Exercises

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1. Reproduce the Friedmann equation

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}G\rho - \frac{k}{R^2} \tag{1}$$

from the Newtonian argument. Then using the (central values of) Planck data

$$\begin{aligned} h_0 &= 0.6704, && (2)\\ \Omega_{rel} &= 4.15\times 10^{-5} h_0^{-2}, && (3)\\ \Omega_m &= \Omega_b + \Omega_c = 0.3183, && (4)\\ \Omega_\Lambda &= 0.6817, && (5) \end{aligned}$$

compute the age of the Universe. Note that $H_0 = 100h_0$ km/s/Mpc, and assume ρ_Λ is constant (vacuum energy).

2. Using the Schwarzschild metric due to a point mass,

$$ds^2 = \left(1 - \frac{r_S}{r}\right)(cdt)^2 - \frac{(dr)^2}{1 - r_S/r} - r^2(d\theta)^2 - r^2\sin^2\theta(d\phi)^2, \tag{6}$$

where $r_S = \frac{2GM}{c^2}$, show that the deflection angle to the first order in M is given by

$$\Delta\theta = 2\frac{r_S}{r_c} \tag{7}$$

where r_c is the closest approach to the point mass.

3. Using the deflection angle from part **2.**, show how a round-shaped object can be seen as multiple images of elongated shapes.

4. Using the conservation of n_b/s and the current value $n_b/n_\gamma = 6.5\times 10^{-10}$, work out the asymmetry in the quark number

$$A_q \equiv \frac{n_q - n_{\bar{q}}}{n_q + n_{\bar{q}}} \tag{8}$$

before the QCD phase transition.

5. Show that only two flavors is sufficient to have CP violation in the seesaw mechanism.