Boundary terms in the decomposition of nucleon spin

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DIS 2015
XXIII International Workshop on
Deep-Inelastic Scattering and
Related Subjects

Dallas, Texas
April 27 – May 1, 2015
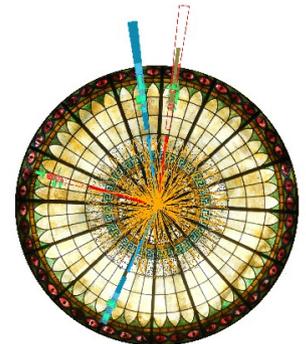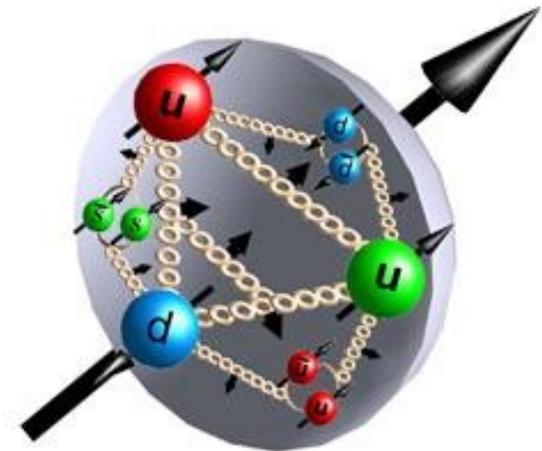
- Many approaches to understanding the proton spin crisis involve decomposing the QCD angular momentum operator J_{QCD}, and forming a sum rule [Jaffe, Manohar (1990); Jaffe (1996); Leader, Lorcé (2014)]:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

$$S_{q/g} = \frac{\langle p, s | S_{q/g}^3 | p, s \rangle}{\langle p, s | p, s \rangle}$$

Is the rest here?

- Sum rules such as these raise important questions:

  – *Is it feasible to measure each of the terms?*

  – *What is their theoretical interpretation?*

  – *Is the sum rule theoretically consistent?*

→ so a solid understanding of these equations is important!

[R. L. Jaffe and A. Manohar, *Nucl. Phys. B* **337**, 509 (1990).]
[R. L. Jaffe, *Phys. Lett. B* **365**, 359 (1996).]
[E. Leader and C. Lorcé, *Phys. Rept.* **541**, 163 (2014).]

- There are in fact many different ways in which one can decompose J_{QCD} [Leader, Lorcé (2014)]:

  – Jaffe-Manohar [Jaffe, Manohar (1990)]:

  $$J_{QCD}^i = S_{q,\text{JM}}^i + L_{q,\text{JM}}^i + S_{g,\text{JM}}^i + L_{g,\text{JM}}^i$$

  – Ji [Ji (1997)]:

  $$J_{QCD}^i = S_{q,\text{Ji}}^i + L_{q,\text{Ji}}^i + J_{g,\text{Ji}}^i$$

  – Wakamatsu [Wakamatsu (1997)]:

  $$J_{QCD}^i = S_{q,\text{Wak}}^i + L_{q,\text{Wak}}^i + S_{g,\text{Wak}}^i + L_{g,\text{Wak}}^i$$

  – etc...

    → *but which of these J_{QCD} decompositions (if any) are correct?*

[E. Leader and C. Lorcé, *Phys. Rept.* **541**, 163 (2014).]
[R. L. Jaffe and A. Manohar, *Nucl. Phys. B* **337**, 509 (1990).]
[X. Ji, *Phys. Rev. Lett* **78**, 610 (1997).]
[M. Wakamatsu, *Int. J. Mod. Phys. A* **29**, 1430012 (2014).]

- The Jaffe-Manohar decomposition of J_{QCD} has a surprisingly simple form:

$$J_{QCD}^i = \epsilon^{ijk} \int d^3x \underbrace{\left[\frac{i}{2} \overline{\psi} \gamma^0 \left(x^j \partial^k \right) \psi + \text{ h.c.} \right]}_{:=L_q^i} + \epsilon^{ijk} \int d^3x \underbrace{\left[\frac{1}{4} \epsilon^{0jkl} \overline{\psi} \gamma_l \gamma^5 \psi \right]}_{:=S_q^i}$$

$$\underbrace{-\epsilon^{ijk} \int d^3x \left[F^{0la} \left(x^j \partial^k \right) A_l^a \right]}_{:=L_g^i} + \underbrace{\epsilon^{ijk} \int d^3x \left[F^{0ka} A^{ja} \right]}_{:=S_g^i}$$

$$\underbrace{-\frac{i}{16} \epsilon^{ijk} \int d^3x \ \partial_l \left[x^j \overline{\psi} \{ \gamma^k, [\gamma^0, \gamma^l] \} \psi \right]}_{:=\mathcal{S}_1^i} + \underbrace{\epsilon^{ijk} \int d^3x \ \partial_l (x^j F^{0la} A^{ka})}_{:=\mathcal{S}_2^i}$$

- But it only holds if these spatial boundary terms vanish

    → *this is a common feature of many of the J_{QCD} decompositions*

- Spatial boundary terms arise frequently in decompositions of J_{QCD} because the angular momentum current $M^{\mu\nu\lambda}$ contains "*superpotentials*" [Jaffe, Manohar (1990)]:

$$\partial_\beta B^{[\mu\beta][\nu\lambda]}$$

→ which in the conserved charge J_{QCD} take the form:

$$\int d^3x \; \partial_i B^{[0i][\nu\lambda]}$$

- Stokes' theorem is then invoked to justify the vanishing of these terms – "*fields vanish at spatial infinity*"

Problem: this is a purely classical argument

→ *applying classical reasoning to the full quantum theory can be problematic!*

[R. L. Jaffe and A. Manohar, *Nucl. Phys. B* **337**, 509 (1990).]

- Quantum fields *φ(x)* are *operator-valued distributions*, and so in general are not point-wise defined

  → *the question of when operators of the form:* $\int d^3x \, \partial_i B^i$ *vanish is therefore more subtle in QFT*

- It *is* possible to rigorously make sense of these operators though, by smearing with test functions [Kastler et al. (1966); Ferrari et al. (1977); Kugo, Ojima (1979)]

  → *originally used in the context of Goldstone's Theorem*

- Once the operator is defined in this way, one can apply general QFT axioms such as local (anti-)commutativity

$$[\Phi(f),\Psi(g)] = 0, \text{ for space-like separated } f \text{ and } g$$

[D. Kastler, D. W. Robinson and J. A. Swieca, *Commun. Math. Phys.* **2**, 108 (1966).]
[R. Ferrari, L. E. Picasso and F. Strocchi, *Il Nuovo Cim.* **39A**, 1 (1977).]
[T. Kugo and I. Ojima, *Prog. Theor. Phys. Suppl.* **66** (1979).]

- It turns out that by using this more rigorous QFT approach one can determine a necessary and sufficient condition for these terms to vanish [Lowdon (2014)]:

$$\int d^3x \ \partial_i B^i \ \ vanishes \ in \ \mathcal{H} \quad \Longleftrightarrow \quad \int d^3x \ \partial_i B^i |0\rangle = 0$$

An interesting feature of this condition is that it only depends on the action of the operator on the vacuum state

…and from this one has the following condition:

$$\text{If} \ \exists |p\rangle \in \mathcal{H} \ \text{s.t:} \ \langle p| \int d^3x \ \partial_i B^i |0\rangle \neq 0 \quad \Longrightarrow \quad \int d^3x \ \partial_i B^i \neq 0$$

→ which can then be applied to the superpotentials $\mathcal{S}_1^i$ and $\mathcal{S}_2^i$

[P. Lowdon, *Nucl. Phys. B* **889**, 801 (2014).]

- So in this case if $\exists |p\rangle \in \mathcal{H}$ $s.t.$ $\langle p|\mathcal{S}_1^i|0\rangle \neq 0$ or: $\langle p|\mathcal{S}_2^i|0\rangle \neq 0$ then $\mathcal{S}_1^i$ or $\mathcal{S}_2^i$ are non-vanishing as operators

- Choosing $|p\rangle = |0\rangle$ one has: $\langle 0|\mathcal{S}_1^i|0\rangle \sim \epsilon^{ijk}\epsilon^{0jkl}\langle 0|\overline{\psi}\gamma^l\gamma^5\psi|0\rangle$

*Evidence* [Pasupathy, Singh (2006)] *to suggest this is non-vanishing*

$\rightarrow$ which suggests: $\boxed{J_{QCD}^i \neq S_q^i + L_q^i + S_g^i + L_g^i}$

- This condition therefore casts doubt on the validity of the Jaffe-Manohar angular momentum operator decomposition

$\rightarrow$ *what's interesting about the apparent failure of this decomposition is that it follows from the non-trivial structure of the QCD vacuum*

[J. Pasupathy and R. K. Singh, *Int. J. Mod. Phys. A* **21**, 5099 (2006).]
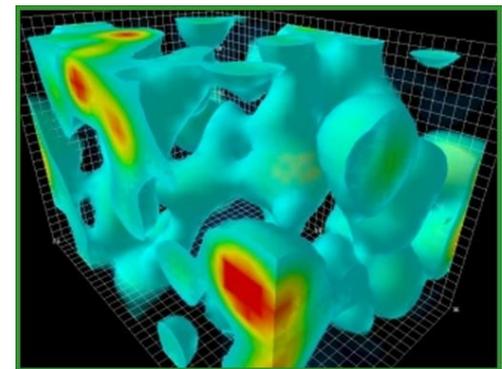
[The University of Adelaide (2015)]

- Although the J_{QCD} decomposition may not hold, it is possible that $\mathcal{S}_1^i$ and $\mathcal{S}_2^i$ vanish when inserted between z-polarised proton states

  → *but it looks as though this is not the case* [Lowdon (2014)]

- Since the spin decomposition appears neither to hold operator-wise nor on a matrix element level, this casts doubt on the validity and physicality of the Jaffe-Manohar spin sum rule

  → the *interpretation* of S_q *as the quark contribution to the proton spin is therefore unclear*

$$\frac{1}{2} \overset{?}{=} S_q + L_q + S_g + L_g$$

[P. Lowdon, *Nucl. Phys. B* **889**, 801 (2014).]

4. Conclusions

- Spatial boundary terms play an important role in the determination of decompositions of J_{QCD}, and the subsequent spin sum rules

- The issue of whether or not these terms vanish is subtle, but can be addressed using a more rigorous QFT approach

- A necessary and sufficient condition for operators of the form: $\int d^3x \, \partial_i B^i$ to vanish is that they must annihilate the vacuum

- Applying this condition to the spatial boundary (*superpotential*) terms that arise in the Jaffe-Manohar J_{QCD} decomposition suggests they are non-vanishing, and hence: $J_{QCD}^i \neq S_q^i + L_q^i + S_g^i + L_g^i$

- Moreover, it appears that the obstruction to such a decomposition arises because of the non-trivial vacuum structure of QCD

- The derivative of distributions is defined by:

$$\int d^4x \; \varphi'(x) f(x) = -\int d^4x \; \varphi(x) f'(x)$$

$\rightarrow$ *there are no boundary terms because distributions are in general not point-wise defined!*

- Distributions only make sense when smeared with suitable test functions:

$$\boxed{\int d^3x \; \partial_i B^i} \qquad \Longleftrightarrow \qquad \boxed{\lim_{R \to \infty} \int d^4x \; \alpha(x_0) f_R(\mathbf{x}) \partial_i B^i(x)}$$

where: $\displaystyle \int dx_0 \; \alpha(x_0) = 1, \qquad f_R(\mathbf{x}) = \begin{cases} 1, & |\mathbf{x}| < R \\ 0, & |\mathbf{x}| > R(1+\varepsilon) \end{cases}$

with $\alpha \in \mathcal{D}(\mathbb{R})$ $(\mathrm{supp}\,(\alpha) \subset [-\delta, \delta], \; \delta > 0)$ and $f_R \in \mathcal{D}(\mathbb{R}^3)$

- Derivation of J_{QCD} from the QCD energy-momentum tensor:

$$T_{QCD}^{\mu\nu} = T_{\text{phys}}^{\mu\nu} - \left\{ Q_B, \ (\partial^\mu \overline{C}^a) A^{\nu a} + (\partial^\nu \overline{C}^a) A^{\mu a} + g^{\mu\nu} \left(\frac{1}{2} \xi \overline{C}^a B^a - (\partial^\rho \overline{C}^a) A_\rho^a \right) \right\}$$

$$T_{\text{phys}}^{\mu\nu} = \frac{1}{2} \overline{\psi} \left(\frac{i}{2} \gamma^\mu (\overrightarrow{\partial^\nu} - \overleftarrow{\partial^\nu}) + g T^a A^{\nu a} \gamma^\mu \right) \psi + (\mu \leftrightarrow \nu) + F_\rho^{\mu a} F^{\rho\nu a} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta}^a F^{\alpha\beta a}$$
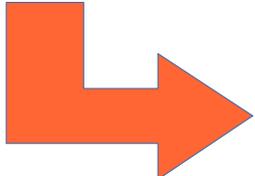
$$M^{\mu\nu\lambda} := x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu}$$

$$M_{QCD}^{\mu\nu\lambda} = x^\nu \left[\frac{1}{2} \overline{\psi} \left(\frac{i}{2} \gamma^\mu (\overrightarrow{\partial^\lambda} - \overleftarrow{\partial^\lambda}) + g T^a A^{\lambda a} \gamma^\mu \right) \psi + (\mu \leftrightarrow \lambda) + F_\rho^{\mu a} F^{\rho\lambda a} + \frac{1}{4} g^{\mu\lambda} F_{\alpha\beta}^a F^{\alpha\beta a} \right]$$

$$- x^\lambda \left[\frac{1}{2} \overline{\psi} \left(\frac{i}{2} \gamma^\mu (\overrightarrow{\partial^\nu} - \overleftarrow{\partial^\nu}) + g T^a A^{\nu a} \gamma^\mu \right) \psi + (\mu \leftrightarrow \nu) + F_\rho^{\mu a} F^{\rho\nu a} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta}^a F^{\alpha\beta a} \right]$$

$$= x^\nu F_\rho^{\mu a} F^{\rho\lambda a} - x^\lambda F_\rho^{\mu a} F^{\rho\nu a} + \frac{1}{4} F_{\alpha\beta}^a F^{\alpha\beta a} (x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu})$$

$$+ \frac{i}{4} \left[x^\nu \overline{\psi} (\gamma^\mu D^\lambda + \gamma^\lambda D^\mu) \psi - (\nu \leftrightarrow \lambda) \right] + \text{h.c.}$$

$$J_{QCD}^i = \epsilon^{ijk} \int d^3x \left[\frac{i}{2} \overline{\psi} \gamma^0 \left(x^j \partial^k \right) \psi + \text{h.c.} \right] + \epsilon^{ijk} \int d^3x \left[\frac{1}{4} \epsilon^{0jkl} \overline{\psi} \gamma_l \gamma^5 \psi \right]$$
$$\underbrace{}_{:=L_q^i} \qquad \underbrace{}_{:=S_q^i}$$

$$- \epsilon^{ijk} \int d^3x \left[F^{0la} \left(x^j \partial^k \right) A_l^a \right] + \epsilon^{ijk} \int d^3x \left[F^{0ka} A^{ja} \right]$$
$$\underbrace{}_{:=L_g^i} \qquad \underbrace{}_{:=S_g^i}$$

$$- \frac{i}{16} \epsilon^{ijk} \int d^3x \ \partial_l \left[x^j \overline{\psi} \{ \gamma^k, [\gamma^0, \gamma^l] \} \psi \right] + \epsilon^{ijk} \int d^3x \ \partial_l (x^j F^{0la} A^{ka})$$
$$\underbrace{}_{:=S_1^i} \qquad \underbrace{}_{:=S_1^i}$$

$$J_{QCD}^i := \frac{1}{2} \epsilon^{ijk} \int d^3x \ M_{QCD}^{0jk}(x)$$

- The matrix elements for explicitly *x*-dependent spatial boundary terms (as in the J_{QCD} decompositions) are given by:

$$\langle p| \int d^3x \; \partial_i \left(x^j B^{k0i}(x)\right) |0\rangle = \begin{cases} \lim_{R\to\infty} \int d^3x \; f_R(\mathbf{x})\langle 0|B^{k0j}(0)|0\rangle, & p = 0 \\ \lim_{R\to\infty} \int d^4x \; \alpha(x_0)f_R(\mathbf{x})e^{ip_\mu x^\mu}\big[\langle p|B^{k0j}(0)|0\rangle & \\ \qquad +ip_i\langle p|x^j B^{k0i}(0)|0\rangle\big], & p \neq 0 \end{cases}$$

- The vacuum expectation values of *superpotential* terms in the Jaffe-Manohar decomposition have the explicit form:

$$\langle 0|\mathcal{S}_1^i|0\rangle = \lim_{R\to\infty} \int d^3x \; \frac{1}{4}f_R(\mathbf{x})\epsilon^{ijk}\epsilon^{0jkl}\langle 0|\overline{\psi}\gamma^l\gamma^5\psi|0\rangle$$

$$\langle 0|\mathcal{S}_2^i|0\rangle = \lim_{R\to\infty} \int d^3x \; f_R(\mathbf{x})\epsilon^{ijk}\langle 0|F^{0ja}A^{ka}|0\rangle$$

- One has the following pion form factor relation which also suggests that the *superpotential* term $\mathcal{S}_1^i$ is non-vanishing:

$$\langle \pi|\overline{\psi}\gamma^l\gamma^5\psi|0\rangle = if_\pi p^l$$