

CGC beyond eikonal accuracy and its applications in pA collisions

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[T. A. , N. Armesto, G. Beuf, A. Moscoso, to appear soon]

[T. A. , N. Armesto, G. Beuf, M. Martinez, C. A. Salgado, JHEP 1407 (2014) 068]



Dilute-Dense Scattering within CGC

High energy scattering within the CGC :

- **Semi-classical approximation :**

- dense target \equiv classical background field $A_a^\mu(x) = O\left(\frac{1}{g}\right)$ at weak coupling g
- dilute projectile \equiv color charge $J_a^\mu(x) = O(g)$

- **Eikonal approximation:**

- take the high energy limit $s \rightarrow \infty$.
- drop power-suppressed contributions.

Coupling between the projectile and the target $\rightarrow \int d^4x J_\mu^a(x) A_a^\mu(x)$

In the semi-classical approximation, the eikonal limit can be obtained by either boosting the projectile or the target or both...

Boosting the target:

$$A_a^\mu(x) \mapsto \begin{cases} \gamma_t A_a^- \left(\gamma_t x^+, \frac{x^-}{\gamma_t}, \mathbf{x} \right) \\ \frac{1}{\gamma_t} A_a^+ \left(\gamma_t x^+, \frac{x^-}{\gamma_t}, \mathbf{x} \right) \\ A_a^i \left(\gamma_t x^+, \frac{x^-}{\gamma_t}, \mathbf{x} \right) \end{cases}$$

- $A_a^- \gg A_a^i \gg A_a^+$ in a generic gauge
- in the light-cone gauge:

$$A_a^\mu(x) = \delta^{\mu-} \delta(x^+) A_a^-(\mathbf{x})$$

target is localized at $x^+ = 0$

independent of x^-

Boosting the projectile :

$$J_a^\mu(x) \mapsto \begin{cases} \frac{1}{\gamma_p} J_a^- \left(\frac{x^+}{\gamma_p}, \gamma_p x^-, \mathbf{x} \right) \\ \gamma_p J_a^+ \left(\frac{x^+}{\gamma_p}, \gamma_p x^-, \mathbf{x} \right) \\ J_a^i \left(\frac{x^+}{\gamma_p}, \gamma_p x^-, \mathbf{x} \right) \end{cases}$$

- $J_a^+ \gg J_a^i \gg J_a^-$
- slow x^+ dependence due to Lorentz time dilation

$$J_a^\mu(x) \propto \delta^{\mu+} \delta(x^-) \rho^a(\mathbf{x})$$

projectile is localized at $x^- = 0$

Corrections beyond eikonal accuracy

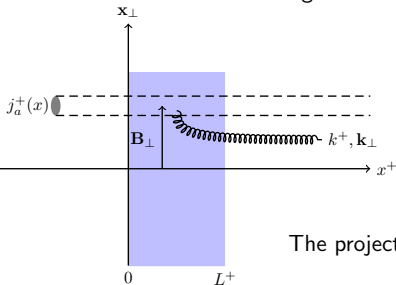
At least three sources of corrections to eikonal approximation :

- 1 other components of the target background field $\mathcal{A}_a^\mu(x)$
 - 2 dynamics of the target : x^- dependence of $\mathcal{A}_a^\mu(x)$
 - 3 Finite width L^+ of the target along x^+
- In the context of jet quenching and in-medium energy loss:
full finite width effects are included, but not the other effects.
→ further approximation (like harmonic potential for BDMPS-Z) required to deal with quantum diffusion of the projectile inside the target.
 - For scattering in the high-energy limit, power-suppressed finite L^+ correction can be calculated systematically without further approximation.

$$\mathcal{A}^\mu = \delta^{\mu-} \delta(x^+) \mathcal{A}^-(\mathbf{x}) \rightarrow \mathcal{A}^\mu = \delta^{\mu-} \mathcal{A}^-(x^+, \mathbf{x})$$

Finite width target : relaxing the eikonal approximation

Consider a finite width target :



The target $\rightarrow \mathcal{A}^\mu(x) \equiv \delta^{\mu-} \mathcal{A}_a^-(x^+, \mathbf{x})$

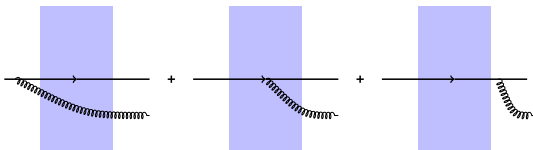
The projectile $\rightarrow j_a^\mu(x) \propto \delta^{\mu+} \delta(x^-) \rho^b(\mathbf{x} - \mathbf{B})$

The single inclusive gluon cross section for pA:

$$(2\pi)^3 (2k^+) \frac{d\sigma}{dk^+ d^2\mathbf{k}}(\mathbf{B}) = \int d^2\mathbf{B} \sum_{\lambda \text{ phys.}} \left\langle \left\langle |\mathcal{M}_\lambda^a(\underline{k}, \mathbf{B})|^2 \right\rangle_P \right\rangle_A$$

↓
gluon production amplitude

$$\mathcal{M}_\lambda^a(\underline{k}, \mathbf{B}) =$$



at LO in g , LSZ reduction formula $\Rightarrow \mathcal{M}_\lambda^a(\underline{k}, \mathbf{B}) = \varepsilon_\lambda^{\mu*} \int d^4x e^{ik \cdot x} \square_x A_\mu^a(x)$

power counting : dense target $\Rightarrow \mathcal{A}_a^-(x^+, \mathbf{x}) = \mathcal{O}(1/g)$

dilute projectile $\Rightarrow j_a^+(\underline{x}) = \mathcal{O}(g)$

perturbative expansion of the classical field : $A_a^\mu(x) = \mathcal{A}_a^\mu(x) + a_a^\mu(x) + \mathcal{O}(g^3)$

$$\downarrow$$

$$-i \int d^4y G_R^{\mu-}(x, y)_{ab} j_b^+(y)$$

In the LC gauge : $A_a^+ = 0$ & $\varepsilon_\lambda^{+*} = 0 \Rightarrow \varepsilon_\lambda^{\mu*} \mathcal{A}_\mu^a(x) = -\varepsilon_\lambda^{i*} a_a^i$

$$\mathcal{M}_\lambda^a(\underline{k}, \mathbf{B}) = \varepsilon_\lambda^{i*} (2k^+) \lim_{x^+ \rightarrow +\infty} \int d^2\mathbf{x} \int dx^- e^{ik \cdot x} \int d^4y G_R^{i-}(x, y)_{ab} j_b^+(y)$$

$G_R^{\mu\nu}(x, y)_{ab}$ is the background retarded gluon propagator

Properties of the background retarded gluon propagator

$G_R^{\mu\nu}(x, y)_{ab}$ is the retarded solution of the linearized Yang-Mills equations around the classical field $\mathcal{A}_a^-(x^+, \mathbf{x})$:

$$\left[g_{\mu\nu} \left(\delta_{ab} \square_x - 2ig \left(\mathcal{A}^-(x^+, \mathbf{x}) \cdot T \right)_{ab} \partial_{x^-} \right) - \delta_{ab} \partial_{x^\mu} \partial_{x^\nu} \right] G_R^{\nu\rho}(x, y)_{bc} = i g_{\mu}{}^\rho \delta_{ac} \delta^{(4)}(x-y)$$

$\mathcal{A}_a^-(x^+, \mathbf{x})$ is independent of the x^- , so it is convenient to introduce the 1-d Fourier transform of $G_R^{\mu\nu}(x, y)_{ab}$

$$G_R^{\mu\nu}(x, y)_{ab} = \int \frac{dk^+}{2\pi} e^{-ik^+(x^- - y^-)} \frac{1}{2(k^+ + i\epsilon)} \mathcal{G}_{k^+}^{\mu\nu}(\underline{x}; \underline{y})_{ab}$$

The ($i-$) component of the $\mathcal{G}_{k^+}^{\mu\nu}(\underline{x}; \underline{y})_{ab}$ can be written as

$$\mathcal{G}_{k^+}^{i-}(\underline{x}; \underline{y})^{ab} = \frac{i}{k^+ + i\epsilon} \partial_{\mathbf{y}^i} \mathcal{G}_{k^+}^{ab}(\underline{x}; \underline{y})$$

$\mathcal{G}_{k^+}^{ab}(\underline{x}; \underline{y})$ is the background scalar propagator.

Properties of the background retarded gluon propagator

The background scalar propagator $\mathcal{G}_{k^+}^{ab}(\underline{x}; \underline{y})$ satisfies the scalar Green's equation :

$$\left[\delta^{ab} \left(i\partial_{x^+} + \frac{\partial_{\underline{x}}^2}{2(k^+ + i\epsilon)} \right) + g \left(\mathcal{A}^-(\underline{x}) \cdot T \right)^{ab} \right] \mathcal{G}_{k^+}^{bc}(\underline{x}; \underline{y}) = i \delta^{ac} \delta^{(3)}(\underline{x} - \underline{y})$$

Schrödinger equation in 2 + 1 dimensions in a space-time dependent matrix potential $-g \left(\mathcal{A}^-(\underline{x}) \cdot T \right)^{ab}$.

The solution can be written formally as a path integral

$$\mathcal{G}_{k^+}^{ab}(\underline{x}; \underline{y}) = \theta(x^+ - y^+) \int_{\mathbf{z}(y^+) = \mathbf{y}}^{\mathbf{z}(x^+) = \mathbf{x}} \mathcal{D}\mathbf{z}(z^+) \exp \left[\frac{ik^+}{2} \int_{y^+}^{x^+} dz^+ \dot{\mathbf{z}}^2(z^+) \right] \\ \times \mathcal{U}^{ab}(x^+, y^+, [\mathbf{z}(z^+)])$$

with the Wilson line

$$\mathcal{U}^{ab}(x^+, y^+, [\mathbf{z}(z^+)]) = \mathcal{P}_+ \exp \left\{ ig \int_{y^+}^{x^+} dz^+ T \cdot \mathcal{A}^-(z^+, \mathbf{z}(z^+)) \right\}^{ab}$$

following the Brownian trajectory $\mathbf{z}(z^+)$.

Discretization of the background propagator

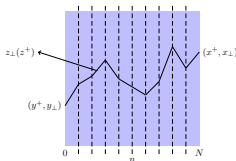
Discretized form of the background propagator :

$$\mathcal{G}_{k^+}^{ab}(\underline{x}; \underline{y}) = \lim_{N \rightarrow +\infty} \theta(x^+ - y^+) \int \left(\prod_{n=1}^{N-1} d^2 \mathbf{z}_n \right) \left(\frac{-i(k^+ + i\epsilon)N}{2\pi(x^+ - y^+)} \right)^N \\ \times \exp \left[\frac{i(k^+ + i\epsilon)N}{2(x^+ - y^+)} \sum_{n=0}^{N-1} (\mathbf{z}_{n+1} - \mathbf{z}_n)^2 \right] \mathcal{U}^{ab}(x^+, y^+, \{\mathbf{z}_n\})$$

with the discretized Wilson line

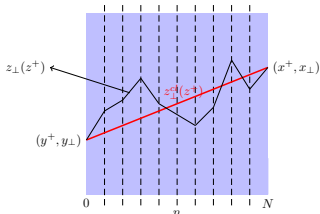
$$\mathcal{U}^{ab}(x^+, y^+, \{\mathbf{z}_n\}) = \mathcal{P}_+ \left\{ \prod_{n=0}^{N-1} \exp \left[ig \frac{(x^+ - y^+)}{N} (\mathcal{A}^-(z_n^+, \mathbf{z}_n) \cdot T) \right] \right\}^{ab}$$

with $z_n^+ = y^+ + \frac{n}{N}(x^+ - y^+)$



Expansion of the background propagator

Perturbative expansion around free classical path:



eikonal limit : $\frac{k^+}{(x^+ - y^+)} \gg Q_1^2$ in the problem

\mapsto large k^+ limit (classical free path!)

\Rightarrow perturbative expansion around the free classical path:

$$\mathbf{z}_n = \mathbf{z}_n^{\text{cl}} + \mathbf{u}_n \text{ with } \mathbf{z}_n^{\text{cl}} = \mathbf{y} + \frac{n}{N}(\mathbf{x} - \mathbf{y})$$

Then

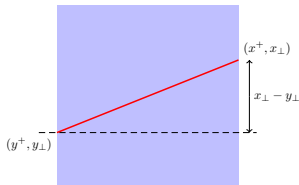
$$\mathcal{G}_{k^+}^{ab}(x; \underline{y}) = 2\pi i \mathcal{G}_{0, k^+}(x; \underline{y}) \frac{(x^+ - y^+)}{k^+} \lim_{N \rightarrow +\infty} \int \left(\prod_{n=1}^{N-1} d^2 \mathbf{u}_n \right) \times \mathcal{P}_+ \prod_{n=0}^{N-1} \left\{ \mathcal{G}_{0, k^+} \left(z_{n+1}^+, \mathbf{u}_{n+1}; z_n^+, \mathbf{u}_n \right) \exp \left[\frac{(x^+ - y^+)}{N} igT \cdot \mathcal{A}^- \left(z_n^+, \mathbf{z}_n^{\text{cl}} + \mathbf{u}_n \right) \right] \right\}$$

- Taylor expand the gauge link around $\mathbf{u}_n = 0$ at each discretization step.
- Take the product of the Taylor expansion of all links and collect terms according to power of \mathbf{u}_n .

Expansion around the initial transverse position :

The first expansion is performed for fixed initial and final positions.

In the large k^+ limit , the result has to be re-expanded since $\mathbf{z}^{cl}(\mathbf{z}^+) - \mathbf{y}$ is small at each step.



After all:

$$\int d^2\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} \mathcal{G}_{k^+}^{ab}(\underline{x}; \underline{y}) = \theta(x^+ - y^+) e^{-i\mathbf{k}\cdot\mathbf{y}} e^{-ik^-(x^+ - y^+)} \left\{ \mathcal{U}(x^+, y^+, \mathbf{y}) \right. \\ + \frac{(x^+ - y^+)}{k^+} \left[\mathbf{k}^i \mathcal{U}_{[0,1]}^i(x^+, y^+, \mathbf{y}) + \frac{i}{2} \mathcal{U}_{[1,0]}(x^+, y^+, \mathbf{y}) \right] \\ \left. + \frac{(x^+ - y^+)^2}{(k^+)^2} \left[\mathbf{k}^i \mathbf{k}^j \mathcal{U}_{[0,2]}^{ij}(x^+, y^+, \mathbf{y}) + \frac{i}{2} \mathbf{k}^i \mathcal{U}_{[1,1]}^i(x^+, y^+, \mathbf{y}) - \frac{1}{4} \mathcal{U}_{[2,0]}(x^+, y^+, \mathbf{y}) \right] \right\}^{ab}$$

Structure of the decorated Wilson lines

$$U_{[0,1]}^j \propto \overline{x^+ \xrightarrow{U} z^+ \xrightarrow{B^j} z^+ \xrightarrow{U} y^+}$$

$$U_{[0,2]}^{ij} \propto \overline{x^+ \xrightarrow{U} z^+ \xrightarrow{B^{ij}} z^+ \xrightarrow{U} y^+} + \overline{x^+ \xrightarrow{U} z_1^+ \xrightarrow{B^i} z_1^+ \xrightarrow{U} z_2^+ \xrightarrow{B^j} z_2^+ \xrightarrow{U} y^+}$$

$$U_{[1,1]}^i \propto \overline{x^+ \xrightarrow{U} z^+ \xrightarrow{B^{ijj}} z^+ \xrightarrow{U} y^+} + \overline{x^+ \xrightarrow{U} z_1^+ \xrightarrow{B^{ij}} z_1^+ \xrightarrow{U} z_2^+ \xrightarrow{B^j} z_2^+ \xrightarrow{U} y^+} + \overline{x^+ \xrightarrow{U} z_1^+ \xrightarrow{B^i} z_1^+ \xrightarrow{U} z_2^+ \xrightarrow{B^j} z_2^+ \xrightarrow{U} z_3^+ \xrightarrow{B^j} z_3^+ \xrightarrow{U} y^+}$$

with

$$\begin{aligned} B^i(z^+, \mathbf{y}) &\equiv igT \cdot \partial_{\mathbf{y}^i} \mathcal{A}^-(z^+, \mathbf{y}), \\ B^{ij}(z^+, \mathbf{y}) &\equiv igT \cdot \partial_{\mathbf{y}^i} \partial_{\mathbf{y}^j} \mathcal{A}^-(z^+, \mathbf{y}), \\ B^{ijl}(z^+, \mathbf{y}) &\equiv igT \cdot \partial_{\mathbf{y}^i} \partial_{\mathbf{y}^j} \partial_{\mathbf{y}^l} \mathcal{A}^-(z^+, \mathbf{y}), \end{aligned}$$

Reduced amplitude and the observables

Defining Fourier transform of the color charge density

$$\rho^a(\mathbf{y} - \mathbf{B}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot(\mathbf{y}-\mathbf{B})} \tilde{\rho}^a(\mathbf{q})$$

and gluon-nucleus reduced amplitude

$$\mathcal{M}_\lambda^a(\underline{k}, \mathbf{B}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{-i\mathbf{q}\cdot\mathbf{B}} \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q}) \tilde{\rho}^b(\mathbf{q})$$

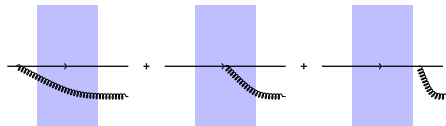
The cross section:

$$k^+ \frac{d\sigma}{dk^+ d^2\mathbf{k}} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \varphi_p(\mathbf{q}) \frac{\mathbf{q}^2}{4} \frac{1}{N_c^2 - 1} \sum_{\lambda \text{ phys.}} \langle \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q})^\dagger \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q}) \rangle_A.$$

The light-front helicity asymmetry:

$$k^+ \frac{d\sigma^+}{dk^+ d^2\mathbf{k}} - k^+ \frac{d\sigma^-}{dk^+ d^2\mathbf{k}} = \int \frac{d^2\mathbf{q}}{(2\pi)^2} \varphi_p(\mathbf{q}) \times \frac{\mathbf{q}^2}{4} \frac{1}{N_c^2 - 1} \sum_{\lambda \text{ phys.}} \lambda \langle \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q})^\dagger \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q}) \rangle_A$$

Total reduced amplitude at next-to-eikonal accuracy



strict eikonal term!

$$\begin{aligned}
 \overline{\mathcal{M}}_{\lambda}^{ab}(\underline{k}; \mathbf{q}) &= i \varepsilon_{\lambda}^{i*} \int d^2 \mathbf{y} e^{i \mathbf{y} \cdot (\mathbf{q} - \mathbf{k})} \left\{ 2 \left(\frac{\mathbf{k}^i}{k^2} - \frac{\mathbf{q}^i}{q^2} \right) \mathcal{U}(L^+, 0; \mathbf{y}) \right. \\
 &+ \left(\frac{L^+}{k^+} \right) \left[\left(\delta^{ij} - 2 \frac{\mathbf{q}^i \mathbf{k}^j}{q^2} \right) \mathcal{U}_{[0,1]}^j(L^+, 0; \mathbf{y}) - i \frac{\mathbf{q}^j}{q^2} \mathcal{U}_{[1,0]}(L^+, 0; \mathbf{y}) \right] \\
 &+ \left(\frac{L^+}{k^+} \right)^2 \left[-2 \frac{\mathbf{q}^i}{q^2} \mathbf{k}^j \mathbf{k}^l \mathcal{U}_{[0,2]}^{jl}(L^+, 0; \mathbf{y}) - i \frac{\mathbf{q}^i \mathbf{k}^j}{q^2} \mathcal{U}_{[1,1]}^j(L^+, 0; \mathbf{y}) + \frac{1}{2} \frac{\mathbf{q}^i}{q^2} \mathcal{U}_{[2,0]}(L^+, 0; \mathbf{y}) \right. \\
 &\left. \left. + \frac{i}{4} (k^2 \delta^{ij} - 2 \mathbf{k}^i \mathbf{k}^j) \mathcal{U}_{(A)}^j(L^+, 0; \mathbf{y}) + \frac{\mathbf{k}^j}{4} \mathcal{U}_{(B)}^{ij}(L^+, 0; \mathbf{y}) + \frac{i}{4} \mathcal{U}_{(C)}^i(L^+, 0; \mathbf{y}) \right] \right\}^{ab}
 \end{aligned}$$

Dipole operators

The usual adjoint dipole

$$\mathcal{O}(\mathbf{r}) = \int d^2\mathbf{b} \frac{1}{N_c^2 - 1} \left\langle \text{tr} \left[\mathcal{U} \left(\mathbf{b} + \frac{\mathbf{r}}{2} \right) \mathcal{U}^\dagger \left(\mathbf{b} - \frac{\mathbf{r}}{2} \right) \right] \right\rangle_A$$

The decorated adjoint dipoles with one decorated Wilson line $\mathcal{U}_{[\alpha,\beta]}^{i\dots j}$

$$\mathcal{O}_{[\alpha,\beta]}^{i\dots j}(\mathbf{r}) = \int d^2\mathbf{b} \frac{1}{N_c^2 - 1} \left\langle \text{tr} \left[\mathcal{U}_{[\alpha,\beta]}^{i\dots j} \left(\mathbf{b} + \frac{\mathbf{r}}{2} \right) \mathcal{U}^\dagger \left(\mathbf{b} - \frac{\mathbf{r}}{2} \right) \right] \right\rangle_A$$

The decorated adjoint dipoles formed from two decorated Wilson lines

$$\mathcal{O}_{[\alpha,\beta];[\gamma,\delta]}^{i\dots j;l\dots m}(\mathbf{r}) = \int d^2\mathbf{b} \frac{1}{N_c^2 - 1} \left\langle \text{tr} \left[\mathcal{U}_{[\alpha,\beta]}^{i\dots j} \left(\mathbf{b} + \frac{\mathbf{r}}{2} \right) \mathcal{U}_{[\gamma,\delta]}^{l\dots m \dagger} \left(\mathbf{b} - \frac{\mathbf{r}}{2} \right) \right] \right\rangle_A$$

Squared Amplitude

Strict eikonal:

$$\frac{1}{N_c^2 - 1} \left\langle \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q})^\dagger \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q}) \right\rangle \Big|_{\text{E}} = \varepsilon_\lambda^{i*} \varepsilon_\lambda^j f^{ij}(\mathbf{k}, \mathbf{q}) \times (\text{adjoint dipole})$$

The next-to-eikonal contribution:

$$\frac{1}{N_c^2 - 1} \left\langle \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q})^\dagger \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q}) \right\rangle \Big|_{\text{NE}} = \varepsilon_\lambda^{i*} \varepsilon_\lambda^j \frac{L^+}{k^+} g^{ij}(\mathbf{k}, \mathbf{q}) \times (\text{decorated dipoles})$$

The next-to-next-to-eikonal contribution:

$$\frac{1}{N_c^2 - 1} \left\langle \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q})^\dagger \overline{\mathcal{M}}_\lambda^{ab}(\underline{k}, \mathbf{q}) \right\rangle \Big|_{\text{NNE}} = \varepsilon_\lambda^{i*} \varepsilon_\lambda^j \left(\frac{L^+}{k^+} \right)^2 l^{ij}(\mathbf{k}, \mathbf{q}) \times (\text{decorated dipoles})$$

$f^{ij}(\mathbf{k}, \mathbf{q})$ and $l^{ij}(\mathbf{k}, \mathbf{q})$ are symmetric under $i \rightarrow j$

$g^{ij}(\mathbf{k}, \mathbf{q})$ is anti-symmetric under $i \rightarrow j$

Back to observables

For the single inclusive gluon cross section:

$$\sum_{\lambda} \varepsilon_{\lambda}^{i*} \varepsilon_{\lambda}^j = \delta^{ij}$$

The next-to-eikonal contributions vanish!

The next-to-next-to-eikonal terms give contribution.

For the light-front helicity asymmetry:

$$\sum_{\lambda} \lambda \varepsilon_{\lambda}^{i*} \varepsilon_{\lambda}^j = i \epsilon^{ij}$$

The strict eikonal and next-to-next-to-eikonal terms vanish!

The next-to-eikonal terms are the leading terms for the light-front helicity.

The single inclusive gluon cross section at next-to-next-to-eikonal accuracy:

$$\begin{aligned}
 k^+ \frac{d\sigma}{dk^+ d^2\mathbf{k}} &= \int \frac{d^2\mathbf{q}}{(2\pi)^2} \varphi_p(\mathbf{q}) \frac{\mathbf{q}^2}{4} \int d^2\mathbf{r} e^{i\mathbf{r}\cdot(\mathbf{q}-\mathbf{k})} \left\{ 4\mathcal{C}^i(\mathbf{k}, \mathbf{q}) \mathcal{C}^i(\mathbf{k}, \mathbf{q}) \mathcal{O}(\mathbf{r}) \right. \\
 &+ \left(\frac{L^+}{k^+} \right)^2 \left[2 \frac{\mathbf{q}^i}{\mathbf{q}^2} \mathcal{C}^i(\mathbf{k}, \mathbf{q}) \left[-4 \mathbf{k}^l \mathbf{k}^m \mathcal{O}_{[0,2]}^{l,m}(\mathbf{r}) - 2i \mathbf{k}^l \mathcal{O}_{[1,1]}^l(\mathbf{r}) + \mathcal{O}_{[2,0]}(\mathbf{r}) \right] \right. \\
 &+ \mathcal{C}^i(\mathbf{k}, \mathbf{q}) \left[i(\mathbf{k}^2 \delta^{il} - 2\mathbf{k}^i \mathbf{k}^l) \mathcal{O}_{(A)}^l(\mathbf{r}) + \mathbf{k}^m \mathcal{O}_{(B)}^{im}(\mathbf{r}) + i \mathcal{O}_{(C)}^i(\mathbf{r}) \right] \\
 &\left. \left. + \tilde{\mathcal{C}}^{li}(\mathbf{k}, \mathbf{q}) \left[\tilde{\mathcal{C}}^{mi}(\mathbf{k}, \mathbf{q}) \mathcal{O}_{[0,1];[0,1]}^{l,m}(\mathbf{r}) + 2i \frac{\mathbf{q}^i}{\mathbf{q}^2} \mathcal{O}_{[0,1];[1,0]}^l(\mathbf{r}) \right] + \frac{1}{\mathbf{q}^2} \mathcal{O}_{[1,0];[1,0]}(\mathbf{r}) \right] \right\}
 \end{aligned}$$

The light front helicity asymmetry:

$$\begin{aligned}
 k^+ \frac{d\sigma^+}{dk^+ d^2\mathbf{k}} - k^+ \frac{d\sigma^-}{dk^+ d^2\mathbf{k}} &= \int \frac{d^2\mathbf{q}}{(2\pi)^2} \varphi_p(\mathbf{q}) \mathbf{q}^2 \int d^2\mathbf{r} e^{i\mathbf{r}\cdot(\mathbf{q}-\mathbf{k})} \\
 &\times \left(\frac{L^+}{k^+} \right) \epsilon^{ij} \mathcal{C}^j(\mathbf{k}, \mathbf{q}) \left\{ i \tilde{\mathcal{C}}^{li}(\mathbf{k}, \mathbf{q}) \mathcal{O}_{[0,1]}^l(\mathbf{r}) + \frac{\mathbf{q}^i}{\mathbf{q}^2} \mathcal{O}_{[1,0]}(\mathbf{r}) \right\}
 \end{aligned}$$

with $\mathcal{C}^i(\mathbf{k}, \mathbf{q}) = \left(\frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{\mathbf{q}^i}{\mathbf{q}^2} \right)$ and $\tilde{\mathcal{C}}^{ij}(\mathbf{k}, \mathbf{q}) = \left(\delta^{ij} - 2 \mathbf{k}^i \frac{\mathbf{q}^j}{\mathbf{q}^2} \right)$

Conclusions and Outlook

- We develop a systematic eikonal expansion of the retarded gluon propagator in a background field.
- We apply this method to single inclusive gluon cross section in order to study the corrections to the CGC beyond eikonal limit. The strict eikonal term provides the usual k_{\perp} -factorization formula, whereas the next-to-eikonal corrections vanish for this particular observable. The first non vanishing correction to the strict eikonal limit appears at next-to-next-to-eikonal accuracy.
- For light front helicity asymmetry the strict eikonal and next-to-next-to-eikonal terms vanish leaving the next-to-eikonal corrections as the leading contribution.
- The same expansion can be applied to DIS and "Hybrid Formula" by considering the quark background propagator [work in progress!]
- Rapidity evolution of the decorated dipoles?[work in progress!]

$$\mathcal{U}_{[0,1]}^i(x^+, y^+; \mathbf{y}) = \mathbb{P}_+ \mathcal{U}(x^+, y^+; \mathbf{y}) \int_{y^+}^{x^+} dz_1^+ \frac{(z_1^+ - y^+)}{(x^+ - y^+)} \mathcal{B}^i(z_1^+, \mathbf{y})$$

$$\begin{aligned} \mathcal{U}_{[0,2]}^{ij}(x^+, y^+; \mathbf{y}) = & \mathbb{P}_+ \mathcal{U}(x^+, y^+; \mathbf{y}) \left\{ \int_{y^+}^{x^+} dz_1^+ \left(\frac{z_1^+ - y^+}{x^+ - y^+} \right)^2 \mathcal{B}^{ij}(z_1^+, \mathbf{y}) \right. \\ & \left. + 2 \int_{y^+}^{x^+} dz_1^+ \int_{z_1^+}^{x^+} dz_2^+ \frac{(z_2^+ - y^+)(z_1^+ - y^+)}{(x^+ - y^+)^2} \mathcal{B}^i(z_2^+, \mathbf{y}) \mathcal{B}^j(z_1^+, \mathbf{y}) \right\} \end{aligned}$$

$$\begin{aligned} \mathcal{U}_{[1,0]}(x^+, y^+; \mathbf{y}) = & \mathbb{P}_+ \mathcal{U}(x^+, y^+; \mathbf{y}) \left\{ \int_{y^+}^{x^+} dz_1^+ \frac{(z_1^+ - y^+)}{(x^+ - y^+)} \delta^{ij} \mathcal{B}^{ij}(z_1^+, \mathbf{y}) \right. \\ & \left. + 2 \int_{y^+}^{x^+} dz_1^+ \int_{y^+}^{z_1^+} dz_2^+ \frac{(z_2^+ - y^+)}{(x^+ - y^+)} \mathcal{B}^i(z_1^+, \mathbf{y}) \mathcal{B}^i(z_2^+, \mathbf{y}) \right\} \end{aligned}$$

$$\begin{aligned}
 \mathcal{U}_{[1,1]}^i(x^+, y^+; \mathbf{y}) = & \mathbb{P}_+ \mathcal{U}(x^+, y^+; \mathbf{y}) \left\{ \int_{y^+}^{x^+} dz_1^+ \left(\frac{z_1^+ - y^+}{x^+ - y^+} \right)^2 \delta^{jl} \mathcal{B}^{ijl}(z_1^+, \mathbf{y}) \right. \\
 & + 2 \int_{y^+}^{x^+} dz_1^+ \int_{z_1^+}^{x^+} dz_2^+ \left[\left(\frac{z_1^+ - y^+}{x^+ - y^+} \right)^2 \mathcal{B}^j(z_2^+, \mathbf{y}) \mathcal{B}^{ij}(z_1^+, \mathbf{y}) \right. \\
 & \quad + \frac{(z_1^+ - y^+)(z_2^+ - y^+)}{(x^+ - y^+)^2} \left(\mathcal{B}^{ij}(z_2^+, \mathbf{y}) \mathcal{B}^i(z_1^+, \mathbf{y}) \right. \\
 & \quad \quad \left. \left. + \frac{1}{2} \mathcal{B}^i(z_2^+, \mathbf{y}) \delta^{jl} \mathcal{B}^{jl}(z_1^+, \mathbf{y}) + \frac{1}{2} \delta^{jl} \mathcal{B}^{jl}(z_2^+, \mathbf{y}) \mathcal{B}^i(z_1^+, \mathbf{y}) \right) \right] \\
 & + 2 \int_{y^+}^{x^+} dz_1^+ \int_{z_1^+}^{x^+} dz_2^+ \int_{z_2^+}^{x^+} dz_3^+ \left[\frac{(z_3^+ - y^+)(z_1^+ - y^+)}{(x^+ - y^+)^2} \mathcal{B}^i(z_3^+, \mathbf{y}) \mathcal{B}^j(z_2^+, \mathbf{y}) \mathcal{B}^j(z_1^+, \mathbf{y}) \right. \\
 & \quad \left. \left. + \frac{(z_2^+ - y^+)(z_1^+ - y^+)}{(x^+ - y^+)^2} \mathcal{B}^j(z_3^+, \mathbf{y}) \left(\mathcal{B}^i(z_2^+, \mathbf{y}) \mathcal{B}^i(z_1^+, \mathbf{y}) + \mathcal{B}^j(z_2^+, \mathbf{y}) \mathcal{B}^i(z_1^+, \mathbf{y}) \right) \right] \right\}
 \end{aligned}$$

Back up slides - Squared Amplitude

$$\frac{1}{N_{\xi}^2 - 1} \left\langle \overline{\mathcal{M}}_{\lambda}^{ab}(\underline{k}, \mathbf{q})^{\dagger} \overline{\mathcal{M}}_{\lambda}^{ab}(\underline{k}, \mathbf{q}) \right\rangle \Big|_{\text{E}} = \varepsilon_{\lambda}^{i*} \varepsilon_{\lambda}^j \int d^2 \mathbf{r} e^{i\mathbf{r} \cdot (\mathbf{q} - \mathbf{k})} 4 \mathcal{C}^i(\mathbf{k}, \mathbf{q}) \mathcal{C}^j(\mathbf{k}, \mathbf{q}) \mathcal{O}(\mathbf{r}). \quad (1)$$

Similarly, the next-to-eikonal contribution can be written as

$$\begin{aligned} & \frac{1}{N_{\xi}^2 - 1} \left\langle \overline{\mathcal{M}}_{\lambda}^{ab}(\underline{k}, \mathbf{q})^{\dagger} \overline{\mathcal{M}}_{\lambda}^{ab}(\underline{k}, \mathbf{q}) \right\rangle \Big|_{\text{NE}} = \varepsilon_{\lambda}^{i*} \varepsilon_{\lambda}^j \int d^2 \mathbf{r} e^{i\mathbf{r} \cdot (\mathbf{q} - \mathbf{k})} 2 \frac{L^+}{k^+} \\ & \times \left\{ \left[\mathcal{C}^j(\mathbf{k}, \mathbf{q}) \tilde{\mathcal{C}}^{li}(\mathbf{k}, \mathbf{q}) - \mathcal{C}^i(\mathbf{k}, \mathbf{q}) \tilde{\mathcal{C}}^{lj}(\mathbf{k}, \mathbf{q}) \right] \mathcal{O}_{[0,1]}^l(\mathbf{r}) - i \left[\mathcal{C}^j(\mathbf{k}, \mathbf{q}) \frac{\mathbf{q}^i}{\mathbf{q}^2} - \mathcal{C}^i(\mathbf{k}, \mathbf{q}) \frac{\mathbf{q}^j}{\mathbf{q}^2} \right] \mathcal{O}_{[1,0]}^j(\mathbf{r}) \right\} \end{aligned} \quad (2)$$

and

$$\begin{aligned} & \frac{1}{N_{\xi}^2 - 1} \left\langle \overline{\mathcal{M}}_{\lambda}^{ab}(\underline{k}, \mathbf{q})^{\dagger} \overline{\mathcal{M}}_{\lambda}^{ab}(\underline{k}, \mathbf{q}) \right\rangle \Big|_{\text{NNE}} = \varepsilon_{\lambda}^{i*} \varepsilon_{\lambda}^j \int d^2 \mathbf{r} e^{i\mathbf{r} \cdot (\mathbf{q} - \mathbf{k})} \left(\frac{L^+}{k^+} \right)^2 \\ & \times \left\{ \left[\mathcal{C}^j(\mathbf{k}, \mathbf{q}) \frac{\mathbf{q}^i}{\mathbf{q}^2} + \mathcal{C}^i(\mathbf{k}, \mathbf{q}) \frac{\mathbf{q}^j}{\mathbf{q}^2} \right] \left[-4 \mathbf{k}^l \mathbf{k}^m \mathcal{O}_{[0,2]}^{l,m}(\mathbf{r}) - 2i \mathbf{k}^l \mathcal{O}_{[1,1]}^l(\mathbf{r}) + \mathcal{O}_{[2,0]}(\mathbf{r}) \right] \right. \\ & + \frac{i}{2} \left[\mathcal{C}^j(\mathbf{k}, \mathbf{q}) (\mathbf{k}^2 \delta^{il} - 2\mathbf{k}^i \mathbf{k}^l) + \mathcal{C}^i(\mathbf{k}, \mathbf{q}) (\mathbf{k}^2 \delta^{jl} - 2\mathbf{k}^j \mathbf{k}^l) \right] \mathcal{O}_{(A)}^l(\mathbf{r}) \\ & + \frac{1}{2} \left[\mathcal{C}^j(\mathbf{k}, \mathbf{q}) \delta^{il} + \mathcal{C}^i(\mathbf{k}, \mathbf{q}) \delta^{jl} \right] \left[\mathbf{k}^m \mathcal{O}_{(B)}^{lm}(\mathbf{r}) + i \mathcal{O}_{(C)}^l(\mathbf{r}) \right] \\ & + \left[\tilde{\mathcal{C}}^{li}(\mathbf{k}, \mathbf{q}) \tilde{\mathcal{C}}^{mj}(\mathbf{k}, \mathbf{q}) \mathcal{O}_{[0,1];[0,1]}^{l,m}(\mathbf{r}) + \frac{\mathbf{q}^i}{\mathbf{q}^2} \frac{\mathbf{q}^j}{\mathbf{q}^2} \mathcal{O}_{[1,0];[1,0]}(\mathbf{r}) \right] \\ & \left. + i \left[\frac{\mathbf{q}^i}{\mathbf{q}^2} \tilde{\mathcal{C}}^{lj}(\mathbf{k}, \mathbf{q}) + \frac{\mathbf{q}^j}{\mathbf{q}^2} \tilde{\mathcal{C}}^{li}(\mathbf{k}, \mathbf{q}) \right] \mathcal{O}_{[0,1];[1,0]}^l(\mathbf{r}) \right\}. \quad (3) \end{aligned}$$