Classical Gluon Production Amplitude for Nucleus-Nucleus Collisions

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Based on:

GA Chirilli, YV Kovchegov, DE Wertepny, Classical Gluon Production Amplitude for Nucleus-Nucleus Collisions: First Saturation Correction in the Projectile, JHEP (2015), arXiv:1501.03106

Nucleus-Nucleus Collisions

- Single inclusive gluon production cross-section for Heavy Nucleus-Nucleus collisions at the classical level using perturbative QCD.

- Problem is too hard at the moment.

- Instead calculate a simpler problem, Heavy-Light Ion collisions.

- Take into account all nucleons in one of the ions (heavy) and only two in the other (light).
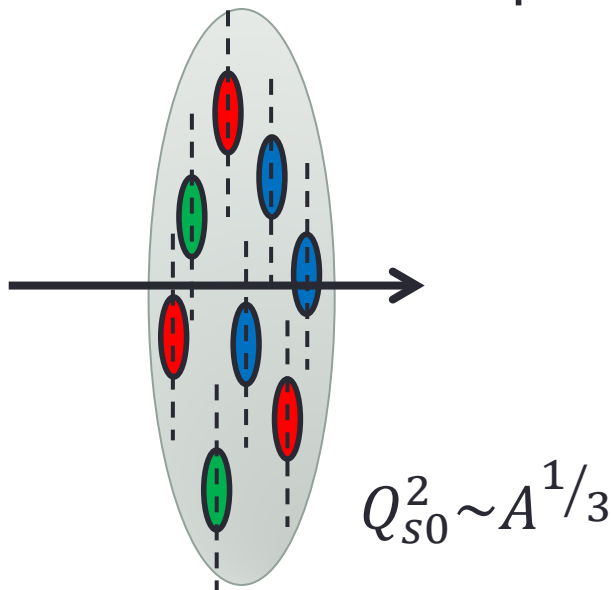
- Calculate the g^3 amplitude.

Proton-Nucleus Collison Case

- Due to the energy of the collision the projectile and target are Lorentz contracted.
- The interaction happens instantaneously compared to gluon emission time.
- View the projectile as emitting gluons which interact with the target instantaneously.
- Model using saturation physics.
- Using the Light-cone gauge, $A^+ = 0$.

x^-

x^+

Projectile

Target
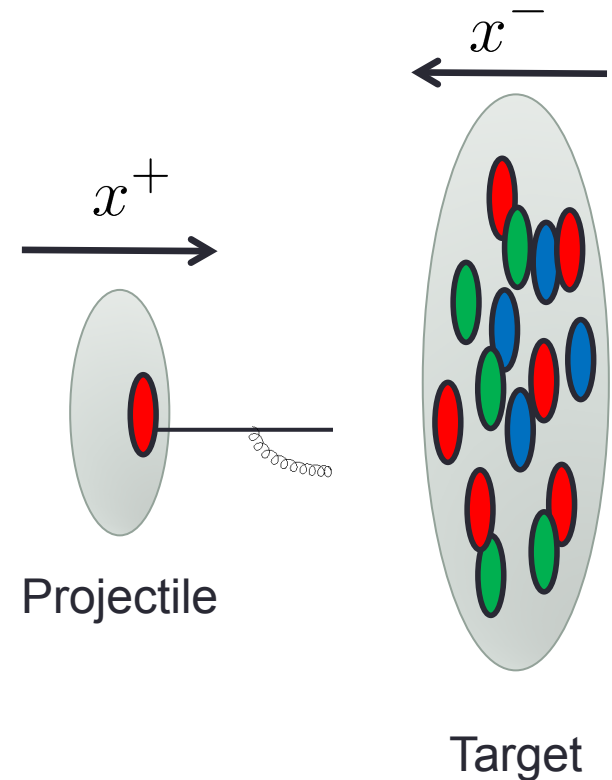
Modeling the Interaction as a Wilson Line

• A quark/gluon propagating through a nucleus at high energy can be modeled as a Wilson line. Recoilless in transverse spatial coordinate it interacts with many different "color patches".

$$U_{\boldsymbol{x}} = \mathrm{P} \exp \left\{ i\, g \int_{-\infty}^{\infty} dx^+ \, \mathcal{A}^-(x^+, x^- = 0, \boldsymbol{x}) \right\}$$

$$Q_{s0}^2 \sim A^{1/3}$$

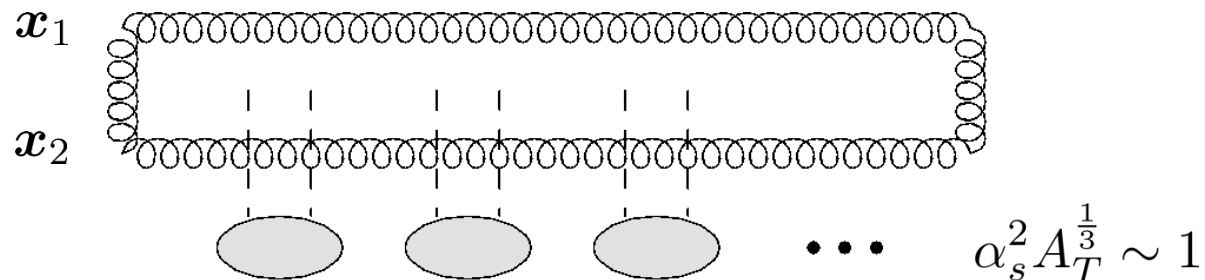$$\mathcal{A}^- = \sum_i T^a A_i^{a-}$$

• In the heavy light collision case:

$$Q_{s\,P} \ll Q_{s\,T}$$

Analysis of the Gluon Dipole – Saturation effects

- Model the gluon dipole as a series of tree level scatterings off many nucleons. The dotted lines represent gluons.

$$S_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y) \equiv \frac{1}{N_c^2 - 1} \left\langle Tr[U_{\boldsymbol{x}_1} U_{\boldsymbol{x}_2}^\dagger] \right\rangle$$

$$\alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

$$S_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y = 0) = \exp\left[-\frac{1}{4} |\boldsymbol{x}_1 - \boldsymbol{x}_2|^2 \, Q_{s0}^2 \left(\frac{\boldsymbol{x}_1 + \boldsymbol{x}_2}{2} \right) \ln\left(\frac{1}{|\boldsymbol{x}_1 - \boldsymbol{x}_2|\Lambda} \right) \right]$$

- The forward scattering amplitude is given by.

$$N_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y = 0) = 1 - S_G(\boldsymbol{x}_1, \boldsymbol{x}_2, y = 0)$$

Diagrams for pA Collisions
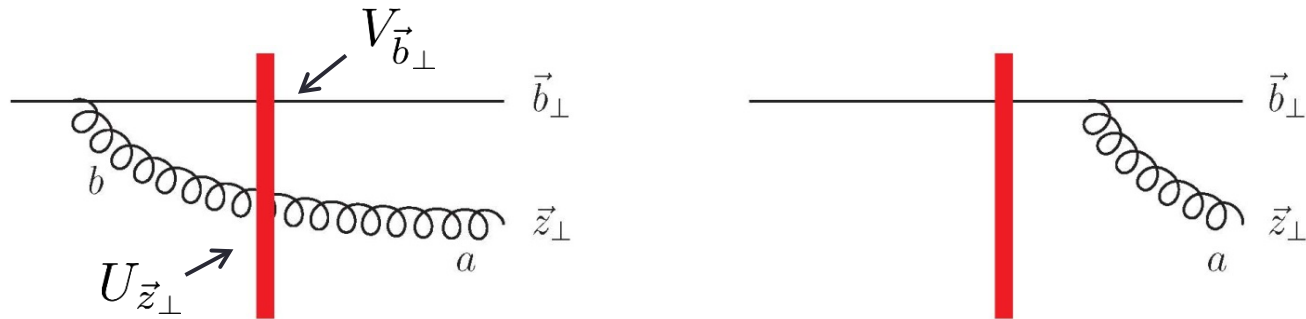
- Consist of a quark-gluon splitting and an interaction with the target, modeled as a shockwave.
- The projectile interacting with the target results in a power counting of

$$|M|^2 \sim \frac{1}{\alpha_s}(\alpha_s^2 A_P^{\frac{1}{3}})$$

- In total the amplitude is

$$M(\vec{z}_\perp, \vec{b}_\perp) = \frac{i\,g}{\pi} \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_\perp)}{|\vec{z}_\perp - \vec{b}_\perp|^2} \left[U_{\vec{z}_\perp}^{ab} - U_{\vec{b}_\perp}^{ab}\right] \left(V_{\vec{b}_\perp} t^b\right)$$

Result for p-A

- The resulting cross section is

$$\frac{d\sigma}{d^2 k_T \, dy} = \frac{\alpha_s \, C_F}{4 \, \pi^4} \int d^2 z \, d^2 z' \, d^2 b \, e^{-i\vec{k}_\perp \cdot (\vec{z}_\perp - \vec{z}_\perp')} \, \frac{\vec{z}_\perp - \vec{b}_\perp}{|\vec{z}_\perp - \vec{b}_\perp|^2} \cdot \frac{\vec{z}_\perp' - \vec{b}_\perp}{|\vec{z}_\perp' - \vec{b}_\perp|^2}$$

$$\times \left[S_G(\vec{z}_\perp, \vec{z}_\perp') - S_G(\vec{b}_\perp, \vec{z}_\perp') - S_G(\vec{z}_\perp, \vec{b}_\perp) + 1 \right]$$

- Overall power counting:

  - From Classical fields:

  $$A_\mu \sim \frac{1}{g} \implies \langle A_\mu A^\mu \rangle \sim \frac{1}{\alpha_s}$$

  - Single nucleon in the projectile:

  $$\alpha_s^2 A_P^{\frac{1}{3}}$$

  - Interactions in the target:

  $$(\alpha_s^2 A_T^{\frac{1}{3}})^N \sim 1$$

- In total:

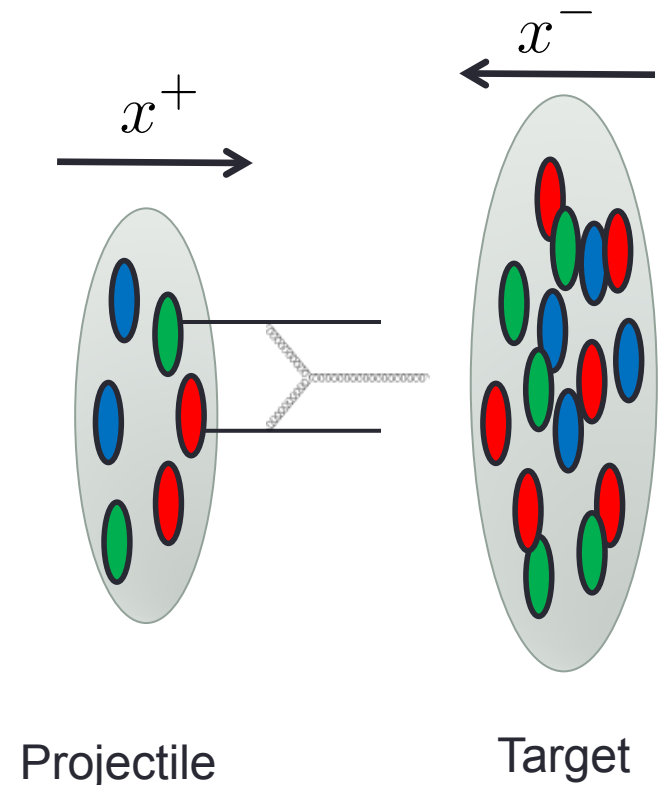$$\frac{1}{\alpha_s} (\alpha_s^2 A_P^{\frac{1}{3}})$$

Heavy-Light Collision Case

- Target nucleus has same power counting as before.
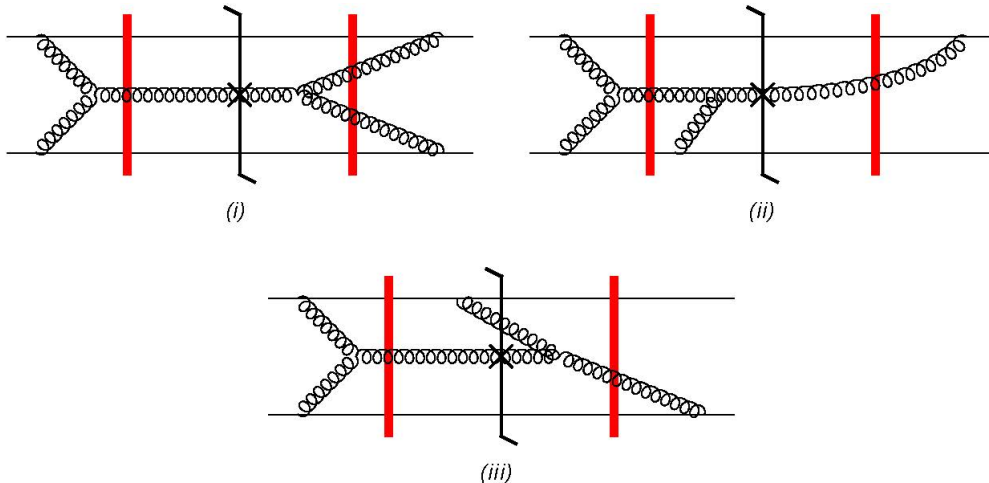- Projectile has many nucleons, but not too many such that

$$\alpha_s^2 A_P^{\frac{1}{3}} \lesssim 1 \qquad \alpha_s^2 A_T^{\frac{1}{3}} \sim 1$$

- Two nucleons from projectile.
- Overall power counting for the cross section,

$$\sim \frac{1}{\alpha_s} (\alpha_s^2 A_P^{\frac{1}{3}})^2 (\alpha_s^2 A_T^{\frac{1}{3}})^N$$

x^+

x^-

Projectile

Target

Types of Diagrams

(i)   *(ii)*

(iii)

- The many diagrams can be separated into three classes:
  i)   Square of order-g^3 amplitudes
  ii) Interference between order-g^5 and order-g amplitudes
  iii) Interference between order-g^4 and order-g^2 amplitudes
- Light-cone gauge, $A^+ = 0$
- All diagrams end up reducing to diagrams representing classical fields.
- Only calculate g^3 amplitude.

$$\frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon} \qquad \text{where} \qquad D_{\mu\nu}(l) = g_{\mu\nu} - \frac{1}{\eta \cdot l}\left(\eta_\mu\, l_\nu + \eta_\nu\, l_\mu\right)$$
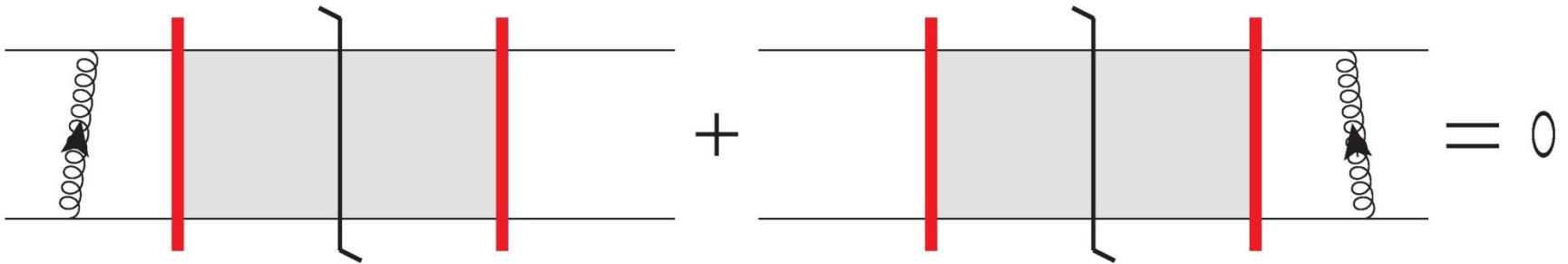
Retarded Green Function
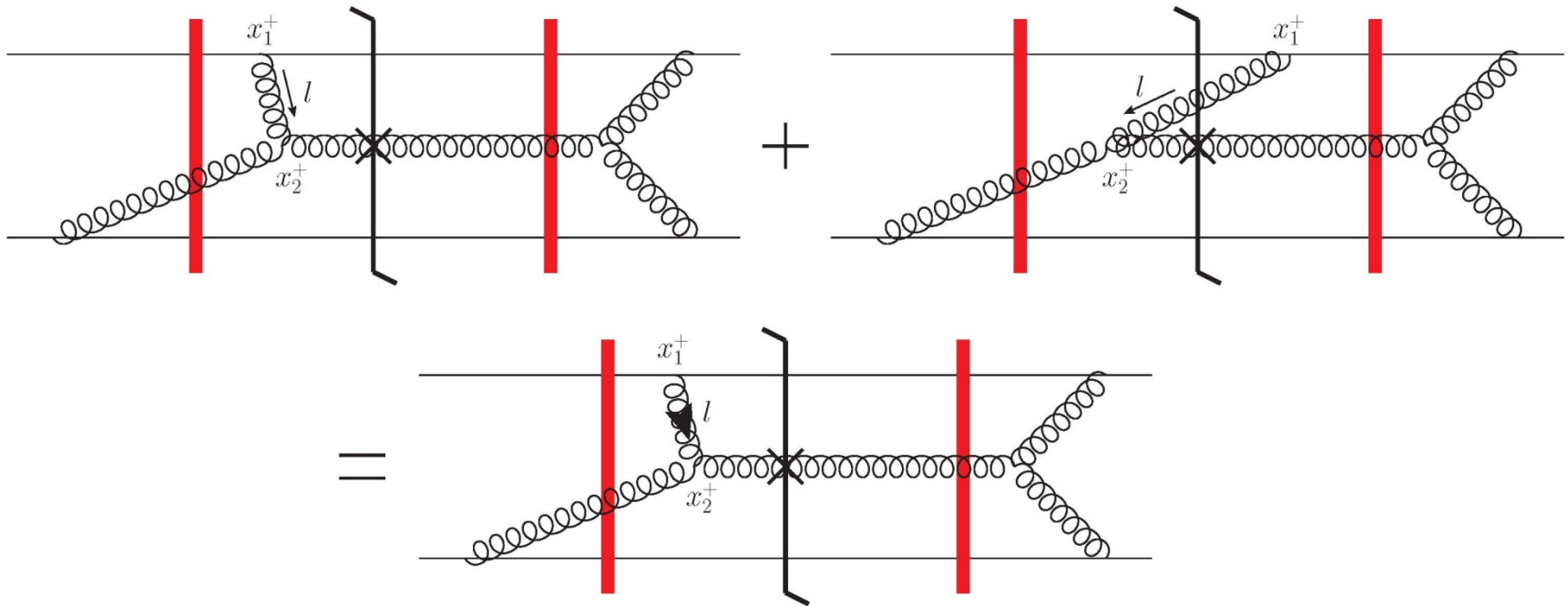
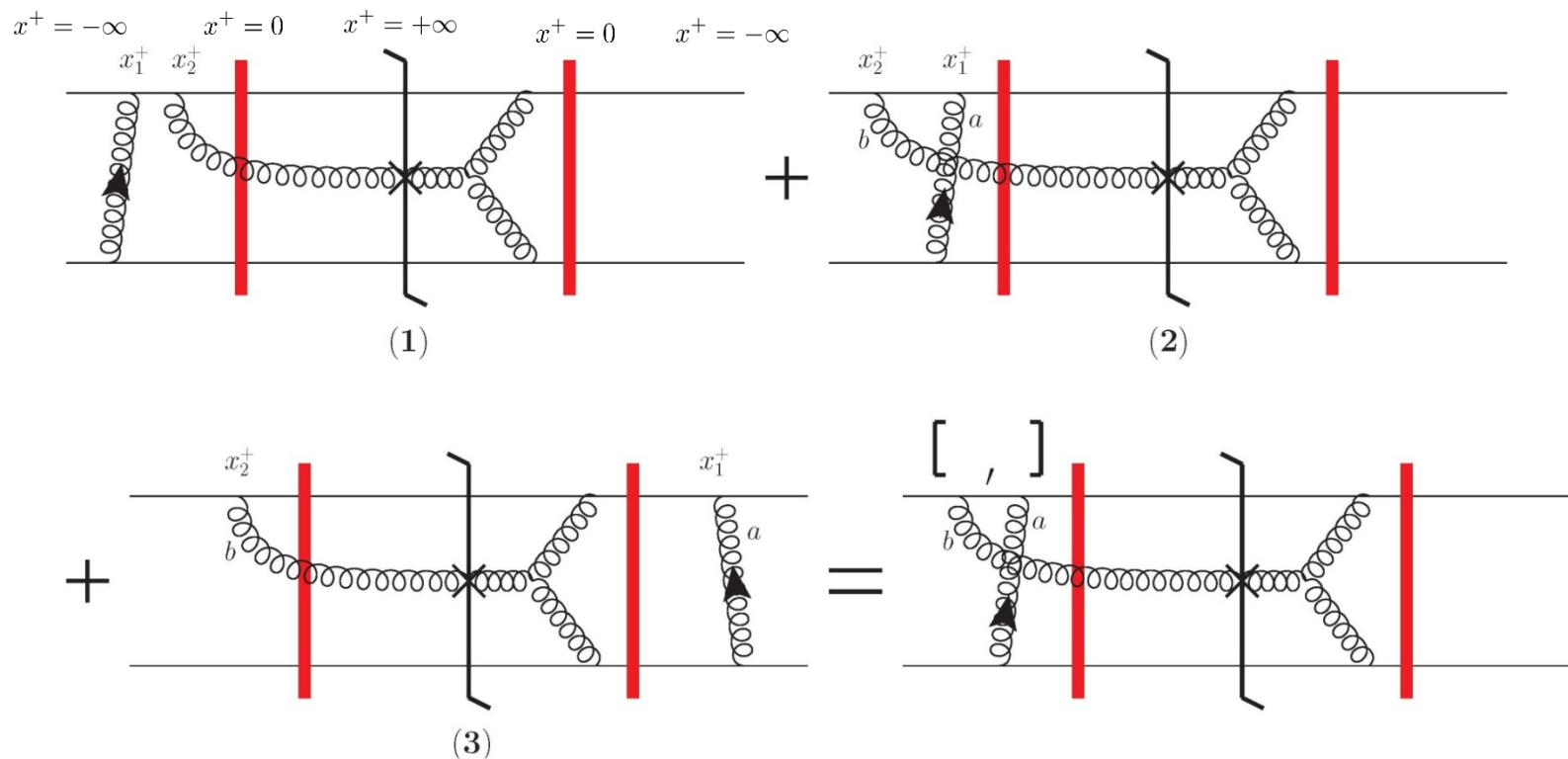- Adding the top two diagrams turns the propagator into a retarded propagator, represented by the arrow.

$$\frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon} + 2\pi\,\theta(-l^+)\,\delta(l^2)\,D_{\mu\nu}(l) = \frac{-i\,D_{\mu\nu}(l)}{l^2 + i\,\epsilon\,l^+}$$

Cancellations

- Shaded region represents any late-time interaction.
- Moving the retarded propagator across the cut gives rise to a minus sign.
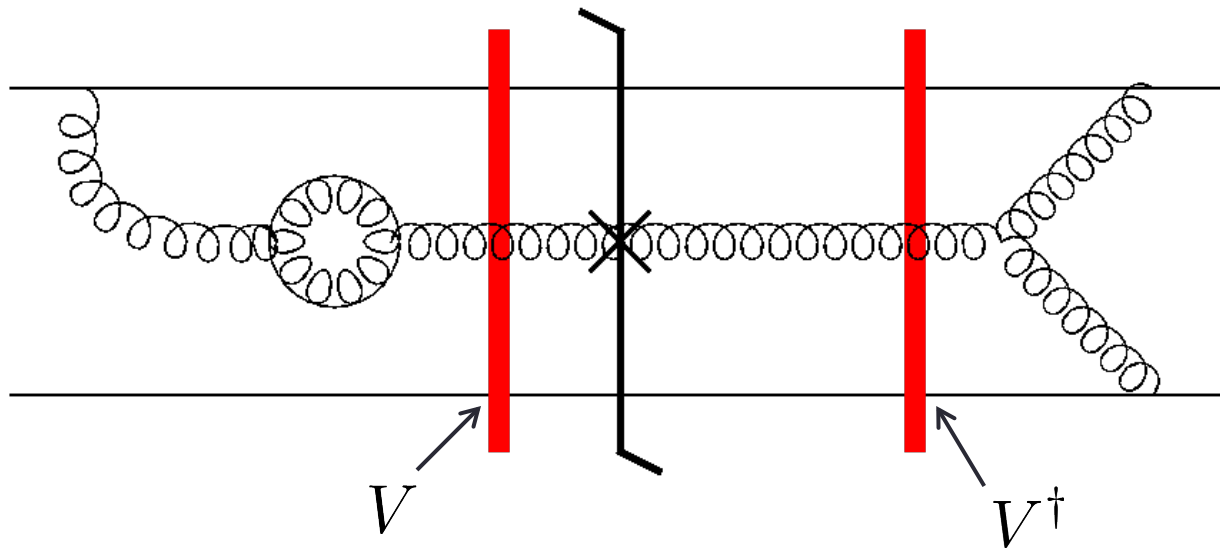- The sum of the diagrams is zero.

Commutators

$x^+ = -\infty$ $x^+ = 0$ $x^+ = +\infty$ $x^+ = 0$ $x^+ = -\infty$

x_1^+ x_2^+

(1)

x_2^+ x_1^+

b a

(2)

$+$

x_2^+ b x_1^+

a

(3)

$[\ ,\ ]$ b a $=$

- Using the cancellation shown previously diagrams (1), (2), and (3) can be combined into a single diagram. Diagram (2) with the color structure of the quark line replaced by a commutator, notated by the square brackets.

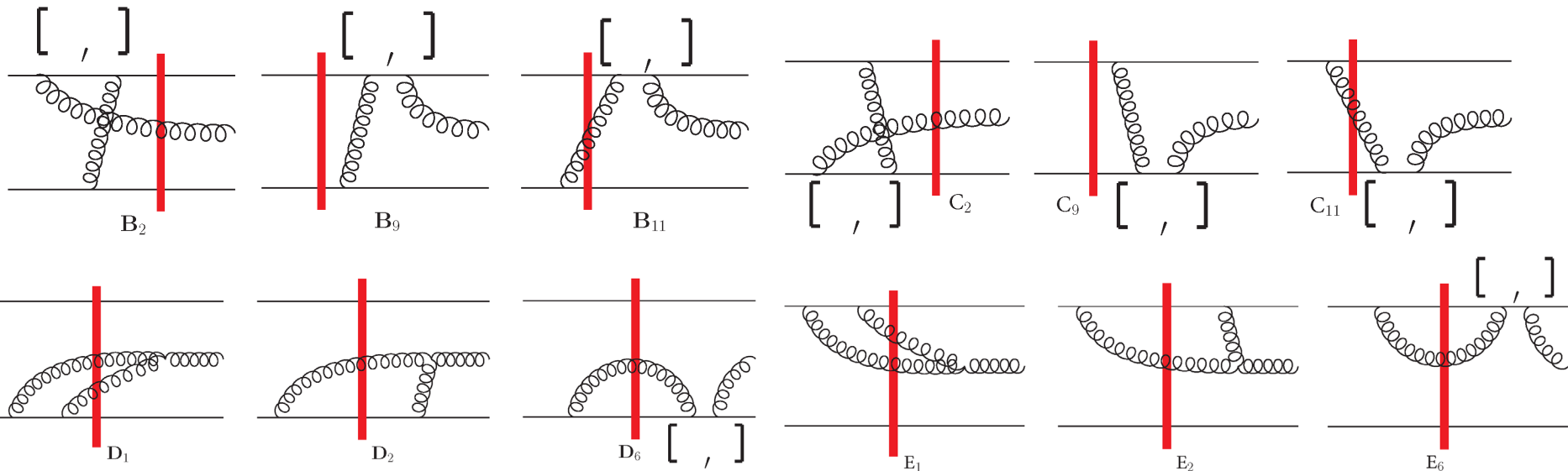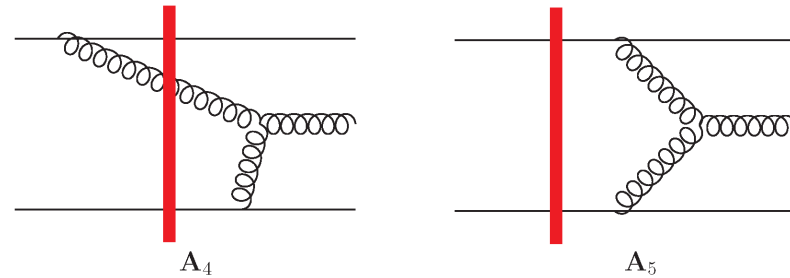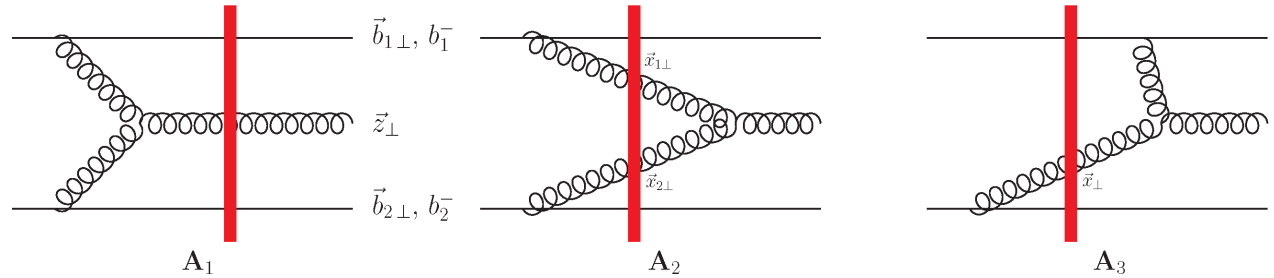$$t^a t^b \rightarrow [t^a, t^b]$$

No Quantum Contributions

- Quantum corrections go away at this order.
- Left with classical fields
- Zero due to color averaging of quark two.

$$tr[t^a V^\dagger V] = tr[t^a] = 0$$

Final Diagrams

There are many more diagrams but they all combine into the diagrams shown here using the simplifications shown previously.

$\vec{b}_{1\perp}, b_1^-$

$\vec{z}_\perp$

$\vec{b}_{2\perp}, b_2^-$

$\vec{x}_{1\perp}$

$\vec{x}_{2\perp}$

$\vec{x}_\perp$

$\mathbf{A}_1$

$\mathbf{A}_2$

$\mathbf{A}_3$

$\mathbf{A}_4$

$\mathbf{A}_5$

$[\quad,\quad]$

$[\quad,\quad]$

$[\quad,\quad]$

$\mathbf{B}_2$

$\mathbf{B}_9$

$\mathbf{B}_{11}$

$[\quad,\quad]$

$\mathbf{C}_2$

$\mathbf{C}_9$ $[\quad,\quad]$

$\mathbf{C}_{11}$ $[\quad,\quad]$

$\mathbf{D}_1$

$\mathbf{D}_2$

$\mathbf{D}_6$ $[\quad,\quad]$

$\mathbf{E}_1$

$\mathbf{E}_2$

$[\quad,\quad]$

$\mathbf{E}_6$

Results: Amplitude – A, B, and C graphs

$$\sum_i A_i + \sum_i B_i + \sum_i C_i$$

$$= -\frac{g^3}{4\pi^4} \int d^2x_1\, d^2x_2\, \delta[(\vec{z}_\perp - \vec{x}_{1\perp}) \times (\vec{z}_\perp - \vec{x}_{2\perp})] \left[\frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{x}_{2\perp} - \vec{x}_{1\perp})}{|\vec{x}_{2\perp} - \vec{x}_{1\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{x}_{1\perp} - \vec{b}_{1\perp})}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_{1\perp}}{|\vec{z}_\perp - \vec{x}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \right.$$

$$\left. - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{x}_{2\perp} - \vec{b}_{2\perp})}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{z}_\perp - \vec{x}_{2\perp}}{|\vec{z}_\perp - \vec{x}_{2\perp}|^2} \right] f^{abc} \left[U_{\vec{x}_{1\perp}}^{bd} - U_{\vec{b}_{1\perp}}^{bd} \right] \left[U_{\vec{x}_{2\perp}}^{ce} - U_{\vec{b}_{2\perp}}^{ce} \right] \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2$$

$$+ \frac{i\,g^3}{4\pi^3} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \int d^2x \left[U_{\vec{b}_{1\perp}}^{bd} \left(U_{\vec{x}_\perp}^{ce} - U_{\vec{b}_{2\perp}}^{ce} \right) \left(\frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right. \right.$$

$$\left. - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right)$$

$$- \left(U_{\vec{x}_\perp}^{bd} - U_{\vec{b}_{1\perp}}^{bd} \right) U_{\vec{b}_{2\perp}}^{ce} \left(\frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \right.$$

$$\left. \left. - \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \right]$$

$$- \frac{i\,g^3}{4\pi^2} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \left[(U_{\vec{z}_\perp}^{bd} - U_{\vec{b}_{1\perp}}^{bd}) U_{\vec{b}_{2\perp}}^{ce} \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{2\perp}|\,\Lambda} - U_{\vec{b}_{1\perp}}^{bd} (U_{\vec{z}_\perp}^{ce} - U_{\vec{b}_{2\perp}}^{ce}) \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{1\perp}|\,\Lambda} \right]$$

$$- \frac{i\,g^3}{4\pi^3} \int d^2x \left[U_{x_\perp}^{ab} - U_{z_\perp}^{ab} \right] f^{bde} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \frac{\vec{\epsilon}_\perp^{\lambda*} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \operatorname{Sign}(b_2^- - b_1^-)$$
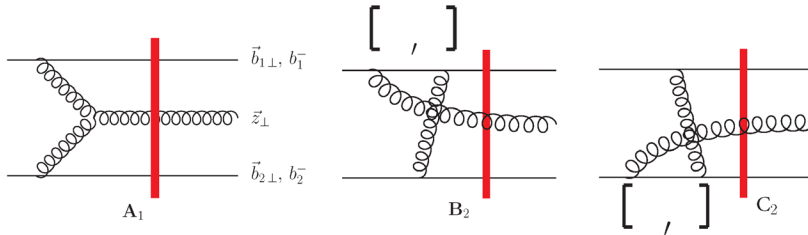

$$\vec{x}_\perp \times \vec{y}_\perp = x_1\, y_2 - x_2\, y_1$$

$$\Lambda = \text{IR cutoff}$$

Results: Amplitude – D graphs

$$\sum_i D_i = -\frac{g^3}{8\,\pi^4}\int d^2x_1\,d^2x_2\,\delta[(\vec{z}_\perp - \vec{x}_{1\perp})\times(\vec{z}_\perp - \vec{x}_{2\perp})]\left[\frac{\vec{\epsilon}_\perp^{\lambda*}\cdot(\vec{x}_{2\perp}-\vec{x}_{1\perp})}{|\vec{x}_{2\perp}-\vec{x}_{1\perp}|^2}\frac{\vec{x}_{1\perp}-\vec{b}_{2\perp}}{|\vec{x}_{1\perp}-\vec{b}_{2\perp}|^2}\cdot\frac{\vec{x}_{2\perp}-\vec{b}_{2\perp}}{|\vec{x}_{2\perp}-\vec{b}_{2\perp}|^2}\right.$$

$$\left.-\frac{\vec{\epsilon}_\perp^{\lambda*}\cdot(\vec{x}_{1\perp}-\vec{b}_{2\perp})}{|\vec{x}_{1\perp}-\vec{b}_{2\perp}|^2}\frac{\vec{z}_\perp-\vec{x}_{1\perp}}{|\vec{z}_\perp-\vec{x}_{1\perp}|^2}\cdot\frac{\vec{x}_{2\perp}-\vec{b}_{2\perp}}{|\vec{x}_{2\perp}-\vec{b}_{2\perp}|^2}+\frac{\vec{\epsilon}_\perp^{\lambda*}\cdot(\vec{x}_{2\perp}-\vec{b}_{2\perp})}{|\vec{x}_{2\perp}-\vec{b}_{2\perp}|^2}\frac{\vec{x}_{1\perp}-\vec{b}_{2\perp}}{|\vec{x}_{1\perp}-\vec{b}_{2\perp}|^2}\cdot\frac{\vec{z}_\perp-\vec{x}_{2\perp}}{|\vec{z}_\perp-\vec{x}_{2\perp}|^2}\right]$$

$$\times f^{abc}\left[U^{bd}_{\vec{x}_{1\perp}}-U^{bd}_{\vec{b}_{2\perp}}\right]\left[U^{ce}_{\vec{x}_{2\perp}}-U^{ce}_{\vec{b}_{2\perp}}\right]\left(V_{\vec{b}_{1\perp}}\right)_1\left(V_{\vec{b}_{2\perp}}t^e\,t^d\right)_2$$

$$+\frac{i\,g^3}{4\,\pi^3}\int d^2x\,f^{abc}\,U^{bd}_{\vec{b}_{2\perp}}\left[U^{ce}_{\vec{x}_\perp}-U^{ce}_{\vec{b}_{2\perp}}\right]\left(V_{\vec{b}_{1\perp}}\right)_1\left(V_{\vec{b}_{2\perp}}t^e\,t^d\right)_2\left(\frac{\vec{\epsilon}_\perp^{\lambda*}\cdot(\vec{z}_\perp-\vec{x}_\perp)}{|\vec{z}_\perp-\vec{x}_\perp|^2}\frac{1}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\right.$$

$$\left.-\frac{\vec{\epsilon}_\perp^{\lambda*}\cdot(\vec{z}_\perp-\vec{b}_{2\perp})}{|\vec{z}_\perp-\vec{b}_{2\perp}|^2}\frac{\vec{z}_\perp-\vec{x}_\perp}{|\vec{z}_\perp-\vec{x}_\perp|^2}\cdot\frac{\vec{x}_\perp-\vec{b}_{2\perp}}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}-\frac{\vec{\epsilon}_\perp^{\lambda*}\cdot(\vec{z}_\perp-\vec{b}_{2\perp})}{|\vec{z}_\perp-\vec{b}_{2\perp}|^2}\frac{1}{|\vec{x}_\perp-\vec{b}_{2\perp}|^2}\right)$$

$$+\frac{i\,g^3}{4\,\pi^2}f^{abc}\,U^{bd}_{\vec{b}_{2\perp}}\left[U^{ce}_{\vec{z}_\perp}-U^{ce}_{\vec{b}_{2\perp}}\right]\left(V_{\vec{b}_{1\perp}}\right)_1\left(V_{\vec{b}_{2\perp}}t^e\,t^d\right)_2\frac{\vec{\epsilon}_\perp^{\lambda*}\cdot(\vec{z}_\perp-\vec{b}_{2\perp})}{|\vec{z}_\perp-\vec{b}_{2\perp}|^2}\ln\frac{1}{|\vec{z}_\perp-\vec{b}_{2\perp}|\,\Lambda}$$

- To get the E graph results switch quark 1 with quark 2 (1 ↔2)
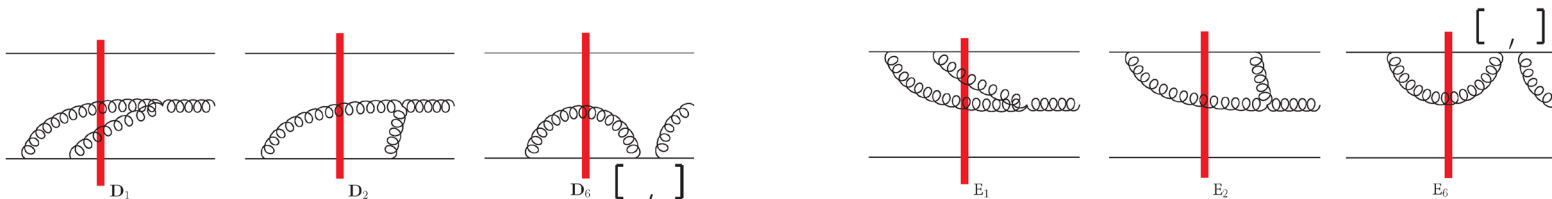
Conclusion

- A serious attempt at analytically calculating the single inclusive gluon production cross-section for Heavy-Light Ion collisions at the classical level.

- Going beyond the p-A result from 1997. [Kovchegov, Mueller]

- Reduced all of the possible diagrams into a few classical field diagrams with a single produced gluon in the amplitude.

- Ended up with a compact result in transverse coordinate space for the g^3 amplitude.

- Similar results in this direction have been obtained by Balitsky (2004).

Outlook
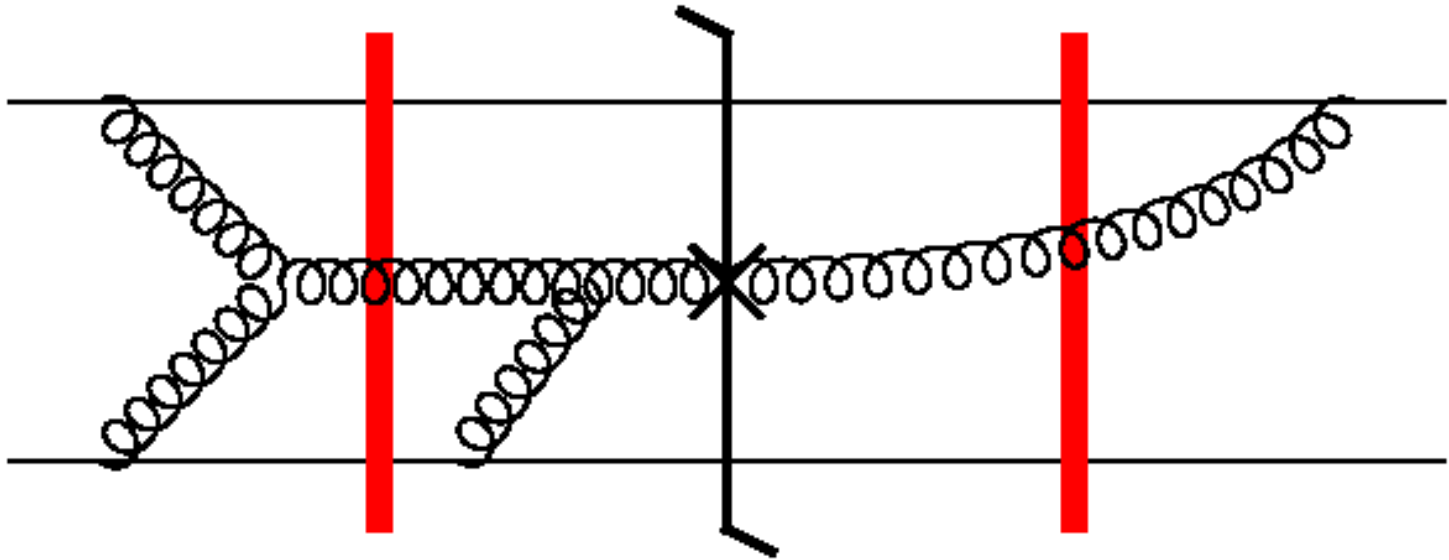
- Need to calculate g^5 amplitude to get the single inclusive gluon production cross-section.
- Work in progress.