

# RESUMMATION OF DOUBLE LOGARITHMS IN THE RAPIDITY EVOLUTION OF COLOR DIPOLES

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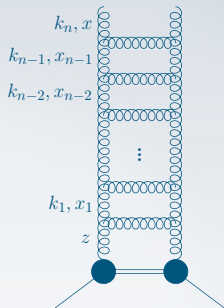
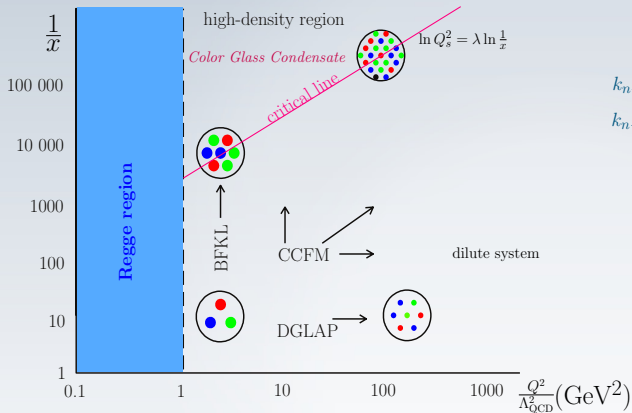


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<sup>†</sup> Based on work in collaboration with E. Iancu, A.H. Mueller, G. Soyez and D.N. Triantafyllopoulos [PLB**744** (2015) 293, [arXiv:1502.05642](https://arxiv.org/abs/1502.05642)]

# Introduction

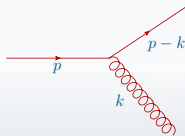
# DGLAP and BFKL Evolutions



[Gribov & Lipatov '72; Dokshitzer '77; Altarelli & Parisi '77]

[Fadin, Kuraev & Lipatov '75,76,77]

[Lipatov '76; Balitsky & Lipatov '78]

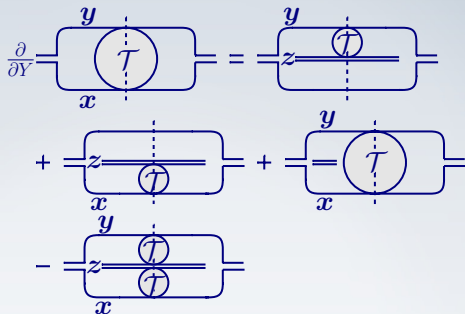
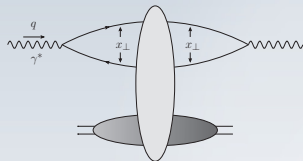


$$k_z = xp \gg k_\perp$$

$$0 < x < 1$$

$$dP_{\text{Brems}} \simeq \frac{\alpha_s \{C_A, C_F\}}{\pi^2} \frac{d^2 k_\perp}{k_\perp^2} \frac{dx}{x}$$

# High-Energy Scattering & the BK Equation



## Dipole Picture [Nikolaev &

Zakharov '91; Mueller '94]

$$S_{xy} = \frac{\text{tr}}{N_c} [V_x^\dagger V_y]$$

- Mixed representation  $\{x_\perp, k^+\}$  well-suited for high-energy scattering (diagonalizes SW interaction)

## Balitsky-Kovchegov (BK) equation

$$\partial_Y S_{xy} = \frac{\bar{\alpha}_s}{2\pi} \int_z \mathcal{M}_{xyz} [S_{xz} S_{zy} - S_{xy}];$$

$$\mathcal{M}_{xyz} = \frac{(x-y)^2}{(x-z)^2 (z-y)^2}$$

[Balitsky '96; Kovchegov '98]

## Balitsky-Kovchegov equation also emerges as mean-field-approximation of JIMWLK formalism

For a gluon crossing a shockwave target, the background field propagator is essentially a **Wilson line**

$$U_x^\dagger = \mathcal{P} \exp \left[ ig \int dx^+ A_a^-(x^+, \mathbf{x}) T^a \right]$$

and then ( $\int dp^+/p^+ \rightarrow \ln(1/x)$ )

$$\Delta H = \ln \frac{1}{r} H_{\text{JIMWLK}}$$

$$H_{\text{JIMWLK}} = \frac{1}{(2\pi)^3} \int \mathcal{K}_{xyz} (U_x^\dagger - U_z^\dagger)^{ab} (U_y^\dagger - U_z^\dagger)^{ac} R_x^b R_y^c$$

$$R_u^a U_x^{R\dagger} = ig \delta_{ux} U_x^{R\dagger} T_R^a$$

$$\mathcal{K}_{xyz} = \mathcal{K}_{xz}^i \mathcal{K}_{yz}^i \int \frac{d^2 k}{(2\pi)^2} e^{ik \cdot (x-z)} \underbrace{\text{diagram}}_{= 2gt^a \frac{\varepsilon_{\lambda k}^a}{k^2}} = \frac{ig t^a}{\pi} \varepsilon_{\lambda}^i \underbrace{\left( \frac{(x-z)^i}{(x-z)^2} \right)}_{\equiv \mathcal{K}_{xz}^i}$$

[Jalilian-Marian, Kovner, McLerran & Weigert '97; J.-Marian, Kovner, Leonidov & Weigert '98; Iancu, Leonidov & McLerran '01]

*Actually, BK and JIMWLK predictions for dipole scattering amplitude turn out to be very similar* [Kuokkanen, Rummukainen & Weigert '08]

# NLO Corrections & Resummation of Collinear Logarithms

- *Tour-de-force* computations of NLO corrections to BFKL [Fadin & Lipatov '98; Camici & Ciafaloni '98], BK [Balitsky & Chirilli '08] and JIMWLK [Balitsky & Chirilli '13; Kovner, Lublinsky & Mulian '14] equations.
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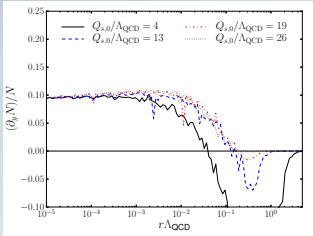
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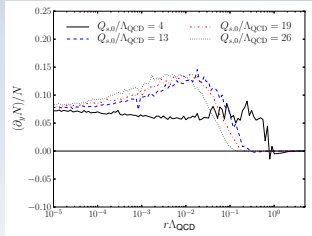


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(a)  $\gamma = 0.6$



(b)  $\gamma = 0.8$

Large corrections and instabilities in NLO BK traced back to double transverse logs [Lappi & Mantysäari '15]:

$$\begin{aligned}
 \frac{d}{d\eta} \text{Tr}\{\hat{U}_x \hat{O}_y^\dagger\} &= \frac{\alpha_s}{2\pi^2} \int d^2z \frac{(x-y)^2}{X^2 Y^2} \left[ 1 + \frac{\alpha_s}{4\pi} \left[ b \ln(x-y)^2 \mu^2 - b \frac{X^2 - Y^2}{(x-y)^2} \ln \frac{X^2}{Y^2} + \left( \frac{67}{9} - \frac{\pi^2}{3} \right) N_c - \frac{10}{9} n_f \right. \right. \\
 &\quad \left. \left. - 2N_c \ln \frac{X^2}{(x-y)^2} \ln \frac{Y^2}{(x-y)^2} \right] \right] [\text{Tr}\{\hat{O}_x \hat{O}_z^\dagger\} \text{Tr}\{\hat{O}_z \hat{O}_y^\dagger\} - N_c \text{Tr}\{\hat{O}_x \hat{O}_y^\dagger\}] \\
 &+ \frac{\alpha_s^2}{16\pi^2} \int d^2z d^2z' \left[ \left( -\frac{4}{(z-z')^4} + \left[ 2 \frac{X^2 Y^2 + X'^2 Y'^2 - 4(x-y)^2(z-z')^2}{(z-z')^4 [X^2 Y^2 - X'^2 Y'^2]} + \frac{(x-y)^4}{X^2 Y^2 - X'^2 Y'^2} \right. \right. \right. \\
 &\quad \times \left. \left. \left[ \frac{1}{X^2 Y^2} + \frac{1}{Y^2 X'^2} \right] + \frac{(x-y)^2}{(z-z')^2} \left[ \frac{1}{X^2 Y^2} - \frac{1}{X'^2 Y'^2} \right] \ln \frac{X^2 Y^2}{X'^2 Y'^2} \right) [\text{Tr}\{\hat{O}_x \hat{O}_z^\dagger\} \text{Tr}\{\hat{O}_z \hat{O}_y^\dagger\} \text{Tr}\{\hat{O}_z \hat{O}_y^\dagger\} \right. \\
 &\quad \left. - \text{Tr}\{\hat{O}_x \hat{O}_z^\dagger \hat{O}_z U_z^\dagger \hat{O}_z \hat{O}_y^\dagger\} - (z' \rightarrow z) \right] + \frac{(x-y)^2}{(z-z')^2} \left[ \frac{1}{X^2 Y^2} + \frac{1}{Y^2 X'^2} \right] - \frac{(x-y)^4}{X^2 Y^2 X'^2 Y'^2} \ln \frac{X^2 Y^2}{X'^2 Y'^2} \\
 &\quad \times \text{Tr}\{\hat{O}_x \hat{O}_z^\dagger\} \text{Tr}\{\hat{O}_z \hat{O}_y^\dagger\} \text{Tr}\{\hat{O}_z \hat{O}_y^\dagger\} + 4n_f \left[ \frac{4}{(z-z')^4} - 2 \frac{X'^2 Y'^2 + Y'^2 X^2 - (x-y)^2(z-z')^2}{(z-z')^4 (X^2 Y^2 - X'^2 Y'^2)} \ln \frac{X^2 Y^2}{X'^2 Y'^2} \right] \\
 &\quad \left. \times \text{Tr}\{r^a \hat{O}_x r^b \hat{O}_y^\dagger\} [\text{Tr}\{r^a \hat{O}_z r^b \hat{O}_z^\dagger\} - (z' \rightarrow z)] \right].
 \end{aligned}$$

# Our Goals

- 1 Identify the diagrammatic origin of double logarithmic corrections and its relation to the 'kinematic constraint'

[Ciafaloni '88; Andersson, Gustafson & Samuelsson '96; Kwieciński, Martin & Sutton '96; Beuf '14].

- 2 Implement directly the collinear resummation in coordinate space, as required by non-linear structure of BK equation.

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# The DLA Limit of the BFKL Equation

# (Naive) DLA Limit of the BFKL Equation

**BFKL Equation** ( $T = 1 - S, T \ll 1$ )

$$\partial_Y T_{\mathbf{x}\mathbf{y}}(Y) = \frac{\bar{\alpha}_s}{2\pi} \int d^2\mathbf{z} \mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{z}} [T_{\mathbf{x}\mathbf{z}}(Y) + T_{\mathbf{z}\mathbf{y}}(Y) - T_{\mathbf{x}\mathbf{y}}(Y)]$$

$\mathbf{z}$ -integration becomes logarithmic when daughter dipoles are much larger than the original one ( $|\mathbf{x} - \mathbf{z}| \simeq |\mathbf{z} - \mathbf{y}| \gg r \equiv |\mathbf{x} - \mathbf{y}|$ )

$\mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{z}} \simeq r^2/(\mathbf{x} - \mathbf{z})^4$  and  $T_{\mathbf{x}\mathbf{z}} \simeq T_{\mathbf{z}\mathbf{y}} \propto z^2$ ; negligible virtual term.

Writing  $T_{\mathbf{x}\mathbf{y}}(Y) \equiv r^2 Q_0^2 \mathcal{A}_{\mathbf{x}\mathbf{y}} \rightarrow r^2 Q_0^2 \mathcal{A}(Y, r^2)$

$$\mathcal{A}(Y, r^2) = \mathcal{A}(0, r^2) + \bar{\alpha}_s \int_0^Y dY_1 \int_{r^2}^{1/Q_0^2} \frac{dz^2}{z^2} \mathcal{A}(Y_1, z^2)$$

**(NAIVE) DLA EQUATION** (resums powers of  $\bar{\alpha}_s Y \rho$ ,  $\rho \equiv \ln[1/r^2 Q_0^2]$  to all orders)

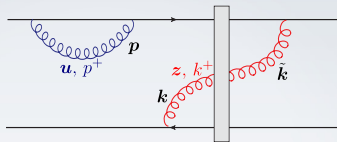
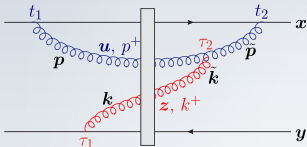
$$\mathcal{A}(Y, \rho) = I_0(2\sqrt{\bar{\alpha}_s Y \rho})$$

# The Diagrammatic Origin of the DLA Equation



# Computation of Time-Ordered Diagrams

- Lifetime of gluon fluctuation  $\tau_p \equiv 2p^+ / p^2 = 1/p^-$
- Eikonal approximation  $p^+ \gg k^+$



$$\begin{aligned}
 & -\frac{g^4 N_c^2}{(2\pi)^2} \int_{uz} S_{xu} S_{uz} S_{zy} \\
 & \times \int_{\vec{p}\vec{k}\vec{\tilde{k}}} e^{i\vec{p}\cdot(\mathbf{u}-\mathbf{x})} e^{i\vec{\tilde{p}}\cdot(\mathbf{x}-\mathbf{u})} e^{i\vec{k}\cdot(\mathbf{z}-\mathbf{y})} e^{i\vec{\tilde{k}}\cdot(\mathbf{u}-\mathbf{z})} \frac{\mathbf{p}\cdot\vec{\tilde{p}}}{p^2 \tilde{p}^2} \frac{\mathbf{k}\cdot\vec{\tilde{k}}}{k^2 \tilde{k}^2} \\
 & \times \int_{q_0^+}^{q^+} \frac{dk^+}{k^+} \int_{k^+}^{q^+} \frac{dp^+}{p^+} \boxed{\frac{p^+}{p^+ + k^+ \frac{p^2}{k^2}} \frac{p^+}{p^+ + k^+ \frac{(\tilde{p}-\tilde{k})^2}{\tilde{k}^2}}}.
 \end{aligned}$$

$$\begin{aligned}
 \frac{p^+}{p^+ + k^+ \frac{p^2}{k^2}} &= \frac{\tau_p}{\tau_p + \tau_k} \\
 &\simeq \begin{cases} 1 & \text{in BFKL} \\ \Theta(\tau_p - \tau_k) & \text{in DLA} \end{cases}
 \end{aligned}$$

## Real-Real Contribution

$$\left(\frac{\bar{\alpha}_s}{2\pi}\right)^2 \int_{q_0^+}^{q^+} \frac{dk^+}{k^+} \int_{k^+}^{q^+} \frac{dp^+}{p^+} \int_{\mathbf{uz}} \mathcal{M}_{xyu} [\mathcal{M}_{uyz} S_{xu} S_{uz} S_{zy} + \mathcal{M}_{xuz} S_{xz} S_{zu} S_{uy}]$$

$$\times \Theta(p^+ \bar{u}^2 - k^+ \bar{z}^2), \quad \bar{u} = \max(|\mathbf{u} - \mathbf{x}|, |\mathbf{u} - \mathbf{y}|); \quad \bar{z} = \max(|\mathbf{z} - \mathbf{x}|, |\mathbf{z} - \mathbf{y}|)$$

## Virtual-Real Contribution

$$-\left(\frac{\bar{\alpha}_s}{2\pi}\right)^2 \int_{q_0^+}^{q^+} \frac{dk^+}{k^+} \int_{k^+}^{q^+} \frac{dp^+}{p^+} \int_{\mathbf{uz}} \mathcal{M}_{xyu} \mathcal{M}_{xyz} S_{xz} S_{zy} \Theta(p^+ \bar{u}^2 - k^+ \bar{z}^2)$$

To DLA accuracy  $\mathcal{M}_{uyz} \mathcal{M}_{xyu} \simeq \frac{r^2}{\bar{u}^2 \bar{z}^4}$  and  $1 - S_{xu} S_{uz} S_{zy} \simeq T_{uz} + T_{zy} \simeq 2T(\bar{z}^2)$  and we generate logarithmic phase space

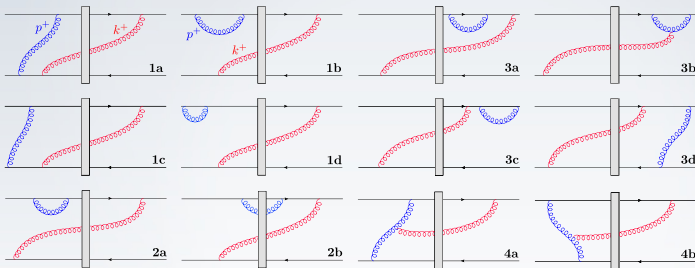
$$\int_{r^2}^{\bar{z}^2} \frac{d\bar{u}^2}{\bar{u}^2} \int_{k^+ \frac{\bar{z}^2}{\bar{u}^2}}^{q^+} \frac{dp^+}{p^+} = \int_{r^2}^{\bar{z}^2} \frac{d\bar{u}^2}{\bar{u}^2} \left( \ln \frac{q^+}{k^+} - \ln \frac{\bar{z}^2}{\bar{u}^2} \right) = Y\rho - \frac{\rho^2}{2}$$

$$Y = \ln \frac{q^+}{k^+}; \quad \rho = \ln \frac{\bar{z}^2}{r^2}$$

# Cancellation of Anti-Time-Ordered Diagrams in DLA

Anti-time ordered graphs, involving factors  $\frac{p^-}{p^-+k^-} \simeq \Theta(\tau_k - \tau_p)$  are also potentially enhanced by double transverse logs

$$\int_{r^2}^{\bar{z}^2} \frac{d\bar{u}^2}{\bar{u}^2} \int_{k+\frac{\bar{z}^2}{\bar{u}^2}}^{q^+} \frac{dp^+}{p^+} = \int_{r^2}^{\bar{z}^2} \frac{d\bar{u}^2}{\bar{u}^2} \ln \frac{\bar{z}^2}{\bar{u}^2} = \frac{\rho^2}{2}$$



However, **double logs cancel in the sum of all ATO diagrams**. This also explains the peculiar way double logs arise in [Balitsky & Chirilli '08].

# DLA Evolution for the Scattering Amplitude and the Lifetime Ordering Constraint

We conclude that perturbative corrections enhanced by double logarithms  $Y\rho$  or  $\rho^2$  can be resummed to all orders by solving a modified DLA equation involving manifest time-ordering

$$\mathcal{A}(q^+, r^2) = \mathcal{A}(0, r^2) + \bar{\alpha}_s \int_{r^2}^{1/Q_0^2} \frac{dz^2}{z^2} \int_{q_0^+}^{q^+ \frac{r^2}{z^2}} \frac{dk^+}{k^+} \mathcal{A}(k^+, z^2)$$

As it stands, this equation is non-local in rapidity

$$\partial_Y \mathcal{A}(Y, \rho) = \bar{\alpha}_s \int_0^\rho d\rho_1 \mathcal{A}(Y - \rho + \rho_1, \rho)$$

# Resummed Kernel for DLA, BFKL, and BK Evolutions

# Towards a Resummed Rapidity-Independent Kernel

- By direct iteration of the modified DLA equation, we get

$$\mathcal{A}(Y, \rho) = \int_0^\rho d\rho_1 f(Y, \rho - \rho_1) \mathcal{A}(0, \rho_1),$$
$$f(Y, \rho) = \delta(\rho) + \Theta(Y - \rho) \underbrace{\sum_{k=1}^{\infty} \frac{\bar{\alpha}_s^k (Y - \rho)^k \rho^{k-1}}{k!(k-1)!}}_{= \sqrt{\frac{\bar{\alpha}_s (Y - \rho)}{\rho}} I_1(2\sqrt{\bar{\alpha}_s (Y - \rho)\rho})}$$

- This can be written in integral representation:

$$f(Y, \rho) = \Theta(Y - \rho) \tilde{f}(Y, \rho);$$

$$\tilde{f}(Y, \rho) = \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\xi}{2\pi i} \exp \left[ \frac{\bar{\alpha}_s}{1-\xi} (Y - \rho) + (1-\xi)\rho \right]$$

# The Local Kernel within DLA Approximation

A change of variables brings this as usual Mellin representation

$$\tilde{f}(Y, \rho) = \int_c \frac{d\gamma}{2\pi i} J(\gamma) \exp[\bar{\alpha}_s \chi_{\text{DLA}}(\gamma) Y + (1 - \gamma)\rho]$$

$$\bar{\alpha}_s \chi_{\text{DLA}}(\gamma) = \frac{1}{2} \left[ -(1 - \gamma) + \sqrt{(1 - \gamma)^2 + 4\bar{\alpha}_s} \right] = \frac{\bar{\alpha}_s}{(1 - \gamma)} - \frac{\bar{\alpha}_s^2}{(1 - \gamma)^3} + \dots$$

$$J(\gamma) = 1 - \bar{\alpha}_s \chi'_{\text{DLA}}(\gamma) = 1 - \frac{\bar{\alpha}_s}{(1 - \gamma)^2} + \dots$$

Mellin representation and exponentiation in  $Y$  ensures the existence of an evolution equation for  $f$  (and thus for  $\mathcal{A}$ ) with an energy-independent kernel  $\mathcal{K}_{\text{DLA}}(\rho)$  defined as inverse Mellin of  $\chi_{\text{DLA}}(\gamma)$

$$\tilde{\mathcal{A}}(Y, \rho) = \tilde{\mathcal{A}}(0, \rho) + \bar{\alpha}_s \int_0^Y dY_1 \int_0^\rho d\rho_1 \mathcal{K}_{\text{DLA}}(\rho - \rho_1) \tilde{\mathcal{A}}(Y_1, \rho_1), \quad Y > \rho$$

$$\mathcal{K}_{\text{DLA}}(\rho) = \frac{J_1(2\sqrt{\bar{\alpha}_s \rho^2})}{\sqrt{\bar{\alpha}_s \rho^2}} = 1 - \frac{\bar{\alpha}_s \rho^2}{2} + \frac{(\bar{\alpha}_s \rho^2)^2}{12} + \dots$$

Coincides with momentum-space kernel proposed by [Sabio Vera '05]; compare with non-local approaches in [Salam '98; Motyka & Stařto '09; Beuf '14].

# The Change in the Initial Condition

Jacobian of Mellin transform induces also resummation in the initial condition ( $\sim$  impact factor):

$$\begin{aligned}\tilde{\mathcal{A}}(0, \rho) &= \int_0^\rho d\rho_1 \tilde{f}(0, \rho - \rho_1) \mathcal{A}(0, \rho_1), \\ \tilde{f}(0, \rho) &= \delta(\rho) - \sqrt{\bar{\alpha}_s} J_1(2\sqrt{\bar{\alpha}_s \rho^2}).\end{aligned}$$

$[\tilde{\mathcal{A}}(Y, \rho)$  coincides with physical amplitude  $\mathcal{A}(Y, \rho)$  for  $Y > \rho]$

$$\tilde{\mathcal{A}}(0, \rho) = \begin{cases} \frac{1}{2} [1 + J_0(\bar{\rho})] & \text{for } \mathcal{A}(0, \rho) = 1, \\ \frac{\rho}{2} \left[ 1 + J_0(\bar{\rho}) + \frac{\pi}{2} \mathbf{H}_0(\bar{\rho}) J_1(\bar{\rho}) - \frac{\pi}{2} \mathbf{H}_1(\bar{\rho}) J_0(\bar{\rho}) \right] & \text{for } \mathcal{A}(0, \rho) = \rho, \end{cases}$$



# Resummed Kernel for BFKL/BK Evolution

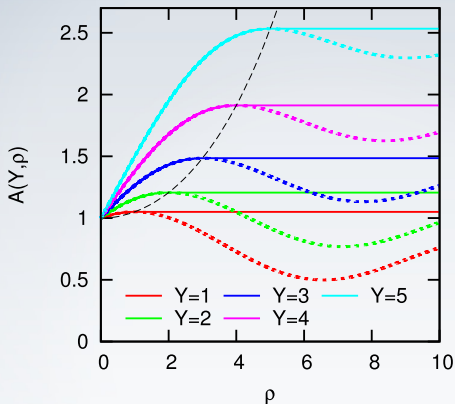
We can now easily promote our local DLA equation to **easily include NLL BFKL/BK**:

- ①  $\tilde{T}(Y, \rho) = e^{-\rho} \tilde{\mathcal{A}}(Y, \rho)$
- ② Return to transverse coordinates:  $\rho = \ln(1/r^2 Q_0^2)$ ;  $\rho - \rho_1 = \ln(z^2/r^2)$ ;  $\tilde{T}(Y, \rho) = \tilde{T}_{\mathbf{x}\mathbf{y}}(Y)$ ;  $2\tilde{T}(Y, z^2) \rightarrow \tilde{T}_{\mathbf{x}\mathbf{z}}(Y) + \tilde{T}_{\mathbf{z}\mathbf{y}}(Y)$
- ③ Restore full dipole kernel  $\frac{r^2}{z^4} dz^2 \rightarrow \frac{1}{\pi} \mathcal{M}_{\mathbf{x}\mathbf{y}\mathbf{z}} d^2\mathbf{z}$
- ④ Introduce the virtual term and remove IR and UV cutoffs in the  $\mathbf{z}$  integration
- ⑤ Replace the argument of  $\mathcal{K}_{\text{DLA}}$  by  $\ln \frac{z^2}{r^2} \rightarrow \sqrt{L_{\mathbf{x}\mathbf{z}r} L_{\mathbf{y}\mathbf{z}r}}$ , with  $L_{\mathbf{x}\mathbf{z}r} \equiv \ln[(\mathbf{x} - \mathbf{z})^2 / (\mathbf{x} - \mathbf{y})^2]$

$$\frac{\partial \tilde{T}_{xy}(Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi} \int d^2\mathbf{z} \mathcal{M}_{xyz} \mathcal{K}_{\text{DLA}} \left( \sqrt{L_{xzr} L_{yzr}} \right) \\ \times \left[ \tilde{T}_{xz}(Y) + \tilde{T}_{zy}(Y) - \tilde{T}_{xy}(Y) - \tilde{T}_{xz}(Y) \tilde{T}_{zy}(Y) \right]$$

# Numerical Results

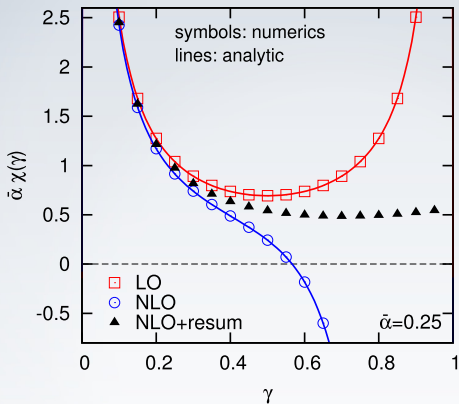
# Identity of Local and Non-Local Solutions



$A(Y, \rho)$ : full lines;  $\tilde{A}(Y, \rho)$ : dashed lines

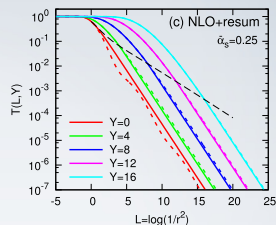
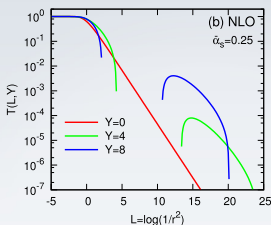
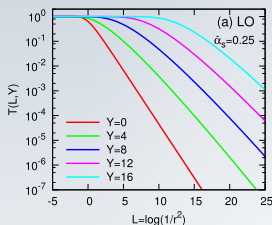
For  $Y > \rho$  both functions coincide; for  $Y < \rho$ ,  $\tilde{A}(Y, \rho)$  shows unphysical oscillations

# Resummed Characteristic Function



All-orders resummation ensures smooth behavior near  $\gamma = 1$ . For  $\bar{\alpha}_s = 0,25$ ,  $\chi(\gamma)$  is essentially flat for  $\gamma \gtrsim 0,5$

# Numerical Solution of Resummed BK

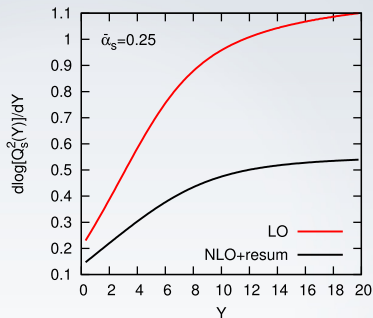
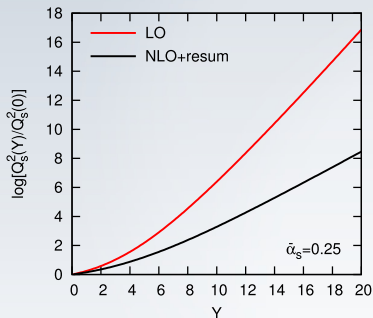


Initial condition of MV type  $\mathcal{A}(0, \rho) = 1$

Reduction of phase-space coming from time-ordering and giving rise to collinear double logs leads to a considerable reduction in the speed of the evolution

For  $\rho > Y$ , expected physical behavior  $T \propto e^{-\rho}$

# Modification of the Rapidity Dependence of the Saturation Momentum



The growth of the saturation scale with  $Y$  is considerably reduced by the resummation: for sufficiently large  $Y$ , the saturation exponent  $\lambda_s \equiv \frac{d\rho_s}{dY}$  smaller by factor 2 compared to LO BFKL (asymptotically,  $\lambda_s \sim 0,55$ ).

# Conclusions & Outlook



# Conclusions

- ① We established clearly through a diagrammatic analysis the origin of double logs as coming from reduction of phase space due to time ordering
- ② We were able to give an evolution equation with all-orders resummation of double logs in terms of an energy-independent kernel very convenient for numerical implementation
- ③ Our resummation is formulated directly in coordinate space allowing us its application to BK equation
- ④ Collinear resummation stabilizes and slows down the evolution. Very important phenomenological consequences expected

# Outlook

- Applications to phenomenology
- Study and resummation of single logs
- Consequences of resummation for initial condition/impact factor

# Back-Up Slides

# Collinear Resummation à la Salam

Double Mellin Representation for BFKL Green's function

$$G(k, k_0, Y) = \frac{1}{k^2} \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{s}{kk_0}\right)^\omega e^{\gamma\rho} \frac{1}{\omega - \kappa(\omega, \gamma)},$$
$$\rho = \ln(k^2/k_0^2); \quad \kappa(\omega, \gamma) = \bar{\alpha}_s \chi(\gamma) + \bar{\alpha}_s^2 \chi_1(\omega, \gamma) + \dots$$

Matching with DGLAP through identification of relevant evolution variable for  $k^2 > k_0^2$  and viceversa:  $\omega$ -shift

$$G(k, k_0, Y) = \frac{1}{k^2} \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{s}{k^2}\right)^\omega e^{(\gamma+\omega/2)\rho} \frac{1}{\omega - \kappa(\gamma, \omega)}$$
$$= \frac{1}{k_0^2} \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i} \int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty} \frac{d\gamma}{2\pi i} \left(\frac{s}{k_0^2}\right)^\omega e^{(1-\gamma+\omega/2)(-\rho)} \frac{1}{\omega - \kappa(\omega, \gamma)}$$

# Dipole Scattering Amplitude

## Glauber-Mueller Formula for Dipole S-Matrix

$$S(r, Y) = \exp \left[ -\frac{r^2 Q_s^2(Y)}{4} \right]$$

( $T(r) \sim 1$  for  $r \gg \frac{1}{Q_s}$  (black disk limit);  $T(r) \sim 0$  for  $r \ll \frac{1}{Q_s}$  (color transparency))

## GBW Model for Dipole Cross Section

$$\sigma^{\text{dip}} = \sigma_0 \left[ 1 - \exp \left( -\frac{r^2 Q_s^2(x)}{4} \right) \right]; \quad Q_s^2(x) = Q_0^2 \left( \frac{x_0}{x} \right)^\lambda$$

## AAMQS Parametrization

$$T(r, b) = 1 - \exp \left[ -\frac{(r^2 Q_{s0}^2(b))^\gamma}{4} \ln \left( \frac{1}{\Lambda r} + e \right) \right]$$

# Saturation Momentum

## Gribov-Levin-Ryskin Estimate

$$Q_s \sim \alpha_s^2 \Lambda_{\text{QCD}} \left( \frac{1}{x} \right)^{\alpha_P - 1}$$

**DLA Estimate of Rapidity Dependence of Dipole Scattering Amplitude** ( $r \ll 1/Q_{s0}$ )

$$T(r, Y) \sim (rQ_{s0})^2 (\bar{\alpha}_s Y)^{1/4} \rho^{-3/4} \exp[2\sqrt{2\bar{\alpha}_s Y \rho}]$$