Exercises in Flavor Physics

A. Consider a model with three quark flavors, u, d, s: u_R^I, d_R^I and s_R^I are each an $SU(2)$-singlet representation (as in the SM); u_L^I and d_L^I form an $SU(2)$-doublet (as in the SM), but s_L^I is an $SU(2)$-singlet (differently from the SM). (The super-index I denotes interaction eigenstates, and distinguish them from the mass eigenstates u, d, s.)

1. Write the most general Yukawa and mass terms.

2. What is the size of the quark mixing matrix (characterizing the W-couplings)?

3. Show that the Z-boson has flavor-changing couplings.

4. Find a relation between the $Z\overline{d_L}s_L$ coupling and the $W\overline{u_L}s_L$ coupling.

5. Show that the Higgs-boson has flavor-changing couplings.

6. Are there tree-level contributions to $K_L \to \mu^+\mu^-$ decay? Draw the diagram and estimate the ratio $\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K^+ \to \mu^+\nu_\mu)$. Based on the measured rates, is the model viable?

B. We here investigate CP violation in B decays. (The quark content of the relevant meson states are the following: $B^0 = \bar{b}d$, $K^0 = \bar{s}d$, $\psi = \bar{c}c$, $\phi = \bar{s}s$.)

1. Write down the underlying quark process for the $B \to \psi K_S$ decay. Draw the tree level diagram and find the leading CKM dependence.

2. Write down the underlying quark process for the $B \to \phi K_S$ decay. Argue that there is no tree level contribution.

3. Draw the diagram(s) and find the leading CKM dependence.

4. Find $S_{\psi K_S}$ and $S_{\phi K_S}$. Obtain the relation between them.

5. If the relation is violated, which of the following explanations is (im)plausible:

   - There are large SM corrections to the leading CKM dependence.
   - There is a new physics contribution to $B^0 - \bar{B}^0$ mixing.
   - There is a new physics contribution to $B \to \psi K_S$.
   - There is a new physics contribution to $B \to \phi K_S$.