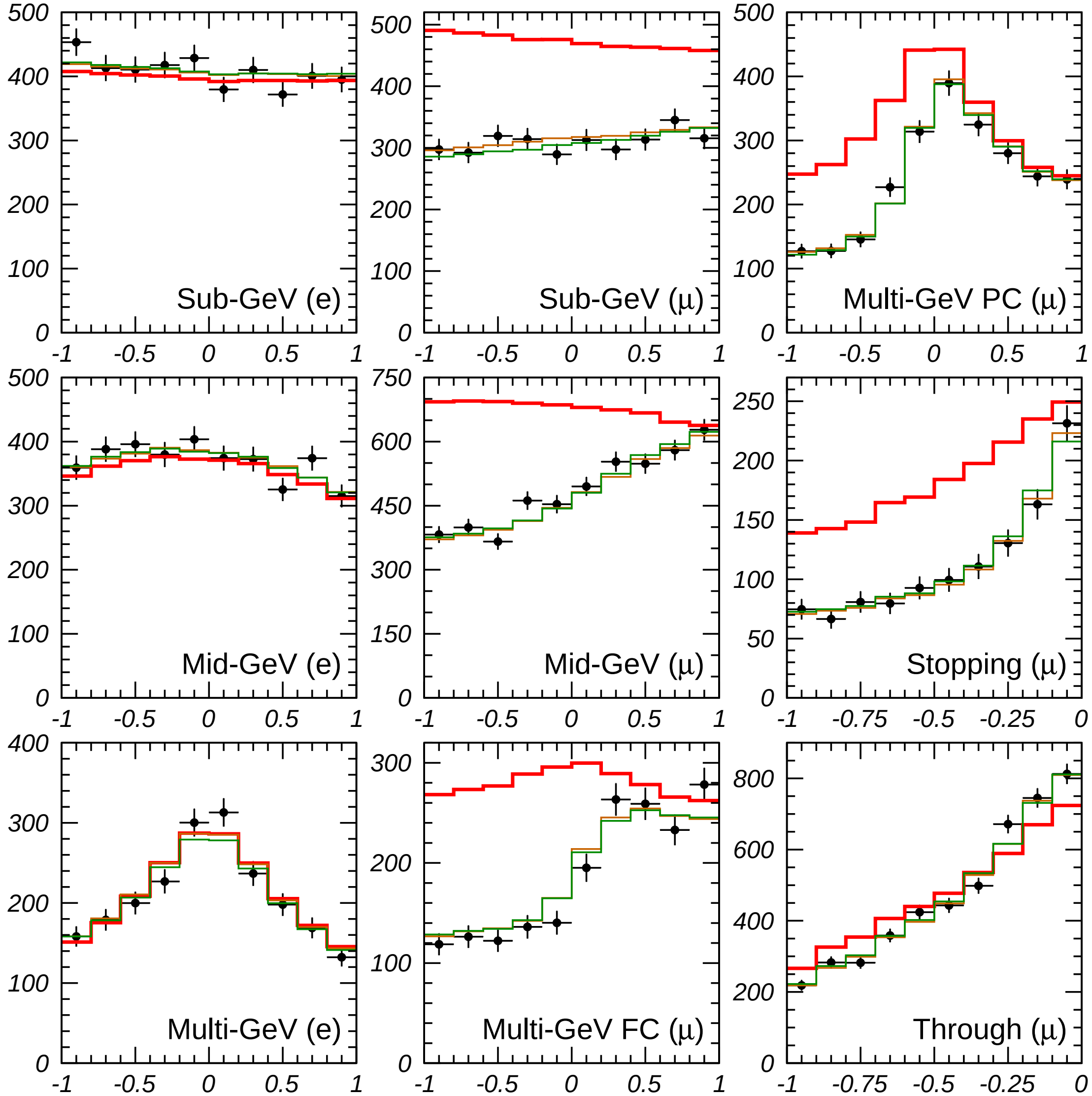
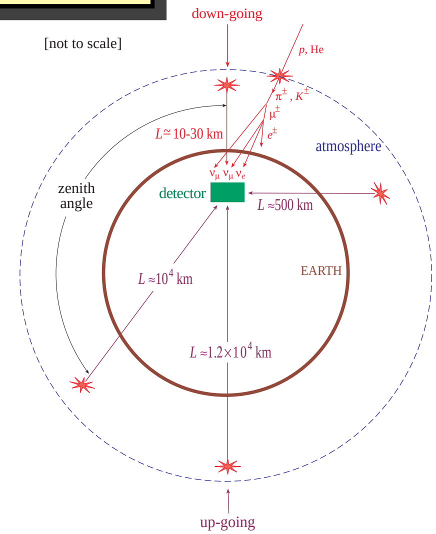
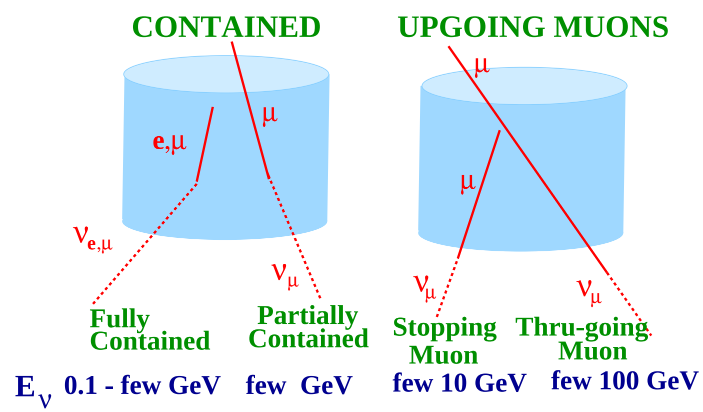
- SKI+II+III+IV data:







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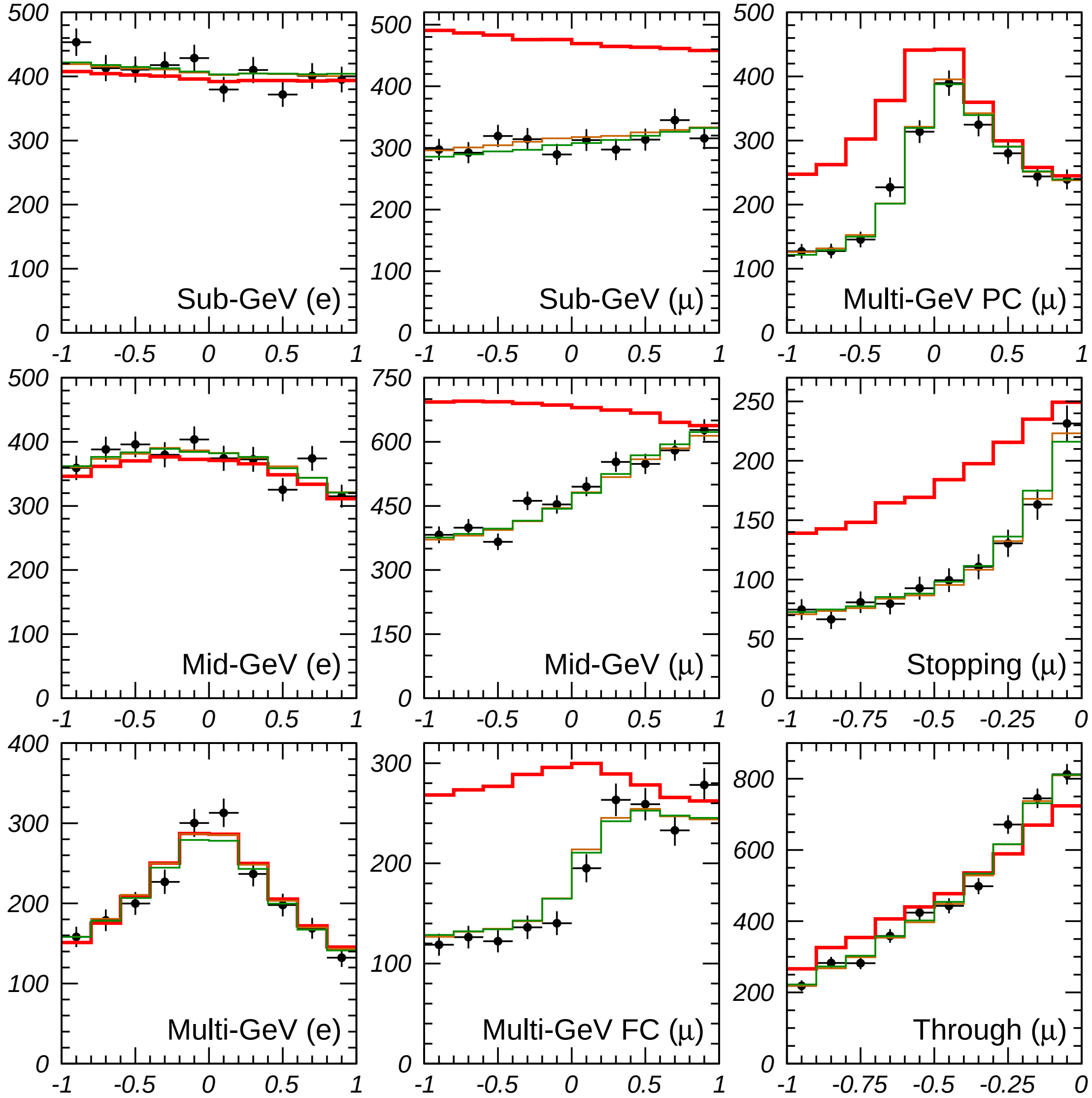

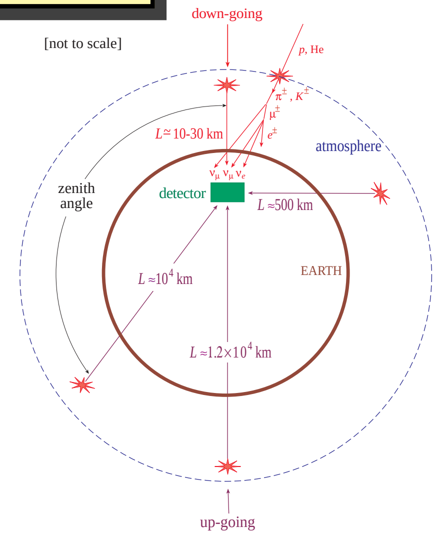

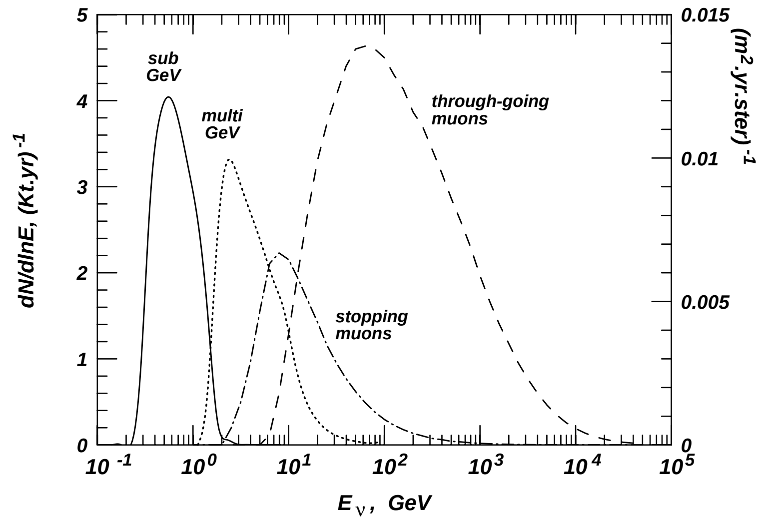

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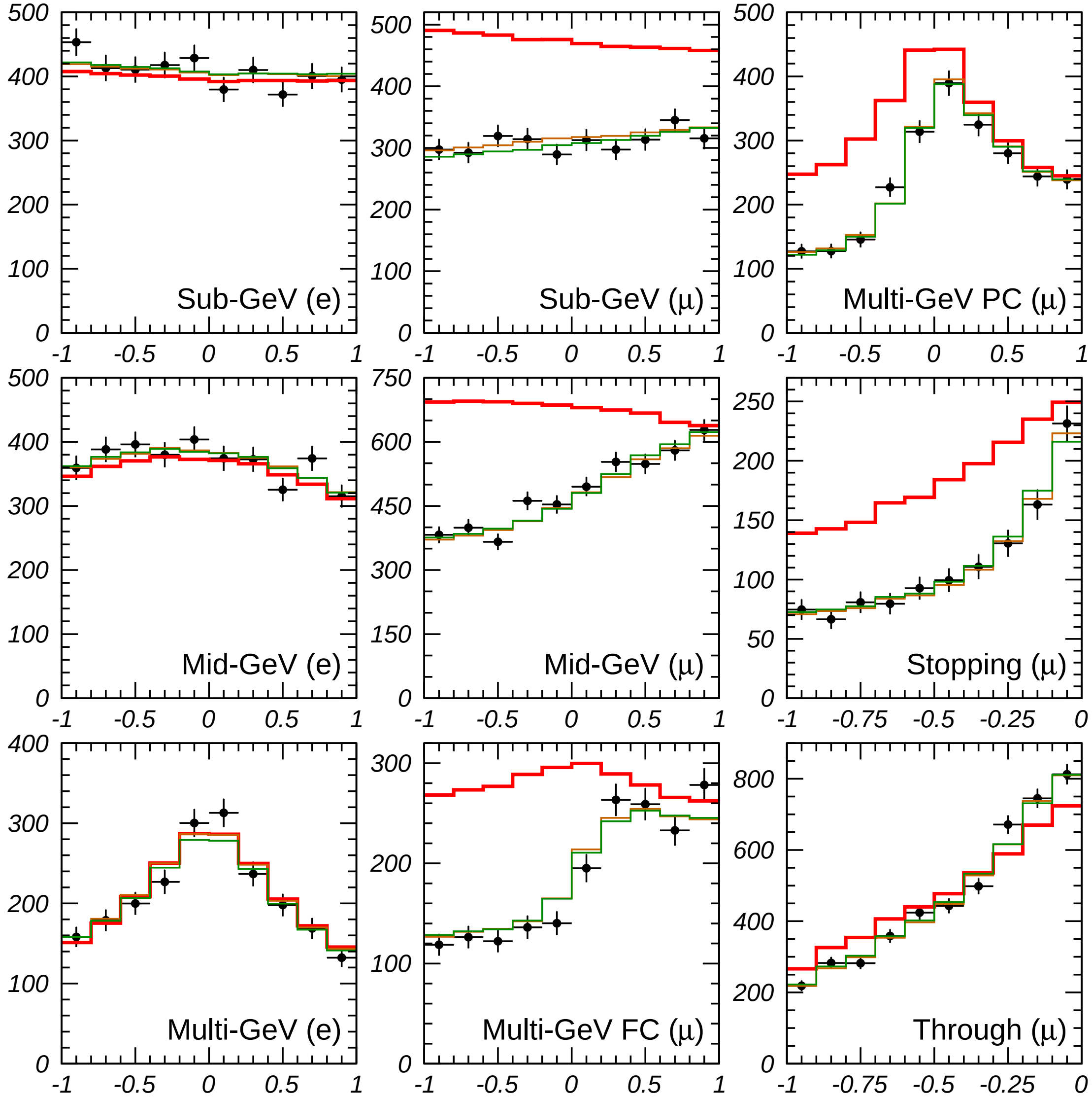

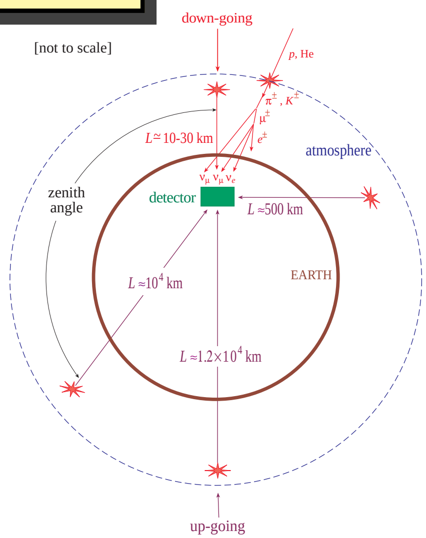

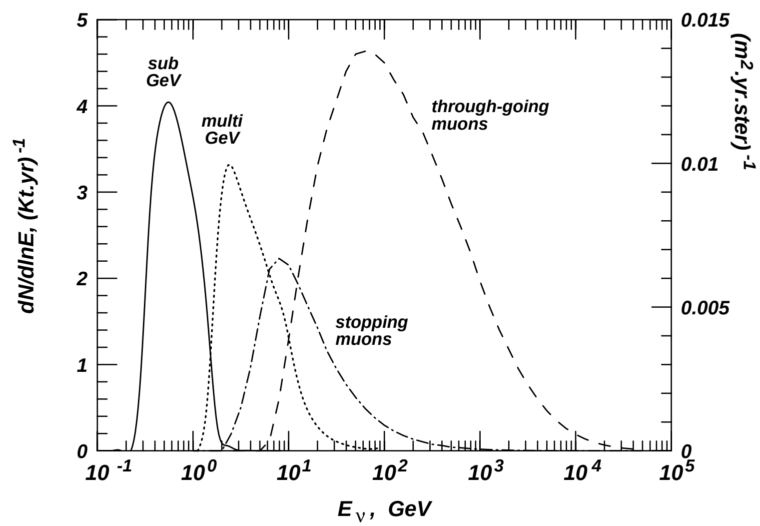

Atmospheric Neutrinos

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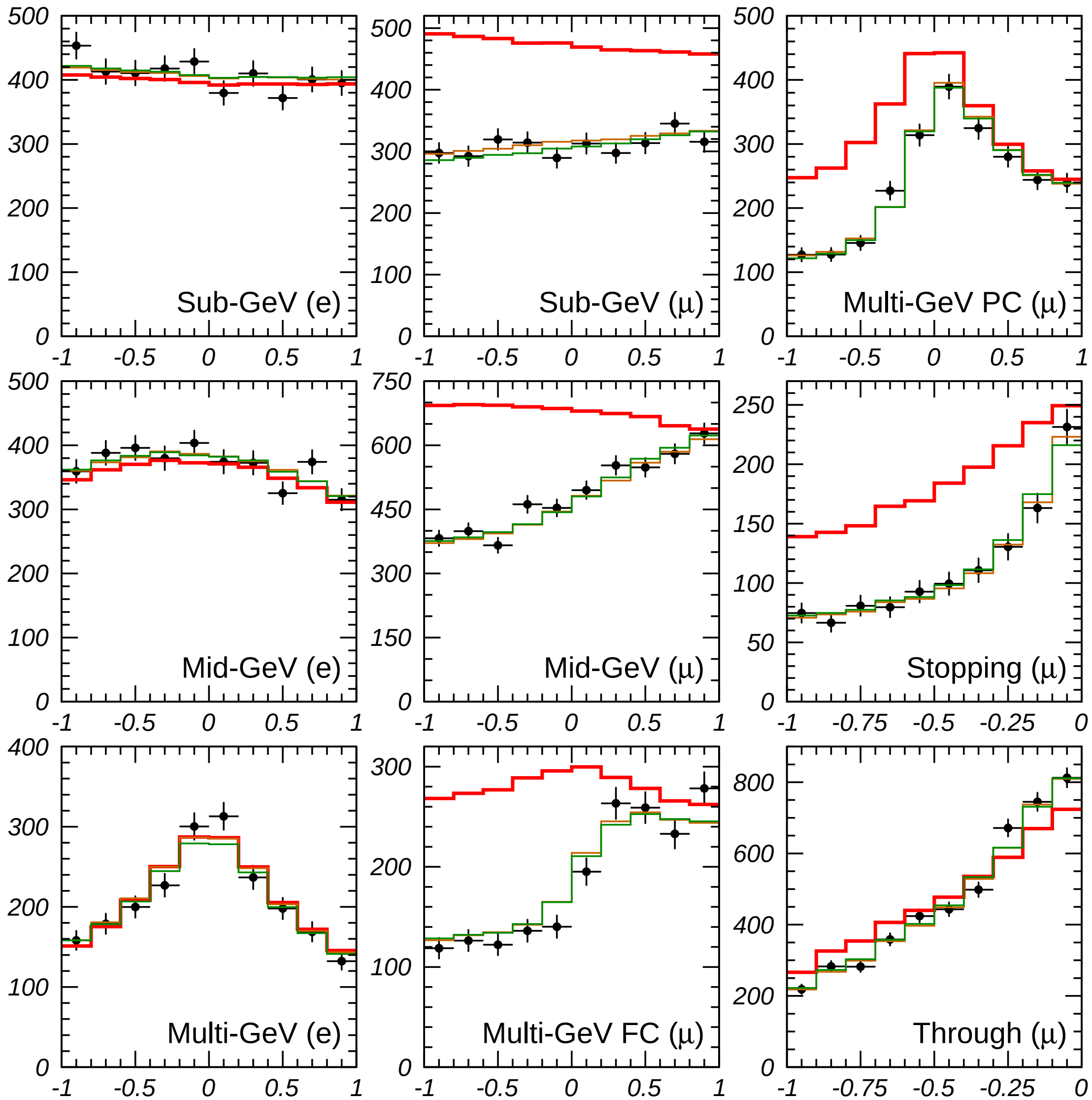
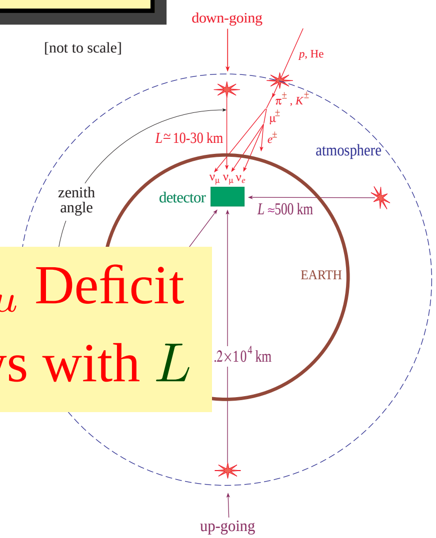
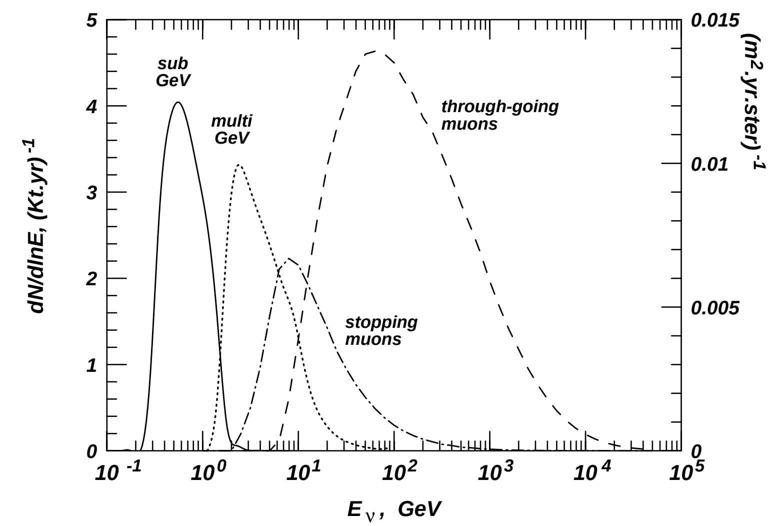
- SKI+II+III+IV data:

ν_e in agreement with SM



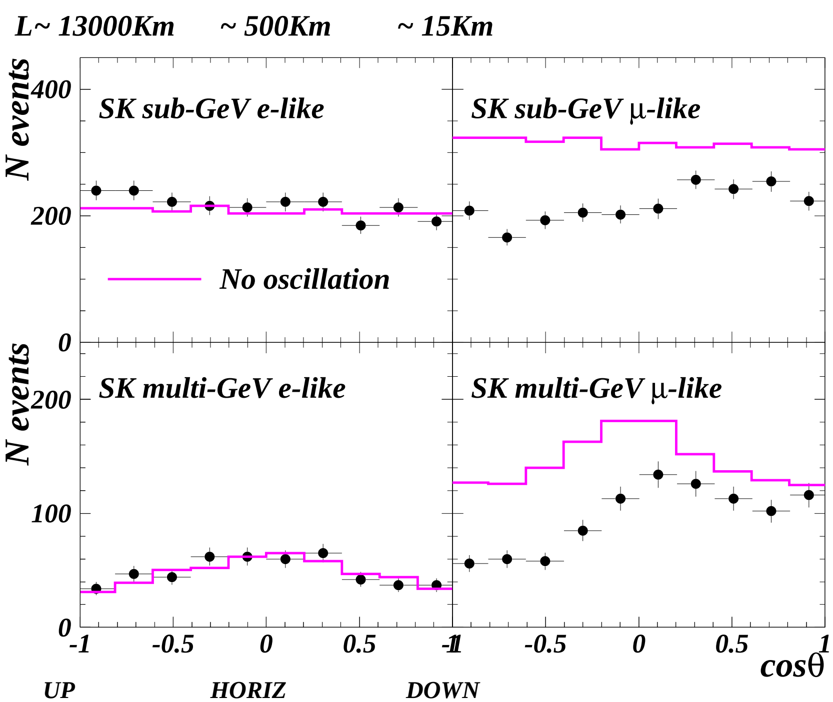


$\rightarrow \nu_\mu$ Deficit grows with L



$\rightarrow \nu_\mu$ Deficit decreases with E

Atmospheric ν Oscillations: Parameter Estimate



- For SubGeV

$$\langle P_{\mu\mu} \rangle = 1 - \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{2E} \sim 0.5 - 0.7$$

$$\Rightarrow \sin^2 2\theta \gtrsim 0.6$$

- For $E \sim$ few GeV deficit at $L \sim 10^2 - 10^4$ Km

$$\frac{\Delta m^2(\mathrm{eV}^2) L(\mathrm{km})}{2E(\mathrm{GeV})} \sim 1$$

$$\Rightarrow \Delta m^2 \sim 10^{-4} - 10^{-2} \mathrm{eV}^2$$

Atmospheric ν Oscillation Analysis

(1) Theoretical Predictions:

– The expected number of contained events

$$R_\alpha(\theta) = \sum_\beta n_t T \int \frac{d^2\Phi_\beta}{dE_\nu d\cos\theta_\nu} P_{\beta\alpha}(E_\nu)\kappa_\beta(h)\frac{d\sigma}{dE_\alpha}\varepsilon(E_\alpha)dE_\nu dE_\alpha d\cos\theta_\nu dh$$

$$\Phi_\beta \equiv \text{Neutrino Flux} \qquad \kappa_\alpha \equiv \text{Neutrino Production Point Distribution}$$

$$\frac{d\sigma}{dE_\alpha} \equiv \text{Neutrino Interaction Cross Section} \qquad \varepsilon(E_\alpha) \equiv \text{Detection Efficiency}$$

– The expected upgoing-μ events:

$$R_\mu(\theta)_{S,T} = \int \frac{d\Phi_\mu(E_\mu,\cos\theta)}{dE_\mu d\cos\theta} A_{S,T}(E_\mu,\theta)dE_\mu$$

$$\frac{d\Phi_\mu}{dE_\mu d\cos\theta} = \int_0^\infty \frac{d\Phi_{\nu_\mu}}{dE_\nu d\cos\theta} P_{\mu\mu}(E_\nu)\frac{d\sigma}{dE_{\mu 0}} F_{rock}(E_{\mu 0}, E_\mu, X) N_A dE_\nu dE_{\nu 0} dX$$

$$A_{S,T}(E_\mu,\theta) \equiv \text{Detector Effective Area} \qquad F_{rock}(E_{\mu 0}, E_\mu, X) \equiv \text{Muon Energy Loss in Rock}$$

Atmospheric ν Oscillation Analysis

(2) Statistical Analysis:

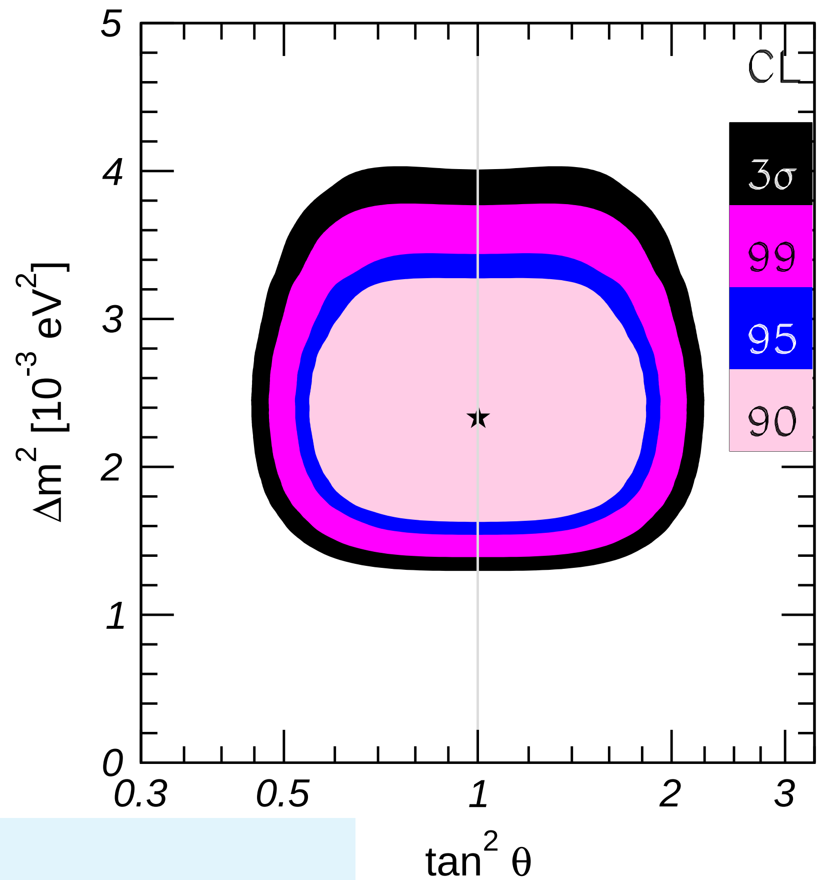
90 (70) data points SKI+II+III+(IV) data:

- Sub-GeV e-like and μ-like: 10+10 points
- Mid-GeV e-like and μ-like: 10+10 points
- Multi-GeV e-like:10 points
- Multi-GeV FC and PC μ-like: 10+10 points
- Stopping and Thrugoing μ's: 10+10 points

Using 3-dim atmospheric fluxes from Honda

Use "pull" approach for theoretical and systematic errors

$\nu_\mu \to \nu_\tau$



$$\chi^2 = \min_{\xi_i}\left[\sum_{n=1}^{90}\left(\frac{R_n^{\rm theo}-\sum_i \xi_i\sigma_n^i - R_n^{\rm exp}}{\sigma_n^{\rm stat}}\right)^2 + \sum_{i,theory}\xi_i^2 + \sum_{i,syt}\xi_i^2\right]$$

$\Delta m^2 \sim 2.4\times 10^{-3}$ eV^2

$\tan^2\theta \sim 1 \Rightarrow \theta \sim \frac{\pi}{4}$

Include *many* sources of theoretical and systematic uncertainties

- Flux Uncertainties:

(1) Total normalization: $\sigma_{\rm norm} = 20\%$

(2) "Tilt" error

$\Phi_\delta(E) = \Phi_0(E)\left(\frac{E}{E_0}\right)^\delta$

$\sigma_\delta = 5\%$ $E_0 = 2$ GeV

(3) ν_μ/ν_e ratio: $\sigma_{\mu/e} = 5\%$

E independent for contained events

(4) Zenith angle dependence:

$\sigma_{\rm zen,i} = 5\%\langle\cos\theta\rangle_i$

- Cross Section Uncertainties:

(5) $\sigma_{\rm norm}^{\sigma_{\rm QE}} = 15\%$

(6) $\sigma_{\rm norm}^{\sigma_{1\pi}} = 15\%$,

(7) $\sigma_{\rm norm}^{\sigma_{\rm DIS}} = 15\%$ for contained

$\sigma_{\rm norm}^{\sigma_{\rm DIS}} = 10\%$ for upward-going μ

(8)–(10) $\sigma_{i,\nu_\mu}^{\rm QE,1\pi,DIS}/\sigma_{i,\nu_e}^{\rm QE,1\pi,DIS} = 0.1$–$1\%$

- Systematic uncert (from SK pub):

(11) Simulation of had int (contained):

$\sigma_{\rm hadron}^{\rm sys} = -0.25$–$1.1$ %

(12) Particle identification (contained):

$\sigma_{\mu/e}^{\rm sys} = -1.1$–$1.6\%$

(13) Ring Counting:

$\sigma_{\rm ring}^{\rm sys} = -0.75$–$5.5\%$

(14) Fiducial Volume:

$\sigma_{\rm f-vol}^{\rm sys} = -0.3$–$1.4\%$

(15) Energy Calibration:

$\sigma_{\rm E-cal}^{\rm sys} = -0.4$–$2\%$

(16) PC/FC norm: (multi-GeV μ)

$\sigma_{\rm PC-nrm}^{\rm sys} = 2.85\%$

(17) Up-μ track reconstruction:

$\sigma_{\rm track}^{\rm sys} = 1.4$–$6.4\%$

(18) Up Effi and Stop/Thru separation:

$\sigma_{\rm up-eff}^{\rm sys} = 1$–$1.4\%$

Long Baseline Experiments: ν_μ Disappearance

K2K/T2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km

Long Baseline Experiments: ν_μ Disappearance

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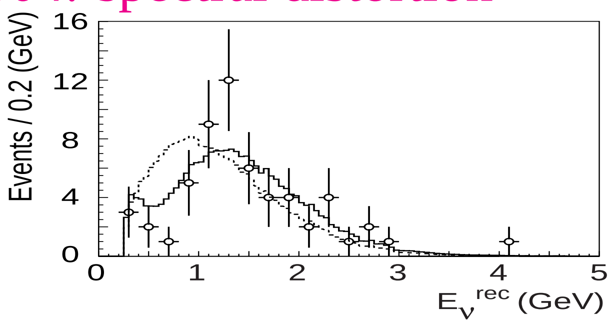
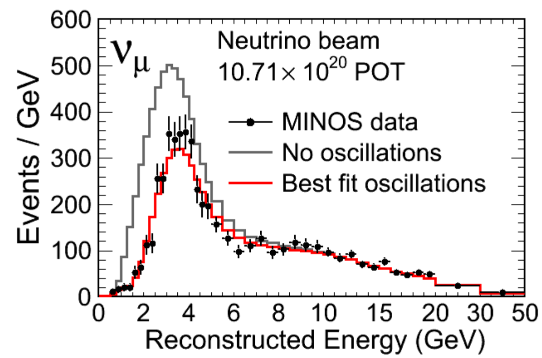
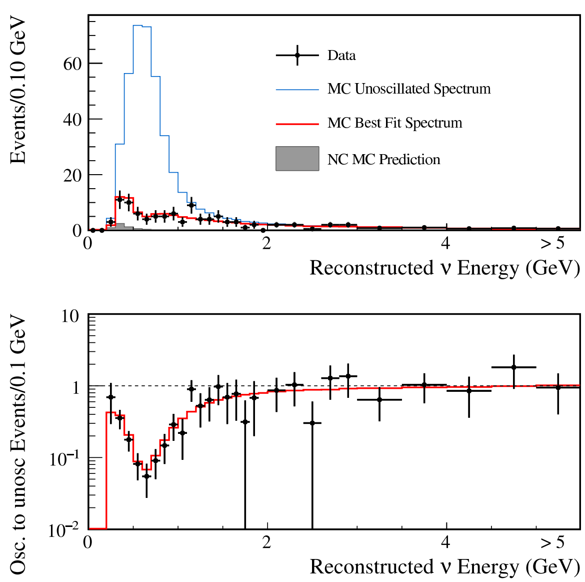
K2K 2004: spectral distortion

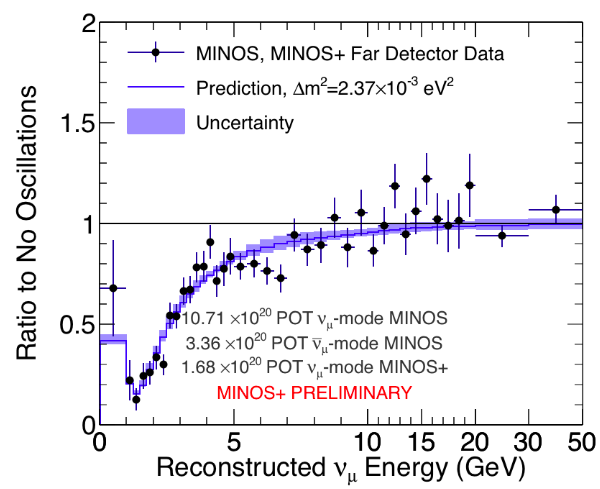


MINOS 2006–: detail spectral distortion



T2K 2010–: spectral distortion





Confirmation of ν_μ oscillations and agreement in mass and mixing with ATM

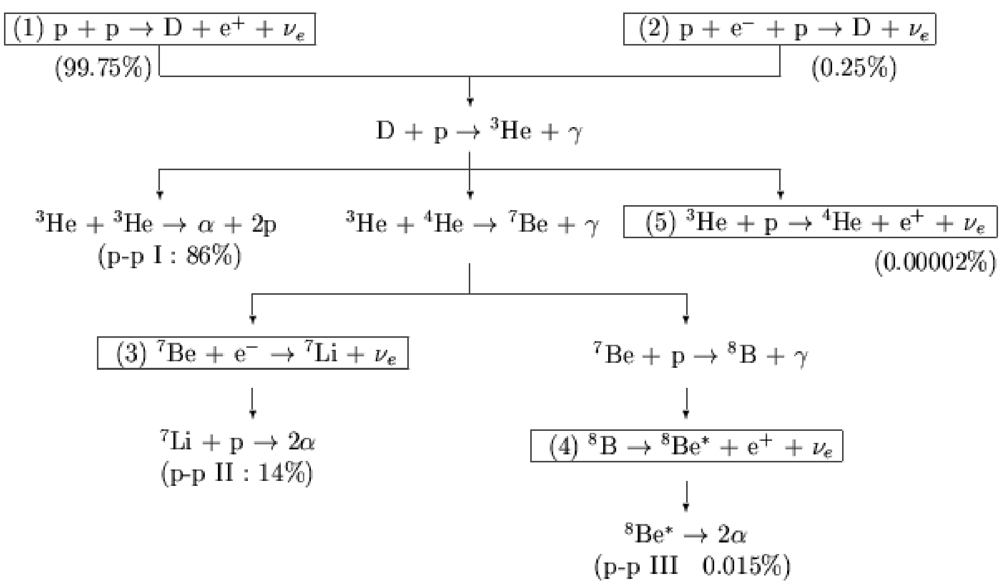
Solar Neutrinos: Fluxes

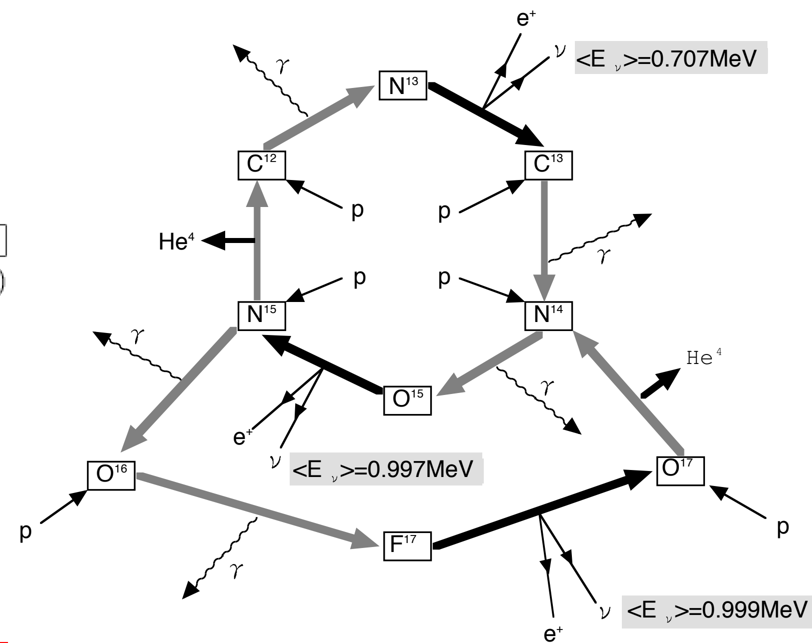
- The Sun shines converting protons into α, e^+ and $\nu's$

  $4\,p \;\rightarrow\; {}^4He \;+\; 2\,e^+ \;+\; 2\,\nu_e \;+\; \gamma$

  $4m_p - m_{^4He} - 2m_e \simeq 26$ MeV Thermal energy mostly in γ
- Two major chains of nuclear reactions

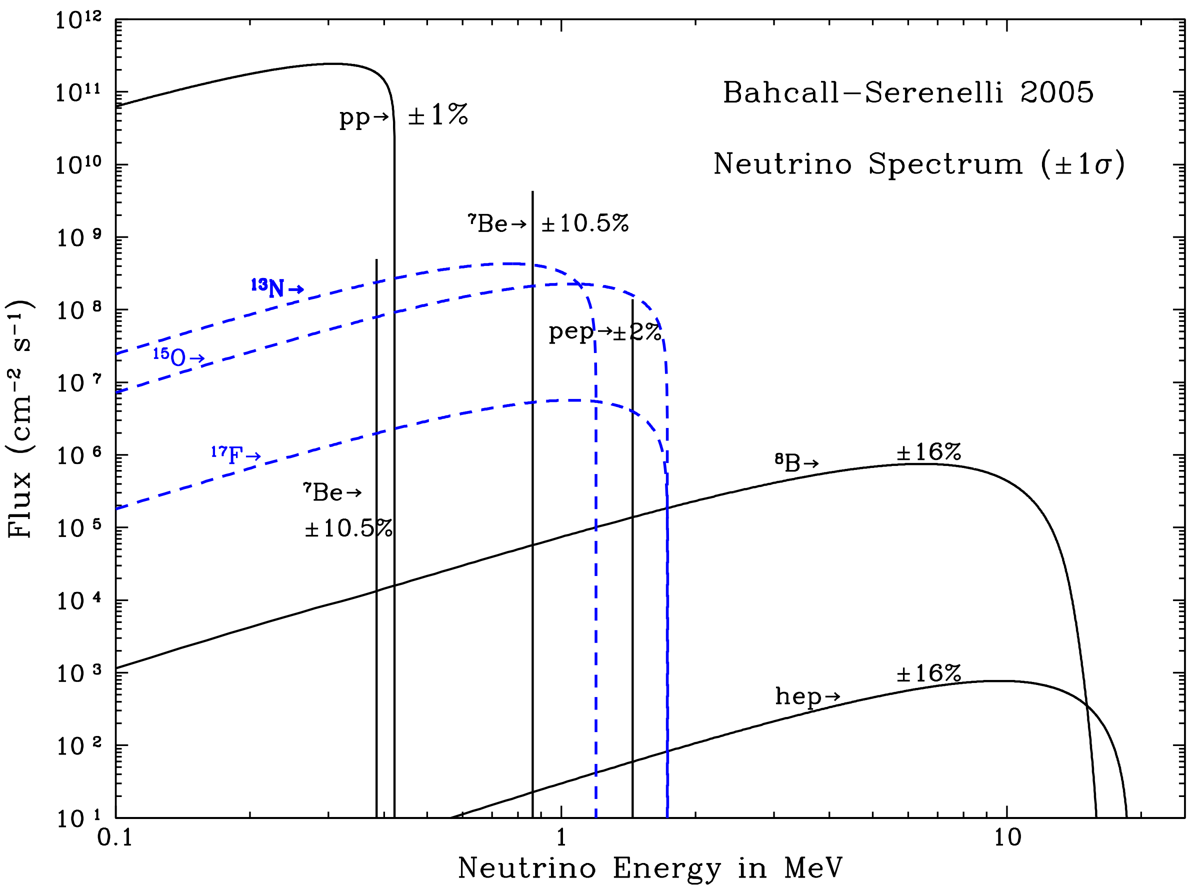
pp chain: CNO cycle:





- Present Solar Model $\Rightarrow$ *pp*-chain dominates by 99%

Solar Neutrinos: Fluxes

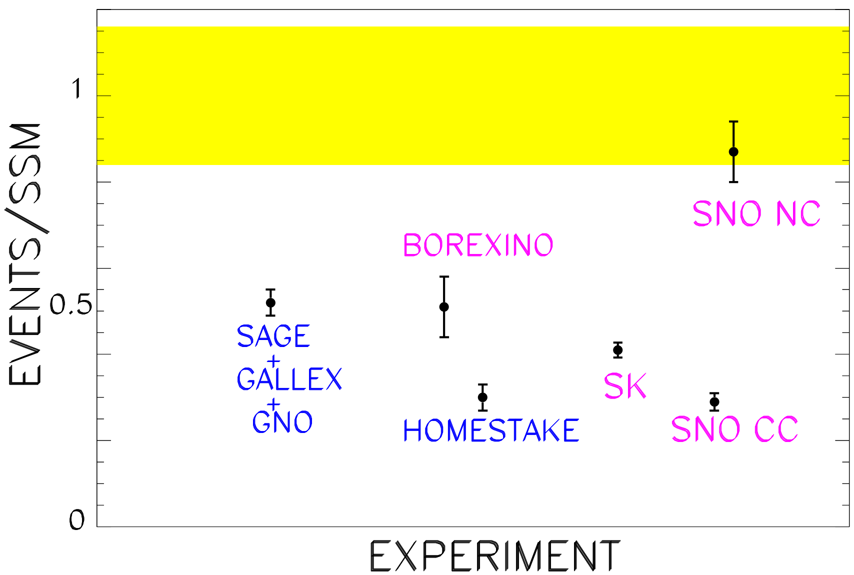


PP CHAIN	E_ν (MeV)
(pp)	
$p+p \to ^2H + e^+ + \nu_e$	≤ 0.42
(pep)	
$p+e^- + p \to ^2H + \nu_e$	1.552
(^{7}Be)	
$^7Be + e^- \to ^7Li + \nu_e$	$0.862(90\%)$
	0.384 (10%)
(hep)	
$^2He + p \to ^4He + e^+ + \nu_e$	≤ 18.77
(^{8}B)	
$^8B \to ^8Be^* + e^+ + \nu_e$	≤ 15
CNO CHAIN	E_ν (MeV)
(^{13}N)	
$^{13}N \to ^{13}C + e^+ + \nu_e$	≤ 1.199
(^{15}O)	
$^{15}O \to ^{15}N + e^+ + \nu_e$	≤ 1.732
(^{17}F)	
$^{17}F \to ^{17}O + e^+ + \nu_e$	≤ 1.74

Concha Gonzalez-Garcia

Solar Neutrinos: Data

	Experiment	Detection	Flavour	$E_{\rm th}$ (MeV)
radio-chemical	Homestake	^{37}Cl$(\nu, e^-)^{37}$Ar	ν_e	$E_\nu > 0.81$
	Sage + Gallex+GNO	^{71}Ga$(\nu, e^-)^{71}$Ge	ν_e	$E_\nu > 0.23$
real time	Kam $\Rightarrow$ SK	ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$ $\left(\frac{\sigma_{\mu\tau}}{\sigma_e} \simeq \frac{1}{6}\right)$	$E_e > 5$
	SNO	CC $\nu_e d \to p p e^-$	ν_e	$T_e > 5$
		NC $\nu_x d \to \nu_x p\, n$	$\nu_e, \nu_{\mu/\tau}$	$T_\gamma > 5$
		ES $\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$	$T_e > 5$
	Borexino	$\nu_x e^- \to \nu_x e^-$	$\nu_e, \nu_{\mu/\tau}$	$E_\nu = 0.862$



Experiments measuring ν_e observe a deficit

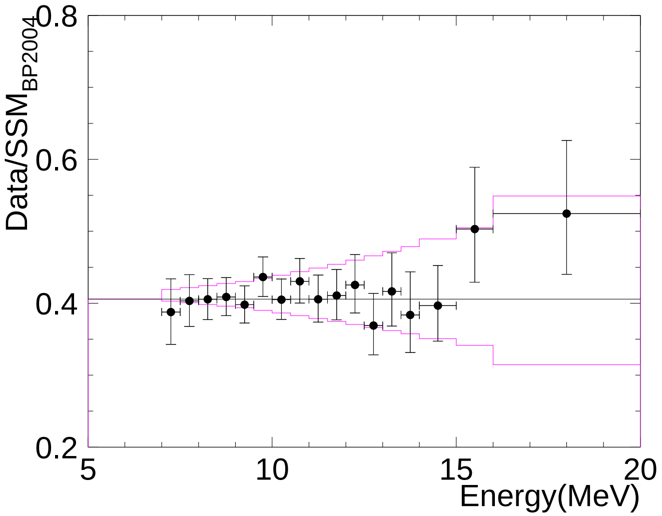
Deficit is energy dependent

Deficit disappears in NC

- Real Time experiments can also give information on Energy and Direction of $\nu' s$ and can search for Energy and Time variations of the effect

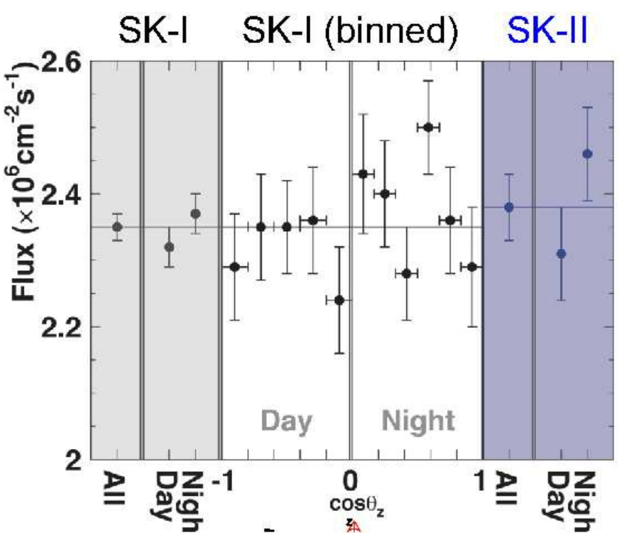
- From SK (also from SNO)

Energy Dependence



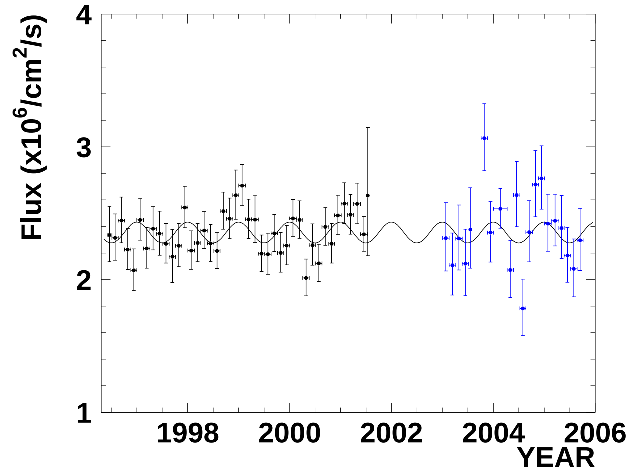
Deficit indep $E_\nu \gtrsim 5$ MeV

Day-Night Variation



Not significant

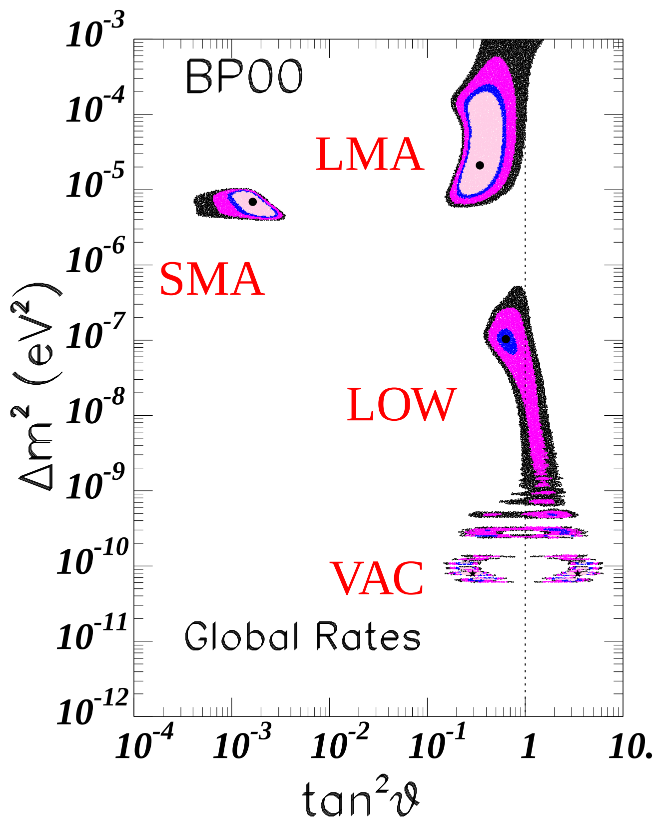
Seasonal Variation

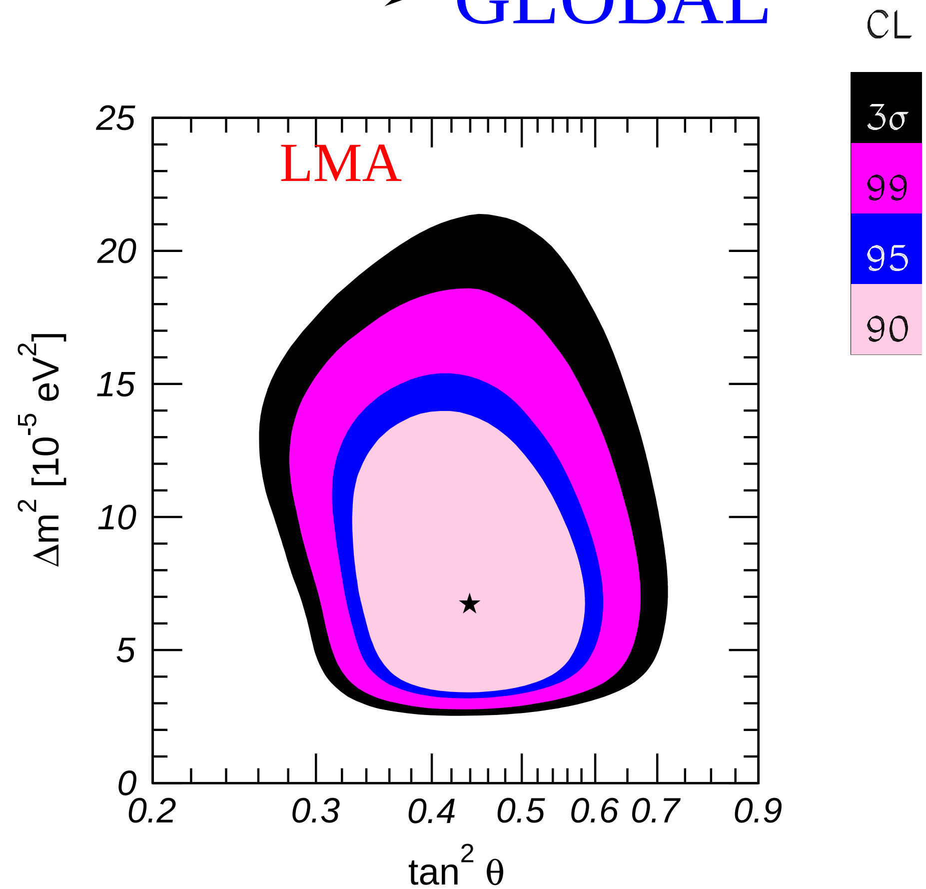


Nothing beyond $\frac{1}{R^2}$

Solar Neutrinos: Oscillation Solutions

RATES ONLY —SK and SNO E and t dependence→ GLOBAL





$\Delta m^2 \sim 7 \times 10^{-5}$ eV^2

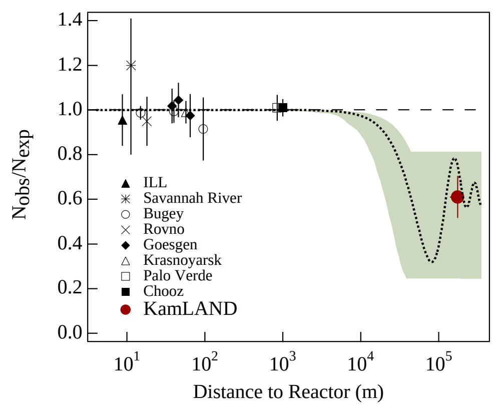
$\tan^2\theta \sim 0.4 \Rightarrow \theta \sim \frac{\pi}{6}$

Different frequency and flavour than ATM and LBL

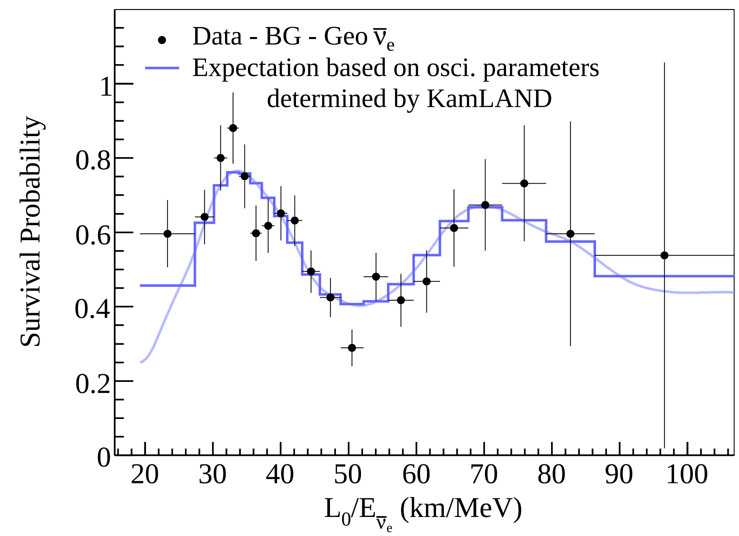
LBL experiment with reactors: KamLAND

- Search for $\overline{\nu_e} \Rightarrow \overline{\nu_e}$ at L$\sim$ 180 km reactors, $E_{\overline{\nu}} \sim$ few MeV:

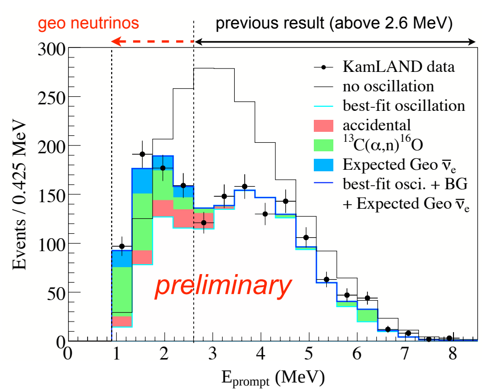
2002: Deficit $R_{\rm KLAND} = 0.611 \pm 0.094$



Oscillation Signal



2004: Significant Energy Distortion



With

$\Delta m^2 \sim 8 \times 10^{-5}$ eV^2

$\tan^2\theta = 0.40$ or $2.2 \Rightarrow \theta \sim \frac{\pi}{6}$ or $\frac{\pi}{3}$

Medium Baseline Reactor Experiments

- Searches for $\bar{\nu}_e \to \bar{\nu}_e$ disapperance at $L \sim$ Km ($E/L \sim 10^{-3}$ eV^2)
- Relative measurement: near and far detectors

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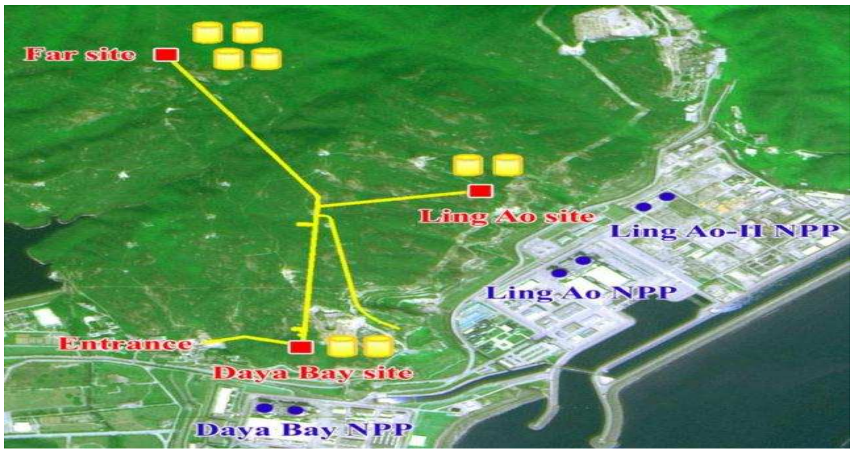
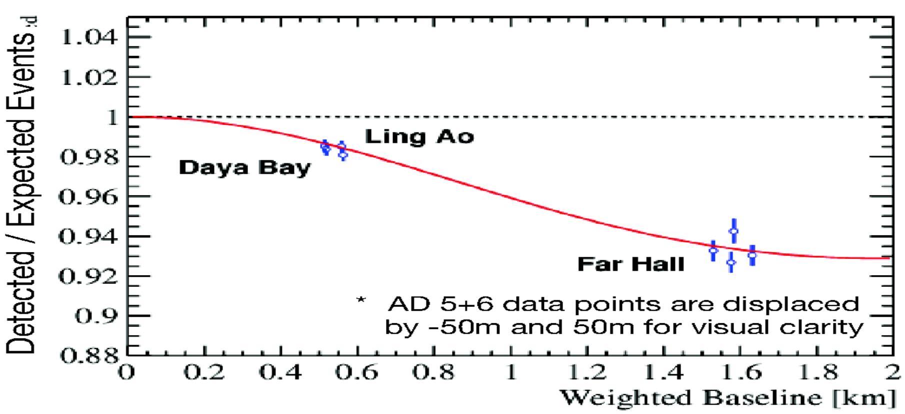
Daya-Bay





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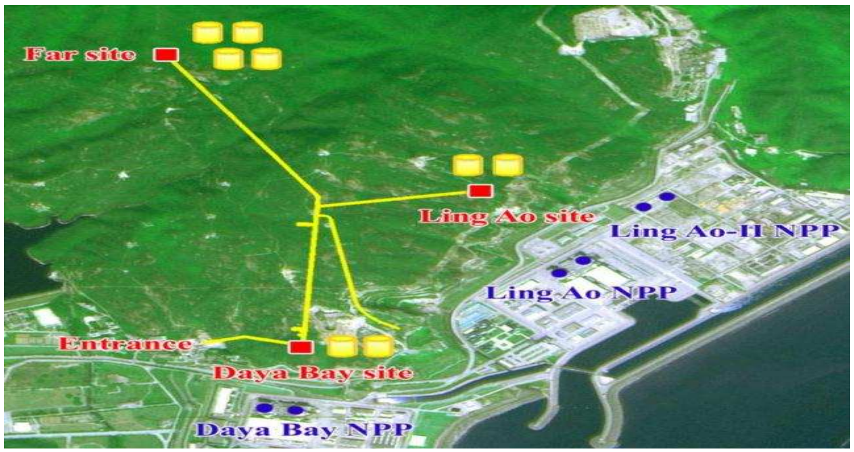
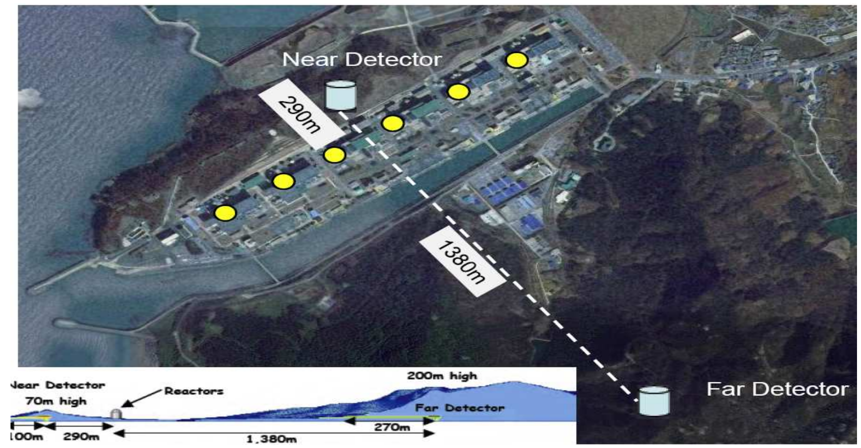
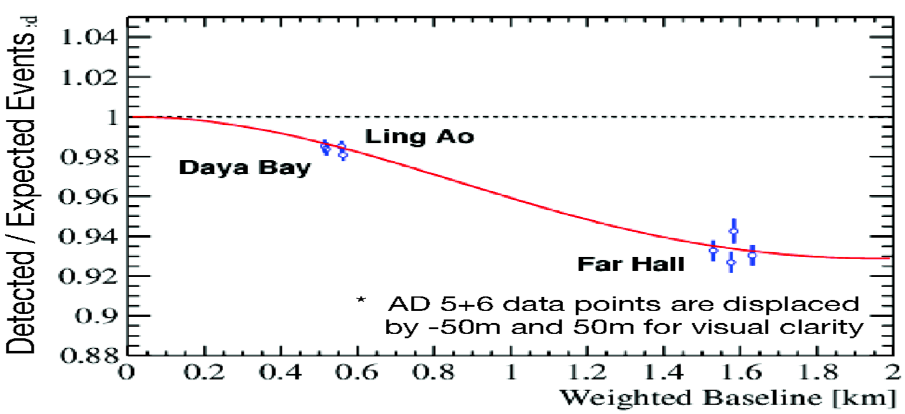
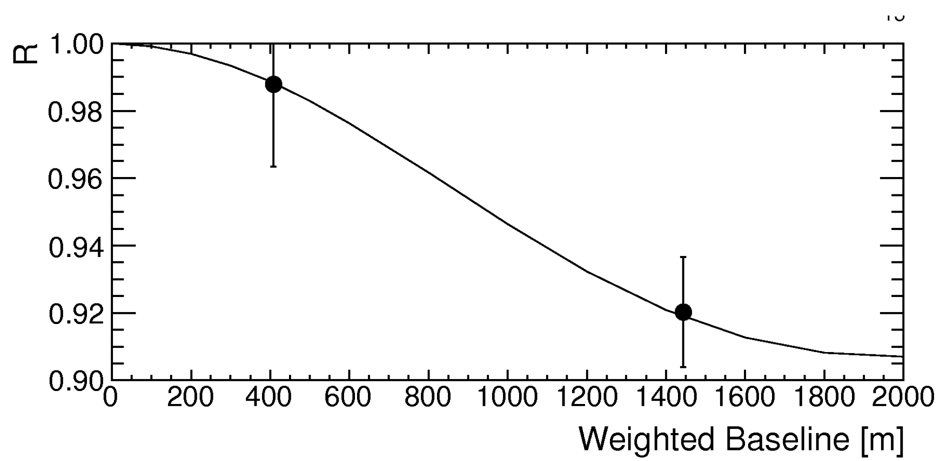


Reno



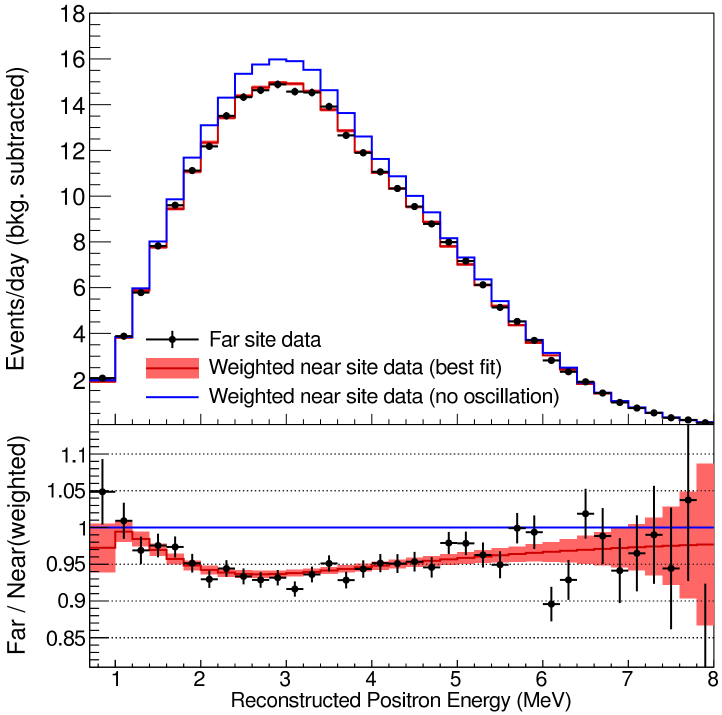




Daya-Bay and Reno Reactor Experiments

- Searches for $\bar{\nu}_e \to \bar{\nu}_e$ disapperance at $L \sim$ Km ($E/L \sim 10^{-3}$ eV^2)
- Relative measurement: near and far detectors

Daya-Bay: 4 Near+ 4 Far



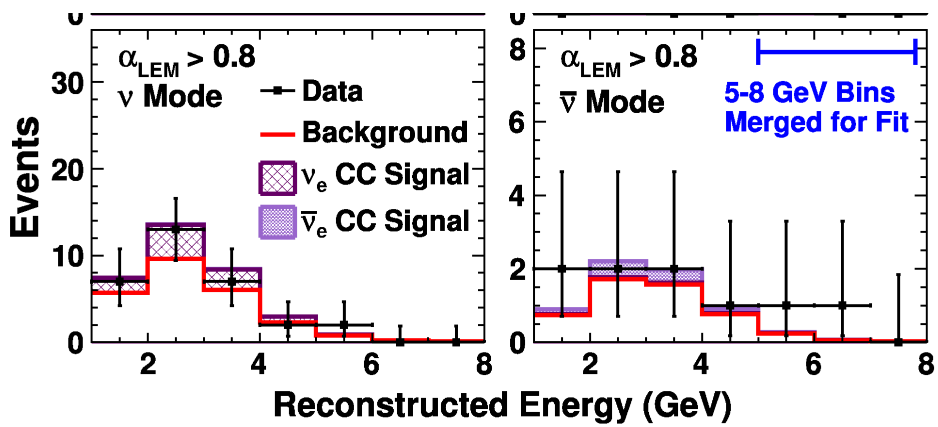
Described with $\Delta m^2 \sim 2.5 \times 10^{-3}$eV^2 (as ν_μ ATM and LBL acc ν_μ disapp) and $\theta \sim 9^\circ$

Long Baseline Experiments: ν_e Appearance

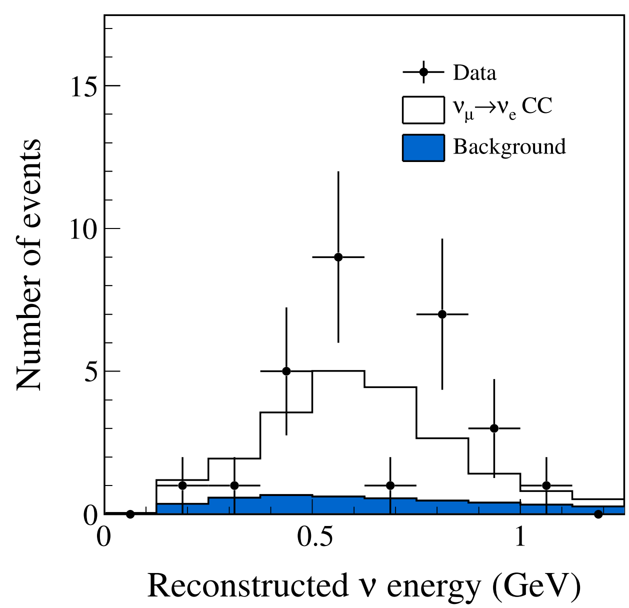
T2K	ν_μ at KEK	SK	L=250 km
MINOS	ν_μ at Fermilab	Soundan	L=735 km

- Observation of $\nu_\mu \to \nu_e$ transitions with $E/L \sim 10^{-3}$ eV^2

MINOS



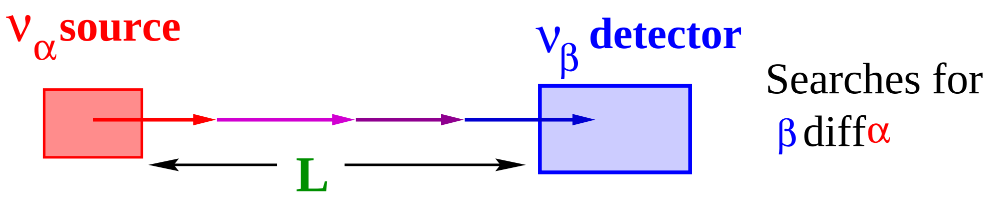
T2K



Results described with $\nu_\mu \to \nu_e$ oscilations with $\Delta m^2 \sim 2 \times 10^{-3}$ eV^2 and $\theta \sim 10°$

ν Oscillations: Lab Searches at Short Distance

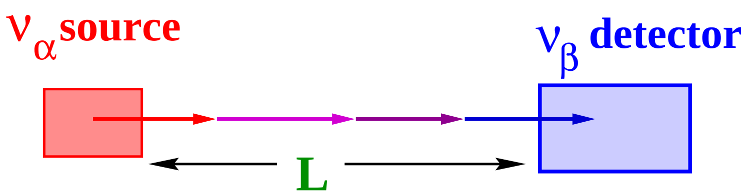
Appearance Experiment



Experiment $\langle\frac{E/MeV}{L/m}\rangle$			α	β
CCFR	100	FNAL	ν_μ, ν_e	ν_τ
E531	25	FNAL	ν_μ, ν_e	ν_τ
Nomad	13	CERN	ν_μ, ν_e	ν_τ
Chorus	13	CERN	ν_μ, ν_e	ν_τ
E776	2.5	BNL	ν_μ	ν_e
Karmen2	2.5	Rutherford	$\bar\nu_\mu$	$\bar\nu_e$
LSND	3	Los Alamos	$\bar\nu_\mu$	$\bar\nu_e$
Miniboone	3	Fermilab	ν_μ $\bar\nu_\mu$	ν_e $\bar\nu_e$
ICARUS	1	Fermilab	ν_μ	ν_e

ν Oscillations: Lab Searches at Short Distance

Appearance Experiment



Searches for
β diff α

Experiment	$\langle \frac{E/MeV}{L/m} \rangle$		α	β
CCFR	100	FNAL	ν_μ, ν_e	ν_τ
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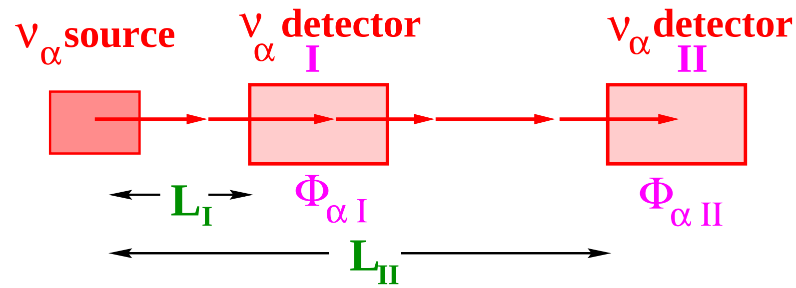
Karmen2	2.5	Rutherford	$\bar\nu_\mu$	$\bar\nu_e$
LSND	3	Los Alamos	$\bar\nu_\mu$	$\bar\nu_e$
Miniboone	3	Fermilab	ν_μ $\bar\nu_\mu$	ν_e $\bar\nu_e$
ICARUS	1	Fermilab	ν_μ	ν_e

Disappearance Experiment

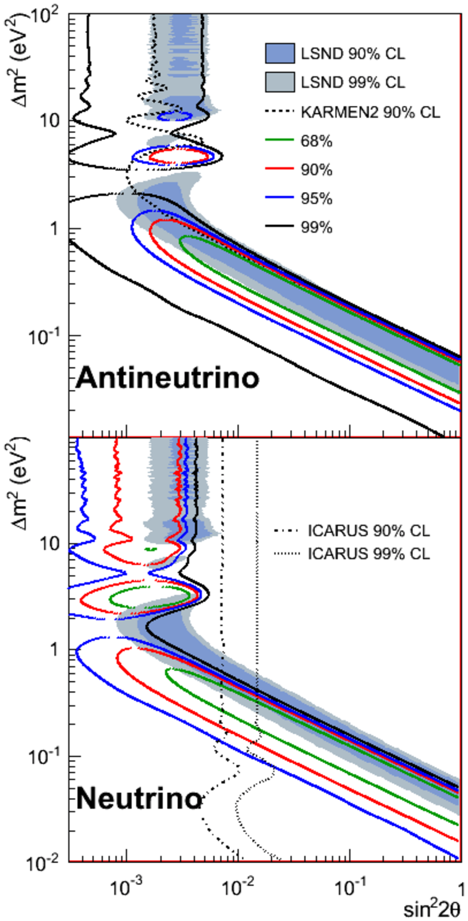


Compares $\Phi_{\alpha\,I}$ and $\Phi_{\alpha\,II}$ to look for loss

Experiment	$\langle \frac{E/MeV}{L/m} \rangle$		α
CDHSW	1.4	CERN	ν_μ
BugeyIII	0.05	Reactor	$\bar\nu_e$
Chooz	0.005	Reactor	$\bar\nu_e$

LSND and MiniBooNE

- LSND: Main signal for $\bar{\nu}_\mu \to \bar{\nu}_e$ with $E_\nu \sim 0.03$ GeV and $L = 30$ m
- MiniBooNE: Search for $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ with $E_\nu = 0.3 - 2$ GeV and $L = 540$ m

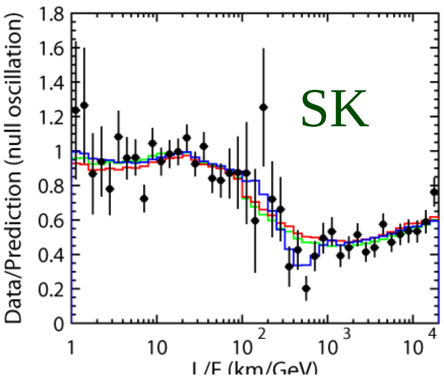


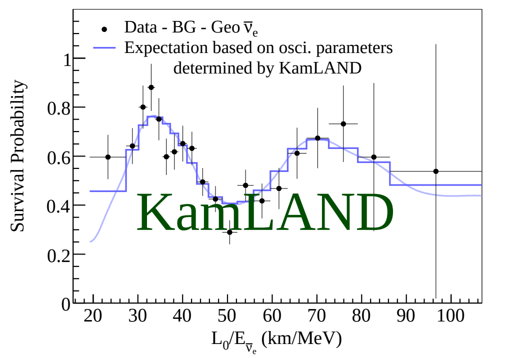
Compatibility (?)
for $\Delta m^2 \sim \text{eV}^2$

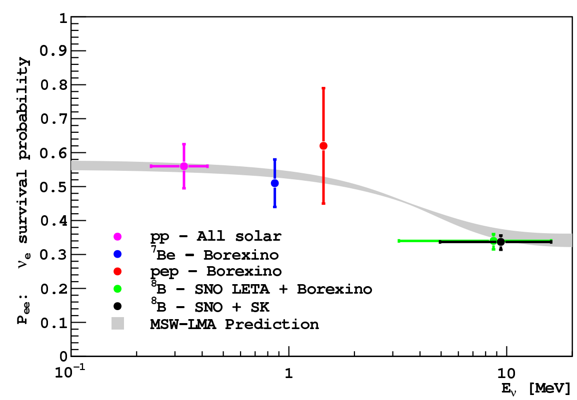
a third osc frequency?

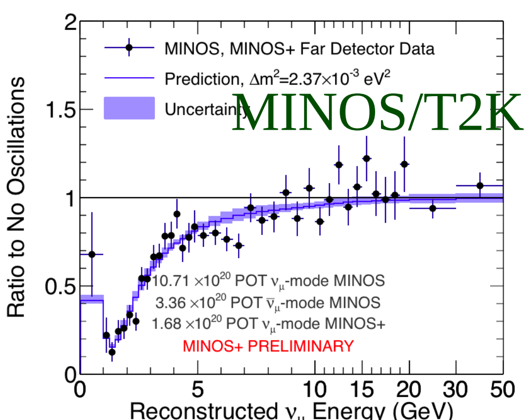
- By 2015 we have observed with high (or good) precision:
  - $*$ Solar ν_e convert to ν_μ/ν_τ (Cl, Ga, SK, SNO, Borexino)
  - $*$ Reactor $\overline{\nu_e}$ disappear at $L \sim 200$ Km (KamLAND)
  - $*$ Atmospheric ν_μ & $\bar{\nu}_\mu$ disappear most likely to ν_τ (SK,MINOS)
  - $*$ Accelerator ν_μ & $\bar{\nu}_\mu$ disappear at $L \sim 250[700]$ Km (K2K,T2K, [MINOS])
  - $*$ Some accel ν_μ appear as ν_e at $L \sim 250[700]$ Km (T2K), [MINOS]
  - $*$ Reactor $\overline{\nu_e}$ disappear at $L \sim 1$ Km (D-Chooz, Daya-Bay, Reno)
- Confirmed: Vacuum oscillation L/E pattern with 2 frequencies

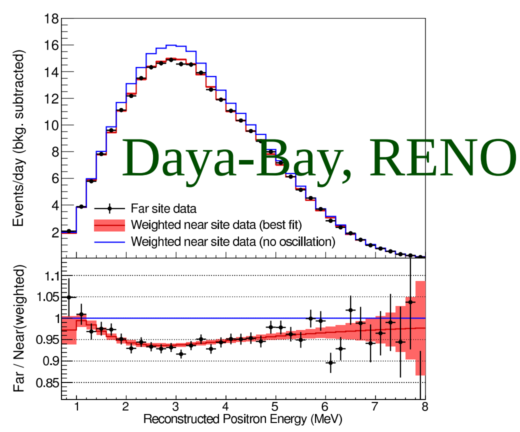
MSW conversion in Sun











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All this implies that neutrinos are massive

and There is Physics Beyond SM

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All this implies that neutrinos are massive

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- The *important* question:

  What is the BSM theory?

- The *difficult* path:

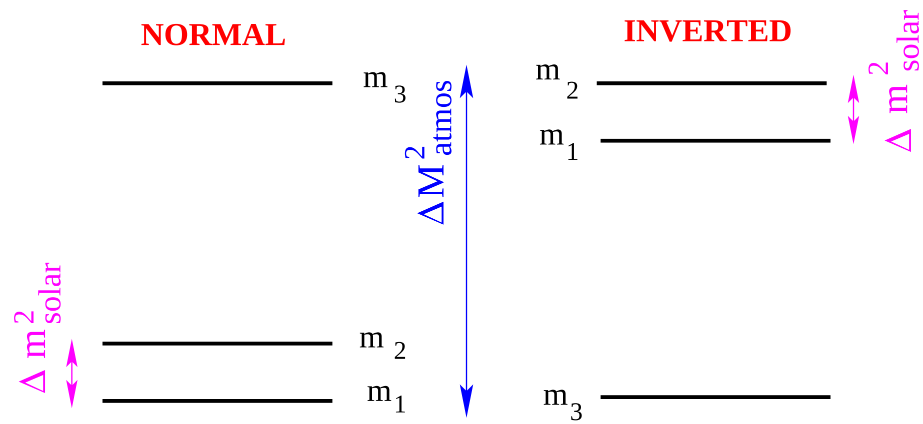
  Detailed determination of the new low energy parametrization

3ν Flavour Parameters

- For for 3 ν's : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

$$U_{\text{LEP}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta_{\text{cp}}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{\text{cp}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Two Possible Orderings

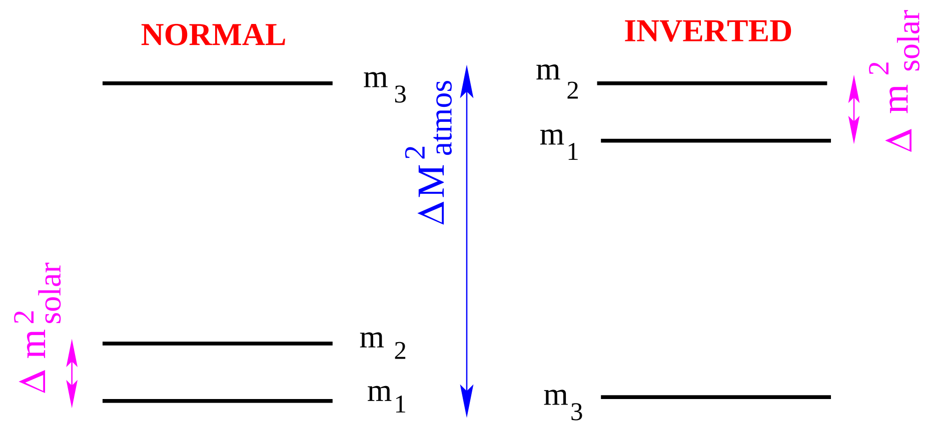


3ν Flavour Parameters

- For for 3 ν's : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

$$U_{\text{LEP}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta_{\text{cp}}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{\text{cp}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\eta_1} & 0 & 0 \\ 0 & e^{i\eta_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Two Possible Orderings

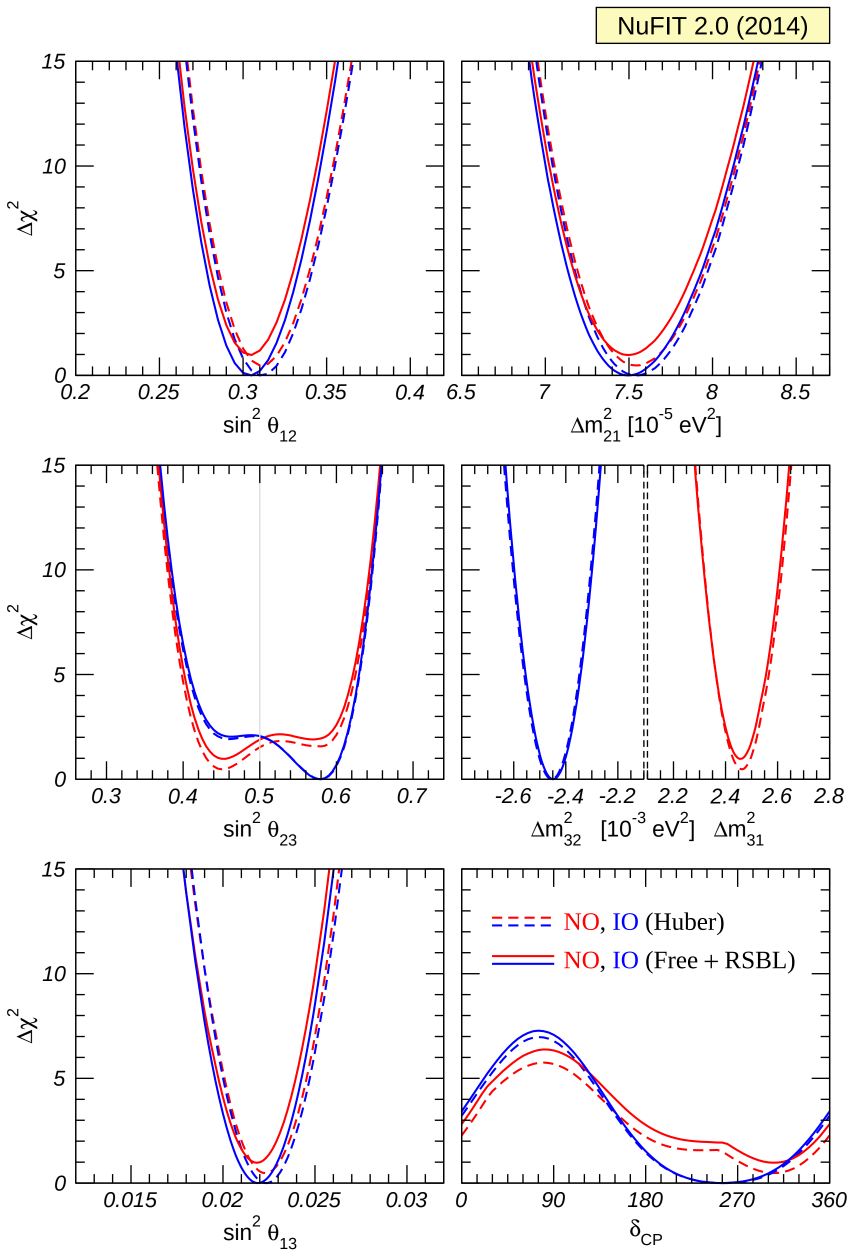


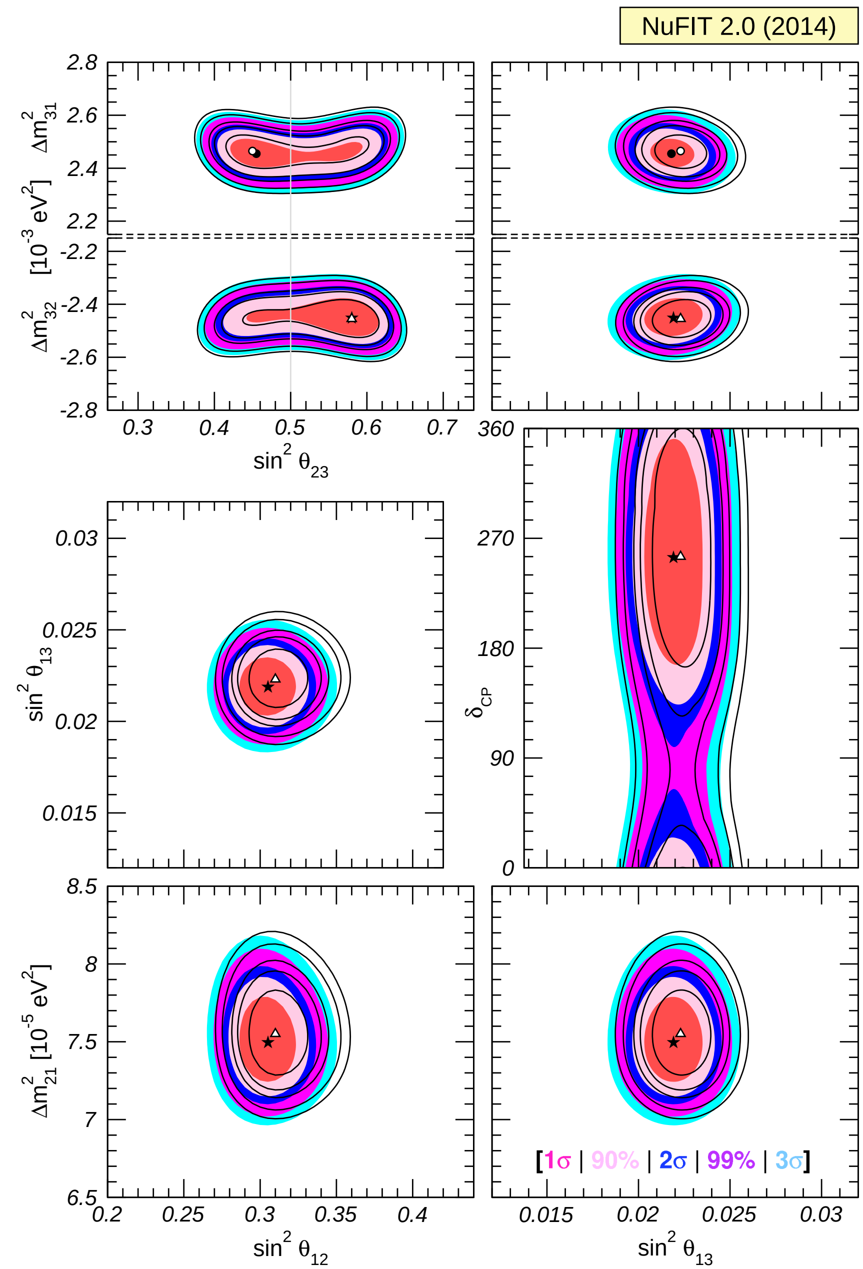
Experiment	Dominant Dependence	Important Dependence
Solar Experiments	$\rightarrow \theta_{12}$	Δm^2_{21} , θ_{13}
Reactor LBL (KamLAND)	$\rightarrow \Delta m^2_{21}$	θ_{12} , θ_{13}
Reactor MBL (Daya-Bay, Reno, D-Chooz)	$\rightarrow \theta_{13}$	Δm^2_{atm}
Atmospheric Experiments	$\rightarrow \theta_{23}$	Δm^2_{atm}, θ_{13} ,δ_{cp}
Accelerator LBL ν_μ Disapp (Minos)	$\rightarrow \Delta m^2_{\text{atm}}$	θ_{23}
Accelerator LBL ν_e App (Minos,T2K)	$\rightarrow \theta_{13}$	δ_{cp} , θ_{23}

3 ν Flavour Parameters: Present Status

Global 6-parameter fit http://www.nu-fit.org (ArXiv:1409.5439)

Maltoni, Schwetz, MCGG





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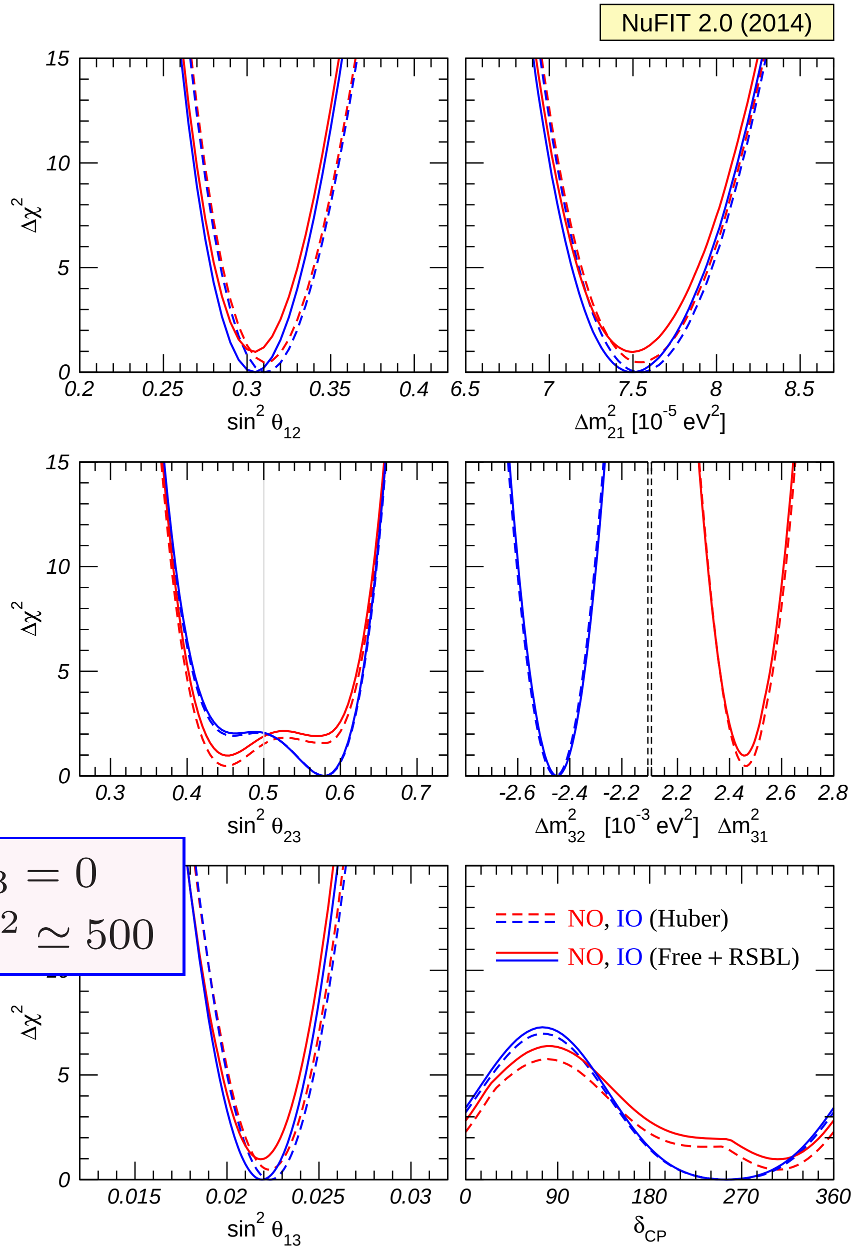
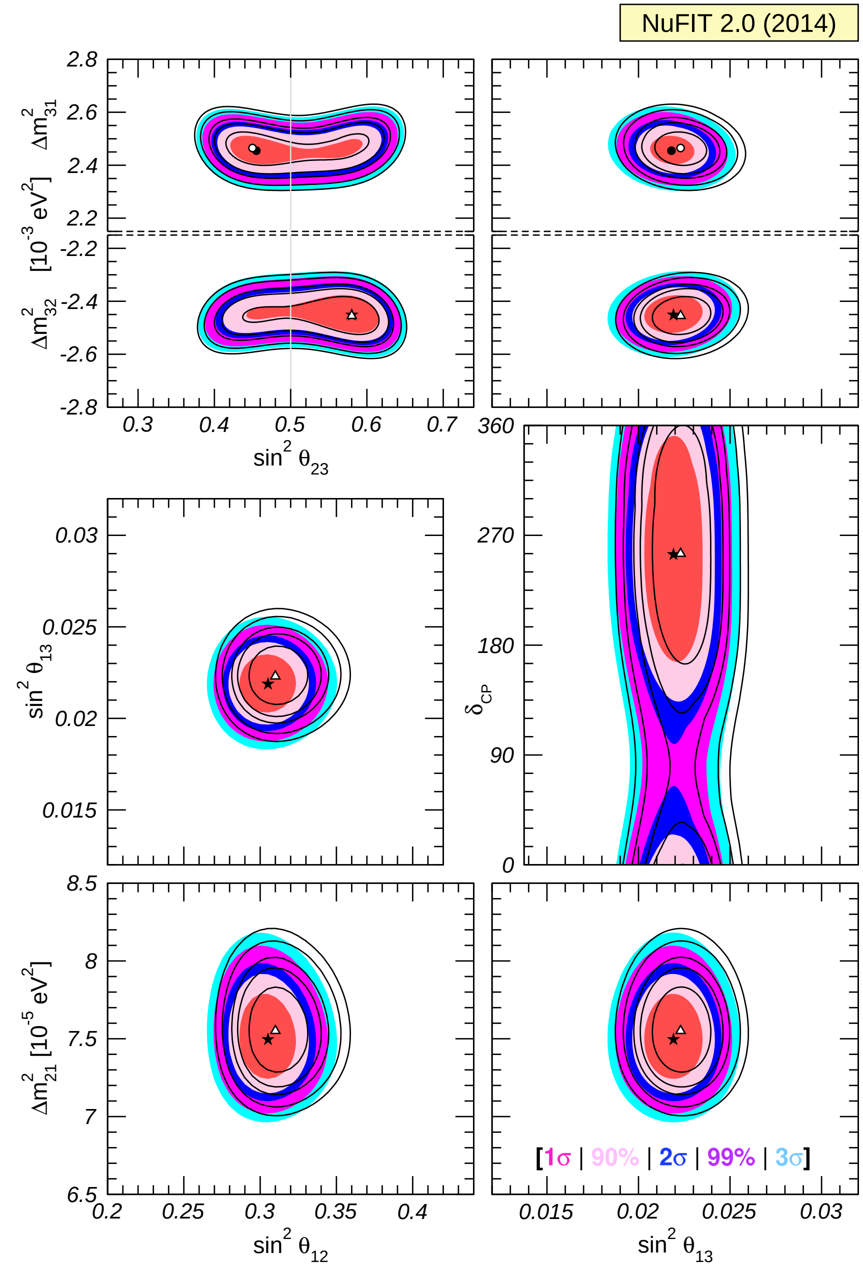


$\theta_{13} = 0$

$\Delta\chi^2 \simeq 500$



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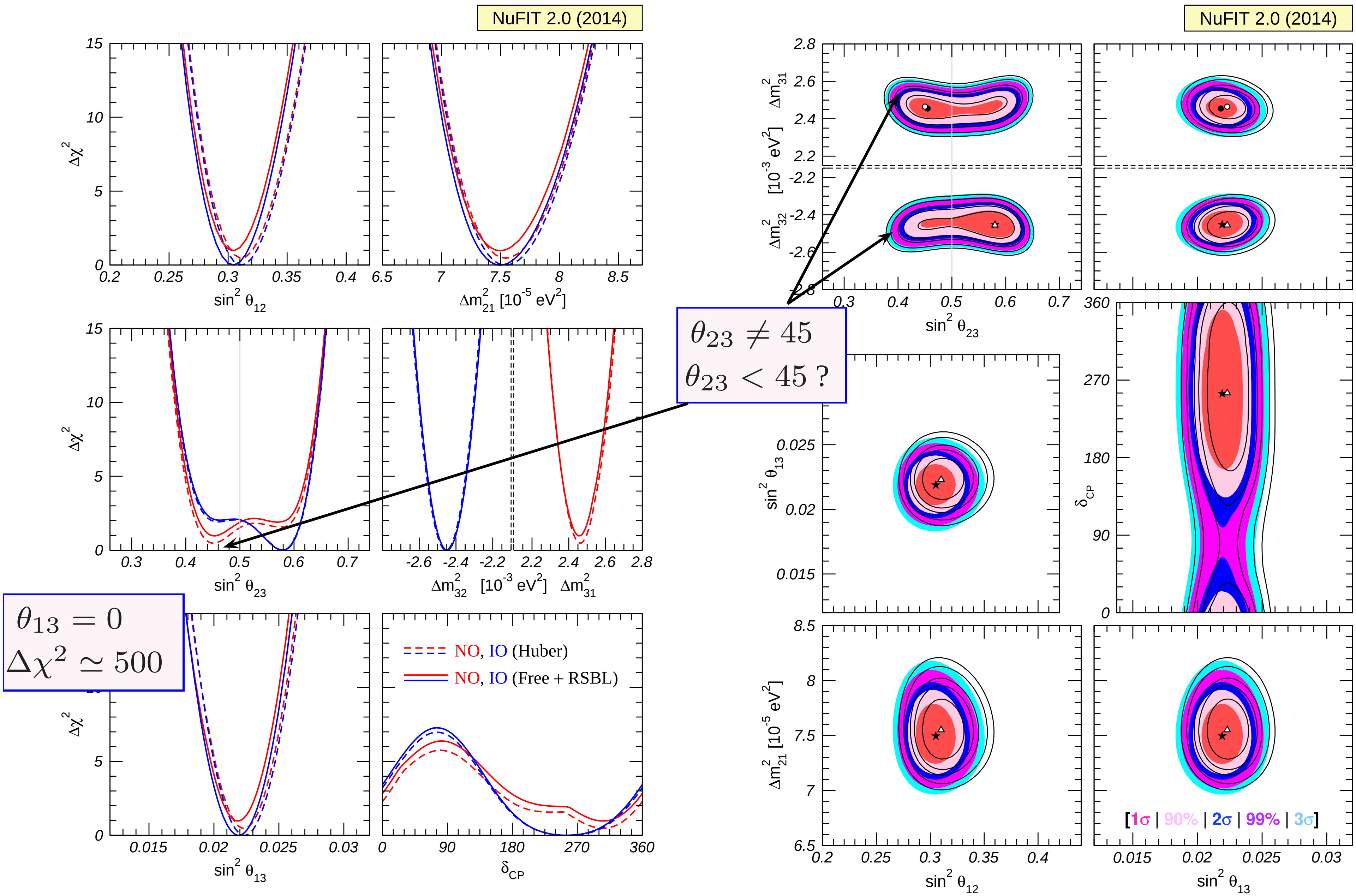

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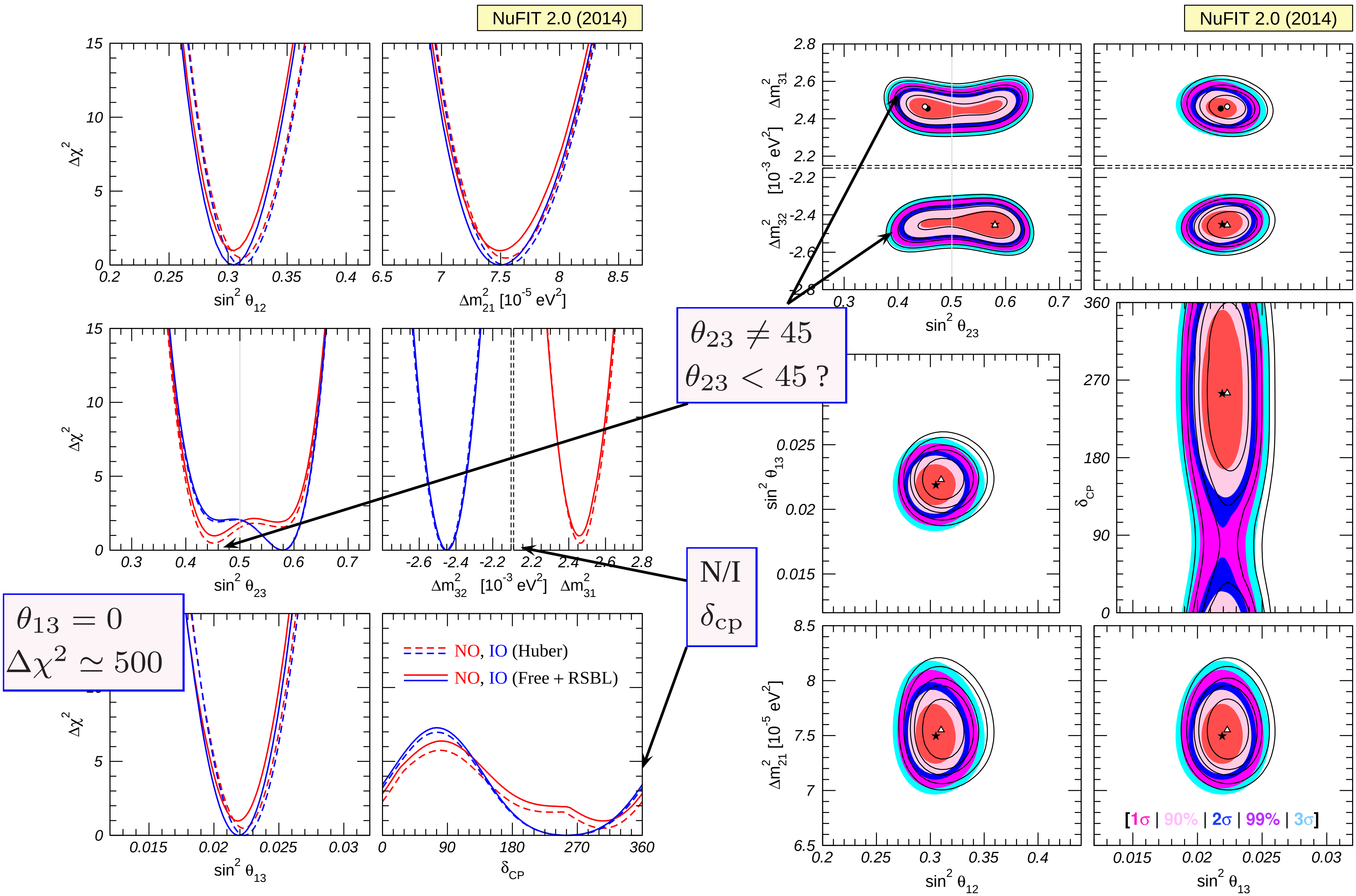

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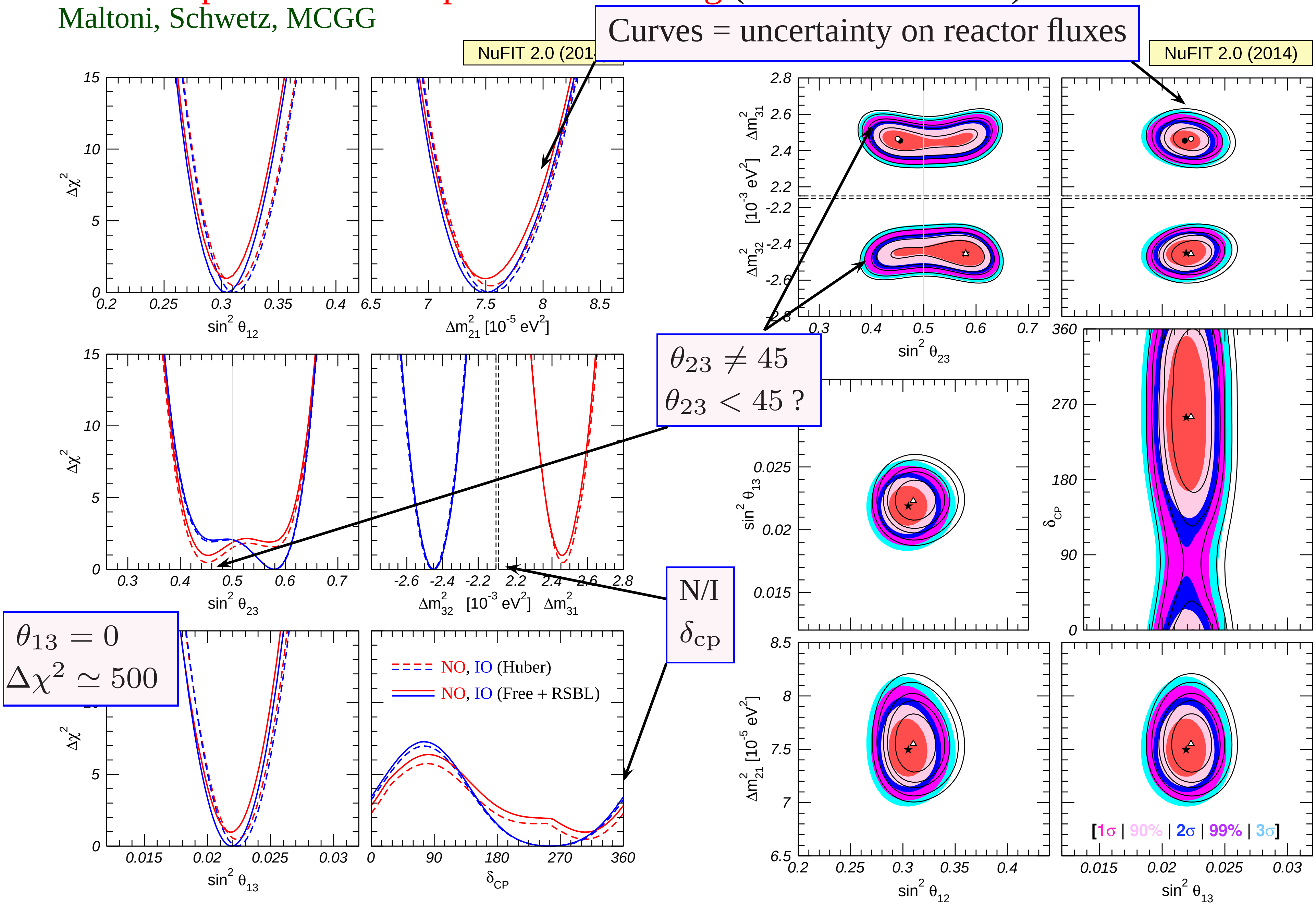
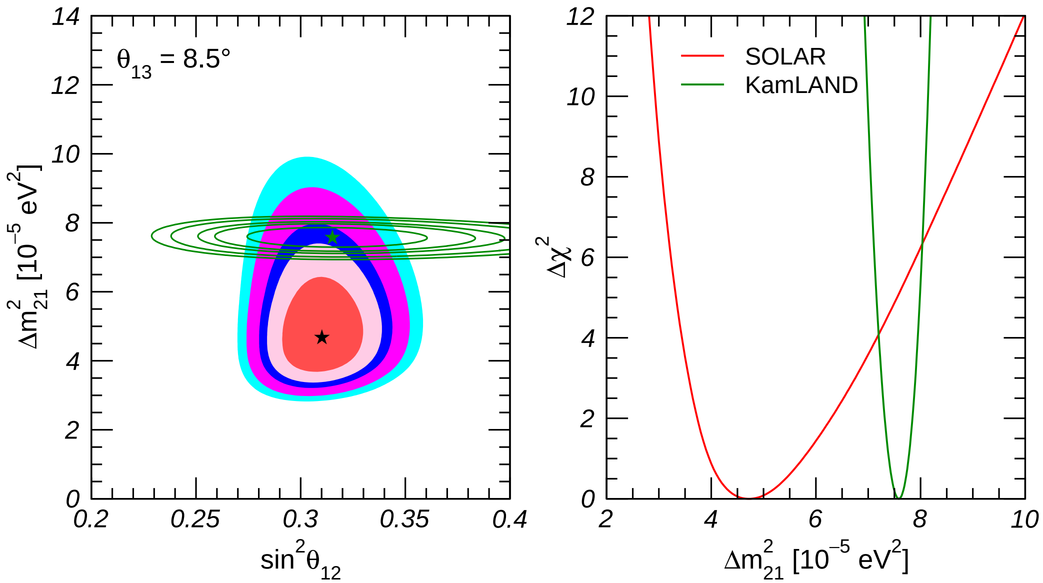
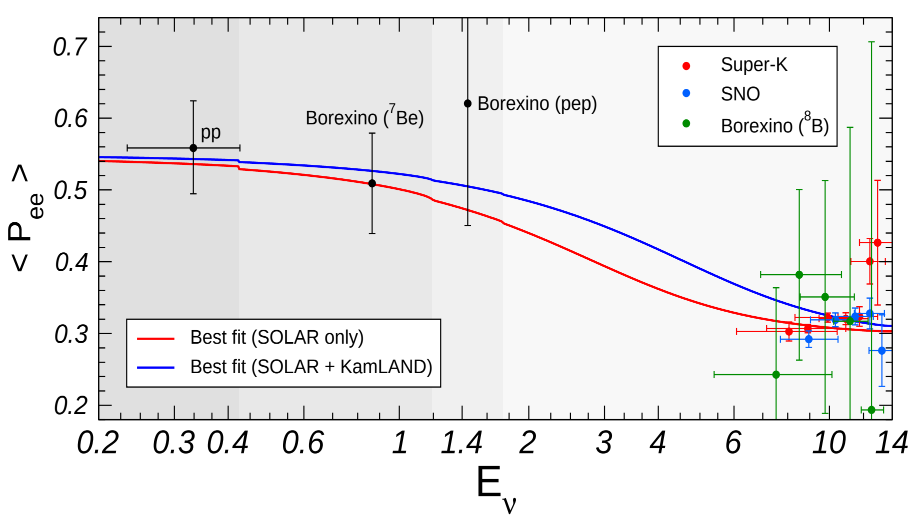


Issues in 3 ν Analysis: Δm^2_{21} KamLAND vs SOLAR

For $\theta_{13} \simeq 9^\circ$ θ_{12} OK. But residual tension on Δm^2_{12} NuFIT 2.0 (2014)



Tension related to: a)"too large" of Day/Night at SK



b) smaller-than-expected low-E turn up from MSW at best global fit

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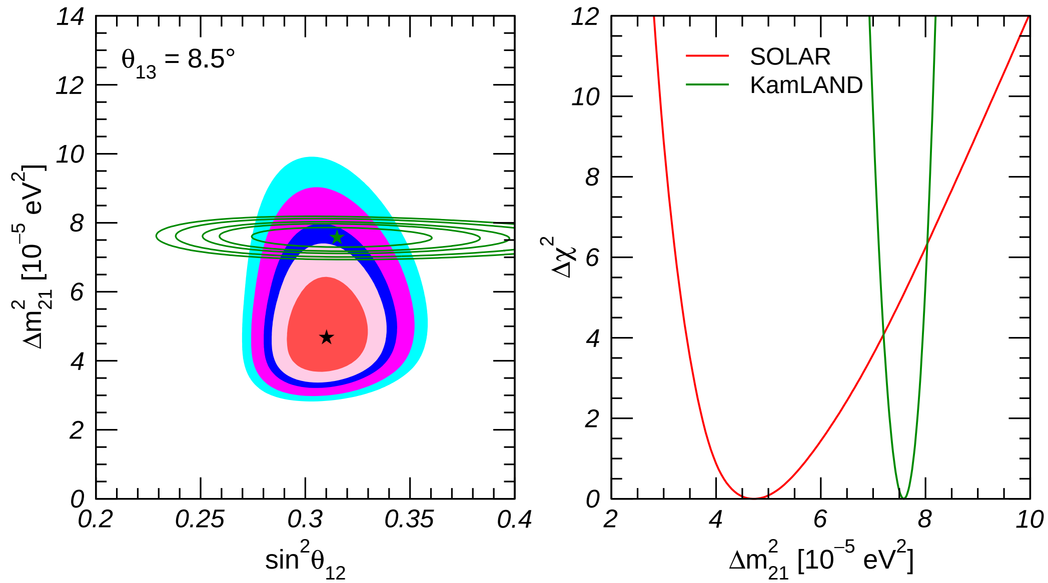

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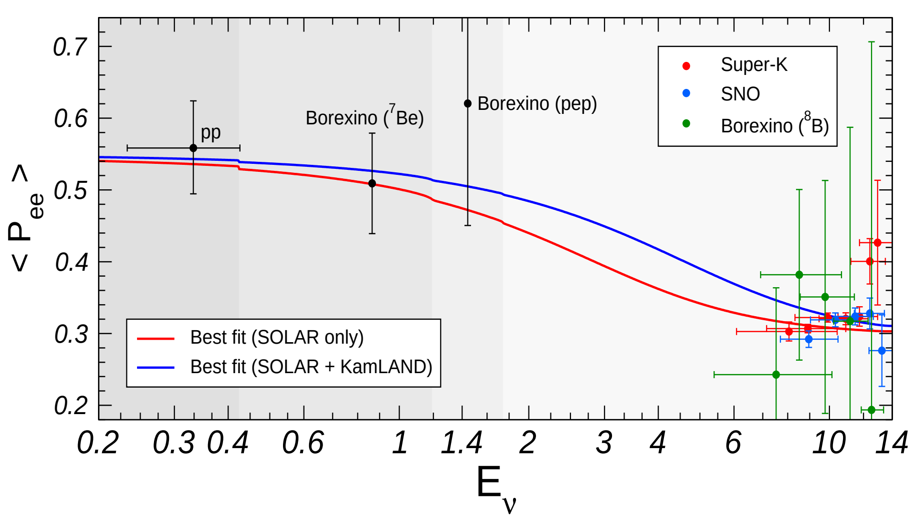


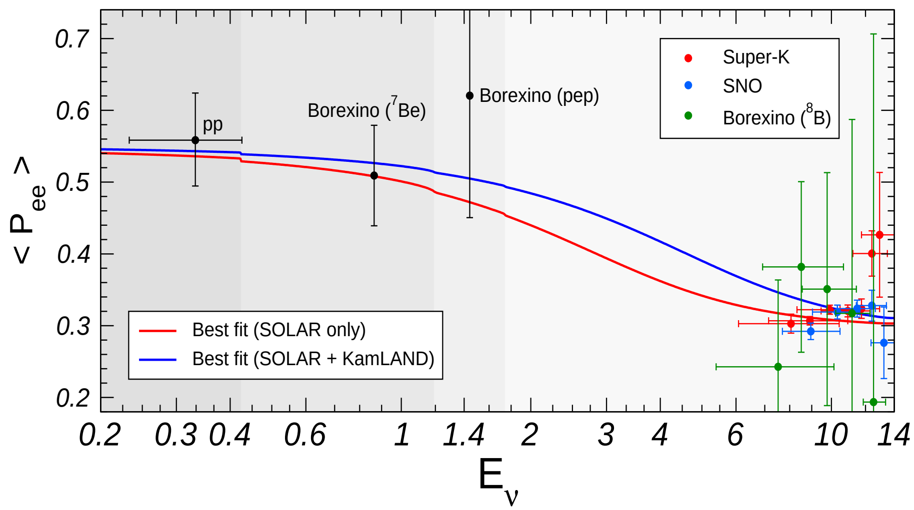
b) smaller-than-expected low-E turn up from MSW at best global fit

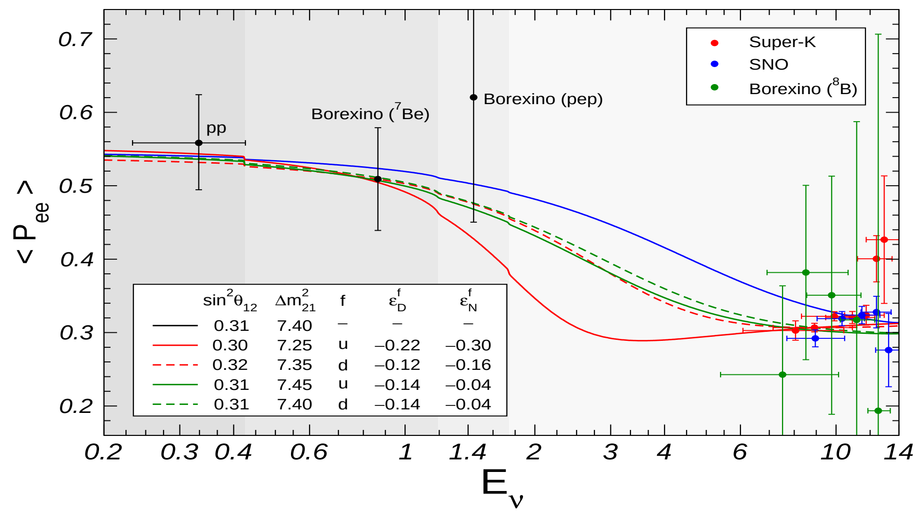
Modified matter potential?

Issues in 3 ν Analysis: Δm^2_{21} KamLAND vs SOLAR

Modified MSW with NSI (non-standard neutrino interactions):

$$\mathcal{L}_{\rm NSI} = -2\sqrt{2} G_F \varepsilon^{fP}_{\alpha\beta} (\bar\nu_\alpha \gamma^\mu \nu_\beta)(\bar f \gamma_\mu f)$$





Better fit with NSI ($\Delta\chi^2 \simeq 5$–7)

Oscillations+NSI: Global Analysis

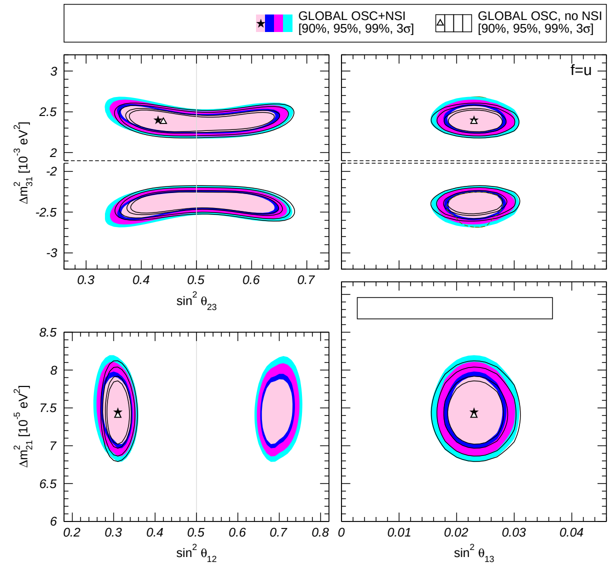
• Bounds on NSI

	90% CL	
Param.	OSC	SCATT
$\|\varepsilon^u_{ee}\|$	0.51–1.19	0.7–1
$\|\varepsilon^u_{\tau\tau}\|$	0.03	1.4–3
$\|\varepsilon^u_{e\mu}\|$	0.09	0.05
$\|\varepsilon^u_{e\tau}\|$	0.15	0.5
$\|\varepsilon^u_{\mu\tau}\|$	0.01	0.05

	90% CL	
Param.	OSC	SCATT
$\|\varepsilon^d_{ee}\|$	0.51–1.17	0.3–0.7
$\|\varepsilon^d_{\tau\tau}\|$	0.03	1.1–6
$\|\varepsilon^d_{e\mu}\|$	0.09	0.05
$\|\varepsilon^d_{e\tau}\|$	0.14	0.5
$\|\varepsilon^d_{\mu\tau}\|$	0.01	0.05

Bounds from global osc fit stronger than scattering ones for $\varepsilon^{u,d}_{\tau\beta}$

• Osc parameter robust (but solar dark side)

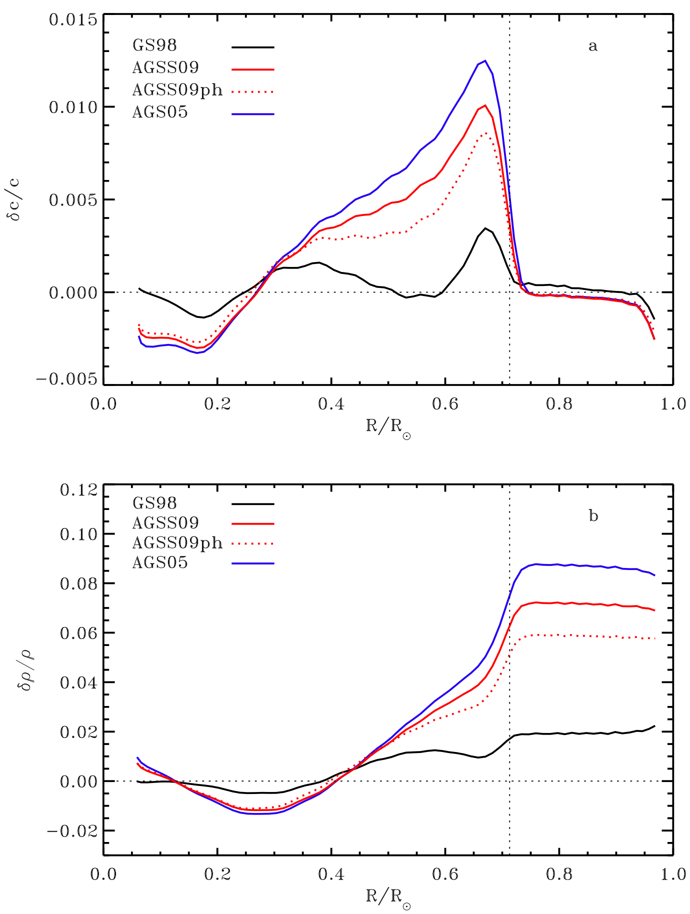


Maltoni, MCG-G, ArXiv:1307.3092

Issues with the Solar Fluxes

– Newer determination of abundance of heavy elements in solar surface give lower values

– Solar Models with these lower metalicities fail in reproducing helioseismology data



– Two sets of SSM:

Starting from Bahcall *etal* 05, Serenelli *etal* 0909.266

GS98 uses older metalicities

AGSXX uses newer metalicities

Flux $cm^{-2}\ s^{-1}$	GS98	AGSS09	Diff (%)
$pp/10^{10}$	5.97	$6.03\ (1 \pm 0.005)$	0.8
$pep/10^{8}$	1.41	$1.44\ (1 \pm 0.010)$	2.1
$hep/10^{3}$	7.91	$8.18\ (1 \pm 0.15)$	3.4
$^{7}Be/10^{9}$	5.08	$4.64\ (1 \pm 0.06)$	8.8
$^{8}B/10^{6}$	5.88	$4.85\ (1 \pm 0.12)$	17.7
$^{13}N/10^{8}$	2.82	$2.07(1^{+0.14}_{-0.13})$	**26.7**
$^{15}O/10^{8}$	2.09	$1.47\ (1^{+0.16}_{-0.15})$	**30.0**
$^{17}F/10^{16}$	5.65	$3.48\ (1^{+0.17}_{-0.16})$	**38.4**

Most difference in CNO fluxes

Issues with the Solar Fluxes

– Two sets of SSM:

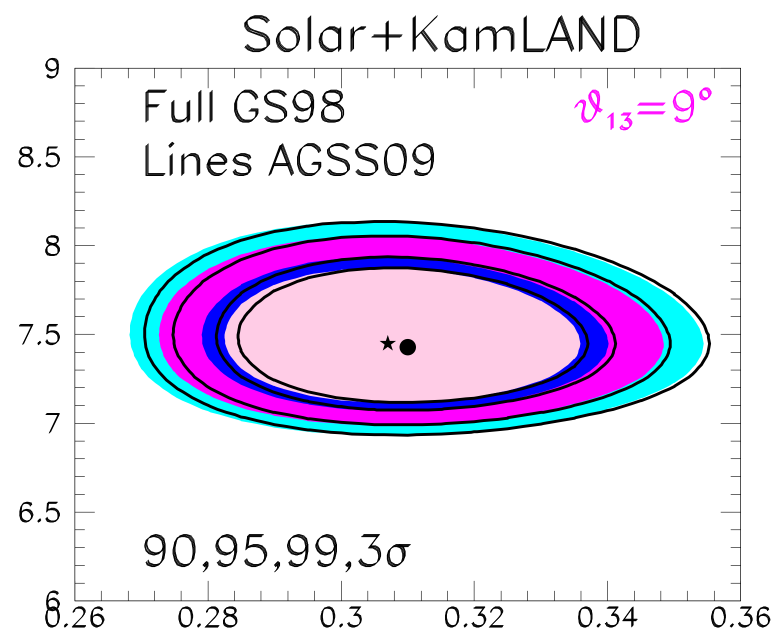
GS98 uses older metalicities

AGSXX uses newer metalicities

∗ What is the effect on the determination of oscillation parameters?

Very small

Impact in Parameter Determination



Learning How the Sun Shines

– Two sets of SSM:

GS98 uses older metalicities

AGSXX uses newer metalicities

∗ What is the effect on the determination of oscillation parameters?

Very small

∗ Which SSM does the solar data favour?

Both model statistically equally prob

Better CNO : *Cleaner* Borexino, SNO+

–Test of Solar Luminosity:

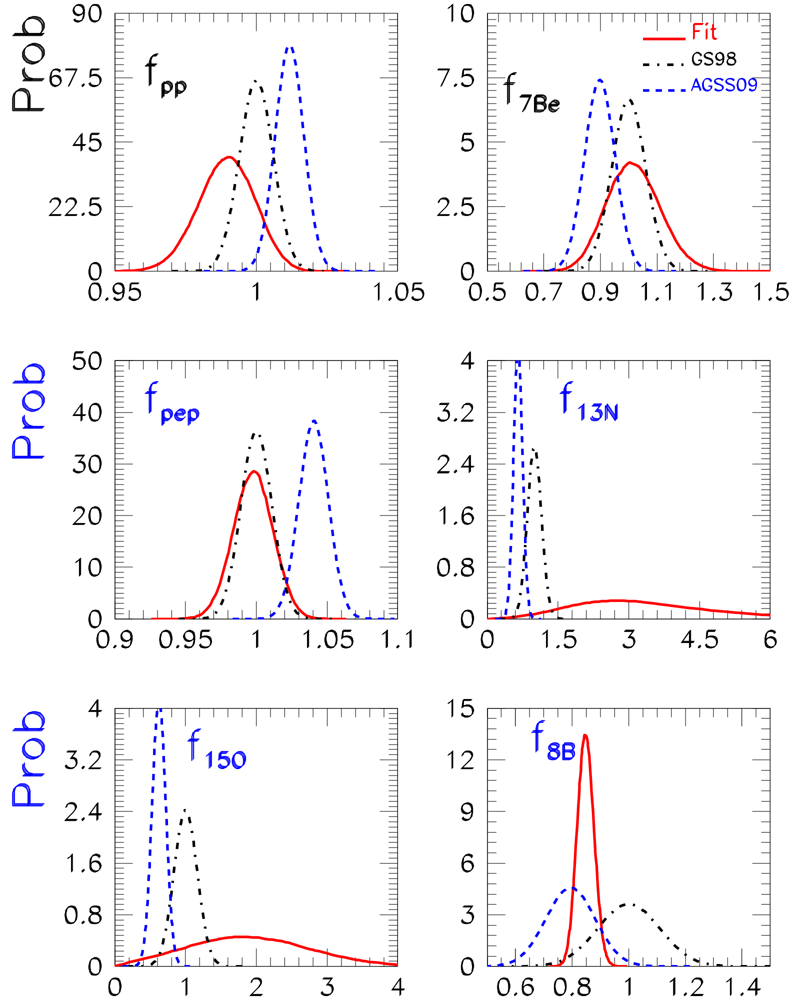
$$\frac{L_{\rm CNO}}{L_\odot} < 3.2\,\%\ (3\sigma)$$

$$\frac{L_\odot(\nu - \text{inferred})}{L_\odot} = 1.0 \pm 0.14\ (1\sigma)$$

3ν oscillation fit with solar fluxes free:

(within luminosity constraint)

Comparison with Models



MCG-G,Maltoni,Salvado JHEP 2010

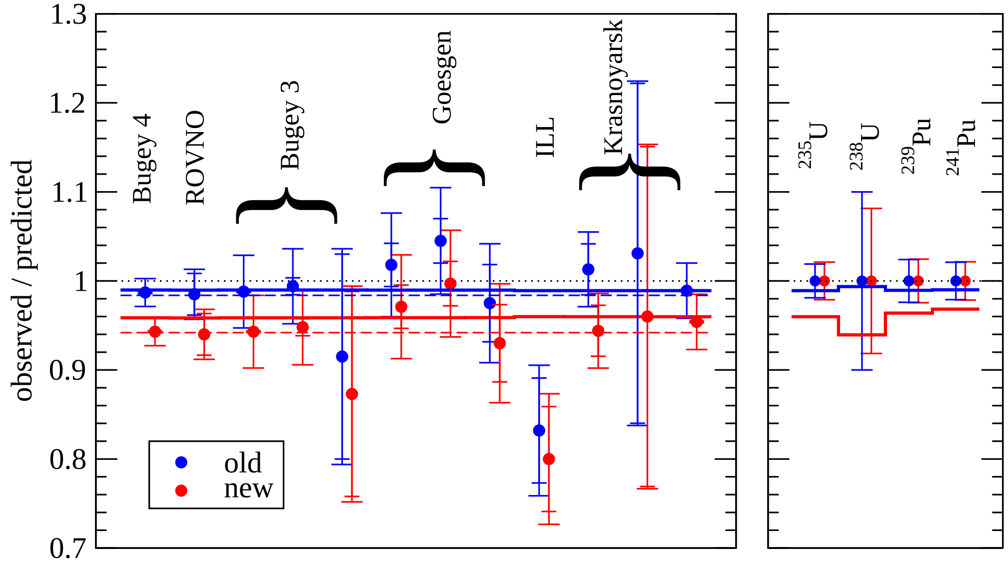
Issues in 3 ν Analysis: Reactor Flux anomaly and θ_{13}

- The reactor $\bar{\nu}_e$ fluxes have been recalculated

  T.A. Mueller et al.,[arXiv:1101.2663].;P. Huber, [arXiv:1106.0687].

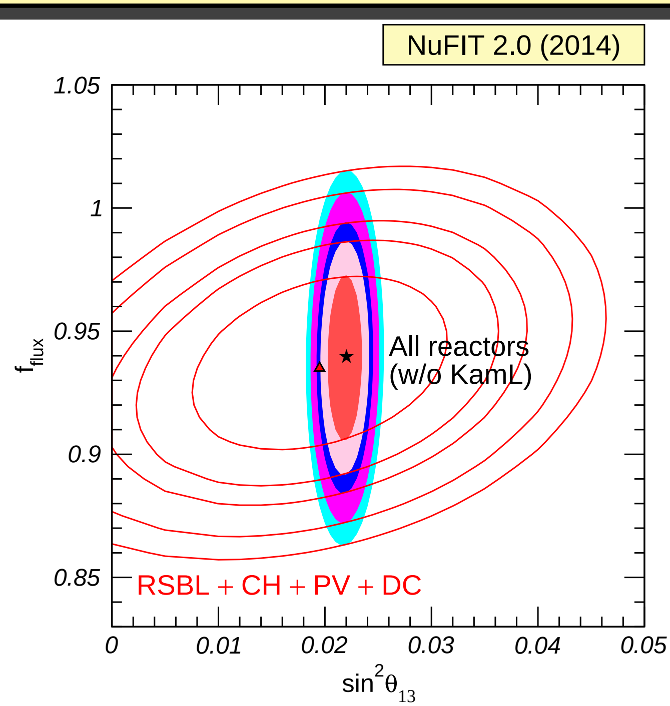
- Both reevaluations find higher fluxes by about 3.5 %

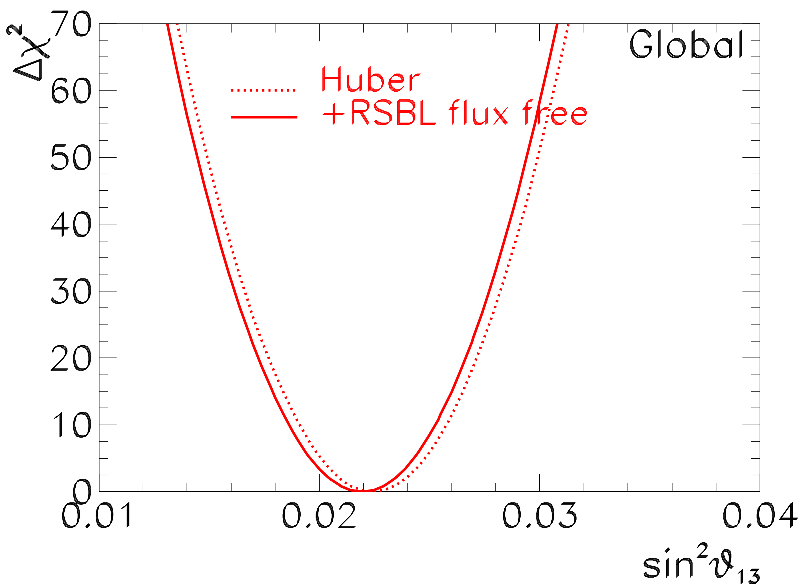
- So *negative* reactor experiments at short baselines (RSBL) indeed observed a deficit



- For 3ν analysis a consistent approach (T. Schwetz et. al. [arXiv:1103.0734]):
  – Fit oscillation parameters and reactor fluxes simultaneously
  – Use theoretical calculation and/or RSBL data as priors

Issues in 3 ν Analysis: Reactor Flux anomaly and θ_{13}





- Experiments without near detector (CHOOZ, Palo-Verde, D-CHOOZ) sensitive to the flux assumptions
- DAYA BAY and RENO Near-Far comparison $\Rightarrow$ results flux independent
- Two extreme priors :

  a) Use fluxes from Huber 1106.0687 without RSBL data

  $$\sin^2\theta_{13} = 0.0223 \pm 0.001$$

  b) Leave flux free and include RSBL

  $$\sin^2\theta_{13} = 0.0218 \pm 0.001$$

  Uncertainty at $\sim 0.5\sigma$ level

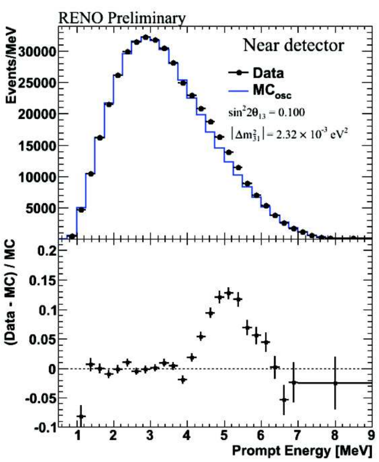
  $\chi^2_{min,a} - \chi^2_{min,b} \sim 7$

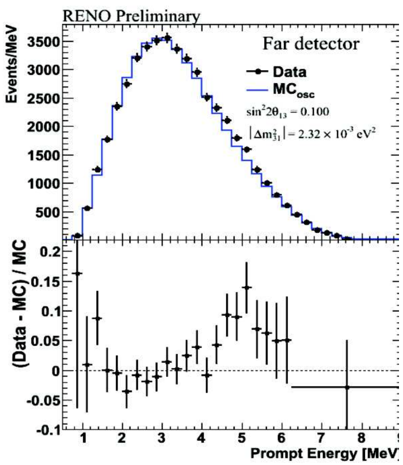
“New” Reactor Anomaly?

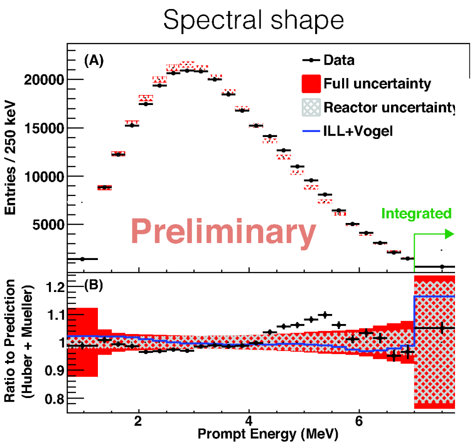
“Bump” at $E \sim 5$ MeV in Near and Far spectra at RENO, Daya Bay and D-Chooz

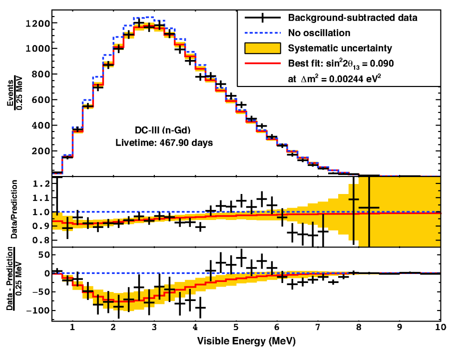
Not understood within present reactor flux calculations

Daya Bay D-Chooz







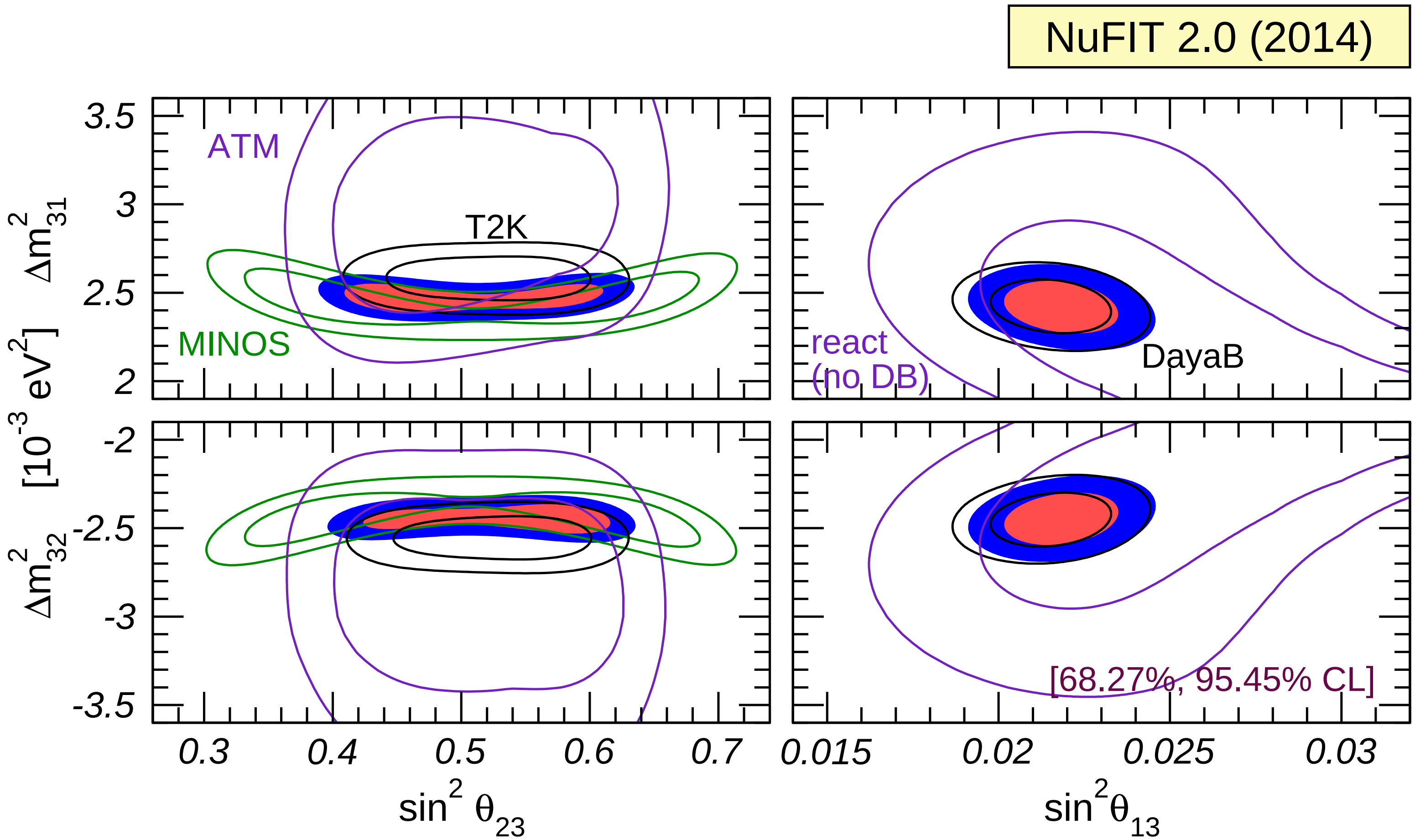


Does not affect to (& unexplained by) oscillations (cancels in near/far)

3 ν Analysis: Long Baseline vs REACT and $|\Delta m^2_{3l}|$

Independent and consistent determination of $|\Delta m^2_{3l}|$ from MBL reactor data

In particular from Daya Bay (also Reno and DC) near/far E Spectrum



3 ν Analysis: Long Baseline vs REACT

- In LBL APP $\nu_\mu \to \nu_e$

$$P_{\mu e} \simeq s^2_{23} \sin^2 2\theta_{13} \left(\frac{\Delta_{31}}{\Delta_{31} \pm V}\right)^2 \sin^2\left(\frac{\Delta_{31} \pm V\, L}{2}\right)$$
$$+8\, J_{\rm CP}^{\rm max} \frac{\Delta_{12}}{V} \frac{\Delta_{31}}{\Delta_{31} \pm V} \sin\left(\frac{V L}{2}\right) \sin\left(\frac{\Delta_{31} \pm V L}{2}\right) \cos\left(\frac{\Delta_{31} L}{2} \pm \delta_{CP}\right)$$

$$J_{\rm CP}^{\rm max} = c^2_{13} s_{13} c_{23} s_{23} c_{12} s_{12}$$

So $\sin^2 2\theta_{\rm APP} = 2 \sin^2 \theta_{23} \sin^2 2\theta_{13}$

- In Reactor $P_{ee} \simeq \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta_{31} L}{2}\right)$

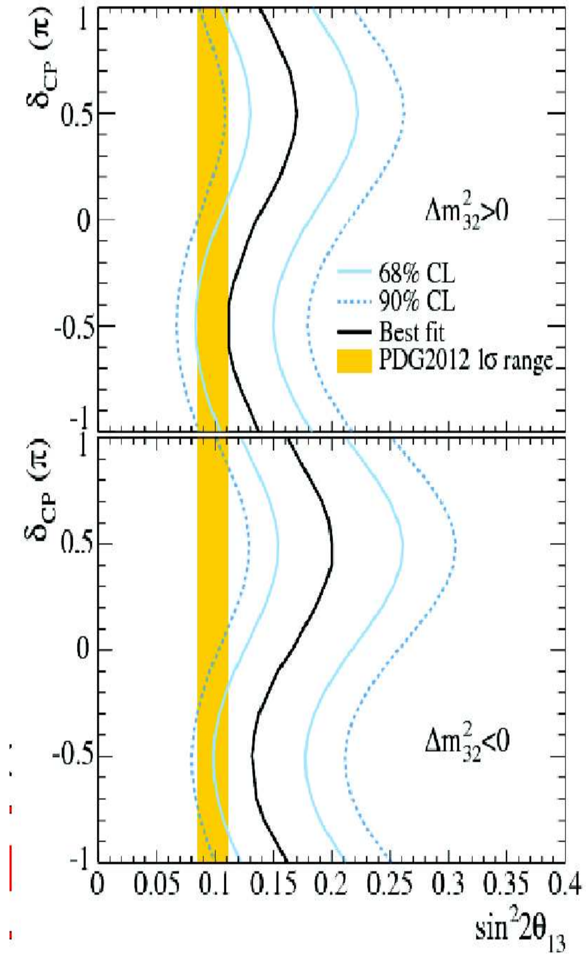
So $\sin^2 2\theta_{\rm REAC} = \sin^2 2\theta_{13}$

–So from first term in $P_{\mu e}$:

$\sin^2 2\theta_{\rm REAC} \leq \sin^2 2\theta_{\rm APP} \quad \Rightarrow \theta_{23} \geq \frac{\pi}{4}$ favoured

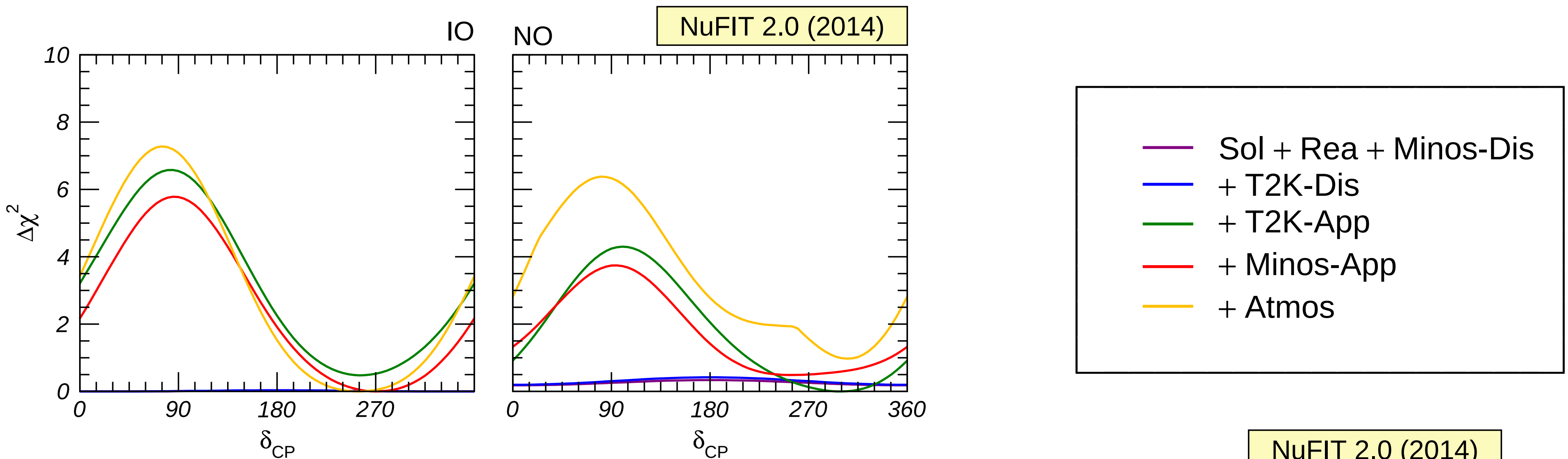
–Or from second term in $P_{\mu e}$:

$\Rightarrow \delta \sim \frac{3\pi}{2} (\equiv -\frac{\pi}{2})$ favoured

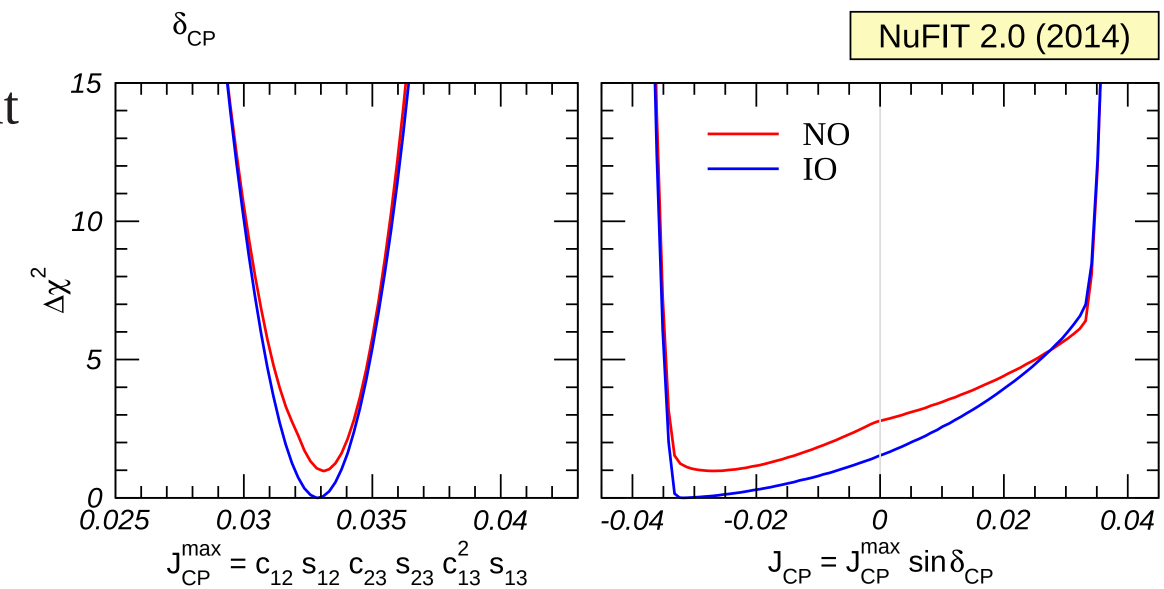


3ν Analysis: Leptonic CP violation

- $\sim 2\sigma$ “Hint” CP phase around $\delta_{CP} = \frac{3\pi}{2}$ driven by the LBL-APP vs REACT θ_{13} (beware of diff notation for δ_{CP} in literature)



- Leptonic Jarslog Determinant



Neutrino Mass Scale: Tritium β Decay

- Fermi proposed a kinematic search of ν_e mass from beta spectra in ^3H beta decay

$$^3H \rightarrow ^3He + e + \overline{\nu}_e$$

- For "allowed" nuclear transitions, the electron spectrum is given by phase space alone

$$K(T) \equiv \sqrt{\frac{dN}{dT}\frac{1}{C p\, E\, F(E)}} \propto \sqrt{(Q-T)\sqrt{(Q-T)^2 - m_{\nu_e}^2}}$$

$T = E_e - m_e$, Q= maximum kinetic energy, (for ^3H beta decay $Q = 18.6$ KeV)

Taking into account mixing $m_{\nu_e} \equiv \sqrt{\sum m_{\nu_j}^2 |U_{ej}|^2}$

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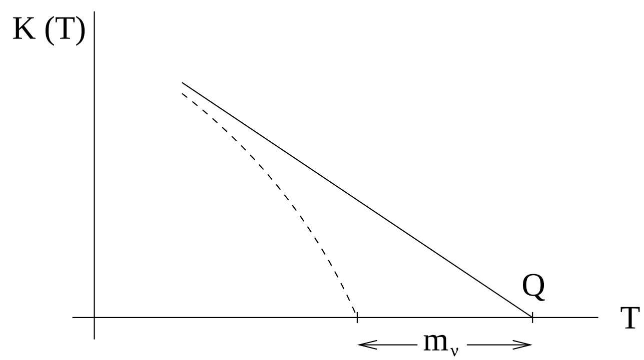
- $m_\nu \neq 0 \Rightarrow$ distortion from the straight-line at the end point of the spectrum

$$m_{\nu_e} = 0 \;\Rightarrow\; T_{\max} = Q$$
$$m_{\nu_e} \neq 0 \;\Rightarrow\; T_{\max} = Q - m_{\nu_e}$$



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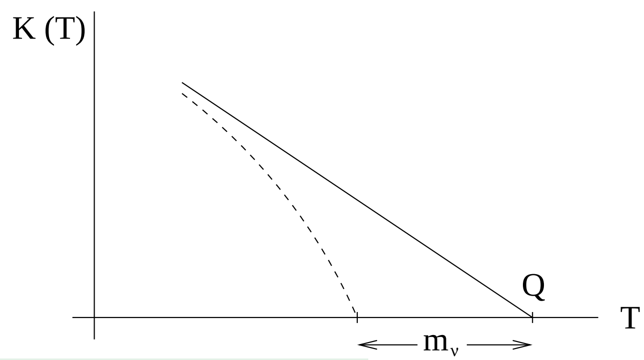

– At present only a bound: $m_{\nu_e} < 2.2$ eV (at 95 % CL) (Mainz & Troisk experiments)

– Katrin operating to improve present sensitivity to $m_{\nu_e} \sim 0.3\,$eV

Neutrino Mass Scale: Other Channels

Muon neutrino mass

• From the two body decay at rest

$$\pi^- \to \mu^- + \overline{\nu}_\mu$$

• Energy momentum conservation:

$$m_\pi = \sqrt{p_\mu^2 + m_\mu^2} + \sqrt{p_\mu^2 + m_\nu^2}$$

$$m_\nu^2 = m_\pi^2 + m_\mu^2 - 2 + m_\mu \sqrt{p^2 + m_\pi^2}$$

• Measurement of p_μ plus the precise knowledge of m_π and $m_\mu \Rightarrow m_\nu$

• The present experimental result bound:

$$m_{\nu_\mu}^{eff} \equiv \sqrt{\sum m_j^2 |U_{\mu j}|^2} < 190 \text{ KeV}$$

Tau neutrino mass

• The τ is much heavier $m_\tau = 1.776$ GeV

$\Rightarrow$ Large phase space $\Rightarrow$ difficult precision for m_ν

• The best precision is obtained from hadronic final states

$$\tau \to n\pi + \nu_\tau \quad \text{with } n \geq 3$$

• Lep I experiments obtain:

$$m_{\nu_\tau}^{eff} \equiv \sqrt{\sum m_j^2 |U_{\tau j}|^2} < 18.2 \text{ MeV}$$

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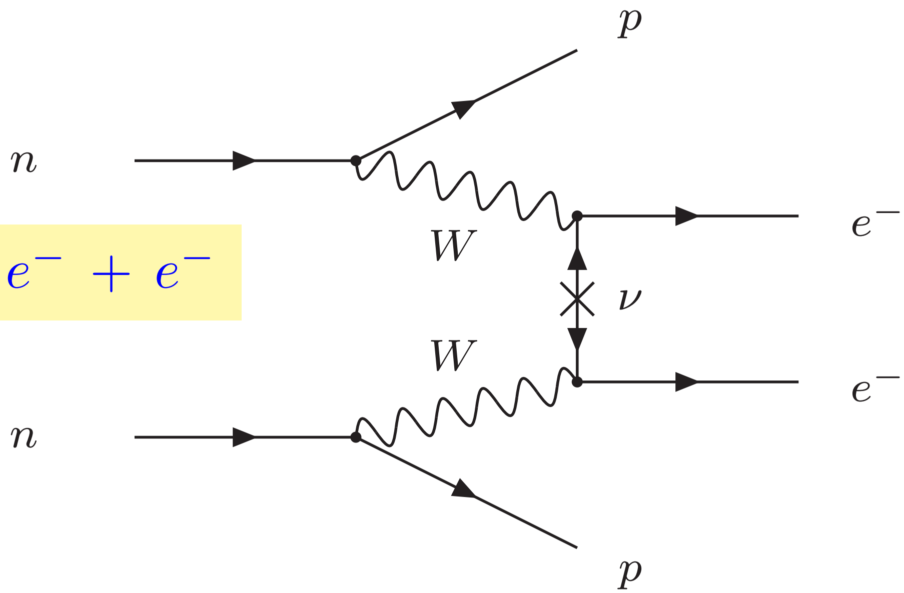
$\Rightarrow$ If mixing angles U_{ej} are not negligible

Best kinematic limit on Neutrino Mass Scale comes from Tritium Beta Decay

ν-less Double-β Decay

ν-less Double-β Decay

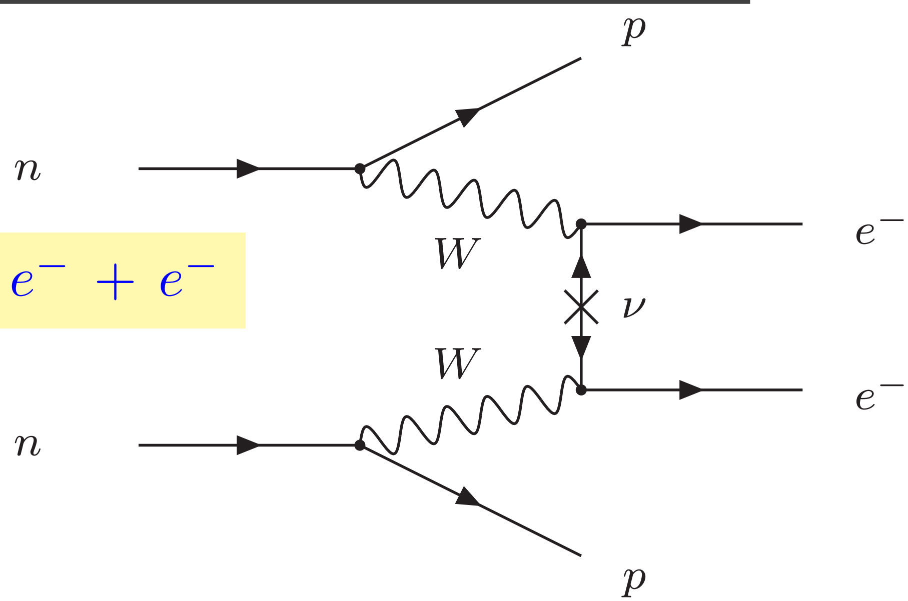
$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$



- Amplitude involves the product of two leptonic currents: $[\overline{e}\gamma^\mu L\nu]\,[\overline{e}\gamma^\mu L\nu]$

ν-less Double-β Decay

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$$



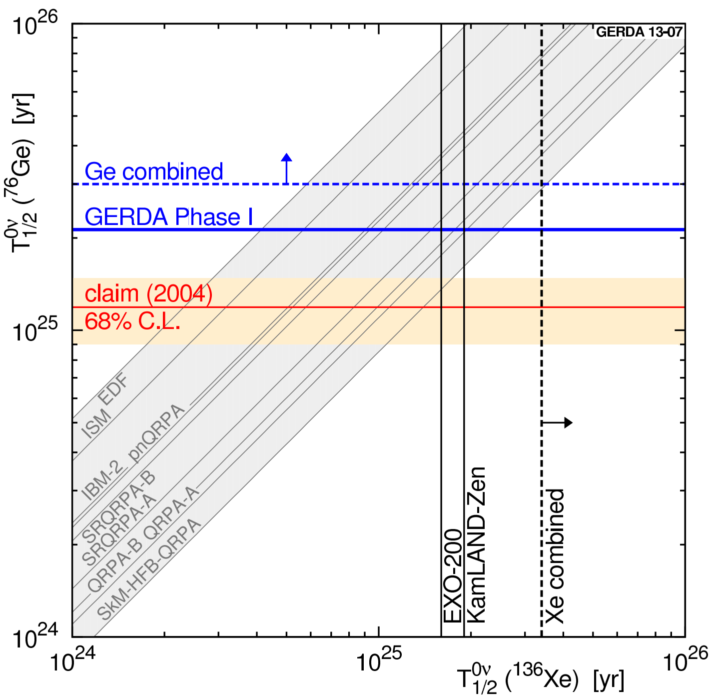
- Amplitude involves the product of two leptonic currents: $\left[\overline{e}\gamma^\mu L\nu\right]\left[\overline{e}\gamma^\mu L\nu\right]$
  - If ν Dirac $\Rightarrow$ ν *annihilates* a *neutrino* and *creates* an *antineutrino*

    $\Rightarrow$ no same state $\Rightarrow$ Amplitude = 0
  - If ν Majorana $\Rightarrow$ $\nu = \nu^c$ *annihilates and creates* a *neutrino=antineutrino*

    $\Rightarrow$ same state $\Rightarrow$ Amplitude $\propto \overbrace{\nu\ (\nu^c)}^{}{}^T \neq 0$
- $(T^{0\nu}_{1/2})^{-1} \propto (m_{ee})^2$ with $|\langle m_{ee}\rangle| = |\sum U^2_{ej} m_j|$
- Complication is uncertainty in the nuclear matter elements

$0\nu\beta\beta$ Decay: Present

Bounds from ^{136}Xe (EXO and KamLAND-ZEN), ^{76}Ge (Gerda) and ^{130}Te (Cuore-0)

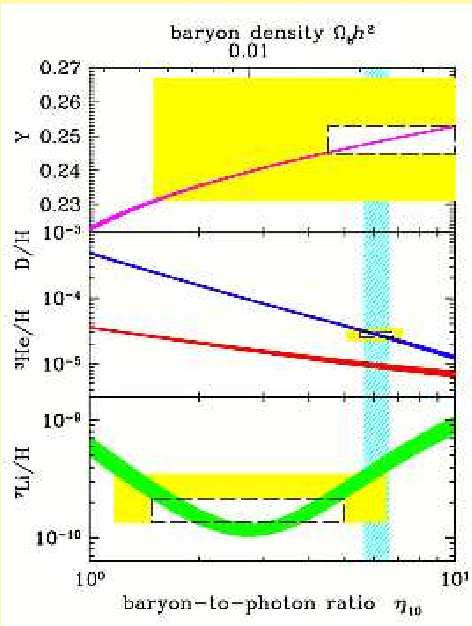
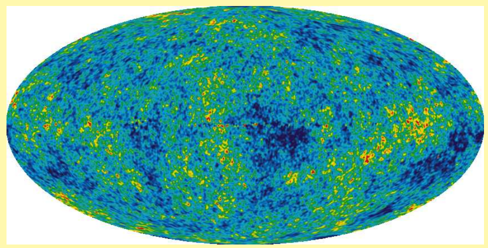
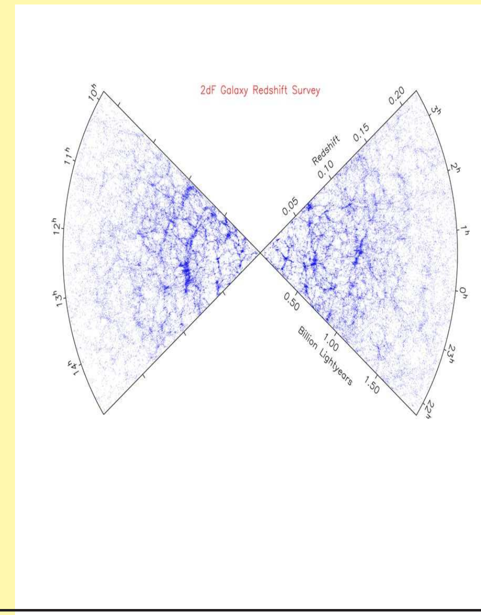


$\Rightarrow$ If neutrinos are Majorana:

$m_{ee} \leq 0.20$–0.40 eV	^{76} Ge	(Gerda+HdM+IGex)
$m_{ee} \leq 0.14$–0.28 eV	^{136}Xe	(KamLAND-ZEN)
$m_{ee} \leq 0.19$–0.45 eV	^{136}Xe	(EXO)
$m_{ee} \leq 0.27$–0.76 eV	^{130}Te	(Cuore-0)

Massive ν in Cosmology

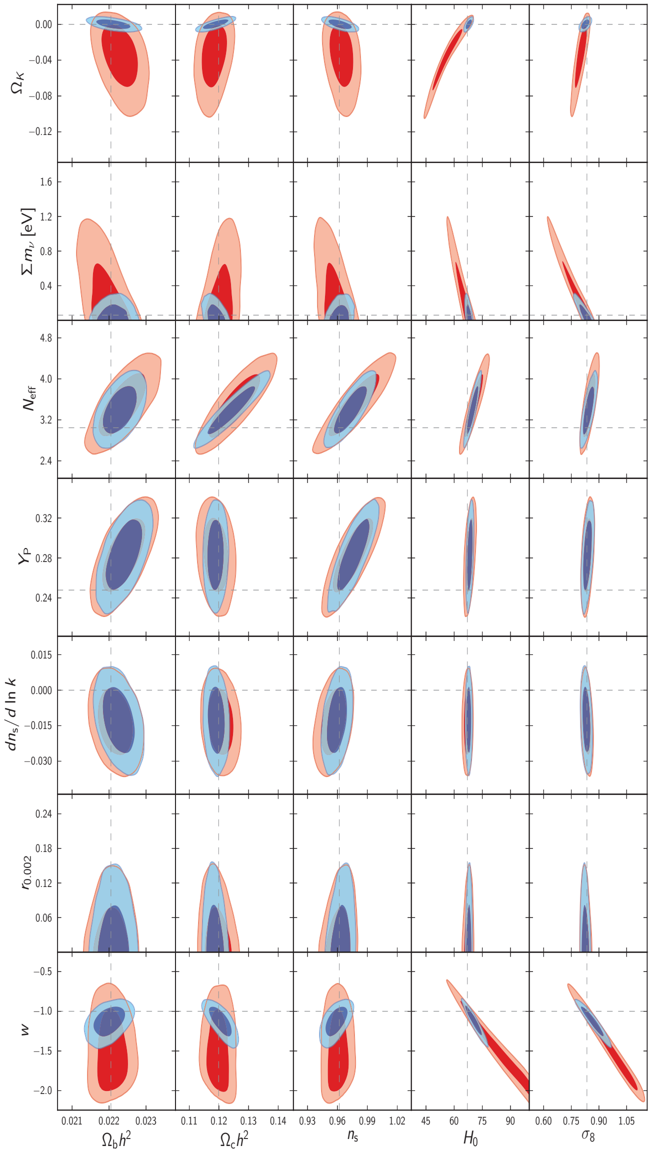
Relic $\nu's$: Effects in several cosmological observations at several epochs

		
Primordial Nucleosynthesis BBN	Cosmic Microwave Background CMB	Large Scale Structure Formation LSS
$T \sim$ MeV	$T \lesssim$ eV	
Number of $\nu's$ ($N_{\rm eff}$)	$N_{\rm eff}$ and $\sum m_\nu$	

Observables also depend on all other cosmological parameters

Cosmological Analysis by Planck (2015)

Correlations



Range of Bounds

Dependence on Data Samples

and Cosmological Model

Model	Observables	Σm_ν (eV) 95%
$\Lambda\text{CDM} + m_\nu$	Planck TT + lowP	≤ 0.72
$\Lambda\text{CDM} + m_\nu$	Planck TT + lowP + lensing	≤ 0.68
$\Lambda\text{CDM} + m_\nu$	Planck TT,TE,EE + lowP+lensing	≤ 0.59
$\Lambda\text{CDM} + m_\nu$	Planck TT,TE,EE + lowP	≤ 0.49
$\Lambda\text{CDM} + m_\nu$	Planck TT + lowP + lensing + BAO + SN + H_0	≤ 0.23
$\Lambda\text{CDM} + m_\nu$	Planck TT,TE,EE + lowP+ BAO	≤ 0.17

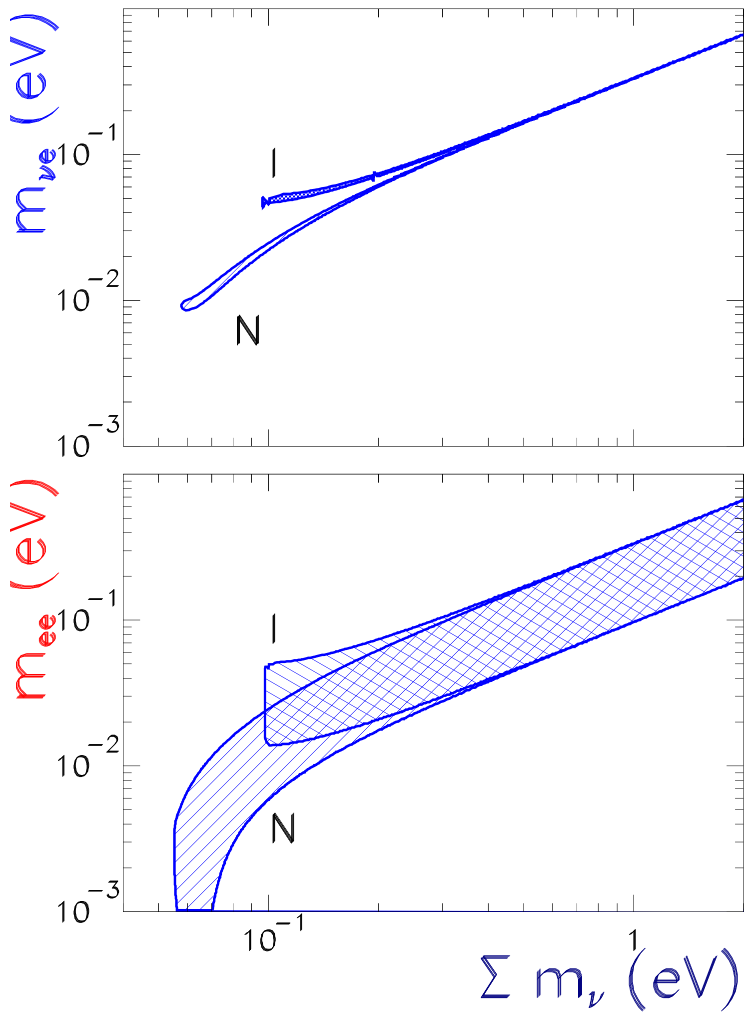
$N_{\rm eff} < 3.7$ (95% Planck TT+lowP+lensing+BAO)

Neutrino Mass Scale: The Cosmo-Lab Connection

Global oscillation analysis

⇒ Correlations m_{ν_e}, m_{ee} and $\sum m_\nu$
(Fogli *et al* (04))

Maltoni, Schwetz,Salvado, MCGG (95%)



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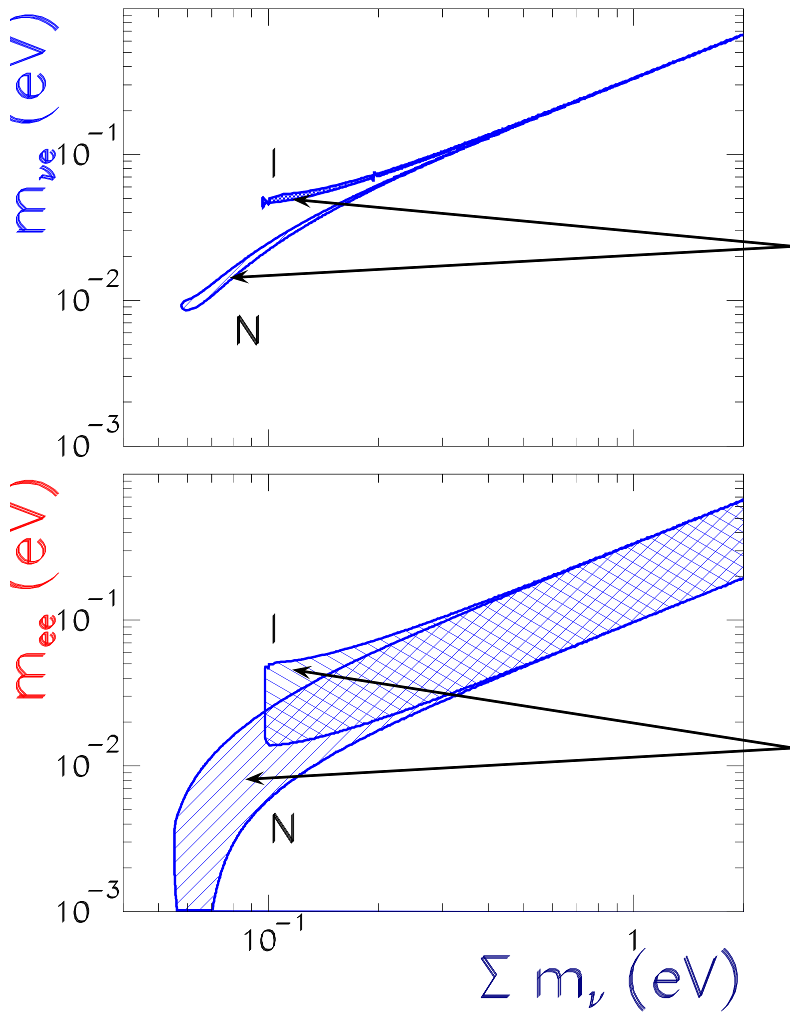


Width due to range in oscillation parameters very narrow

High precision determination of m_{ν_e} and $\sum m_i$ can give information on ordering

Wide band due to unknown Majorana phases

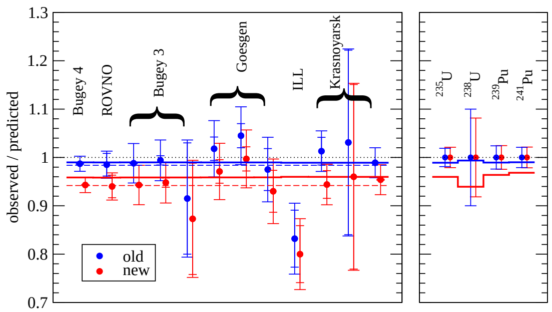
Light Sterile Neutrinos

- Several Observations which can be Interpreted as Oscillations with $\Delta m^2 \sim \text{eV}^2$

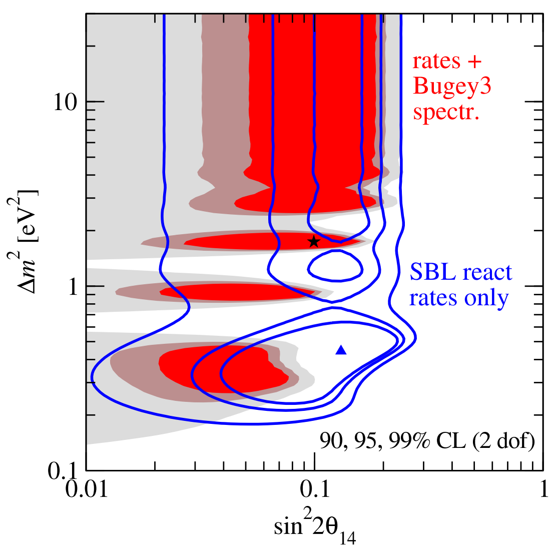
Reactor Anomaly

New reactor flux calculation

$\Rightarrow$ Deficit in data at $L \lesssim 100$ m



Explained as ν_e disappearance



Kopp etal, ArXiv 1303.3011

Gallium Anomaly

Acero, Giunti, Laveder, 0711.4222
Giunti, Laveder,1006.3244

Radioactive Sources (^{51}Cr, ^{37}Ar)

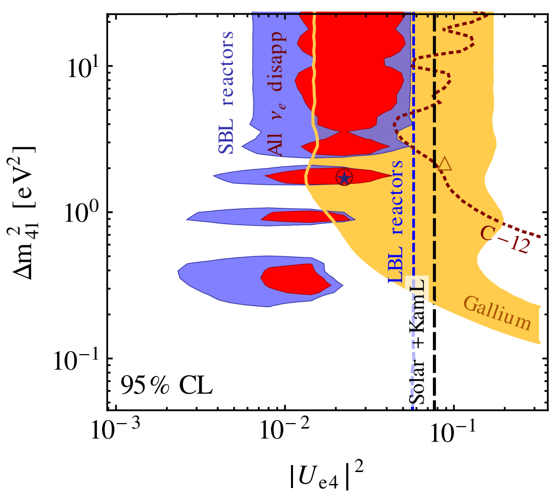
in callibration of Ga Solar Exp;

$\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$

Give a rate lower than expected

$$R = \frac{N_{\rm obs}}{N^{\rm th}_{\rm Bahc}} = 0.86 \pm 0.05 \; (2.8\sigma)$$

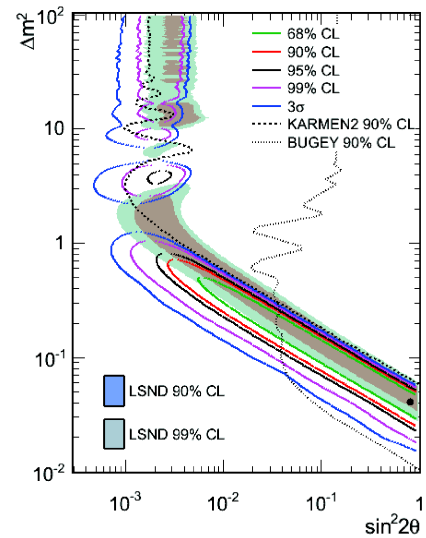
Explained as ν_e disappearance



Kopp etal, ArXiv 1303.3011

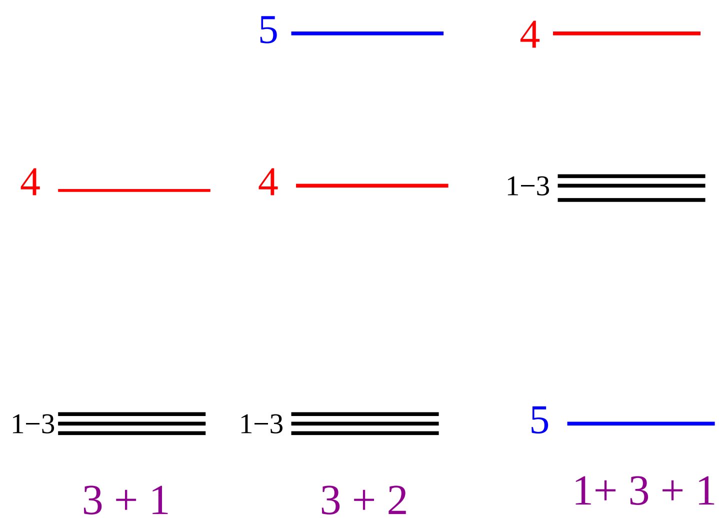
LSND, MiniBoone

$\nu_\mu \rightarrow \nu_e$ and $\bar\nu_\mu \rightarrow \bar\nu_e$



Light Sterile Neutrinos

- These explanations require 3+N_s mass eigenstates $\rightarrow$ N_s sterile neutrinos



- Problem: fit together
  - $\nu_e \rightarrow \nu_e$ disapp (REACT,Gallium,Solar, LSND/KARMEN)
  - $\nu_\mu \rightarrow \nu_e$ app (LSND,KARMEN,NOMAD,MiniBooNE,E776,ICARUS)
  - $\nu_\mu \rightarrow \nu_\mu$ disapp (CDHS,ATM,MINOS,MiniBooNE)

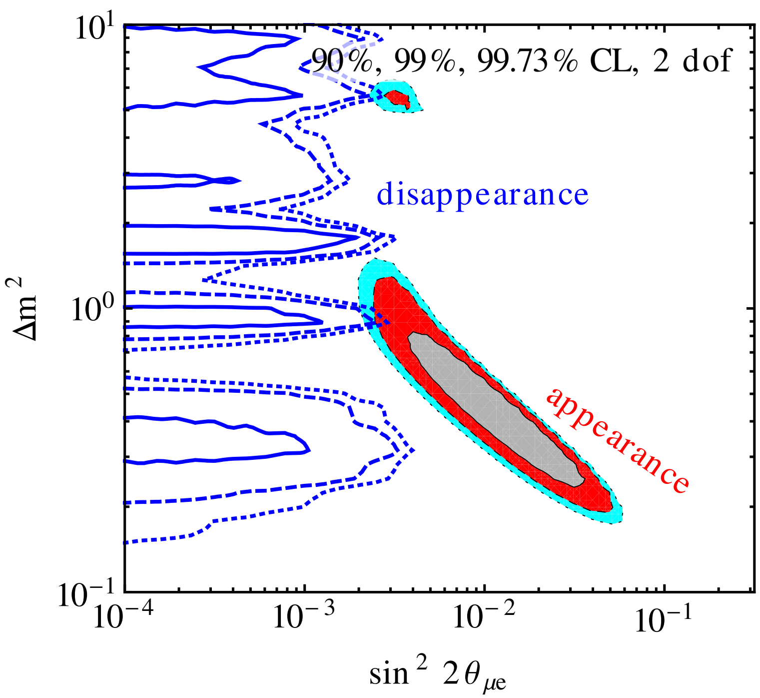
- Generically: $P(\nu_e \rightarrow \nu_\mu) \sim |U_{ei}^* U_{\mu i}|$ [i =heavier state(s)]

  But $|U_{ei}|$ constrained by $P(\nu_e \rightarrow \nu_e)$ disappearance data
  And $|U_{\mu i}|$ constrained by $P(\nu_\mu \rightarrow \nu_\mu)$ disappearance data
  $\Big\}\Rightarrow$ Severe tension

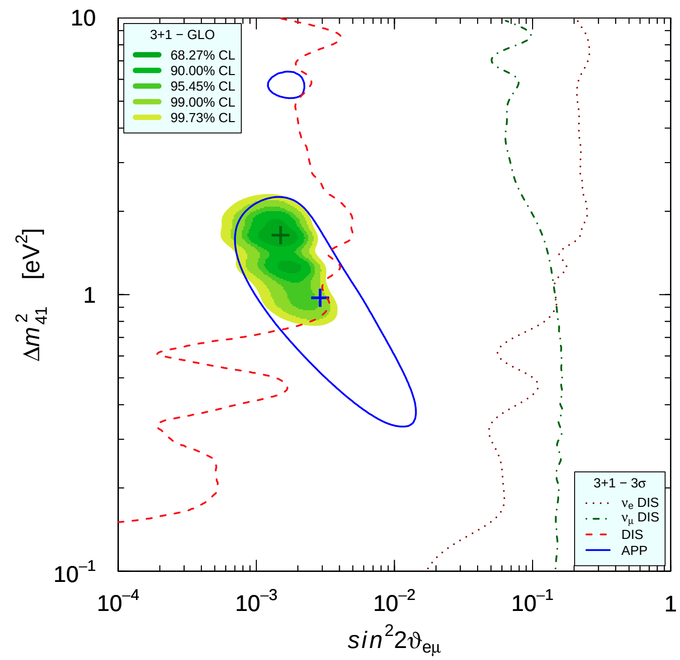
Light Sterile Neutrinos:3+1

- Comparing the parameters required to explain signals with bounds from disapp

Kopp etal, ArXiv 1303.3011



Giunti etal, ArXiv 1308.5288



– Difference in the analysis of both appearance and disappearance

– Somewhat different conclusions

- Adding more steriles (3+2 or 1+3+1): not much improvement

  Also tension with cosmology

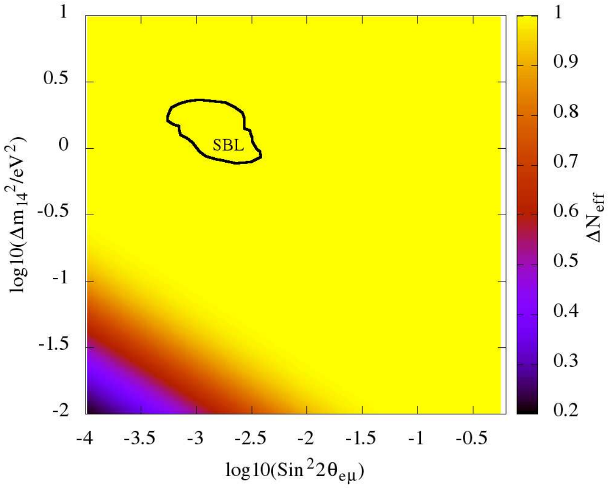
Light Sterile Neutrinos in Cosmology

One light ν_s mixed with 3 $\nu_a's$ contributes to ρ as N_{eff}.

From evol eq for $3+1$ esemble one finds

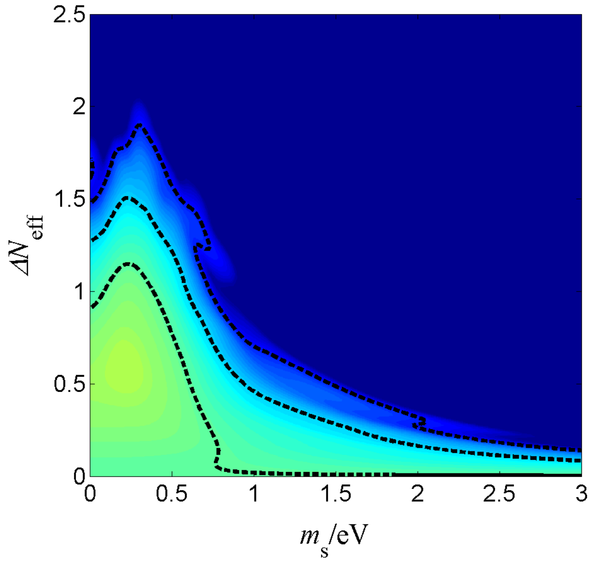
$\Rightarrow$ So if "explaination" to SBL anomalies

1 ν_s contributes as much as 1 ν_a



But analysis of cosmo data in ΛCDM+$r+\nu_s$ tells us

Plank+WP+high-l+BAO



J. Bergstrom, etal, ArXiv:1407.3806

Implications

The two arising questions

- Why are neutrinos so light?

  The Origin of Neutrino Mass

- Why are lepton mixing so different from quark's?

  The Flavour Puzzle

Implications: New Physics

A fermion mass can be seen as at a Left-Right transition

$$m_f \overline{f_L} f_R$$ (this is not $SU(2)_L$ gauge invariant)

If the SM is *the fundamental theory*:

– All terms in lagrangian (including masses) must be $\left\{ \begin{array}{l} \text{gauge invariant} \\ \text{renormalizable (dim} \leq 4\text{)} \end{array} \right.$

– A gauge invariant fermion mass is generated
by interaction with the Higgs field $\lambda_f \overline{f_L} \phi f_R \rightarrow m_f = \lambda_f v$
($v \equiv$ Higgs vacuum expectation value ~ 250 GeV)

– But there are no right-handed neutrinos
$\Rightarrow$ No renormalizable gauge-invariant operator for tree level ν mass

– SM gauge invariance also implies the accidental symmetry
$G_{\rm SM}^{\rm global} = U(1)_{\rm B} \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \Rightarrow m_\nu = 0$ *to all orders*

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Thus the most striking implication of ν masses:

There is New Physics Beyond the SM

To go further one has to make assumptions…

Light Neutrino Mass: Type I See-Saw

- Introduce ν_{R_i} $(i=1,m)$ and write all Lorentz and $SU(2)_L$ invariant mass term

$$\mathcal{L}_Y^{(\nu)} = -\lambda^\nu_{ij}\overline{\nu_{R,i}}L_{L,j}\tilde{\phi}^\dagger - \frac{1}{2}\overline{\nu_{R,i}}M^\nu_{N,ij}\nu^c_{R,j} + \text{h.c.}$$

- After spontaneous symmetry-breaking

$$\mathcal{L}^{(\nu)}_{\text{mass}} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2}\overline{\nu_R}M_N\nu^c_R + \text{h.c.} \equiv -\frac{1}{2}\overline{\vec{\nu}^c}M^\nu\vec{\nu} + \text{h.c.}$$

$$\text{with } \vec{\nu} = \begin{pmatrix}\nu_L \\ \nu^c_R\end{pmatrix} \text{ and } M^\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N\end{pmatrix}$$

- $\mathcal{L}^{(\nu)}_{\text{mass}} = -\sum_k \frac{1}{2}m_k\overline{\nu}^M_k\nu^M_k$ where $V^{\nu T}M^\nu V^\nu = \text{diag}(m_1, m_2, \ldots, m_{3+m})$

- In general if $M_N \neq 0 \Rightarrow$ 3+m Majorana neutrino states

$$\nu^M = V^{\nu\dagger}\nu_L + (V^{\nu\dagger}\nu_L)^c \quad \text{(verify } \nu^{M\,c}_i = \nu^M_i\text{)}$$

$\Rightarrow$ Total Lepton Number is not conserved

Type I See-Saw

- Add m ν_{R_i} so

$$\mathcal{L}^{(\nu)}_{\rm mass} = -\overline{\nu}_R M_D \nu_L - \frac{1}{2}\overline{\nu_R} M_N \nu^c_R + \text{h.c.} \equiv -\frac{1}{2}\overline{\vec{\nu}^c} M^\nu \vec{\nu} + \text{h.c.}$$

with $\vec{\nu} = \begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix}$ and $M^\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix}$

- Assume $M_N \gg m_D \Rightarrow$
  - 3 light neutrinos ν's of mass $m_{\nu_l} \simeq M_D^T M_N^{-1} M_D$
  - m Heavy ν's of mass $m_{\nu_H} \simeq M_N$
  - The heavier ν_H the lighter $\nu_l \Rightarrow$ See-Saw Mechanism

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  - The heavier ν_H the lighter ν_l $\Rightarrow$ See-Saw Mechanism
  - Natural explanation to $m_\nu \ll m_l, m_q$
  - Arises in many extensions of the SM: SO(10) GUTS, Left-right...

Light Neutrino Mass: Type II See-Saw

- Add a $SU(2)$ triplet Scalar $\Delta \equiv (1,3)_1$
- One can build a Gauge Invariant Yukawa Coupling

$$-\mathcal{L} = f_{\Delta ij}\overline{L_{Li}}\,\Delta\,L^C_{Lj} + h.c.$$

- The scalar potential:

$$V(\phi,\Delta) = \lambda\,|\phi|^4 - \mu^2\,|\phi|^2 + M^2_\Delta\,|\Delta|^2 + (\kappa\,\phi^T\,\Delta^\dagger\,\phi + h.c.)$$

it is minimum at $\langle\phi\rangle = \frac{v}{\sqrt{2}} = \frac{\mu}{\sqrt{2\lambda}}$ and $\langle\Delta\rangle = \frac{\kappa\,v^2}{2M^2_\Delta}$

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$\Rightarrow$ $M_\nu = f_\Delta \frac{\kappa\, v^2}{M^2_\Delta}$ The heavier Δ the lighter ν_L $\Rightarrow$ See-Saw Mechanism

- If $M^2_\Delta/\kappa \gg v$ $\langle\Delta\rangle \ll v \Rightarrow$ Natural explanation to $m_\nu \ll m_l, m_q$

ν Mass from Non-Renormalizable Operator

If SM is an effective low energy theory, for $E \ll \Lambda_{\rm NP}$

– The same particle content as the SM and same pattern of symmetry breaking

– But there can be non-renormalizable (dim$>$ 4) operators

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_n \frac{1}{\Lambda_{\rm NP}^{n-4}} \mathcal{O}_n$$

First NP effect $\Rightarrow$ dim=5 operator

There is only one!

$$\mathcal{L}_5 = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} \left(\overline{L_{L,i}} \tilde{\phi}\right) \left(\tilde{\phi}^T L^C_{L,j}\right)$$

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which after symmetry breaking

induces a ν Majorana mass

$$(M_\nu)_{ij} = Z^\nu_{ij} \frac{v^2}{\Lambda_{\rm NP}}$$

Implications:

– It is natural that ν mass is the first evidence of NP

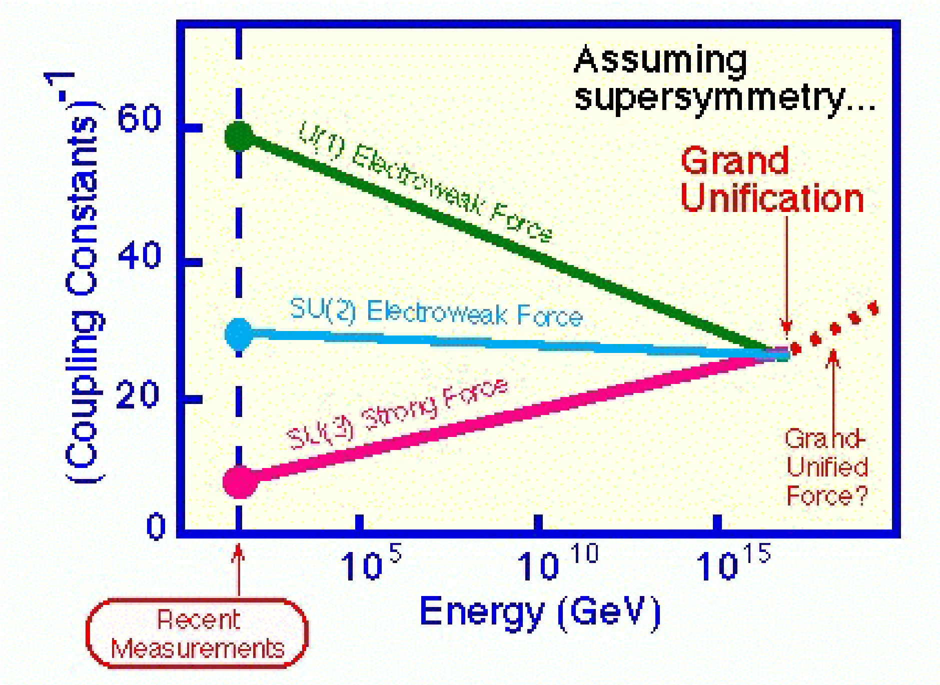
– Naturally $m_\nu \ll$ other fermions masses $\sim \lambda^f v$ if $\Lambda_{\rm NP} >> v$

– See-saw with heavy fermions or scalar integrated out is a particular example of this

Implications: The Scale of New Physics

$$m_\nu > \sqrt{\Delta m^2_{\rm atm}} \sim 0.05\text{eV} \Rightarrow 10^{10} < \Lambda_{\rm NP} < 10^{15}\text{GeV}$$

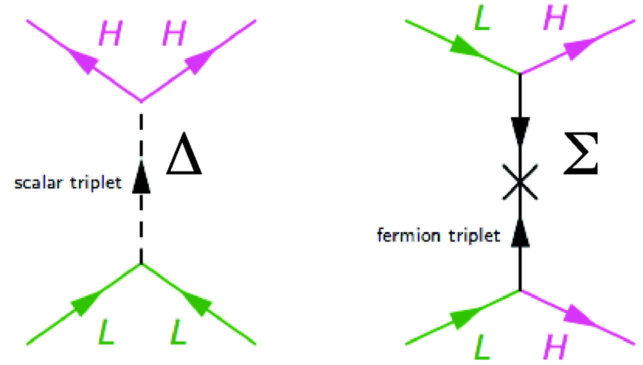
New Physics Scale close to Grand Unification scale



Λ_{NP} in See-Saw Models

Mohapatra,Senjanovich; Foot,Lew,He,Joshi

$\mathcal{O}_5$ can be generated by tree-level exchange of singlet ($N_i \equiv (1,1)_0$) (Type-I) or triplet fermions ($N_i \equiv \Sigma_i \equiv (1,3)_0$) (Type-III) or a scalar triplet $\Delta \equiv (1,3)_1$ (Type-II)



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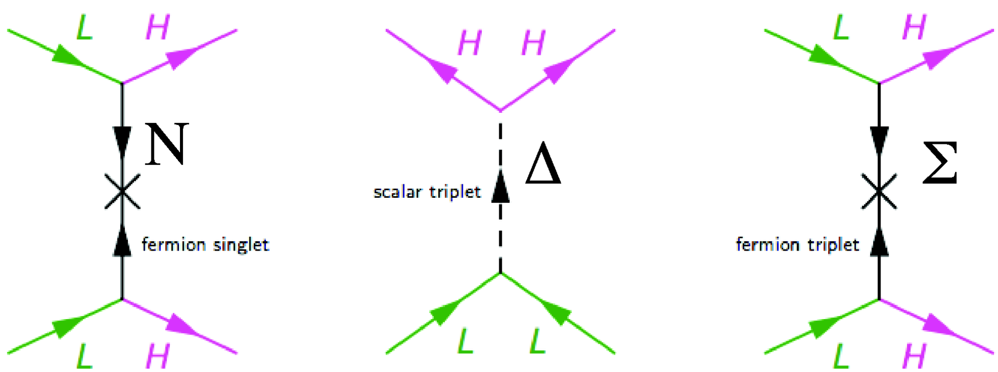

•For fermionic see-saw

$$-\mathcal{L}_{\rm NP} = -i\overline{N_i}\not{D}N_i + \frac{1}{2}M_{N ij}\overline{N_i^c}N_j + \lambda^\nu_{\alpha j}\overline{L_\alpha}\tilde{\phi}N_j[.\tau]$$

$$\Rightarrow \mathcal{O}_5 = \frac{(\lambda^{\nu T}\lambda^\nu)_{\alpha\beta}}{\Lambda_{\rm NP}}\left(\overline{L_\alpha}\tilde{\phi}\right)\left(\tilde{\phi}^T L^C_\beta\right) \text{ with } \Lambda_{\rm NP} = M_N$$

• For scalar see-saw

$$-\mathcal{L}_{\rm NP} = f_{\Delta\alpha\beta}\overline{L_\alpha}\,\Delta\,L^C_\beta + M^2_\Delta\,|\Delta|^2 + \kappa\,\phi^T\,\Delta^\dagger\,\phi\ \ldots$$

$$\Rightarrow \mathcal{O}_5 = \frac{f_{\Delta\alpha\beta}}{\Lambda_{NP}}\left(\overline{L_\alpha}\tilde{\phi}\right)\left(\tilde{\phi}^T L^C_\beta\right) \text{ with } \Lambda_{\rm NP} = \frac{M^2_\Delta}{\kappa}$$

Very different physics, but same ν parameters: How to proceed?

Λ_{NP} in See-Saw Models

Mohapatra,Senjanovich; Foot,Lew,He,Joshi

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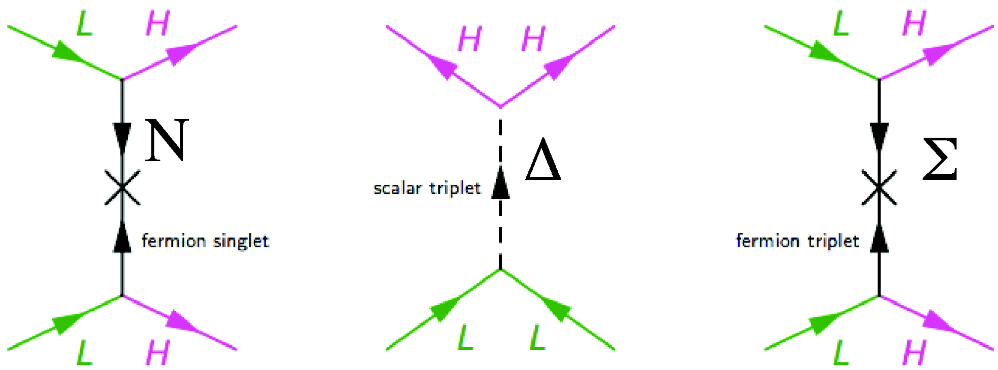


How to proceed?

– Top-down: Assume some specific model and work out the relations

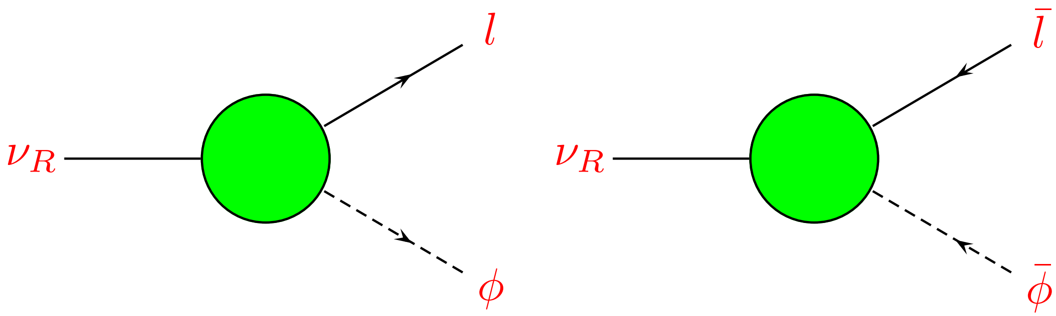
– Still Bottom-up: Hope for additional information from charged LFV, collider signals . . .

Implications: *We are here*

Implications: *We are here* (Leptogenesis)

- Majorana $m_\nu \Rightarrow \not{L} \Rightarrow$ Baryon asymmetry can be generated

- How? In the Early Universe via decay of heavy N Fukugita and Yanagida



– If $\not{C}$P : $\Gamma(N \to \phi\, l_L) \neq \Gamma(N \to \overline{\phi}\, \overline{l_L})$

– And decay is out of equilibrium:

($\Gamma_N \ll$ Universe expansion rate)

} Δ L is generated

Sphaleron processes $\Rightarrow$ Δ L is transformed in $\Delta B \simeq -\frac{\Delta L}{2}$

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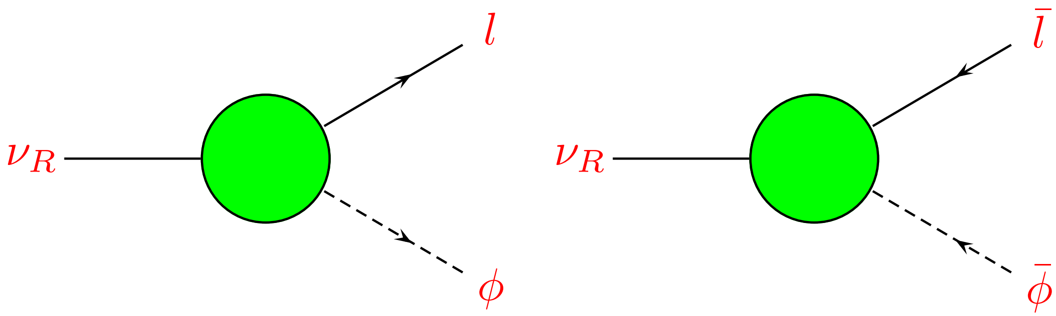

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} Δ L is generated

Sphaleron processes $\Rightarrow$ Δ L is transformed in $\Delta B \simeq -\frac{\Delta \mathrm{L}}{2}$

- Details are model dependent

  In simplest scenario $M \gtrsim 10^{10}$ GeV, $\sum m_\nu \lesssim 0.5$ eV

  But also scenarios for leptogenesis with M ∼O(TeV) $\Rightarrow$ collider signals

Summary

- Neutrino oscillation searches have shown us

$\Delta m^2_{21} = 7.50 \times 10^{-5}\ \text{eV}^2\ (2.3\%) \quad |\Delta m^2_{3\ell}| = 2.45 \times 10^{-3}\ \text{eV}^2 \quad (1.9\%)$

$\sin^2\theta_{12} = 0.3\ (4\%) \qquad \sin^2\theta_{23} = 0.58\ [0.44]\ \text{IO}\ [\text{NO}]\ (8.5\%) \qquad \sin^2\theta_{13} = 0.0219\ (4.8\%)$

$\Rightarrow U_{\rm LEP}$ Very different from $U_{\rm CKM}$

- Still ignore or not significantly determined

Majorana/Dirac? m_ν scale leptonic $\not{C}$P? Normal/Inverted?

Standing Puzzles: SBL anomalies light sterile ν's?

$\Rightarrow$ New experiments needed to answer these questions

- $m_\nu \neq 0 \Rightarrow$ Need to extend SM

NP *breaking* total L $\longrightarrow$ Majorana $\nu : \nu = \nu^C$

NP *conserving* total L $\longrightarrow$ Dirac $\nu : \nu \neq \nu^C$

- Majorana $\nu' s$: *generic* if SM is LE effective theory and explain ν lightness

$\Lambda_{NP} \sim 10^{15}$ GeV Fits OK in GUT

Leptogenesis may explain the baryon asymmetry

Possible scenarios with $\Lambda_{NP} \sim \mathcal{O}(\text{TeV})$ reachable at LHC

ν masses are BSM physics effects to be put together with *all other NP effects*:
from charged LFV, Collider signals, Cosmo-astroparticle. . . to establish
the Next Standard Model

Comment on Theoretical Uncertainties

- Flux Uncertainties:

(1) Total normalization: $\sigma_{\rm norm} = 20\%$

(2) "Tilt" error

$\Phi_\delta(E) = \Phi_0(E) \left(\frac{E}{E_0}\right)^\delta$

$\sigma_\delta = 5\%$ $E_0 = 2$ GeV

(3) ν_μ/ν_e ratio: $\sigma_{\mu/e} = 5\%$

E independent for contained events

(4) Zenith angle dependence:

$\sigma_{\rm zen,i} = 5\% \langle \cos\theta \rangle_i$

- Cross Section Uncertainties:

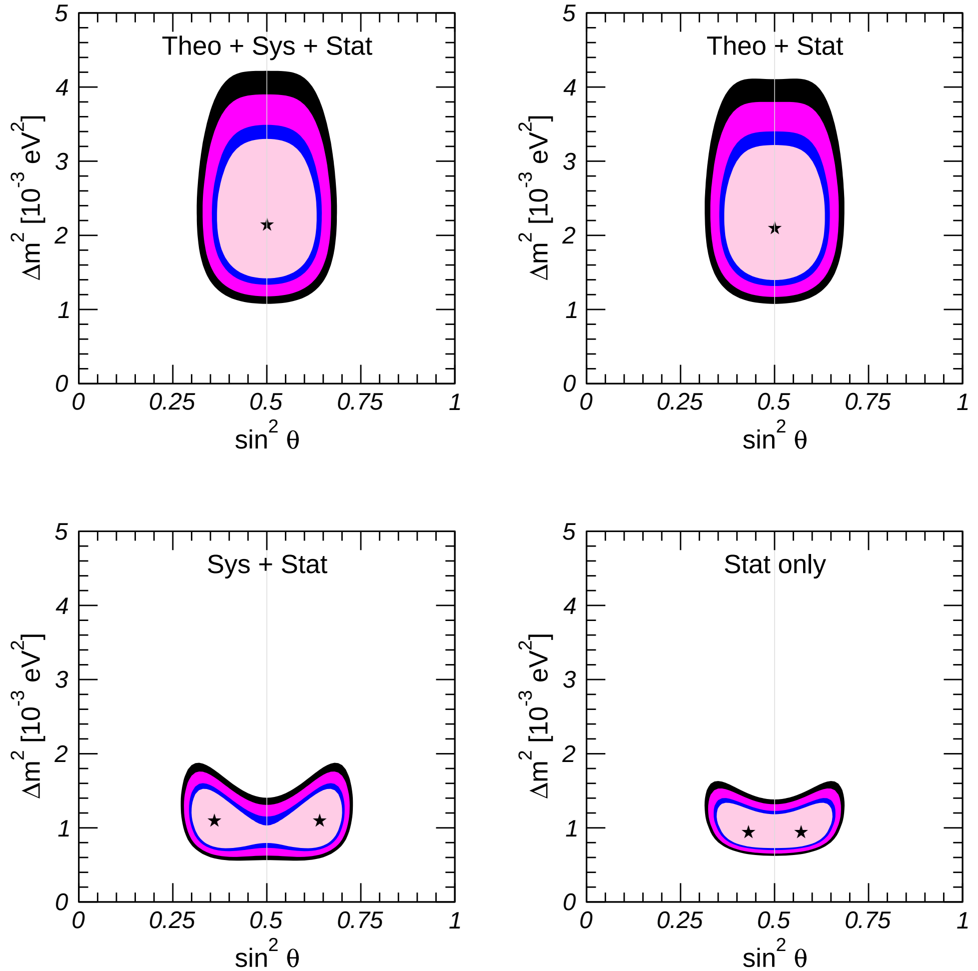
(5) $\sigma^{\sigma_{\rm QE}}_{\rm norm} = 15\%$

(6) $\sigma^{\sigma_{1\pi}}_{\rm norm} = 15\%$,

(7) $\sigma^{\sigma_{\rm DIS}}_{\rm norm} = 15\%$ for contained

$\sigma^{\sigma_{\rm DIS}}_{\rm norm} = 10\%$ for upward-going μ

(8)–(10) $\sigma^{{\rm QE},1\pi,{\rm DIS}}_{i,\nu_\mu}/\sigma^{{\rm QE},1\pi,{\rm DIS}}_{i,\nu_e} = 0.1$–$1\%$



Neutrinos in Matter:Effective Potentials

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- In SM the characteristic ν-p interaction cross section

$$\sigma \sim \frac{G_F^2 E^2}{\pi} \sim 10^{-43}\mathrm{cm}^2 \quad \text{at } \mathrm{E}_\nu \sim \mathrm{MeV}$$

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  so it seems that for neutrinos *matter does not matter*

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  Interference of scattered and unscattered ν waves

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• Coherence ⇒ decoupling of ν evolution equation from eqs of medium.

• The effect of the medium is described by an effective potential depending on density and composition of matter

- Lets consider ν_e in a medium with e, p, and n. The effective low-energy Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}}[J^{(+)\alpha}(x)J_\alpha^{(-)}(x) + \frac{1}{4}J^{(N)\alpha}(x)J_\alpha^{(N)}(x)]$$

CC Int $J_\alpha^{(+)}(x) = \overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)e(x)$ $J_\alpha^{(-)}(x) = \overline{e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x)$

NC Int $J_\alpha^{(N)}(x) = \overline{\nu_e}(x)\gamma_\alpha(1-\gamma_5)\nu_e(x) - \overline{e}(x)[\gamma_\alpha(1-\gamma_5) - s_W^2\gamma_\alpha]e(x)$

$$+\overline{p}(x)[\gamma_\alpha(1-g_A^{(p)}\gamma_5) - 4s_W^2\gamma_\alpha]p(x) - \overline{n}(x)[\gamma_\alpha(1-g_A^{(n)}\gamma_5) - 4s_W^2\gamma_\alpha]n(x)$$

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- Example: The effect of CC with the e medium. The effective CC Hamiltonian:

$$H_C^{(e)} = \frac{G_F}{\sqrt{2}}\int d^3p_e f(E_e,T)\Big\langle\langle e(s,p_e)|\overline{e}\gamma^\alpha(1-\gamma_5)\nu_e\overline{\nu_e}\gamma_\alpha(1-\gamma_5)|e(s,p_e)\rangle\Big\rangle$$

Fierz rearrange

$$= \frac{G_F}{\sqrt{2}}\overline{\nu_e}\gamma_\alpha(1-\gamma_5)\nu_e\int d^3p_e f(E_e,T)\Big\langle\langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle\Big\rangle$$

$f(E_e,T)$ statistical energy distribution of e in *homogeneous and isotropic* medium.

$$\int d^3p_e f(E_e,T) = 1$$

$\Big\langle \ldots \Big\rangle \equiv$ averaging over electron spinors and summing over all e.

coherence $\Rightarrow s, p_e$ same for initial and final e

- Expanding the electron fields e in plane waves

$$\langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle = \frac{1}{V}\langle e(s,p_e)|\overline{u_s}(p_e)a_s^\dagger(p_e)\gamma_\alpha(1-\gamma_5)a_s(p_e)u_s(p_e)|e(s,p_e)\rangle$$

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• Since $a_s^\dagger(p_e)a_s(p_e) = \mathcal{N}_e^{(s)}(p_e)$ (number operator) and assuming that there are the same number of electrons with spin 1/2 and -1/2

$$\frac{1}{V}\left\langle \langle e(s,p_e)|a_s^\dagger(p_e)a_s(p_e)|e(s,p_e)\rangle \right\rangle \equiv N_e(p_e)\frac{1}{2}\sum_s$$

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$$\left\langle \langle e(s,p_e)|\overline{e}\gamma_\alpha(1-\gamma_5)e|e(s,p_e)\rangle \right\rangle = \frac{N_e(p_e)}{2}\sum_s \overline{u_{(s)}}(p_e)\gamma_\alpha(1-\gamma_5)u_{(s)}(p_e)$$

$$= \frac{N_e(p_e)}{2}\sum_s Tr\left[\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)u_s(p_e)\right] = \frac{N_e(p_e)}{2}\sum_s Tr\left[u_s(p_e)\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)\right]$$

$$= \frac{N_e(p_e)}{2} Tr\sum_s\left[u_s(p_e)\overline{u_s}(p_e)\gamma_\alpha(1-\gamma_5)\right] = \frac{N_e(p_e)}{2} Tr\left[\frac{m_e+\not{p}}{2E_e}\gamma_\alpha(1-\gamma_5)\right] = N_e(p_e)\frac{p_e^\alpha}{E_e}$$

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• Isotropy $\Rightarrow \int d^3p_e\vec{p_e}f(E_e,T) = 0$

• Also $\int d^3p_e f(E_e,T)N_e(p_e) = N_e$ electron number density

- The effective charged current Hamiltonian due to electrons in matter is then:

$$H_C^{(e)} = \frac{G_F N_e}{\sqrt{2}} \overline{\nu_e}(x)\gamma_0(1-\gamma_5)\nu_e(x)$$

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$$\begin{aligned} V_C &= \langle \nu_e | \int d^3x H_C^{(e)} | \nu_e \rangle \\ &= \frac{G_F N_e}{\sqrt{2}} \langle \nu_e | \int d^3x \overline{\nu_e}(x)\gamma_0(1-\gamma_5)\nu_e(x) | \nu_e \rangle \\ &= \frac{G_F N_e}{\sqrt{2}} \frac{1}{V} 2 \int d^3x \; u_\nu^\dagger u_\nu = \sqrt{2} G_F N_e \end{aligned}$$

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$$V_C = \sqrt{2} G_F N_e$$

- for $\overline{\nu_e}$ the sign of V is reversed

- Other potentials for ν_e $(\overline{\nu}_e)$ due to different particles in medium

medium	V_C	V_N
e^+ and e^-	$\pm\sqrt{2}G_F(N_e - N_{\bar{e}})$	$\mp\frac{G_F}{\sqrt{2}}(N_e - N_{\bar{e}})(1 - 4\sin^2\theta_W)$
p and $\bar{p}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_p - N_{\bar{p}})(1 - 4\sin^2\theta_W)$
n and $\bar{n}$	0	$\mp\frac{G_F}{\sqrt{2}}(N_n - N_{\bar{n}})$
Neutral $(N_e = N_p)$	$\pm\sqrt{2}G_F N_e$	$\mp\frac{G_F}{\sqrt{2}}N_n$

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• Estimating typical values:

$$V_C = \sqrt{2}G_F N_e \simeq 7.6\, Y_e \frac{\rho}{10^{14}\text{g/cm}^3}\ \text{eV}$$

$Y_e = \frac{N_e}{N_p+N_n} \equiv$ relative number density

$\rho \equiv$ matter density

– At the solar core $\rho \sim 100$ g/cm^3 $\Rightarrow$ $V \sim 10^{-12}$ eV

– At supernova $\rho \sim 10^{14}$ g/cm^3 $\Rightarrow$ $V \sim$ eV

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- Evolution of Φ is given by the Dirac Equations [$\beta = \gamma_0$, $\alpha_x = \gamma_0\gamma_x$ (assuming 1 dim)]

$$E_1\,\Phi_1 = \left[-i\,\alpha_x \frac{\partial}{\partial x} + \beta\, m_1\right]\Phi_1$$
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- We decompose $\Phi_i(x) = \nu_i(x)\phi_i$ ϕ_i is the Dirac spinor part satisfying:

$$\left(\alpha_x\left\{E_i^2 - m_i^2\right\}^{1/2} + \beta\,m_i\right)\phi_i = E\phi_i \qquad (1)$$

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- ϕ_i have the form of free spinor solutions with energy E_i
- Using (1) in Dirac Eq. we can factorize ϕ_i and α_x and get:

$$-i\frac{\partial\nu_1(x)}{\partial x} = \left\{E_1^2 - m_1^2\right\}^{1/2} \nu_1(x)$$
$$-i\frac{\partial\nu_2(x)}{\partial x} = \left\{E_2^2 - m_2^2\right\}^{1/2} \nu_2(x)$$

- In the relativistic limit and first order in mass $\sqrt{E^2 - m_i^2} \simeq E - \frac{m_i^2}{2E}$

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix} E - \frac{m_1^2}{2E} & 0 \\ 0 & \frac{E - m_2^2}{2E}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

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• In weak ($\equiv$ flavour) basis $\nu_\alpha = U_{\alpha i}(\theta)\nu_i$

$$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = \left[E - \frac{m_1^2+m_2^2}{2E}\right] I - \begin{pmatrix}-\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

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$$\Rightarrow \quad -i\frac{\partial}{\partial x}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = -\begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}$$

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Can be rewritten as

$$\begin{aligned} \ddot{\nu}_\alpha + \omega^2 \nu_\alpha &= 0 \\ \ddot{\nu}_\beta + \omega^2 \nu_\beta &= 0 \end{aligned} \qquad \text{with} \qquad \omega = \frac{\Delta m^2}{4E}$$

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- And the flavour transition probability

$$P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(L)|^2 = B_1^2 + B_2^2 + 2B_1 B_2 \cos(2\omega L) = \sin^2(2\theta)\,\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Neutrinos in Matter: Evolution Equation

Evolution Eq. for $|\nu\rangle = \nu_1|\nu_1\rangle + \nu_2|\nu_2\rangle \equiv \nu_e|\nu_e\rangle + \nu_X|\nu_X\rangle$ $(X = \mu, \tau, \text{sterile})$

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(a) In vacuum in the mass basis: $$-i\frac{\partial}{\partial x}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = E - \begin{pmatrix}\frac{m_1^2}{2E} & 0\\ 0 & \frac{m_2^2}{2E}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

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(b) In vacuum in the weak basis

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(c) In matter (e, p, n) in weak basis

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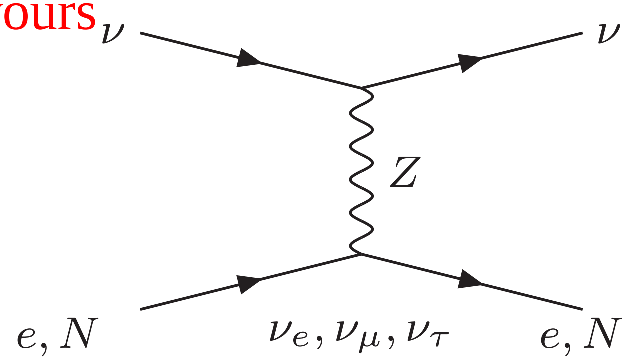
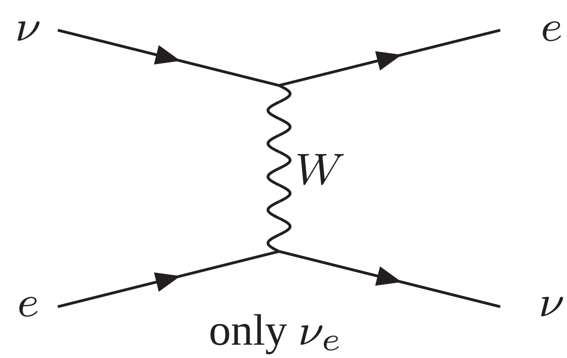
(c)$\neq$ (b) because different flavours have different interactions

For example $X = \mu, \tau$:

$$V_{CC} = V_e - V_X = \sqrt{2}G_F N_e$$

(opposite sign for $\overline{\nu}$)





$\Rightarrow$ Effective masses and mixing are different than in vacuum

$\Rightarrow$ If matter density varies along ν trajectory the effective masses and mixing vary too

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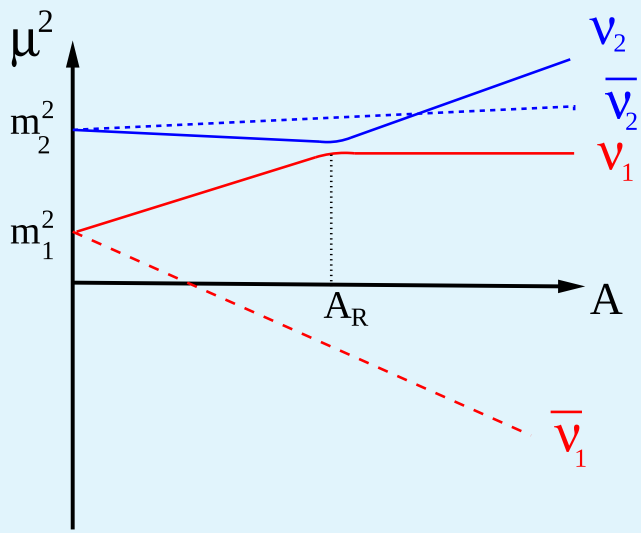
The effective masses: ($A = 2E(V_e - V_X)$)

$$\mu_{1,2}(x) = \frac{m_1^2 + m_2^2}{2} + E(V_e + V_X)$$
$$\pm \frac{1}{2}\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}$$



At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

Minimum $\Delta\mu^2 = \mu_2^2 - \mu_1^2$

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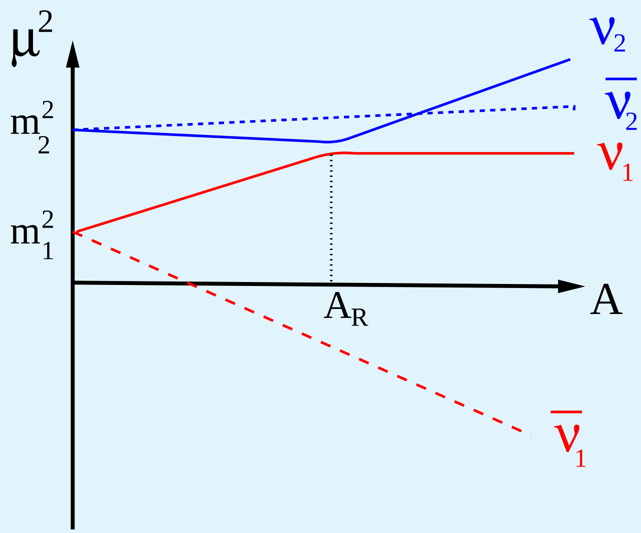

At *resonant* potential: $A_R = \Delta m^2 \cos 2\theta$

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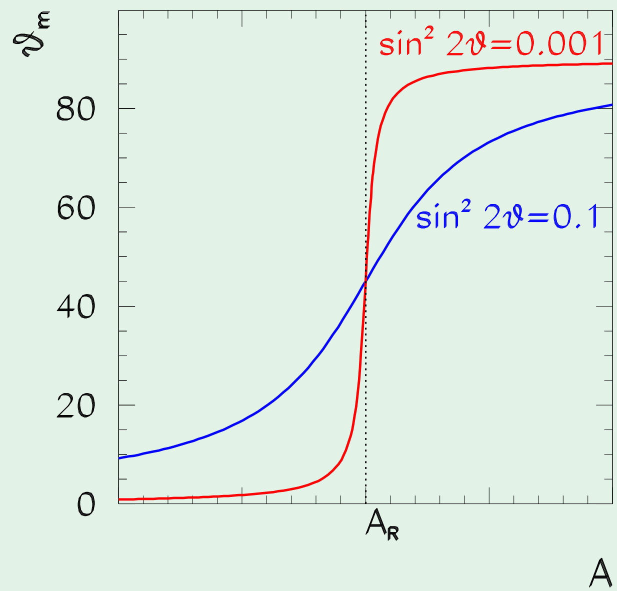
The mixing angle in matter

$$\tan 2\theta_m = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - A}$$



- $*$ At $A = 0$ (vacuum) $\Rightarrow \theta_m = \theta$
- $*$ At $A = A_R \Rightarrow \theta_m = \frac{\pi}{4}$
- $*$ At $A >> A_R \Rightarrow \theta_m \rightarrow \frac{\pi}{2}$

The oscillation length in vacuum

$$L_0^{osc} = \frac{4\pi E}{\Delta m^2}$$

The oscillation length in matter

$$L^{osc} = \frac{L_0^{osc}}{\sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2 \sin 2\theta)^2}} \equiv \frac{4\pi E}{\Delta \mu^2}$$

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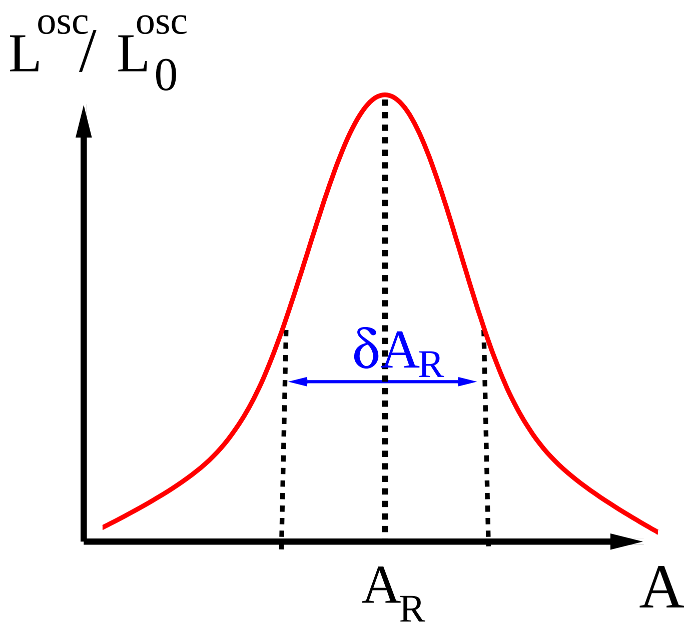
L^{osc} presents a resonant behaviour



At the resonant point

$$L_R^{osc} = \frac{L_0^{osc}}{\sin 2\theta}$$

The width of the resonance in potential:

$$\delta V_R = \frac{\Delta m^2 \sin 2\theta}{E}$$

The width of the resonance in distance:

$$\delta r_R = \frac{\delta V_R}{|\frac{dV}{dr}|_R}$$

- In terms of the mass eigenstates in matter: $$\begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix} = U[\theta_m(x)] \begin{pmatrix} \nu_1^m(x) \\ \nu_2^m(x) \end{pmatrix}$$

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- For constant potential θ_m and μ_i are constant along ν evolution

  $\Rightarrow$ the evolution is determined by masses and mixing in matter as

$$P_{osc} = \sin^2(2\theta_m)\,\sin^2\left(\frac{\Delta\mu^2 L}{2E}\right)$$

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$\Rightarrow$ the evolution equation in flavour basis (removing diagonal part)

$$i \begin{pmatrix} \dot\nu_e \\ \dot\nu_X \end{pmatrix} = \frac{1}{2E} \begin{pmatrix} A - \frac{\Delta m^2}{2}\cos 2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & \frac{\Delta m^2}{2}\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_X \end{pmatrix}$$

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$\Rightarrow$ the evolution equation in instantaneous mass basis

$$i\begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{2E}U^\dagger(\theta_m)\begin{pmatrix} A - \frac{\Delta m^2}{2}\cos 2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & \frac{\Delta m^2}{2}\cos 2\theta \end{pmatrix} U(\theta_m)\begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix} - i\,U^\dagger\dot U(\theta_m)\begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

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$$\Rightarrow\quad i \begin{pmatrix} \dot\nu_1^m \\ \dot\nu_2^m \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\Delta\mu^2(x) & -4\,i\,E\,\dot\theta_m(x) \\ 4\,i\,E\,\dot\theta_m(x) & \Delta\mu^2(x) \end{pmatrix} \begin{pmatrix} \nu_1^m \\ \nu_2^m \end{pmatrix}$$

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The adiabaticity condition

$$\frac{1}{V}\frac{dV}{dx}\Big|_R \ll \frac{\Delta m^2}{2E}\frac{\sin^2 2\theta}{\cos 2\theta} \equiv 2\pi\,\delta r_R \gg L_R^{osc}$$

$\Rightarrow$ Many oscillations take place in the resonant region

Neutrinos in The Sun : MSW Effect

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- Solar neutrinos are ν_e produced in the core ($R \lesssim 0.3 R_\odot$) of the Sun

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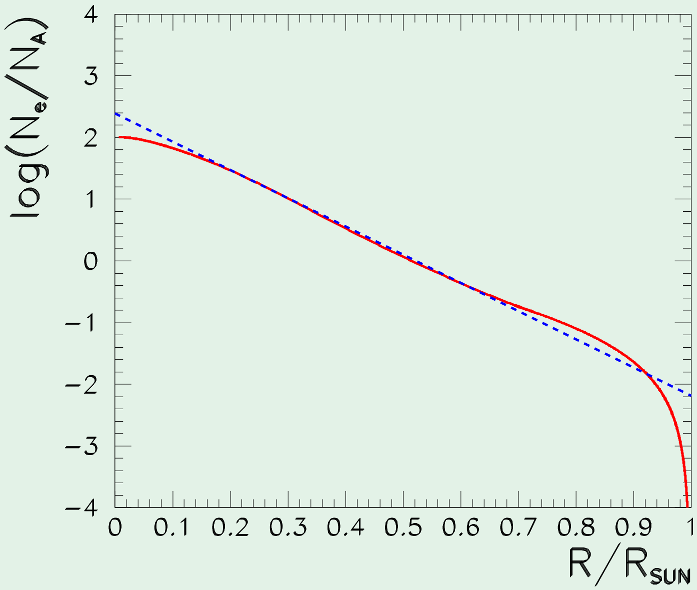
The solar matter density



$$V_{CC} = \sqrt{2} G_F N_e \sim 10^{-14} \frac{N_e}{N_A} \text{ eV}$$

At core: $V_{CC,0} \sim 10^{-14}$–10^{-12} eV

Neutrinos in The Sun : MSW Effect

- Solar neutrinos are ν_e produced in the core ($R \lesssim 0.3 R_\odot$) of the Sun

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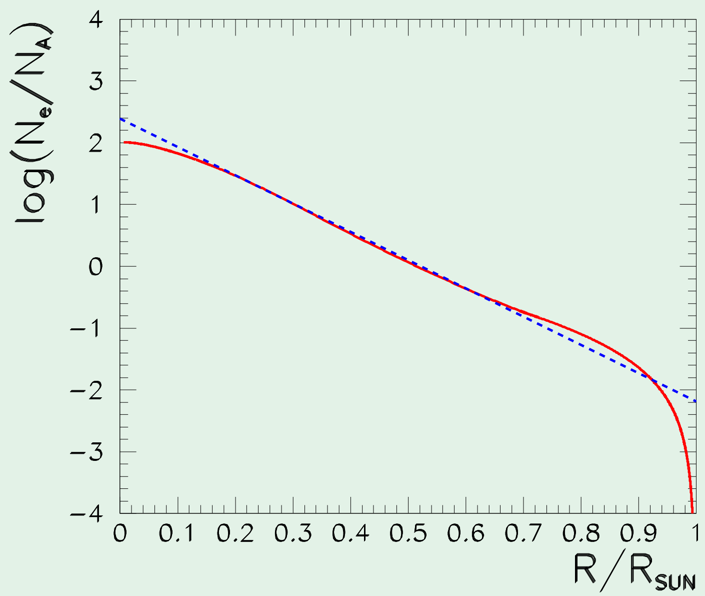

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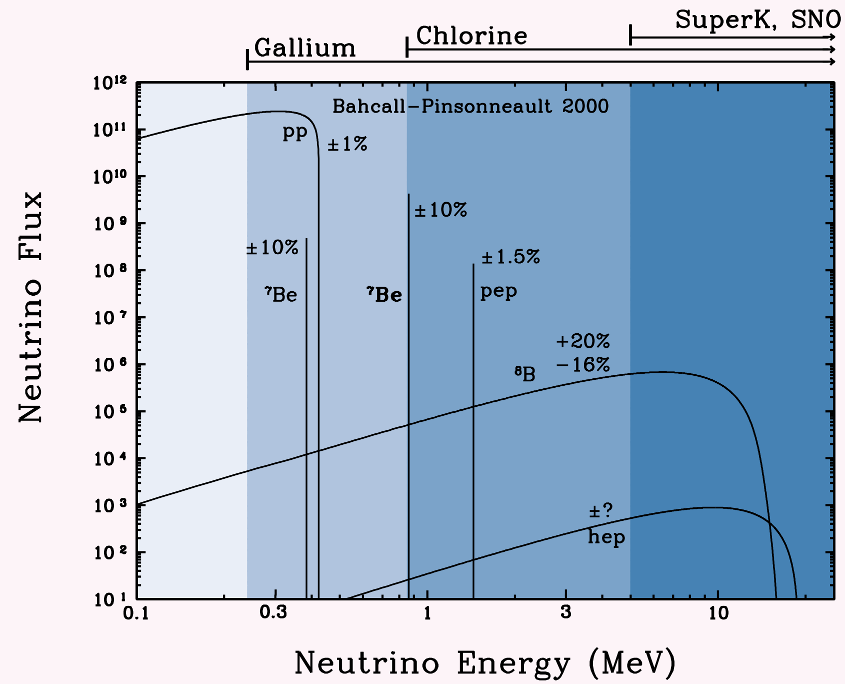

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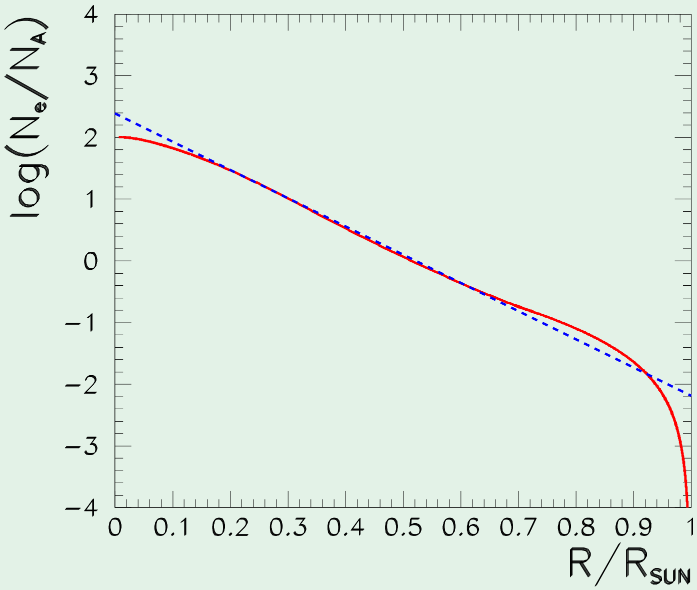

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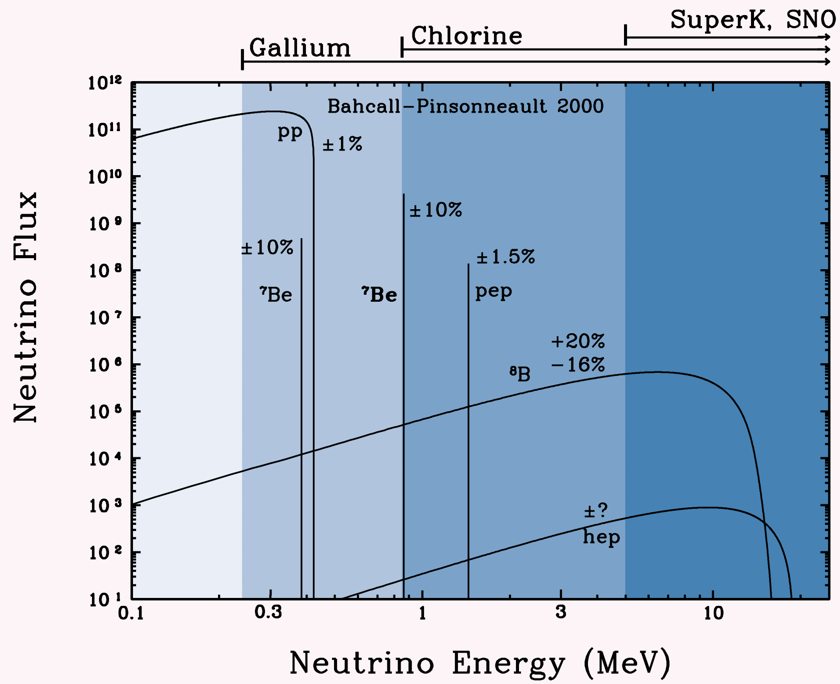

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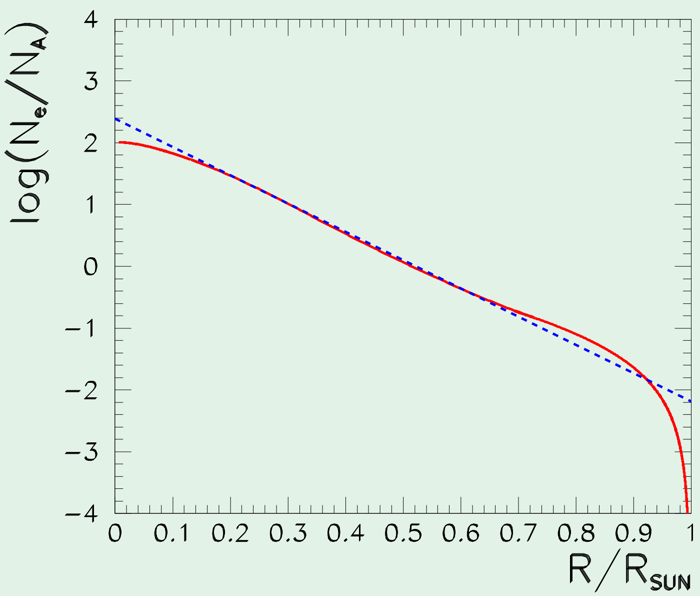

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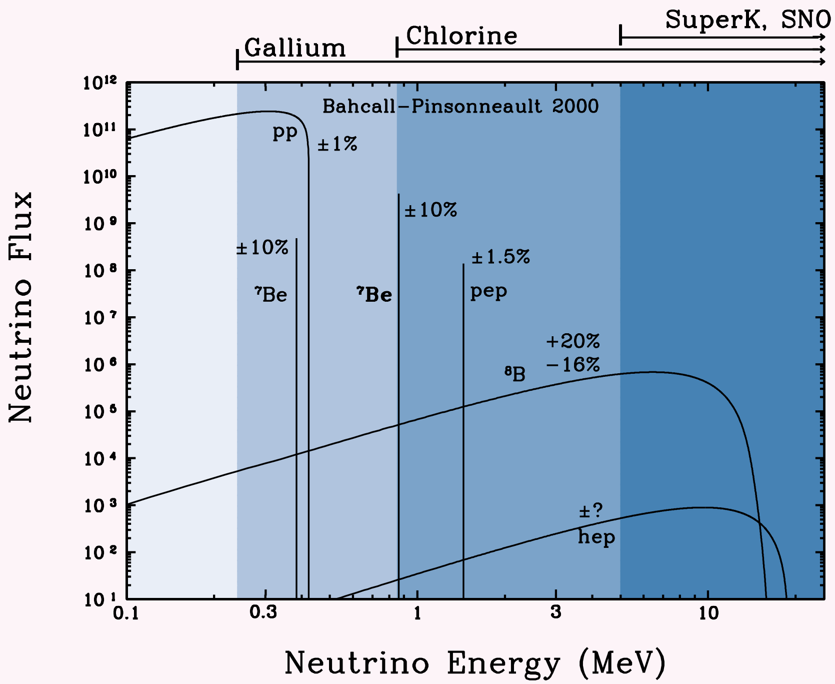

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$\Rightarrow$ ν can cross resonance condition in its way out of the Sun

For $\theta \ll \frac{\pi}{4}$: In vacuum $\nu_e = \cos\theta\,\nu_1 + \sin\theta\,\nu_2$ is mostly ν_1

In Sun core $\nu_e = \cos\theta_{m,0}\,\nu_1 + \sin\theta_{m,0}\,\nu_2$ is mostly ν_2

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If $\frac{(\Delta m^2/\mathrm{eV}^2)\sin^2 2\theta}{(E/\mathrm{MeV})\cos 2\theta} \gg 3\times 10^{-9}$

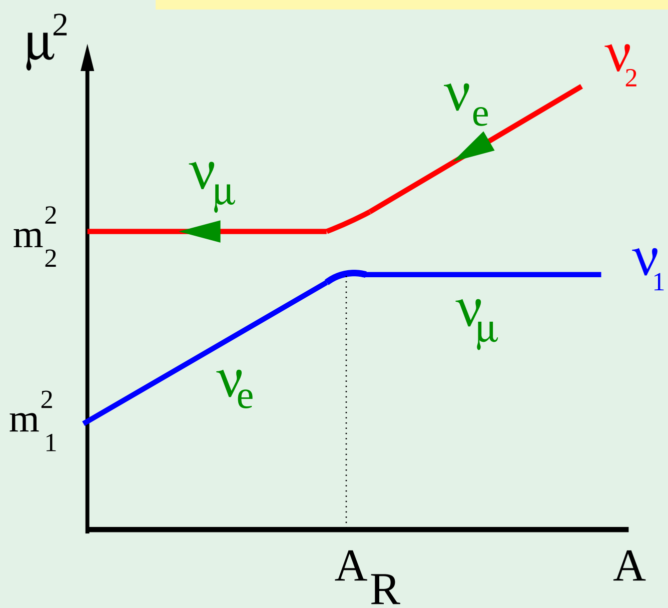
$\Rightarrow$ Adiabatic transition

$*$ ν is mostly ν_2 before and after resonance

$*$ $\theta_m \downarrow$ *dramatically* at resonance

$\Rightarrow \nu_e$ component $\downarrow \Rightarrow P_{ee}\downarrow$

This is the MSW effect



$$P_{ee} = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right]$$

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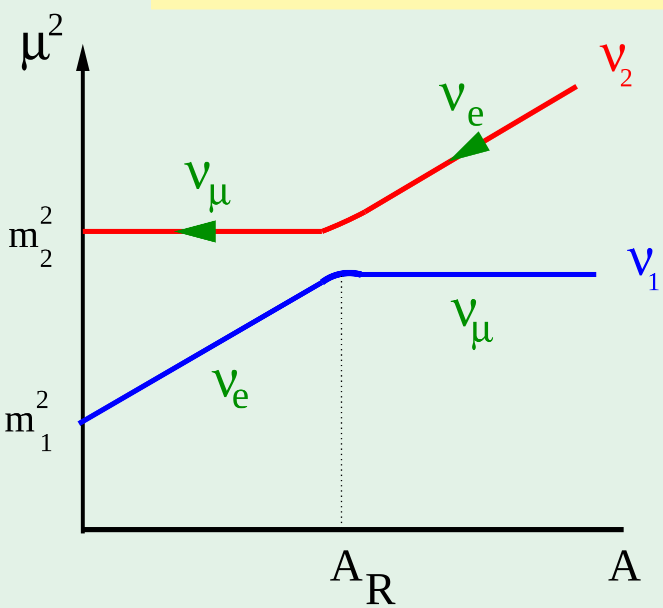

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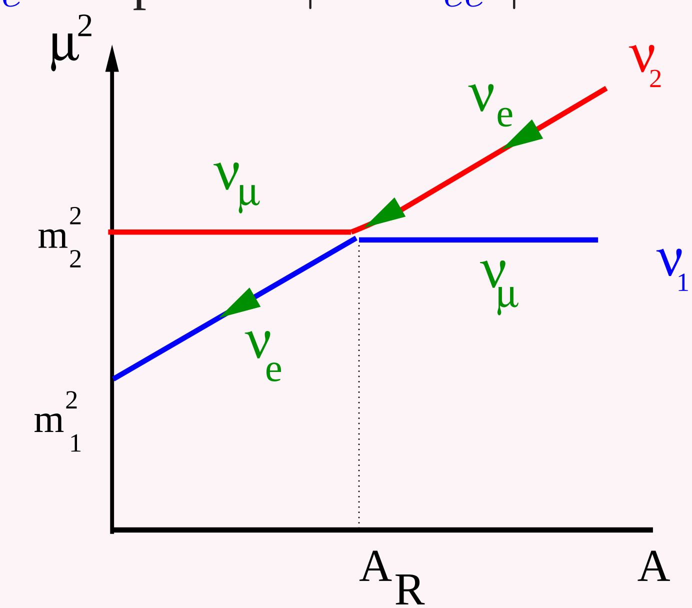
$\Rightarrow$ Non-Adiabatic transition

$*$ ν is mostly ν_2 till the resonance

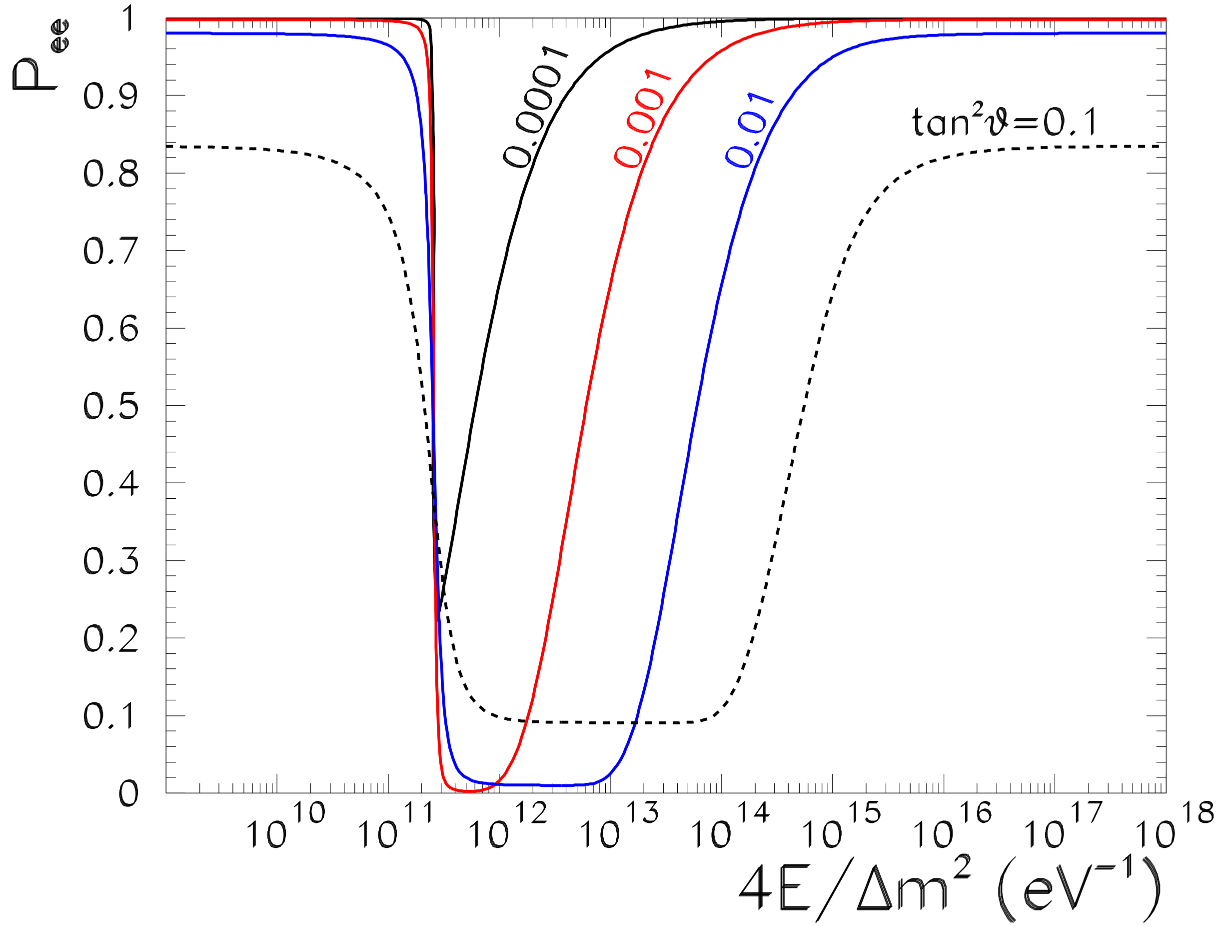
$*$ At resonance the state can jump into ν_1 (with probability P_{LZ})

$\Rightarrow \nu_e$ component $\uparrow \Rightarrow P_{ee} \uparrow$



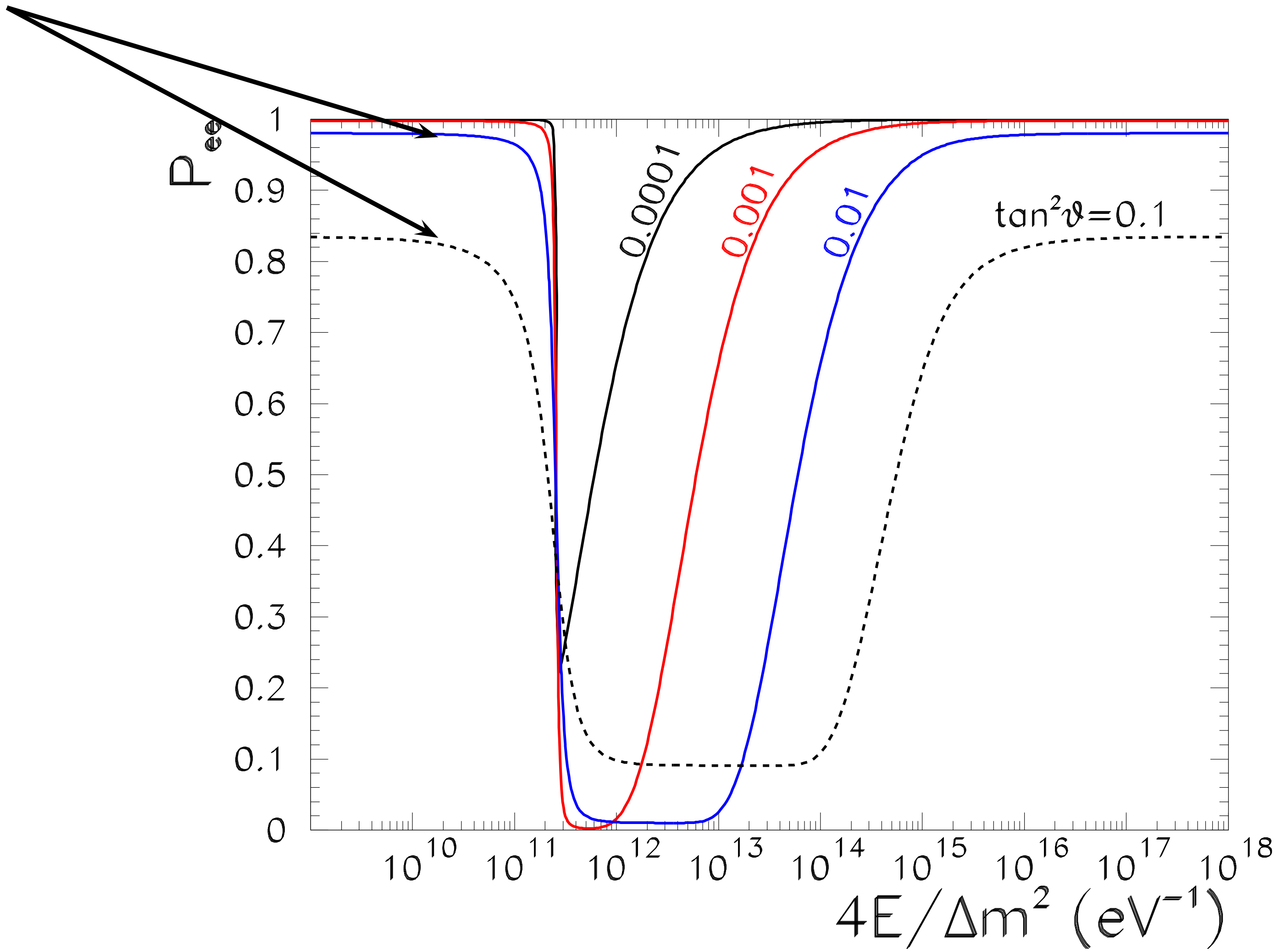
$$P_{ee} = \frac{1}{2}\left[1 + (1-2P_{LZ})\cos 2\theta_{m,0}\cos 2\theta\right]$$

Neutrinos in The Sun : MSW Effect



Neutrinos in The Sun : MSW Effect

ν does not cross resonance: $P_{ee} = 1 - \frac{1}{2}\sin^2 2\theta > \frac{1}{2}$



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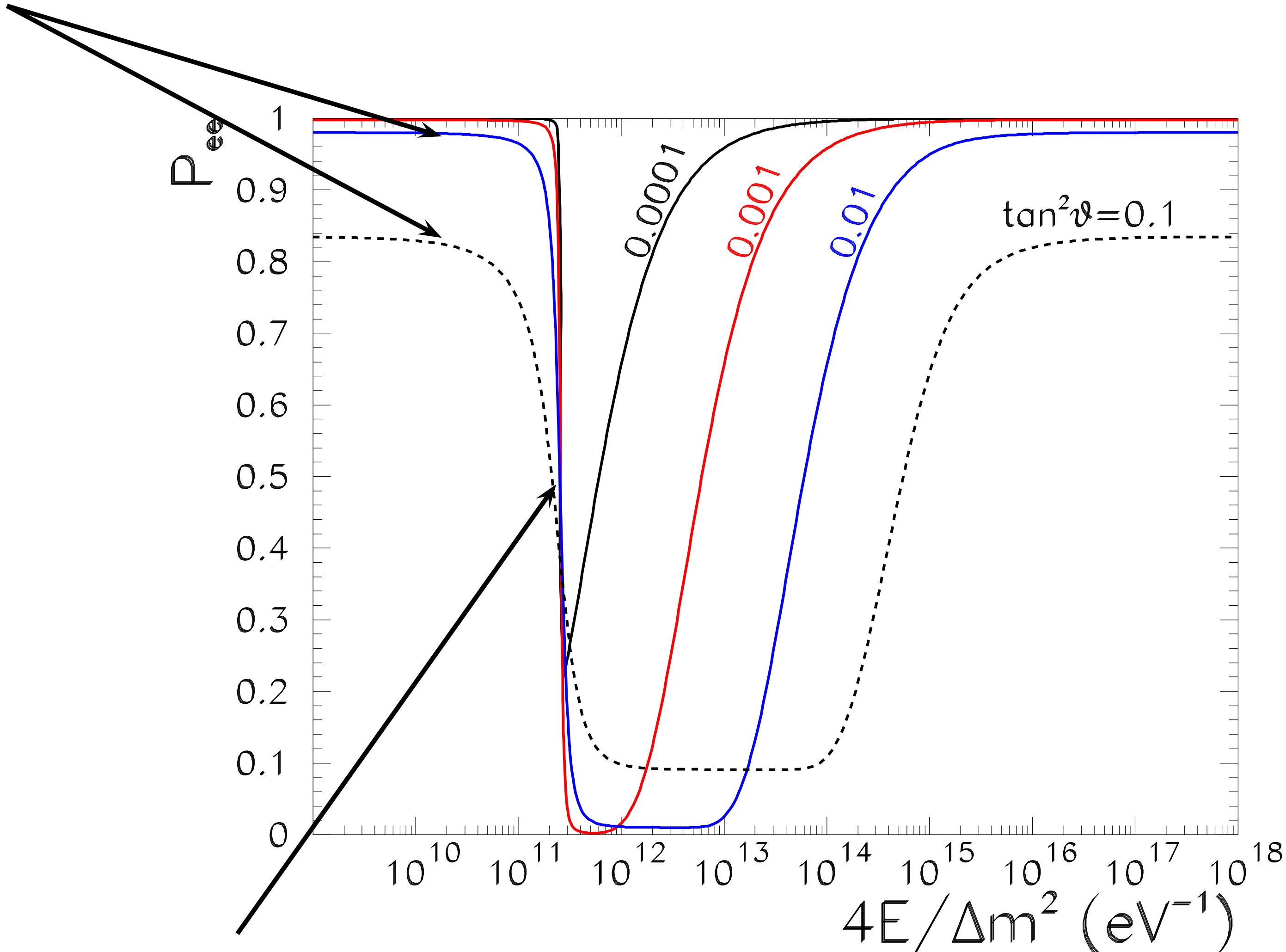

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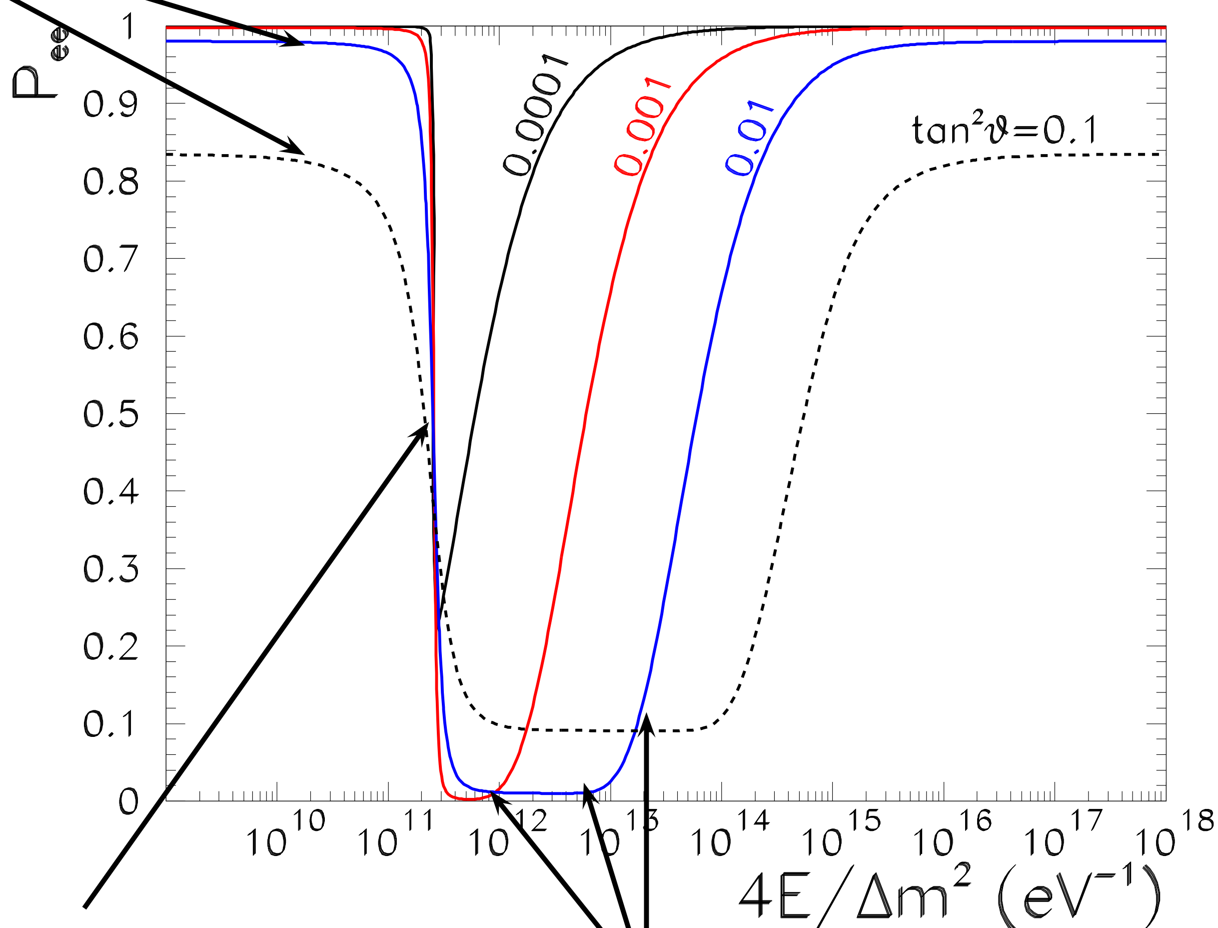

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Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

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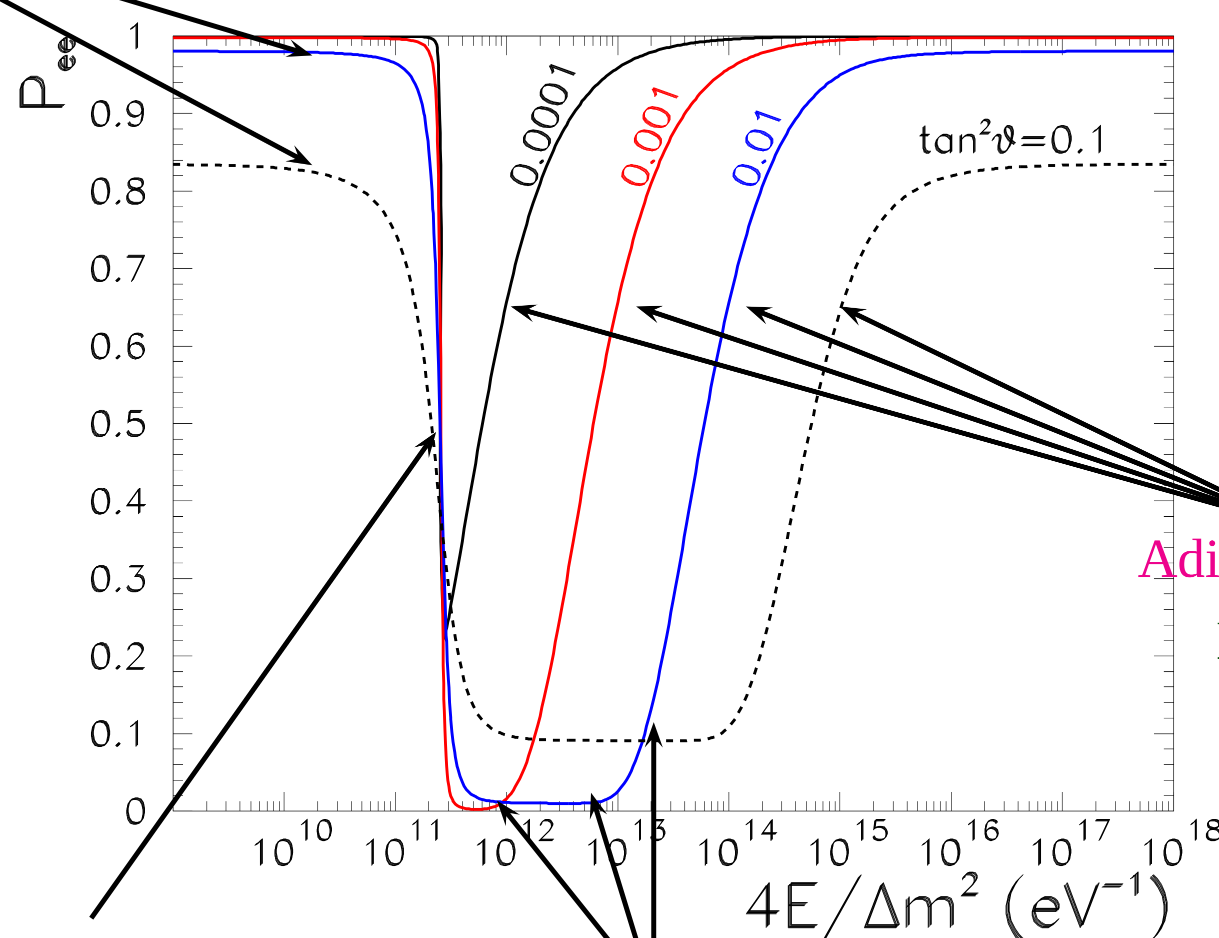


Adiabacity breaking

Effect of P_{LZ}

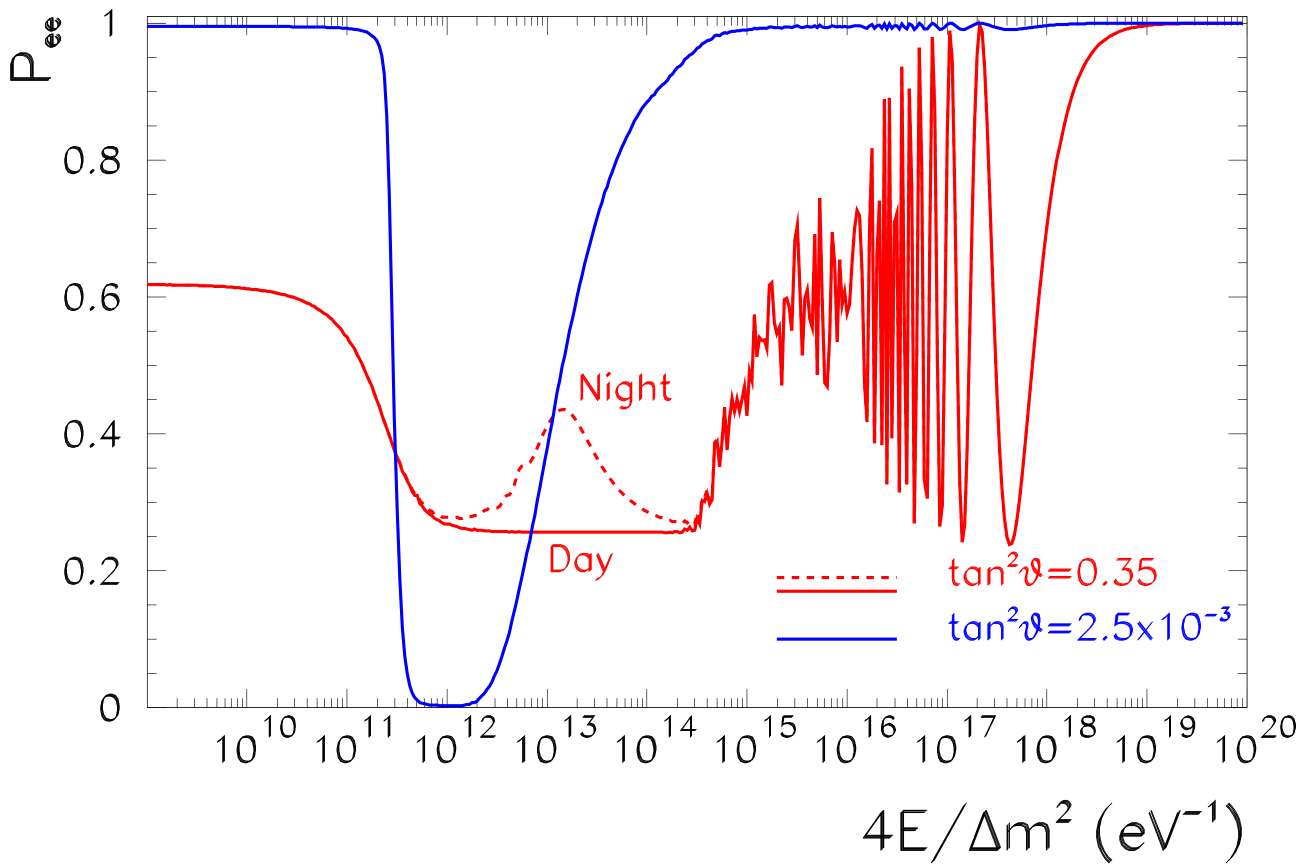
ν crosses resonance

MSW effect

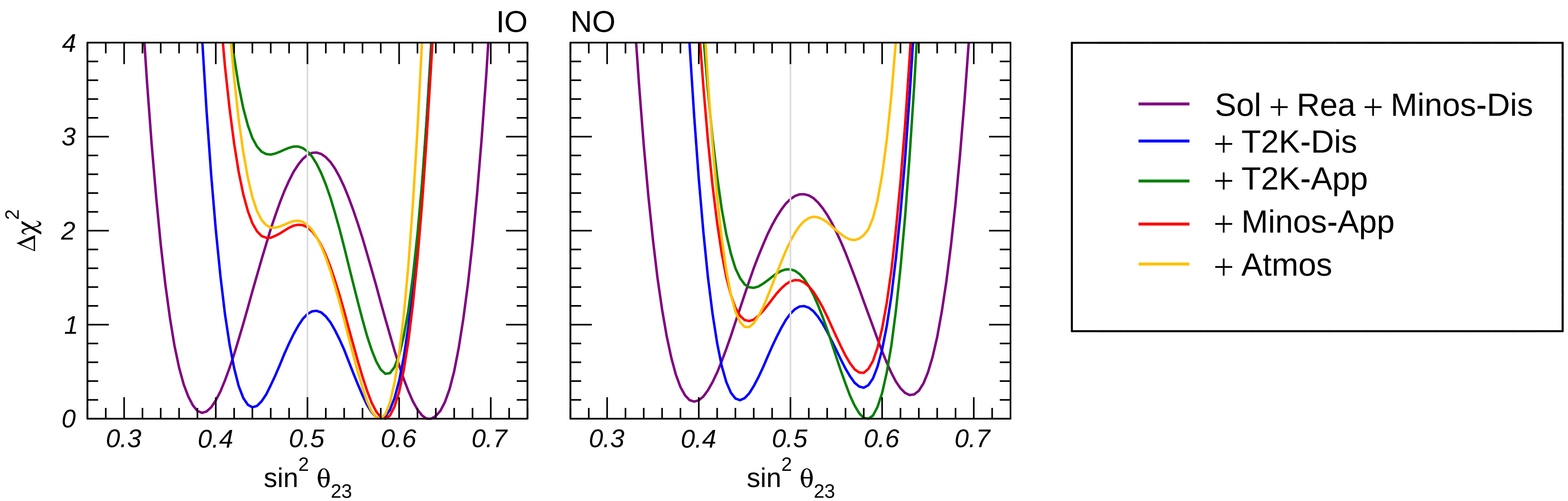
Adiabatic MSW transition $P_{ee} = \sin^2\theta < \frac{1}{2}$

Neutrinos from The Sun : The Full Story

$$A(\nu_e \to \nu_e) = A_{Sun}(\nu_e \to \nu_1) \times A_{\rm vac}(\nu_1 \to \nu_1) \times A_{Earth}(\nu_1 \to \nu_e)$$
$$+A_{Sun}(\nu_e \to \nu_2) \times A_{\rm vac}(\nu_2 \to \nu_2) \times A_{Earth}(\nu_2 \to \nu_e)$$



3 ν: θ_{23} Octant and Mass Ordering



- Determination of Octant of θ_{23}:
  - $\theta_{23} = 45$ Disfavoured at 1.5 σ

    Mostly driven by MINOS ν_μ DIS
  - IO: $\theta_{23} > 45$ Favoured at 1.7 σ

    Driven by T2K-APP+REACT
  - NO: $\theta_{23} < 45$ Favoured at 1 σ

    Driven by SK I–IV ATM Sub-GeV ν_e excess

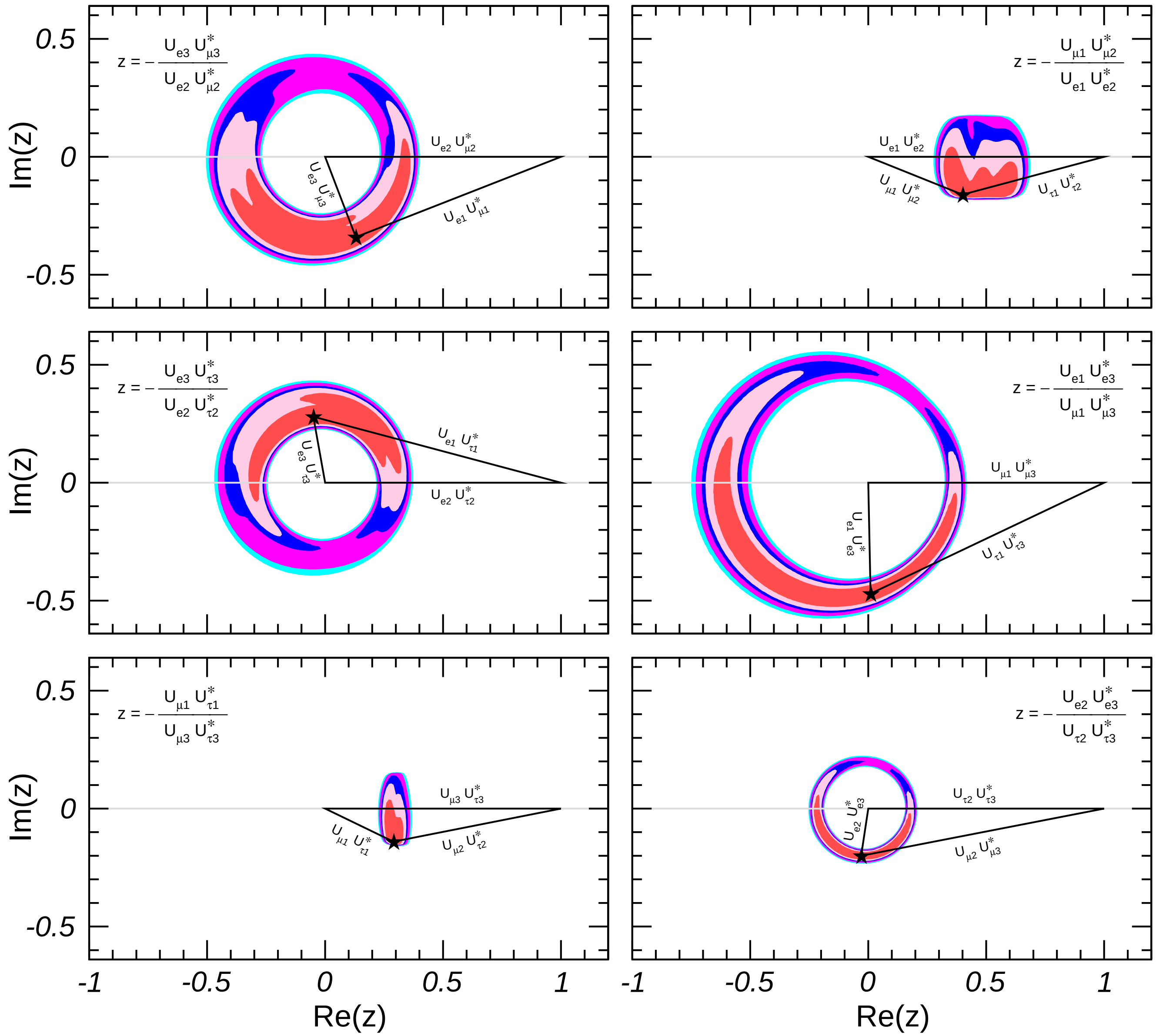
- Determination of Mass Ordering:
  - No significant difference NO vs IO

    IO favoured at 1 σ

- Sign and size of these 1-1.5σ “hints” vary among analysis

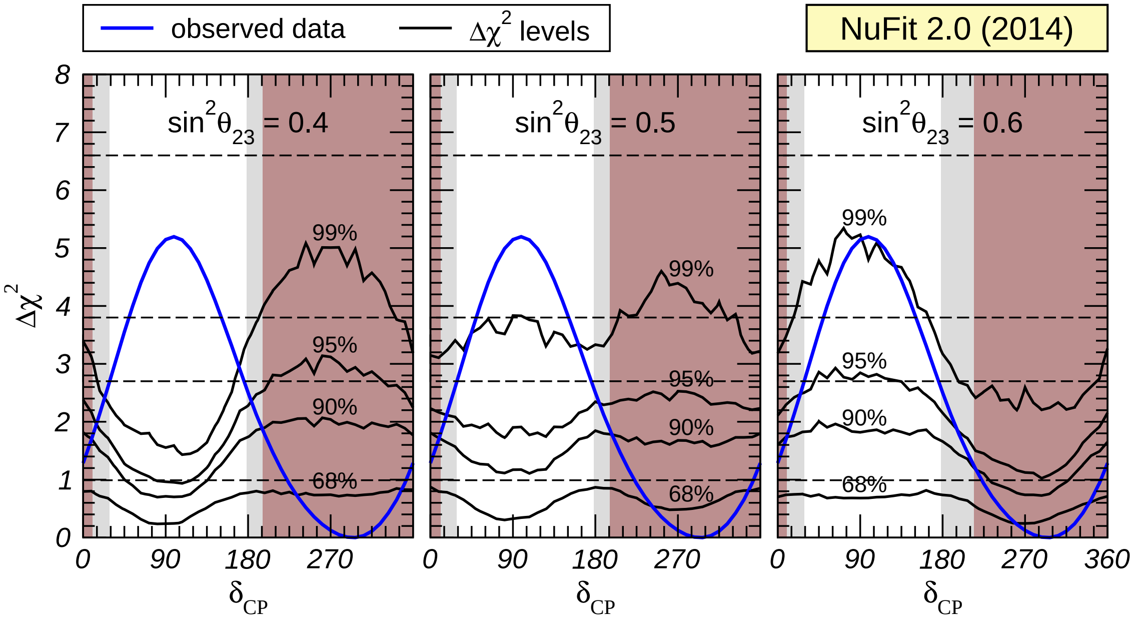
3ν Analysis: Leptonic Unitarity Triangles

NuFIT 2.0 (2014)



3ν Analysis: CL of CP hints

MC generation of probability distribution of $\Delta\chi^2(\delta_{\rm cp})$ for T2K+DB



– Prob_{MC} smaller than $\mathrm{Prob}_{\chi^2-1dof}$

– "Allowed" interval at given CL smaller than assuming χ^2 distribution

– *Strong* dependence on true θ_{23} due to degeneracy θ_{23}-octant/sig$[\sin(\delta_{cp})]$

$$P_{\mu e} \simeq s^2_{23}\sin^2 2\theta_{13}\left(\frac{\Delta_{31}}{B_\mp}\right)^2 \sin^2\left(\frac{B_\mp L}{2}\right)$$
$$+\tilde{J}\,\frac{\Delta_{12}}{V_E}\,\frac{\Delta_{31}}{B_\mp}\,\sin\left(\frac{V_E L}{2}\right)\sin\left(\frac{B_\mp L}{2}\right)\cos\left(\frac{\Delta_{31}L}{2}\pm\delta_{CP}\right)$$

$B_\pm = \Delta_{31} \pm V_E$ $\quad$ $\tilde{J} = c_{13}\sin 2\theta_{13}\sin 2\theta_{23}\sin 2\theta_{12}$

Implications: LFV & Collider Signatures

- ν oscillation $\Rightarrow$ Lepton Flavour is not conserved

  If only $\mathcal{O}_5$ $\Rightarrow Br(\tau \to \mu\gamma) \sim 10^{-41}$ too small!

- But dim=6 operators are LN conserving but LFV (f.e. $O_6 \sim \bar{L}_\alpha \bar{L}_\beta L_\gamma L_\rho$).

  So may be

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{c_{5\alpha\beta}}{\Lambda_{LN}} \left(\overline{L_\alpha}\tilde{\phi}\right)\left(\tilde{\phi}^T L^C_\beta\right) + \sum_i \frac{c_{6,i}}{\Lambda^2_{LF}} \mathcal{O}_{6,i}$$

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  New Physics scale Λ_{LN} responsible for the small m_ν from

  New Physics scale Λ_{LF} ($\ll \Lambda_{LN}$) controlling of LFV

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  If $M \sim \Lambda_{LF} \sim$ TeV ($\ll \Lambda_{LN}$) motivation of light ν OK

  Furthermore if $c_{6,i} \propto c_5{}^{\text{some power}}$ $\Rightarrow$ LFV and coll signals directly related to M_ν

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Minimal Lepton Flavour Violation

Cirigliano, Grinstein, Isidori, Wise(05); Davidson, Palorini (06); Gavela, Hambye, Hernandez,Hernandez (09)
Alonso, Isidori, Merlo, Munoz, Nardi(11)