

Improved effective TMD factorization for forward dijets in pA collisions

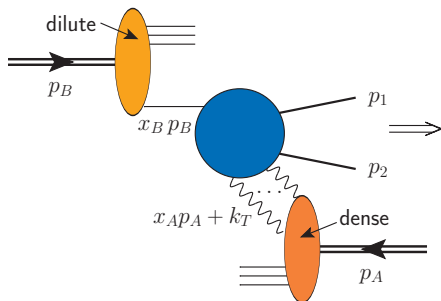
Piotr Kotko

Penn State University

based on arXiv:1503.03421,
P.K., K. Kutak, C. Marquet, E. Petreska,
S. Sapeta, A. van Hameren

supported by:
DEC-2011/01/B/ST2/03643
DE-FG02-93ER40771

Dijets in dilute-dense collision at high energy

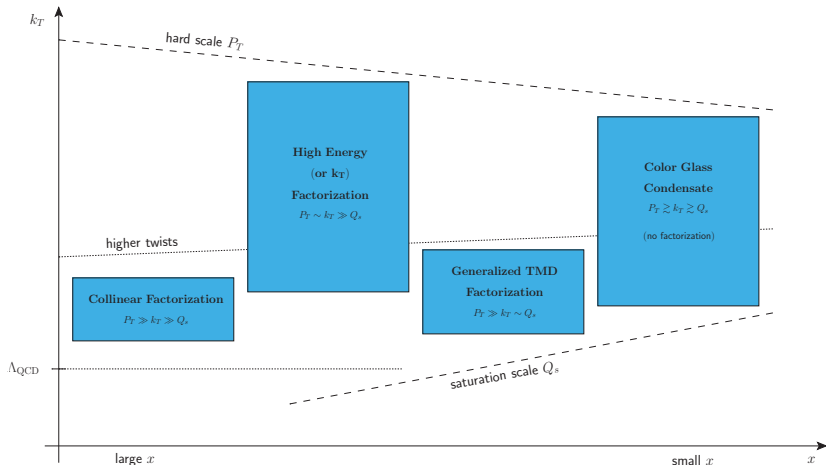


forward dijets with transverse momentum imbalance

$$|\vec{p}_{T1} + \vec{p}_{T2}| = |\vec{k}_T| = k_T$$

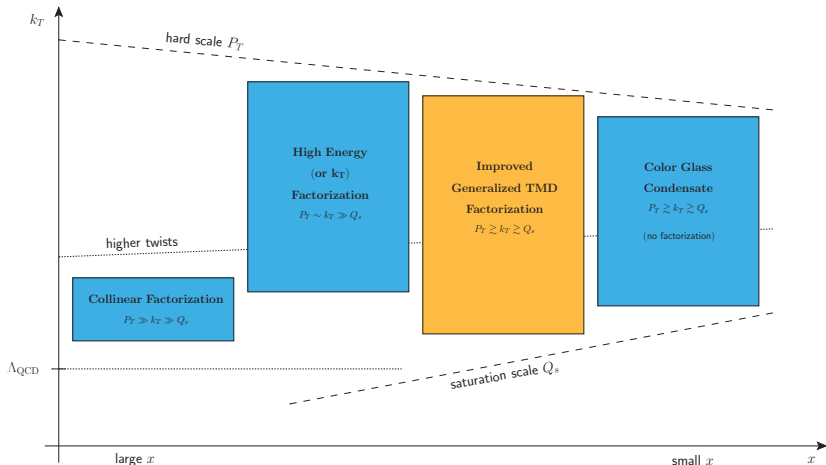
- Hybrid approach: $x_B \gg x_A \Rightarrow$ parton in hadron B is 'collinear'
- Actual approach depends on the relative magnitude of the three scales involved:
 - hard scale P_T (of the order of the average transverse momentum of jets)
 - transverse momentum imbalance k_T
 - saturation scale Q_s (increasing with energy)

Regimes and approaches



- All-order QCD theorem for factorization exists only for collinear case.
- No TMD factorization due to violation of universality (other problems at higher orders may occur).
- However, at small x and large N_c the universality is recovered leading to an 'effective' Generalized TMD factorization valid at leading log.

Regimes and approaches



- All-order QCD theorem for factorization exists only for collinear case.
- No TMD factorization due to violation of universality (other problems at higher orders may occur).
- However, at small x and large N_c the universality is recovered leading to an 'effective' Generalized TMD factorization valid at leading log.

Factorization formulae

Collinear Factorization

$$d\sigma_{AB} = \sum_{a,b} \int dx_A dx_B f_{a/A}(x_A, \mu^2) f_{b/B}(x_B, \mu^2) d\sigma_{ab}(x_A, x_B, \mu^2) + \dots$$

The hard partonic cross section σ_{ab} is computed with partons a, b on-shell.
The off-shellness $k_T \ll \mu$ is integrated over inside the PDF.

High Energy Factorization

[L.V. Gribov, E.M. Levin, M.G. Ryskin, Phys.Rept. 100 (1983) 1-150]
[S. Catani, M. Ciafaloni, F. Hautmann, Nucl.Phys. B366 (1991) 135-188]
[J.C. Collins, R.K. Ellis, Nucl.Phys. B360 (1991) 3-30]

$$d\sigma_{AB} = \sum_b \int dx_A dx_B \int dk_T^2 \mathcal{F}(x_A, k_T^2, \mu^2) f_{b/B}(x_B, \mu^2) d\sigma_{g^*b}(x_A, x_B, k_T^2, \mu^2) + \dots$$

The partonic cross section σ_{g^*b} is computed (in a gauge-invariant way) with off-shell incoming gluon.

$\mathcal{F}(x_A, k_T^2, \mu^2)$ – **Unintegrated Gluon Distribution** evolving via BFKL (or similar).

Generalized (small x) TMD Factorization

[F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan Phys.Rev. D 83 (2011) 105005]

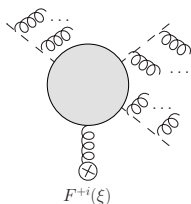
$$d\sigma_{AB} = \sum_b \int dx_A dx_B f_{b/B}(x_B, \mu^2) \sum_i \int dk_T^2 \phi_{bg}^{(i)}(x_A, k_T^2, \mu^2) d\sigma_{gb}^{(i)}(x_A, x_B, k_T^2, \mu^2) + \dots$$

The hard partonic cross sections $\sigma_{gb}^{(i)}$ are computed with partons a, b on-shell.

$\phi_{bg}^{(i)}(x_A, k_T^2, \mu^2)$ – TMD Gluon Distributions depending on the process.

$$\phi_{bg}(x, k_T) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_A p_A^- \xi^+ - i\vec{k}_T \cdot \vec{\xi}_T} \langle p_A | \text{Tr} \{ F^{+i}(\xi) [\xi, 0]_{C_1} F^{+i}(0) [0, \xi]_{C_2} \} | p_A \rangle$$

The **Wilson lines** $[\xi, 0]_{C_i}$ depend on the process-dependent paths.

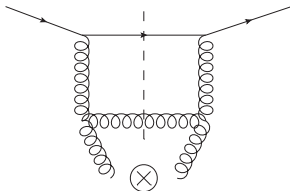


Wilson lines come from resummation of collinear gluons attached to the external lines. Then they are 'glued' by the color structure of the hard process to give $[\xi, 0]_{C_i}$.

Generalized (small x) TMD Factorization (cont.)

Example: TMD for a particular diagram

[C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J.C. 47, 147 (2006)]



$$\langle p_A | \text{Tr} \{ F(\xi) \mathcal{U}^{[+] \dagger} F(0) \left[\frac{\text{Tr} \mathcal{U}^{[0] \dagger}}{N_c} \mathcal{U}^{[+]} + \mathcal{U}^{[-]} \right] \} | p_A \rangle$$

with the following definitions of Wilson lines and loops:

$$\mathcal{U}^{[\pm]} = U(0, \pm\infty; 0_T) U(\pm\infty, \xi^+; \xi_T) \quad \mathcal{U}^{[0]} = \mathcal{U}^{[+]} \mathcal{U}^{[-] \dagger} = \mathcal{U}^{[-]} \mathcal{U}^{[+] \dagger}$$

where $U(a, b; x_T) = \mathcal{P} \exp \left[ig \int_a^b dx^+ A_a^-(x^+, x_T) t^a \right]$.

Generalized (small x) TMD Factorization (cont.)

All TMD structures for dijets

$$\mathcal{F}_{qg}^{(1)} \sim \langle p_A | \text{Tr} \{ F(\xi) \mathcal{U}^{[-]\dagger} F(0) \mathcal{U}^{[+]} \} | p_A \rangle \quad \mathcal{F}_{qg}^{(2)} \sim \langle p_A | \text{Tr} \{ F(\xi) \frac{\text{Tr} \mathcal{U}^{[0]}}{N_c} \mathcal{U}^{[+]\dagger} F(0) \mathcal{U}^{[+]} \} | p_A \rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \langle p_A | \text{Tr} \{ F(\xi) \frac{\text{Tr} \mathcal{U}^{[0]}}{N_c} \mathcal{U}^{[-]\dagger} F(0) \mathcal{U}^{[+]} \} | p_A \rangle \quad \mathcal{F}_{gg}^{(2)} \sim \frac{1}{N_c} \langle p_A | \text{Tr} \{ F(\xi) \mathcal{U}^{[0]\dagger} \} \text{Tr} \{ F(0) \mathcal{U}^{[0]} \} | p_A \rangle$$

$$\mathcal{F}_{gg}^{(3)} \sim \langle p_A | \text{Tr} \{ F(\xi) \mathcal{U}^{[+]\dagger} F(0) \mathcal{U}^{[+]} \} | p_A \rangle \quad \mathcal{F}_{gg}^{(4)} \sim \langle p_A | \text{Tr} \{ F(\xi) \mathcal{U}^{[-]\dagger} F(0) \mathcal{U}^{[-]} \} | p_A \rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \langle p_A | \text{Tr} \{ F(\xi) \mathcal{U}^{[0]\dagger} \mathcal{U}^{[+]\dagger} F(0) \mathcal{U}^{[0]} \mathcal{U}^{[+]} \} | p_A \rangle \quad \mathcal{F}_{gg}^{(6)} \sim \langle p_A | \text{Tr} \{ F(\xi) \mathcal{U}^{[+]\dagger} F(0) \mathcal{U}^{[+]} \} \frac{\text{Tr} \mathcal{U}^{[0]}}{N_c} \frac{\text{Tr} \mathcal{U}^{[0]}}{N_c} | p_A \rangle$$

In the large N_c limit $\mathcal{F}_{qg}^{(1)}, \mathcal{F}_{gg}^{(3)}$ are the only necessary TMDs:

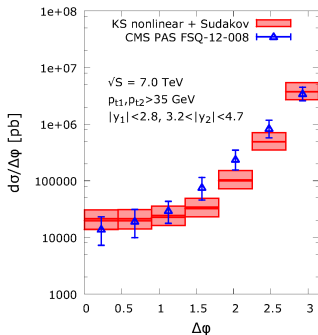
- $\mathcal{F}_{qg}^{(1)} = xG_1$ is the Weizsacker-Wiliams gluon distribution with parton number interpretation (calculable in CGC but decoupled from most simple processes; appears also in dijet production in DIS)
- $\mathcal{F}_{qg}^{(3)} = xG_2$ is so-called 'dipole' gluon distribution probed in inclusive processes at small x

Azimuthal Decorrelations

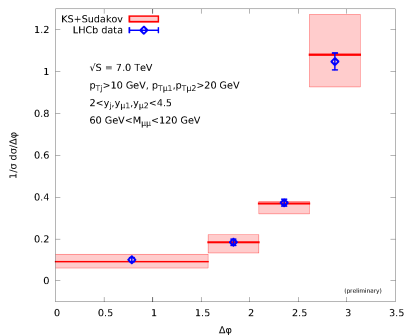
- The previous approach is valid in the correlation limit $k_T \ll P_T$.
- In reality, forward jets are decorrelated \Rightarrow decorrelations are nicely described by the High Energy Factorization.

[A. van Hameren, PK, K. Kutak, S. Sapeta, Phys.Lett. B737 (2014) 335-340]

forward-central dijets



forward Z_0 + jet



- We can improve TMD factorization by introducing off-shellness to the hard factors.

Improved Generalized TMD Factorization

[P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, arXiv:150342]

- We revise the calculation of TMDs using color decomposition of amplitudes

$$\mathcal{M}^{a_1 \dots a_N}(\varepsilon_1^{\lambda_1}, \dots, \varepsilon_N^{\lambda_N}) = \sum_{\sigma \in \mathcal{S}_{N-1}} \text{Tr}(t^{a_1} t^{a_{\sigma 2}} \dots t^{a_{\sigma N}}) \mathcal{M}(1^{\lambda_1}, \sigma_2^{\lambda_{\sigma 2}}, \dots, \sigma_N^{\lambda_{\sigma N}})$$

a_i - color indices, $\varepsilon_i^{\lambda_i}$ - polarization vectors with helicity λ_i , \mathcal{S}_{N-1} - set of noncyclic permutations.

We conclude that there are only two independent TMDs $\Phi^{(i)}$, $i = 1, 2$ (being a combination of $\mathcal{F}_{qg}^{(1)}$'s) needed for each channel.

- We calculate off-shell color-ordered helicity amplitudes.

Methods for gauge invariant off-shell amplitudes:

[E. Antonov, L. Lipatov, E. Kuraev, I. Cherednikov, Nucl.Phys. B721 (2005) 111-135]

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029; JHEP 1301 (2013) 078]

[A. van Hameren, JHEP 1407 (2014) 138], [PK, JHEP 1407 (2014) 128]

see the talk of M. Serino → [A. van Hameren, M. Serino, arXiv:1504.00315]

Improved Generalized TMD Factorization (cont.)

Color-ordered off-shell helicity amplitudes

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029]

In spinor formalism, the non-zero off-shell helicity amplitudes have the form of the MHV amplitudes with certain modification of spinor products:

$$\mathcal{M}_{g^+g \rightarrow gg}(1^*, 2^-, 3^+, 4^+) = 2g^2 \rho_1 \frac{\langle 1^*2 \rangle^4}{\langle 1^*2 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41^* \rangle}$$

$$\mathcal{M}_{g^+g \rightarrow gg}(1^*, 2^+, 3^-, 4^+) = 2g^2 \rho_1 \frac{\langle 1^*3 \rangle^4}{\langle 1^*2 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41^* \rangle}$$

$$\mathcal{M}_{g^+g \rightarrow gg}(1^*, 2^+, 3^+, 4^-) = 2g^2 \rho_1 \frac{\langle 1^*4 \rangle^4}{\langle 1^*2 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41^* \rangle}$$

where $\langle ij \rangle = \langle k_i - |k_j \rangle$ with spinors defined as $|k_i \pm \rangle = \frac{1}{2} (1 \pm \gamma_5) u(k_i)$. Modified spinor products involve only longitudinal component of the off-shell momentum $\langle 1^*i \rangle = \langle p_A i \rangle$. Similar expressions can be derived for quarks.

We construct hard factors $K^{(i)}$ corresponding to $\Phi^{(i)}$. The final factorization formula reads:

$$\frac{d\sigma_{AB}}{dP.S.} \sim \sum_{i=1,2} \left[f_{q/B} \otimes \Phi_{gq \rightarrow qg}^{(i)} \otimes K_{gq \rightarrow qg}^{(i)} + f_{g/B} \otimes \left(\Phi_{gg \rightarrow q\bar{q}}^{(i)} \otimes K_{gg \rightarrow q\bar{q}}^{(i)} + \Phi_{gg \rightarrow gg}^{(i)} \otimes K_{gg \rightarrow gg}^{(i)} \right) \right]$$

Improved Generalized TMD Factorization (cont.)

Hard off-shell factors and corresponding TMDs

$$K_{gq \rightarrow gq}^{(1)} = -\frac{\bar{u}(\bar{s}^2 + \bar{u}^2)}{2\bar{t}\hat{s}} \left(1 + \frac{\bar{s}\hat{s} - \bar{t}\hat{t}}{N_c^2 \bar{u}\hat{u}} \right)$$

$$K_{gq \rightarrow gq}^{(2)} = -\frac{C_F}{N_c} \frac{\bar{s}(\bar{s}^2 + \bar{u}^2)}{\bar{t}\hat{u}}$$

$$\Phi_{gq \rightarrow gq}^{(1)} = \mathcal{F}_{qg}^{(1)}$$

$$\Phi_{gq \rightarrow gq}^{(2)} = \frac{1}{N_c^2 - 1} \left(-\mathcal{F}_{qg}^{(1)} + N_c^2 \mathcal{F}_{qg}^{(2)} \right)$$

$$K_{gg \rightarrow q\bar{q}}^{(1)} = \frac{1}{2N_c} \frac{(\bar{t}^2 + \bar{u}^2)(\bar{u}\hat{u} + \bar{t}\hat{t})}{\bar{s}\hat{s}\hat{t}\hat{u}}$$

$$K_{gg \rightarrow q\bar{q}}^{(2)} = \frac{1}{4N_c^2 C_F} \frac{(\bar{t}^2 + \bar{u}^2)(\bar{u}\hat{u} + \bar{t}\hat{t} - \bar{s}\hat{s})}{\bar{s}\hat{s}\hat{t}\hat{u}}$$

$$\Phi_{gg \rightarrow q\bar{q}}^{(1)} = \frac{1}{N_c^2 - 1} \left(N_c^2 \mathcal{F}_{gg}^{(1)} - \mathcal{F}_{gg}^{(3)} \right)$$

$$\Phi_{gg \rightarrow q\bar{q}}^{(2)} = -N_c^2 \mathcal{F}_{gg}^{(2)} + \mathcal{F}_{gg}^{(3)}$$

$$K_{gg \rightarrow gg}^{(1)} = \frac{N_c}{C_F} \frac{(\bar{s}^4 + \bar{t}^4 + \bar{u}^4)(\bar{u}\hat{u} + \bar{t}\hat{t})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$$

$$K_{gg \rightarrow gg}^{(2)} = -\frac{N_c}{2C_F} \frac{(\bar{s}^4 + \bar{t}^4 + \bar{u}^4)(\bar{u}\hat{u} + \bar{t}\hat{t} - \bar{s}\hat{s})}{\bar{t}\hat{t}\bar{u}\hat{u}\bar{s}\hat{s}}$$

$$\Phi_{gg \rightarrow gg}^{(1)} = \frac{1}{2N_c^2} \left(N_c^2 \mathcal{F}_{gg}^{(1)} - 2\mathcal{F}_{gg}^{(3)} + \mathcal{F}_{gg}^{(4)} + \mathcal{F}_{gg}^{(5)} + N_c^2 \mathcal{F}_{gg}^{(6)} \right)$$

$$\Phi_{gg \rightarrow gg}^{(2)} = \frac{1}{N_c^2} \left(N_c^2 \mathcal{F}_{gg}^{(2)} - 2\mathcal{F}_{gg}^{(3)} + \mathcal{F}_{gg}^{(4)} + \mathcal{F}_{gg}^{(5)} + N_c^2 \mathcal{F}_{gg}^{(6)} \right)$$

$\hat{s}, \hat{t}, \hat{u}$ – ordinary Mandelstam variables, $\hat{s} + \hat{t} + \hat{u} = k_T^2$

$\bar{s}, \bar{t}, \bar{u}$ – off-shell momentum $k_A = x_A p_A + k_T$ replaced by $x_A p_A$, $\bar{s} + \bar{u} + \bar{t} = 0$

Concluding remarks

Summary

- At small x , in the large N_c and dijet correlation limit the cross section can be calculated in terms of two transverse momentum dependent gluon distributions G_1 , G_2 which appear in several processes. In this limit of leading twist the CGC match the TMD approach.
- On the other hand, the High Energy (or k_T) Factorization, which goes beyond correlation limit (beyond the leading twist), can be derived from the CGC in the dilute limit.
- We have matched the both limits.

Future plans

- The phenomenology is under development. The gluon distributions G_1 , G_2 can be obtained from models.
- Consistence of the approach with the particular evolution scenario and radiative corrections remains to be proven.
- NLO corrections to the hard off-shell proces itself can be possibly done along the lines of the dipole formalism for jets with massive partons