Update on the phenomenology of collinear Dihadron FFs

DIS 2015

Aurore Courtoy

IFPA-Université de Liège (Belgium)
DCI-Universidad de Guanajuato (Mexico)

in collaboration with A. Bacchetta, M Radici and M. Guagnelli in Pavia
Extraction of Transversity

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State-of-the-art: Extractions of transversity

- **TMD extraction** [Anselmino et al, Kang et al]
- **Collinear extraction** [Pavia]
- **GPD extraction** [Goldstein et al]
State-of-the-art: Extractions of transversity

- TMD extraction [Anselmino et al, Kang et al]
- Collinear extraction [Pavia]
- GPD extraction [Goldstein et al]
Processes

Semi-inclusive processes
\( \sigma \rightarrow \text{PDF} \times d\sigma \times \text{Fragmentation Function} \)

Exclusive processes
\( \sigma \rightarrow \text{Generalized PDF} \times d\sigma \times \text{Meson Amplitude} \)
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Pavia Fit: What's new?

- COMPASS data for identified pions
- Two Values for $\alpha_s(M_Z^2)$
- Replica methods for both pol. DiFF & transversity

C. Braun
SIDIS production of pion pairs

\[ A_{DIS}(x, z, M_h^2, Q^2) = -C_y \frac{\sum_q e_q^2 h_1^q(x, Q^2) |\mathcal{R}|}{\sum_q e_q^2 f_1^q(x, Q^2)} \frac{H_{1,sp}^{q \rightarrow \pi^+ \pi^-}(z, M_h^2, Q^2)}{D_1^{q \rightarrow \pi^+ \pi^-}(z, M_h^2, Q^2)} \]

[Bacchetta, A.C., Radici, PRL 107 (2011)]
SIDIS production of pion pairs

Knowledge on DiFFs leads to $h_1(x, Q^2)$

[Bacchetta, A.C., Radici, PRL 107 (2011)]
SIDIS production of pion pairs

\[ A_{\text{DIS}}(x, z, M_h^2, Q^2) = -C_y \sum_q e_q^2 h_1^q(x, Q^2) \frac{H_{1,sp}^q \rightarrow \pi^+\pi^- (z, M_h^2, Q^2)}{D_{1}^q \rightarrow \pi^+\pi^- (z, M_h^2, Q^2)} \]

Knowledge on DiFFs leads to \( h_1^q(x, Q^2) \)

Choose error treatment and functional form

[Bacchetta, A.C., Radici, PRL 107 (2011)]
Reminder: Functional Form biases

flexible functional form

rigid functional form
Reminder: Functional Form biases

**flexible functional form**

\[ x h_1^{u(x)} - x h_1^{d(x)/4} \]

fit

data HERMES

data COMPASS

**rigid functional form**

\[ x h_1^{u(x)} - x h_1^{d(x)/4} \]

fit

data HERMES

data COMPASS
Reminder: Functional Form biases

**Flexible functional form**

- Data HERMES
- Data COMPASS

**Rigid functional form**

Only constrained by Soffer bound
Reminder: Functional Form biases

Flexible functional form

Rigid functional form

Only constrained by Soffer bound

2013 ⇒ Replica method to make up for small errors at low- and large-x
**Pavia fitter: 2 steps' approach**

1. SIDIS production of pion pairs

\[
A_{\text{DIS}}(x, z, M_h^2, Q^2) = -C_y \sum_q e_q^2 h_1^q(x, Q^2) \frac{|R|_{M_h}}{M_h} H_{1,sp}^{q \rightarrow \pi^+ \pi^-}(z, M_h^2, Q^2) \frac{D_{1}^{q \rightarrow \pi^+ \pi^-}(z, M_h^2, Q^2)}{D_{1}^{q \rightarrow \pi^+ \pi^-}(z, M_h^2, Q^2)}
\]

Knowledge on DiFFs leads to \(h_1(x, Q^2)\)

2. SI pion pairs production in e+ e- annihilation @ Belle

\[
A_{e^+e^-}(z, M_h^2, \bar{z}, \bar{M}_h^2) \propto \sum_q e_q^2 H_{1,sp}^{q \rightarrow \pi^+ \pi^-}(z, M_h^2) \frac{\bar{H}_{1,sp}^{q \rightarrow \pi^+ \pi^-}(\bar{z}, \bar{M}_h^2)}{\sum_q e_q^2 D_1^{q \rightarrow \pi^+ \pi^-}(z, M_h^2) \frac{D_1^{q \rightarrow \pi^+ \pi^-}(\bar{z}, \bar{M}_h^2)}{D_1^{q \rightarrow \pi^+ \pi^-}(z, M_h^2) D_1^{q \rightarrow \pi^+ \pi^-}(\bar{z}, \bar{M}_h^2)}}
\]
Pavia fitter: 2 steps' approach

1. SIDIS production of pion pairs

\[ A_{\text{DIS}}(x, z, M_h^2, Q^2) = -C_y \sum_q e_q^2 h_1^q(x, Q^2) \frac{|R|}{M_h} \frac{H_{1,sp}^{q\rightarrow\pi^+\pi^-}(z, M_h^2, Q^2)}{D_{1}^{q\rightarrow\pi^+\pi^-}(z, M_h^2, Q^2)} \]

Knowledge on DiFFs leads to \( h_1(x, Q^2) \)

2. SI pion pairs production in e+ e- annihilation

\[ A_{e^+e^-}(z, M_h^2, \bar{z}, \bar{M}_h^2) \propto \sum_q e_q^2 H_{1,sp}^{q\rightarrow\pi^+\pi^-}(z, M_h^2) H_{1,sp}^{q\rightarrow\pi^+\pi^-}(\bar{z}, \bar{M}_h^2) \]

\[ \sum_q e_q^2 D_{1}^{q\rightarrow\pi^+\pi^-}(z, M_h^2) D_{1}^{q\rightarrow\pi^+\pi^-}(\bar{z}, \bar{M}_h^2) \]

Now both \( h_1 \) and \( H_1^< \) with replica method!

[A.C., Bacchetta, Radici, Bianconi, Phys.Rev. D85]
The Replica Approach

Too small errors w.r.t. ABSENCE of data

• generate $n$ sets of data with gaussian noise (@1σ) → $n$ replicas

• redo the fit $n$ times

• keep the 1σ distributed resulting “transversities”, at each data point

• the error band is now made by 68% of the $n$ replica point by point
The Replica Approach

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Effect of $\alpha_S(M_{Z^2})$ value

$$R(z, M_h) = \left| \frac{R}{M_h} \frac{H_{1,s}^{<u}(z, M_h; Q_0^2)}{D_1^{u}(z, M_h; Q_0^2)} \right|$$

D$_1$ unchanged (good statistics)

$\alpha_S(M_{Z^2}) = 0.125$

$\alpha_S(M_{Z^2}) = 0.139$
Tiny effects on
- the Chi2 distribution
- the value of $n_q^\uparrow$

$\alpha_s(M_Z^2) = 0.125$
$\alpha_s(M_Z^2) = 0.139$

$n_q^\uparrow(Q^2) = \int dz \, dM_h \, \left| \frac{R}{M_h} \right| \, H^{q_{\text{sp}}}_{1,sp}(z, M_h; Q^2)$

Effect of $\alpha_s(M_Z^2)$ value
SIDIS production of pion pairs

TRIPTIC plot

Deuteron Data

Proton Data

COMPASS range: 0.2<z<1 & 0.29<M_{hh}<1.29 GeV
SIDIS production of pion pairs

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Fig. 4: Deuteron and proton asymmetries, integrated over the angle \( q \), as a function of \( x, \langle xz \rangle^2 \) and \( M_{hh} \), for the data taken with the \(^6\)LiD (top) and NH\(_3\) target (bottom), respectively. The open data points in both asymmetry distributions vs. \( M_{hh} \) include all hadron pairs with an invariant mass of \( M_{hh} \leq 1.5 \text{ GeV}/c^2 \). These pairs are discarded for the two other distributions, which are integrated over \( M_{hh} \). The grey bands indicate the systematic uncertainties, where the last bin in \( M_{hh} \) is not fully shown. The curves show the comparison of the extracted asymmetries to predictions [37, 38] made using the transversity functions extracted in Ref. [15] (solid lines) or a pQCD based counting rule analysis (dotted lines).

Discussion of Results

The resulting asymmetries are shown in Fig. 4 as a function of \( x, \langle xz \rangle \) and \( M_{hh} \) for the \(^6\)LiD (top) and NH\(_3\) targets, respectively. For \(^6\)LiD, no significant asymmetry is observed in any variable. For NH\(_3\), large negative asymmetries are observed in the region \( x > 0.03 \), which implies that both transversity distributions and polarised two-hadron interference fragmentation functions do not vanish. For \( x < 0.03 \), the asymmetries are compatible with zero. Over the measured range of the invariant mass \( M_{hh} \) and \( \langle xz \rangle \), the asymmetry is negative and shows no strong dependence on these variables.

When comparing the results on the NH\(_3\) target to the published HERMES results on a transversely polarised proton target [28], the larger kinematic region in \( x \) and \( M_{hh} \) is evident. However, both results cannot be directly compared for several reasons: (1) The opposite sign is due to the fact that in the extraction of the asymmetries the phase \( p \) in the angle \( f_{RS} \) is used in the COMPASS analysis; (2) COMPASS calculates asymmetries in the photon-nucleon system, while HERMES published them in the lepton-nucleon system; both agree reasonably well when including \( D_{nn} \) corrections for HERMES; (3) HERMES uses identified \( p^+ + p^+ \) pairs and COMPASS \( h^+ + h^+ \) pairs; (4) COMPASS applies a minimum cut on \( z \), removing a possible dilution due to contributions from target fragmentation.

A naive interpretation of our data, based on Eq. (7) and on isospin symmetry and charge conjugation, yields \( D_1^u = D_1^d \) and \( H_1^u = H_1^d \) [27]. When considering only valence quarks, the asymmetry \( A_{UT, p} \) is proportional to \( \langle h_u^1 + h_d^1 \rangle H_1^u \) for the deuteron target, while for the proton target \( A_{UT, d} \) is proportional to \( \langle 4 h_u^1 h_d^1 \rangle H_1^u \).

Therefore, like in the case of the Collins asymmetry, the small asymmetries observed for the deuteron target range: 0.2<z<1 & 0.29<M_{hh}<1.29 GeV

COMPASS range: 0.2<z<1 & 0.29<M_{hh}<1.29 GeV
SIDIS production of pion pairs

TRIPTIC plot

Deuteron Data

Proton Data

\[ \langle A_{UU} \rangle \sin \theta \]

\[ \langle A_{UD} \rangle \sin \theta \]

\[ \langle A_{DU} \rangle \sin \theta \]

\[ \langle A_{DD} \rangle \sin \theta \]

\( x \)-dependence only from Transversity

\((z, M_{hh})\)-dependence determined by DiFF

COMPASS range: 0.2<z<1 & 0.29<M_{hh}<1.29\text{ GeV}
Therefore, like in the case of the Collins asymmetry, the small asymmetries observed for the deuteron are proportional to yields. A naive interpretation of our data, based on Eq. (7) and on isospin symmetry and charge conjugation, indicates a possible dilution due to contributions from target fragmentation.

Identified system; both agree reasonably well when including contributions from target fragmentation. asymmetries in the photon-nucleon system, while HERMES published them in the lepton-nucleon system. The phase behavior cannot be directly compared for several reasons: (1) The opposite sign is due to the fact that in the extraction of the asymmetries of polarised proton target [28], the larger kinematic region in the COMPASS range: 0.2<z<1 & 0.29<M_{hh}<1.29 GeV.

When comparing the results on the NH asymmetry is negative and shows no strong dependence on these variables. The resulting asymmetries are shown in Fig. 4 as a function of sinφ.

\[ A_{DIS}(x, Q^2) = -C_y \sum_q e_q^2 \frac{h_1^q(x, Q^2)}{f_1^q(x, Q^2)} \frac{n_{u,q}^+(Q^2)}{n_{d,q}^+(Q^2)} \]
Transversity from $A_{UT} \sin(\Phi_R + \Phi_S) \sin \theta$

\[
A_{DIS}^i(x, Q^2) = -C_y \sum_q e_q^2 h_i^q(x, Q^2) n_q(Q^2) \sum_q e_q^2 f_i^q(x, Q^2) n_q(Q^2)
\]

$i=1,..,n$

Using symmetries for DiFFs:

\[
H_{1,u}^{q,u} = -H_{1,d}^{q,d} = -H_{1,u}^{q,d} = H_{1,d}^{q,u}
\]

\[
D_{1,u} = D_{1,d} = D_{1,s} = D_{1,c} = D_{1}^u
\]

Proton

\[
x h_1^{u,v}(x, Q^2) - \frac{1}{4} x h_1^{d,v}(x, Q^2) \propto -A_{DIS}^i(x, Q^2) \frac{n_u(Q^2)}{n_u(Q^2)} \sum_{q=u,d,s} e_q^2 x f_i^{q+\bar{q}}(x, Q^2)
\]

Deuteron

\[
x h_1^{u,v}(x, Q^2) + x h_1^{d,v}(x, Q^2) \propto -\frac{5}{3} A_{DIS}^i(x, Q^2) \frac{n_u(Q^2)}{n_u(Q^2)} x \left( f_i^{u+\bar{u}} + f_i^{d+d} + \frac{2}{5} f_i^{s+s} \right)
\]

and combinations of both ...
Transversity from $A_{UT} \sin(\Phi_R + \Phi_S) \sin \theta$

$$A^i_{DIS}(x, Q^2) = -C_y \sum_q e_q^2 h^q_1(x, Q^2) n^>_q(Q^2) \over \sum_q e_q^2 f^q_1(x, Q^2) n_q(Q^2)$$

$i=1,..,n$

Using symmetries for DiFFs:

$$H^{u,d}_1 = -H^{u,d}_1 = \overline{H}^{u,d}_1 = \overline{H}^{u,d}_1$$

$$D^u_1 = D^d_1 = \overline{D}^u_1 = \overline{D}^d_1$$

$$D^s_1 = \overline{D}^s_1, \quad D^c_1 = \overline{D}^c_1$$

Proton

$$xh^u_1(x, Q^2) - \frac{1}{4} xh^d_1(x, Q^2) \propto -A^i_{DIS}(x, Q^2) \frac{n_u(Q^2)}{n_u(Q^2)} \sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f^{q+\bar{q}}_1(x, Q^2)$$

Deuteron

$$xh^u_1(x, Q^2) + xh^d_1(x, Q^2) \propto -\frac{5}{3} A^i_{DIS}(x, Q^2) \frac{n_u(Q^2)}{n_u(Q^2)} x \left( f^{u+\bar{u}}_1 + f^{d+d\bar{d}}_1 + \frac{2}{5} f^{s+s\bar{s}}_1 \right)$$

and combinations of both ...

Now for $i=j=1,..,n$ results for the replica method
The Functional Form

\[ x h_1^{\text{eq}}(x) = \tanh \left( x^{1/2} (A_q + B_q x + C_q x^2 + D_q x^3) \right) \left( x S^q(x) + x S^q(x) \right) \]

1st order polynomial

\[ A_q + B_q x \]

2nd order polynomial

\[ A_q + B_q x + C_q x^2 \]

3rd order polynomial

\[ A_q + B_q x + C_q x^2 + D_q x^3 \]
The Functional Form

2nd order polynomial

\[ A_q + B_q x \]

3rd order polynomial

\[ A_q + B_q x + C_q x^2 + D_q x^3 \]

Rigid version

Flexible version

Extra-flexible version

\[ x h_{1q}^{q\nu}(x) = \tanh \left( x^{1/2} (A_q + B_q x + C_q x^2 + D_q x^3) \right) (x S B^q(x) + x S B^\bar{q}(x)) \]
The Functional Form

\[ x h_1^{qV}(x) = \tanh \left( x^{1/2} (A_q + B_q x + C_q x^2 + D_q x^3) \right) \left( x SB^q(x) + x SB^\bar{q}(x) \right) \]

1st order polynomial

\[ A_q + B_q x \]

Rigid version

2nd order polynomial

\[ A_q + B_q x + C_q x^2 \]

Flexible version

3rd order polynomial

\[ A_q + B_q x + C_q x^2 + D_q x^3 \]

Extra-flexible version

\[
\begin{array}{|c|c|c|}
\hline
\chi^2/\text{d.o.f.} & \alpha_s(M_Z^2) = 0.125 & \alpha_s(M_Z^2) = 0.139 \\
\hline
\text{rigid} & 1.42 & 1.46 \\
\text{flexible} & 1.65 & 1.71 \\
\text{extraflexible} & 1.97 & 2.07 \\
\hline
\end{array}
\]
Old and New Fits

Figure 4. The up (left) and down (right) valence transversities coming from the present analysis evolved to $Q^2 = 2.4 \text{ GeV}^2$. From top row to bottom, results with the rigid, flexible, and extra-flexible scenarios are shown, respectively. The dark thick solid lines are the So$\nu$er bound. The uncertainty band with solid boundaries is the best fit in the standard approach at 1, whose central value is given by the central thick solid line. The uncertainty band with dashed boundaries is the 68% of all fitting replicas obtained in the Monte Carlo approach. As a comparison, the uncertainty band with short-dashed boundaries is the transversity extraction from the Collins effect \cite{15}. Of the Collins effect, from which the other parametrization of ref. \cite{15} is extracted. As a matter of fact, this is the only source of significant discrepancy between the two extractions, which otherwise show a high level of compatibility despite the fact that they are obtained from very different procedures. Note that if the So$\nu$er bound is saturated at some scale, it is likely to be significantly violated at a lower scale \cite{46}. Therefore, if we want to maintain

\[ \alpha_s(M_Z^2) = 0.125 \quad \alpha_s(M_Z^2) = 0.139 \]

- Flexible version

OLD 1$\sigma$ error band from replicas @2.4 GeV$^2$

NEW 1$\sigma$ error band from replicas @2.4 GeV$^2
Comparison with Single-hadron extr.

Discrepancy in the d distribution

New proton data don't change that!
Comparison with Single-hadron extr.

Discrepancy in the d distribution
New proton data don't change that!

1σ error band from replicas @2.4 GeV²
flexible scenario 0.125
We show in the left panel, namely the truncated first Mellin moment of the valence transversity. The integral is computed for $x$, where there are differences therein). Moreover, we believe that our error analysis, based on the replica method with $d$ and $q^2$ respectively, here explored with $H$. In Ref. [4.3], which also shows that we have no clue on the transversity for large $x$, $v$. In order to compare with the results of Ref. [4.2], the replicas in $e^+$ represent the current most realistic estimate of the uncertainties on transversity. It err in a less arbitrary way than the choice made in Eq. (4.3). 

\[
\delta q = \int_{6.4 \times 10^{-3}}^{0.28} dx \, h_1^{qv}(x)
\]
1. Anselmino et al Phys.Rev. D87

2. rigid old

3. flexible old

4. extraflexible old

5. rigid 0.125

6. flexible 0.125

7. extraflexible 0.125

\[ \delta q = \int_{0}^{1} dx \, h_{1}^{q_{v}}(x) \]
Tensor Charge's Application

Probe New Fundamental Interactions from Beta Decay

$$N(p_n) \rightarrow P(p_p)e^- (p_e)\bar{\nu}_e (p_{\nu})$$

[Cirigliano et al, JHEP 1302 (2013) 046]
Tensor Charge's Application

Probe New Fundamental Interactions from Beta Decay

\[ N(p_n) \rightarrow P(p_p) e^- (p_e) \bar{\nu}_e (p_{\nu}) \]

can be sketched as

\[ \left[ \frac{d \Gamma}{d \nu} u e^- (p_e) \bar{\nu}_e (p_{\nu}) \right] \otimes \left[ \langle P | \bar{u} \Gamma d | N \rangle \right] \]
Tensor Charge's Application

Probe New Fundamental Interactions from Beta Decay

\[ N(p_n) \rightarrow P(p_p)e^-(p_e)\bar{\nu}_e(p_\nu) \]

can be sketched as

\[ \left[ \int \frac{d}{\Gamma} u e^-(p_e)\bar{\nu}_e(p_\nu) \right] \otimes \left[ \langle P|\bar{u}\Gamma d|N \rangle \right] \]

**Standard Model**

\[ M = -i \frac{G_F}{\sqrt{2}} \bar{u}_e \gamma_\mu (1 - \gamma^5) v_\nu \langle p|\bar{u}\gamma^\mu (1 - \gamma^5) d|n \rangle \cos \theta_c \]

**EW: V-A**

Structural: \( g_V \) & \( g_A \)

[Cirigliano et al, JHEP 1302 (2013) 046]
Tensor Charge's Application

Probe New Fundamental Interactions from Beta Decay

\[ N(p_n) \rightarrow P(p_p)e^-(p_e)\bar{\nu}_e(p_{\nu}) \]

can be sketched as

\[
\left[ d \frac{\Gamma}{\nu} u e^-(p_e)\bar{\nu}_e(p_{\nu}) \right] \otimes \left[ \langle P|\bar{u}\Gamma d|N \rangle \right]
\]

**Standard Model**

\[
M = -i \frac{G_F}{\sqrt{2}} \bar{u}_e \gamma_\mu (1 - \gamma^5) \nu_\nu \langle p|\bar{u}\gamma^\mu (1 - \gamma^5) d|n \rangle \cos \theta_c
\]

**4-fermion interaction**

**BSM**

EW: V-A

Structural: \( g_V \) & \( g_A \)

New: S, T, P

Structural: \( g_S \), \( g_T \) & \( g_P \)

[Cirigliano et al, JHEP 1302 (2013) 046]
Isovector Tensor Charge

\[ g_T = \delta u_v - \delta d_v \]

\[ g_T = 0.81 \pm 0.44 \]

at \( Q^2 = 4 \text{ GeV}^2 \)

New Pavia flexible 0.125

Various Lattice QCD results

[Radici, Courtoy, Bacchetta, Guagnelli, 1503.03495]
Isovector Tensor Charge

\[ g_T = \delta u_v - \delta d_v \]

New Pavia flexible 0.125

\[ g_T = 0.81 \pm 0.44 \]

at \( Q^2 = 4 \text{ GeV}^2 \)

Various Lattice QCD results

[Courtoy, Baessler, Gonzalez-Alonso, Liuti, 1503.06814]

[Radici, Courtoy, Bacchetta, Guagnelli, 1503.03495]
Can it constrain New Physics interaction?

Effective theories approach

\[ \Delta \mathcal{L}_{\text{eff}} = -\frac{G_F V_{ud}}{\sqrt{2}} \epsilon_T \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) d \cdot \bar{e} \sigma_{\mu\nu} (1 - \gamma_5) e \]

Nucleon effective coupling from Beta Decay Exp.

\[ C_T = \frac{4 G_F V_{ud}}{\sqrt{2}} g_T \epsilon_T \Rightarrow \left| g_T \epsilon_T \right| < 6 \cdot 10^{-4} \]

[Pattie et al, Phys.Rev. C88]
[Wauters et al, Phys.Rev. C89]
Can it constrain New Physics interaction?

Effective theories approach

\[
\Delta \mathcal{L}_{\text{eff}} = -\frac{G_F V_{ud}}{\sqrt{2}} \epsilon_T \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) d \cdot \bar{e} \sigma_{\mu\nu} (1 - \gamma_5) \nu_e
\]

Nucleon effective coupling from Beta Decay Exp.

\[
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\]

Pavia flexible
GGL
Torino 2013
Bhattacharya et al Lattice
Bali et al lattice
Can it constrain New Physics interaction?

**Effective theories approach**

\[
\Delta L_{\text{eff}} = -\frac{G_F V_{ud}}{\sqrt{2}} \epsilon_T \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) d \cdot \bar{e} \sigma_{\mu\nu} (1 - \gamma_5) \nu_e
\]

**Nucleon effective coupling from Beta Decay Exp.**

\[
C_T = \frac{4}{\sqrt{2}} \frac{G_F V_{ud}}{g_T} \epsilon_T \quad \Rightarrow \quad |g_T \epsilon_T| < 6 \cdot 10^{-4}
\]

[Pattie et al, Phys.Rev. C88]
[Wauters et al, Phys.Rev. C89]

[Courtoy, Baessler, Gonzalez-Alonso, Liuti, 1503.06814]

Dotted curves:
Projection of NEW error after JLab@12

Pavia flexible
GGL
Torino 2013
Bhattacharya et al Lattice
Bali et al lattice
Conclusion

Extraction of valence transversities from collinear framework

- NEW fit in the REPLICA method for
  - $H_1^<$
  - $h_1$
- NEW COMPASS data on proton + identified pions
  - lower distribution for $u_V$, no drastic change for $d_V$
- Two values for $\alpha_s(M_Z^2)$
  - no/mild dependence from the output
Conclusion

*Extraction of valence transversities from collinear framework*

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Waiting for data from CLAS12 and SoLID (JLab@12)!
Conclusion

*Extraction of valence transversities from collinear framework*

- NEW fit in the REPLICA method for
  - $H_1^<$
  - $h_1$
- NEW COMPASS data on proton + identified pions
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Beyond the fit...

- Impact of tensor charge on New Physics?  \cite{1503.06814}
- DiFF and twist-3 observables: Analysis of BSA at CLAS & extraction of $e(x)$ \cite{1405.7659}
- $P^{\uparrow}\!\!-P$ at RHIC (to be considered in the future) \cite{1504.00415}

Waiting for data from CLAS12 and SoLID (JLab@12)!
Back-up slides
Comparison with extraction

**PROTON**

\[ x h_1^{u(x)} - x h_1^{d(x)/4} \]

- fit
- data HERMES
- data COMPASS

**DEUTERON**

\[ x h_1^{u(x)} + x h_1^{d(x)} \]

- fit
- data HERMES
- data COMPASS

**flexible functional form**

**rigid functional form**
Monte Carlo Approach: some illustrations

Can we find “unforeseen” replica?
Monte Carlo Approach:

Can we find “unforeseen” replica?

Yes, here at 1GeV$^2$
State-of-the-art: Extractions of transversity
State-of-the-art: Extractions of transversity

Anselmino et al [Phys.Rev. D87]
Kang et al [Phys.Rev. D91]
this plot we also show theoretical computations without (solid lines) and

\[ N \]

is performed by using the functional form. Note that our resulting errors correspond to the

\[ \chi \]

errors. The difference between these computations represents a well established and robust theoretical context.

State-of-the-art: Extractions of transversity

**Talk by A. Prokudin**

Anselmino et al [Phys.Rev. D87]
Kang et al [Phys.Rev. D91]
Anselmino et al [Phys.Rev. D87]

In Refs. where we also show theoretical computations without NLL (solid lines) and Collins fragmentation function at two different scales. In Table I, we present the fitted parameters of the transversity quark distribution, considering scales, in Refs. where our resulting dependence of functions by allowing a flavor dependent more parameters does not improve it. We estimate flavor dependence of asymmetry in $h_{1}^{u}$ and $h_{1}^{d}$, which our results are produced, represents a well established and robust theoretical context.

The agreement between our result for the total $h_{1}$ within the $68\%$ band at $Q^{2} = 10$ GeV$^{2}$ is encouraging: while the dihadron SIDIS data are a subset of the single-hadron ones, the theoretical frameworks used to in different. Nevertheless, we point out that the collinear framework, in general, also tends to saturate the upper bound at $x > 0.2$.

Talk by A. Prokudin

State-of-the-art: Extractions of transversity

Anselmino et al [Phys.Rev. D87]
Kang et al [Phys.Rev. D91]
Referees package to perform the fit. The resulting parameters are

\[
\begin{align*}
\chi^2 &= n - \text{d.o.f.} \\
\chi^2 &= 0.0065 \pm 0.35
\end{align*}
\]

and

\[
\begin{align*}
Q^2 &= 10 \text{ GeV}^2 \\
Q^2 &= 1000 \text{ GeV}^2
\end{align*}
\]

diminishes when we include higher orders, it means that the errors correspond to the

So agreement with the other extraction based on the Collins et al. [35,36]. We plot the extracted transversity and and

\[
Q^2 \leq 500 \text{ GeV}
\]

the study of the Collins et al. 85, and the valence down transversity tend to saturate the lower limit of the So

that it is not an artifact of the chosen functional form. As a matter of fact, our replicas for

the outcome of the Collins et al. recent transversity extraction from the Collins et al. (x, Q^2) from the BELLE

\[
Q^2 < 20, 35, 36\]

SIDIS. Apart from the range

\[
0 < x < 1
\]

we define the follow-

\[
\delta_p \pi
\]

Figure 8 (color online). Collins asymmetries measured by the

\[
\delta_p \pi
\]

In Fig.

\[
\delta_p \pi
\]

Finally, we present an estimate at 90% confidence level

\[
\delta_p \pi
\]

Talk by A. Prokudin

This talk

State-of-the-art:
Extractions of transversity

Anselmino et al [Phys.Rev. D87]
Kang et al [Phys.Rev. D91]
Two complementary approaches

- partner of Collins FF
- convolution

\[ \int d^2 p_T d^2 k_T \delta^2 (k_T + q_T - p_T) h_1(x, k_T) H_1^+(z, p_T) \]

- QCD evolution: TMD evolution
- ongoing progresses
  [Rogers, Aybat, Prokudin, Bacchetta,...]

- need input Functional Form of the transversity

- partner of chiral-odd DiFF
- simple product

\[ h_1(x) H_1^{\Delta}(z, M_h) \]

- QCD evolution: DGLAP evolution
- known
  [Bacchetta, Radici, Ceccopieri]

- no need for input Functional Form of the transversity
- direct extraction point by point
Frameworks for DiFFs
Frameworks for DiFFs

SIDIS on $p$

lepton

lepton

proton

2 pions
Frameworks for DiFFs

SIDIS on $p^+$

Talks by
N. Makke
C. Braun
S. Gliske
Frameworks for DiFFs

SIDIS on $p^+$

Talks by
N. Makke
C. Braun
S. Gliske

e$^+e^-$ to pion pairs

2 pions
Frameworks for DiFFs

SIDIS on $p^-$

lepton → proton → 2 pions → lepton

Talks by
N. Makke
C. Braun
S. Gliske

$e^+e^-$ to pion pairs

e- → e+ → 2 pions → 2 pions

Talk by
I. Garzia
**Frameworks for DiFFs**

1. **SIDIS on p⁺**
   - Lepton
   - Proton
   - 2 pions

2. **e⁻e⁺ to pion pairs**
   - Electron
   - Positron
   - 2 pions

3. **pp to pion pairs**
   - Proton
   - Proton
   - 2 pions

*Talks by*
- N. Makke
- C. Braun
- S. Gliske

*Talk by*
- I. Garzia
SIDIS production of pion pairs

Chiral-odd DiFF:

Distribution of hadrons inside the jet
is related to the

Direction of the transverse polarization of the fragmenting quarks

\[ A_{\text{DIS}}(x, z, M_h^2, Q^2) = -C_y \sum_q e_q^2 h_1^q(x, Q^2) \left| \frac{R}{M_h} \right| H_{1,sp}^{q\rightarrow\pi^+\pi^-}(z, M_h^2, Q^2) \frac{\sum_q e_q^2 f_1^q(x, Q^2)}{D_1^{q\rightarrow\pi^+\pi^-}(z, M_h^2, Q^2)} \]
SIDIS production of pion pairs

Chiral-odd DiFF:

**Distribution of hadrons inside the jet**

is related to the

Direction of the transverse polarization of the fragmenting quarks

$$A_{\text{DIS}}(x, z, M_h^2, Q^2) = -C_y \frac{\sum q e_q^2 h_1^q(x, Q^2) |\mathcal{R}|}{\sum q e_q^2 f_1^q(x, Q^2)} \frac{H_{1, sp}^q(\pi^+ \pi^-) (z, M_h^2, Q^2)}{D_{1}^{\pi^+ \pi^-}(z, M_h^2, Q^2)}$$

Knowledge on DiFFs leads to $h_1(x, Q^2)$
Fitting the Valence Transversities
Fitting the Valence Transversities

Constraints from first principles

✦ Soffer bound

\[ 2|h_1^q(x, Q^2)| \leq |f_1^q(x, Q^2) + g_1^q(x, Q^2)| \equiv 2 \text{SB}^q(x, Q^2) \]

✦ \( h_1(x=1) = 0 \); the parton model predicts \( h_1(x=0) = 0 \) but too restrictive in QCD
Fitting the Valence Transversities

Constraints from first principles

✦ **Soffer bound**

\[ 2|h_1^q(x, Q^2)| \leq |f_1^q(x, Q^2) + g_1^q(x, Q^2)| \equiv 2 \text{SB}^q(x, Q^2) \]

✦ **h_1(x=1)=0** ; the parton model predicts h_1(x=0)=0 but too restrictive in QCD

QCD evolution with HOPPET code

✦ **of the Soffer bound**: LO evolution of f_1(x) from MSTW08 & g_1(x) from DSS

✦ **of the DiFF & h_1**: LO as in previous papers
Fitting the Valence Transversities

Constraints from first principles

✧ Soffer bound

\[ 2|h_1^q(x, Q^2)| \leq |f_1^q(x, Q^2) + g_1^q(x, Q^2)| \equiv 2 \text{SB}^q(x, Q^2) \]

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QCD evolution with HOPPET code

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✧ of the DiFF \& \( h_1 \): LO as in previous papers

Choice of Functional Form
Fitting the Valence Transversities

Constraints from first principles

✦ Soffer bound

\[ 2|h_1^q(x, Q^2)| \leq |f_1^q(x, Q^2) + g_1^q(x, Q^2)| \equiv 2 \text{SB}^q(x, Q^2) \]

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QCD evolution with HOPPET code

✦ of the Soffer bound: LO evolution of \( f_1(x) \) from MSTW08 & \( g_1(x) \) from DSS

✦ of the DiFF & \( h_1 \): LO as in previous papers

Choice of Functional Form

the CRUCIAL point for further uses
Fitting the Valence Transversities

Constraints from first principles

✧ Soffer bound

\[ 2|h_1^q(x, Q^2)| \leq |f_1^q(x, Q^2) + g_1^q(x, Q^2)| \equiv 2 \text{SB}^q(x, Q^2) \]

✧ \( h_1(x=1)=0 \); the parton model predicts \( h_1(x=0)=0 \) but too restrictive in QCD

QCD evolution with HOPPET code

✧ of the Soffer bound: LO evolution of \( f_1(x) \) from MSTW08 & \( g_1(x) \) from DSS

✧ of the DiFF & \( h_1 \): LO as in previous papers

Choice of Functional Form

\[ \text{the CRUCIAL point for further uses} \]
Fitting the Valence Transversities

Constraints from first principles

✦ Soffer bound

\[ 2|h_1^q(x, Q^2)| \leq |f_1^q(x, Q^2) + g_1^q(x, Q^2)| \equiv 2\text{SB}^q(x, Q^2) \]

✦ \( h_1(x=1)=0 \); the parton model predicts \( h_1(x=0)=0 \) but too restrictive in QCD

QCD evolution with HOPPET code

✦ of the Soffer bound: LO evolution of \( f_1(x) \) from MSTW08 & \( g_1(x) \) from DSS

✦ of the DiFF & \( h_1 \): LO as in previous papers

Choice of Functional Form

\[ x \ h_1^{qV}(x, Q_0^2) = FF(\text{param}, x, Q_0^2) \ (x \text{SB}^q(x, Q_0^2) + x \text{SB}^\bar{q}(x, Q_0^2)) \]

with FF defined [-1,1]

the CRUCIAL point for further uses
Fitting the Valence Transversities

Constraints from first principles

✦ Soffer bound

\[ 2|h_1^q(x, Q^2)| \leq |f_1^q(x, Q^2) + g_1^q(x, Q^2)| \equiv 2 \text{SB}^q(x, Q^2) \]

✦ \( h_1(x=1)=0 \); the parton model predicts \( h_1(x=0)=0 \) but too restrictive in QCD

QCD evolution with HOPPET code

✦ of the Soffer bound: LO evolution of \( f_1(x) \) from MSTW08 & \( g_1(x) \) from DSS

✦ of the DiFF & \( h_1 \): LO as in previous papers

Choice of Functional Form

\[ x h_1^{qV}(x, Q_0^2) = FF(\text{param}, x, Q_0^2) \left( x \text{SB}^q(x, Q_0^2) + x \bar{\text{SB}}^q(x, Q_0^2) \right) \]

with FF defined [-1,1]
Transversity from $e p \rightarrow e' (\pi^+\pi^-) X @ HERMES$

\[ x h_{1u}^u(x, Q^2) - \frac{1}{4} x h_{1d}^d(x, Q^2) = -C_y^{-1} A_{DIS}(x, Q^2) \left( \frac{n_u(Q^2)}{n_u(Q^2)} \right) \sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_{1q+\bar{q}}(x, Q^2) \]

with 1-to-100 GeV$^2$ evolution correction: small corrections

HERMES range: $-0.259^{-1}$ (± 25% theo. err.) from fit integrated in mean values
**Transversity from $e^+p \rightarrow e'^+ (\pi^+\pi^-) X$ @ HERMES**

\[
x h_{1u}^u(x, Q^2) - \frac{1}{4} x h_{1d}^d(x, Q^2) = -C_y^{-1} A_{DIS}(x, Q^2) \left( \frac{n_u(Q^2)}{n_u^{(1)}(Q^2)} \right) \sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_{1q^+(\bar{q})}(x, Q^2)
\]

with 1-to-100 GeV² evolution correction: small corrections

**HERMES range:** \(-0.259^{-1}\) (± 25% Theo. err.) from fit

integrated in mean values

---

**Transversity from $e^+p \rightarrow e'^+ (\pi^+\pi^-) X$ @ COMPASS 2007**

\[
x h_{1u}^u(x, Q^2) - \frac{1}{4} x h_{1d}^d(x, Q^2) = -C_y^{-1} A_{DIS}(x, Q^2) \left( \frac{n_u(Q^2)}{n_u^{(1)}(Q^2)} \right) \sum_{q=u,d,s} \frac{e_q^2}{e_u^2} x f_{1q^+(\bar{q})}(x, Q^2)
\]

with 1-to-100 GeV² evolution correction: negligible corrections

**COMPASS range:** \(-0.208^{-1}\) (± 19% Theo. err.) from fit
Our Flexible Functional Form  

2nd order polynomial

Band: Torino 2009 transversity

Best fit central curve @2.4 GeV$^2$ and standard 1-σ error band

Soffer Bound @ 2.4 GeV$^2$
Our Flexible Functional Form  2nd order polynomial

Band: Torino 2009 transversity

Best fit central curve @2.4 GeV$^2$ and standard 1-$\sigma$ error band

Flexible version
Our Flexible Functional Form  2nd order polynomial

Band: Torino 2009 transversity

Best fit central curve @2.4 GeV² and standard 1-σ error band

Flexible version
The Error Analysis:  

the Monte Carlo approach

1st order polynomial

Figure 4. The up (left) and down (right) valence transversities coming from the present analysis evolved to $Q^2 = 2.4$ GeV$^2$. From top row to bottom, results with the rigid, flexible, and extra-flexible scenarios are shown, respectively. The dark thick solid lines are the So...
The Error Analysis: the Monte Carlo approach
1st order polynomial

Figure 4. The up (left) and down (right) valence transversities coming from the present analysis evolved to $Q^2 = 2.4 \text{ GeV}^2$. From top row to bottom, results with the rigid, flexible, and extra-flexible scenarios are shown, respectively. The dark thick solid lines are the Soffer bound. The uncertainty band with solid boundaries is the best fit in the standard approach at 1,$\sigma$, whose central value is given by the central thick solid line. The uncertainty band with dashed boundaries is the 68% of all fitting replicas obtained in the Monte Carlo approach. As a comparison, the uncertainty band with short-dashed boundaries is the transversity extraction from the Collins effect [15].

Of the Collins effect, from which the other parametrization of ref. [15] is extracted. As a matter of fact, this is the only source of significant discrepancy between the two extractions, which otherwise show a high level of compatibility despite the fact that they are obtained from very different procedures. Note that if the Soffer bound is saturated at some scale, it is likely to be significantly violated at a lower scale [46]. Therefore, if we want to maintain

– 15 –

Rigid version

1$\sigma$ error band from replicas @2.4 GeV$^2$

Best fit central curve @2.4 GeV$^2$
and standard 1$\sigma$ error band

Rigid version
ESTIMATES FROM EXPERIMENTAL PROJECTIONS

- old Pavia fit with artificial data in future range
- includes both CLAS12 on proton and SoLID on neutron
- to be up-dated with new Pavia fit

[A.C., González-Alonso, Liuti, in progress]
Our Rigid Functional Form

1st order polynomial

Band: Torino 2009 transversity

Soffer Bound @ 2.4 GeV^2

Best fit central curve @2.4 GeV^2 and standard 1-σ error band
Our Rigid Functional Form

1st order polynomial

Best fit central curve @2.4 GeV² and standard 1-σ error band

Soffer Bound @ 2.4 GeV²

Band: Torino 2009 transversity

Rigid version
Our Rigid Functional Form \(1\text{st order polynomial}\)

- **Band: Torino 2009 transversity**
- **Best fit central curve @2.4 GeV\(^2\) and standard 1-\(\sigma\) error band**

Rigid version
Dihadron SIDIS
Dihadron SIDIS

Collinear factorization

\[ D_{1}^{q \rightarrow h_{1}h_{2}} (z_1, z_2, R_{T}^{2}) \]

Here:

\[ D_{1}^{q \rightarrow \pi^{+}\pi^{-}} (z, M_{h}) \]

\[ z = z_1 + z_2 \]

\[ 2 |R| = \sqrt{M_{h}^{2} - 4m_{\pi}^{2}} \]
Dihadron SIDIS

\[ D_{1}^{q \rightarrow h_{1} h_{2}} (z_{1}, z_{2}, R_{T}^{2}) \]

\[ h \quad 2R_{T} \]

Here:

\[ D_{1}^{\pi^{+} \pi^{-}} (z, M_{h}) \]

\[ z = z_{1} + z_{2} \]

\[ 2 |R| = \sqrt{M_{h}^{2} - 4m_{\pi}^{2}} \]

\[ H_{1}^{< q \rightarrow H_{1} H_{2}} (z_{1}, z_{2}, R_{T}^{2}) \]

transverse pol. of the fragm. quark ↔ angular distribution of hadron pairs in the transverse plane