

QCD Analysis of the Combined HERA Inclusive Data together with HERA Jet and Charm Data

HERAPDF2.0Jets

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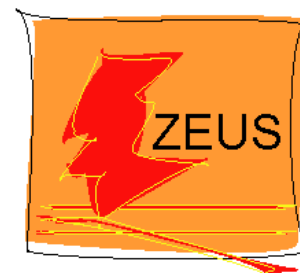
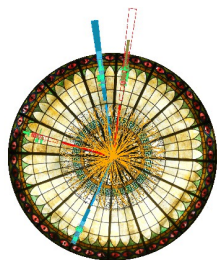
on behalf of the

H1 and ZEUS Collaborations

DIS 2015

XXIII International Workshop on
Deep-Inelastic Scattering and
Related Subjects

Dallas, Texas
April 27 – May 1, 2015



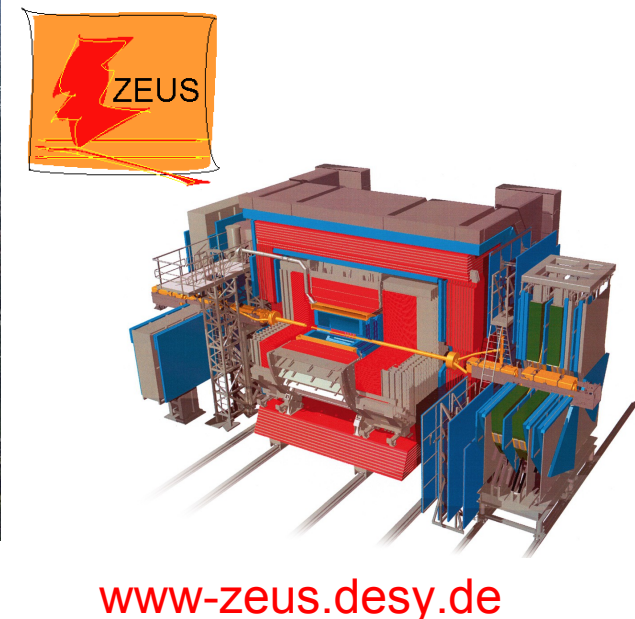
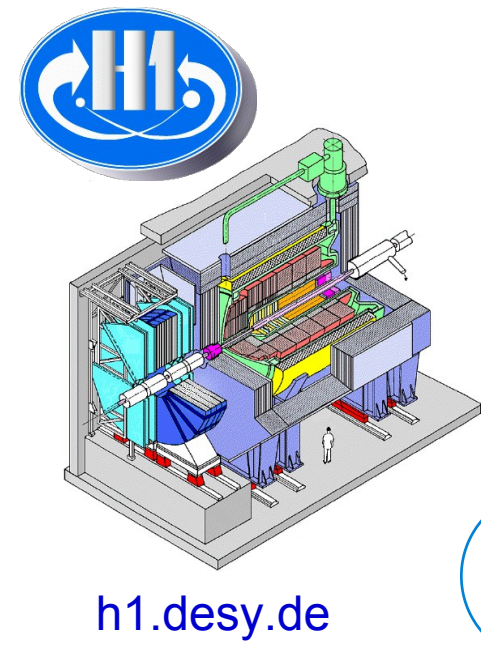
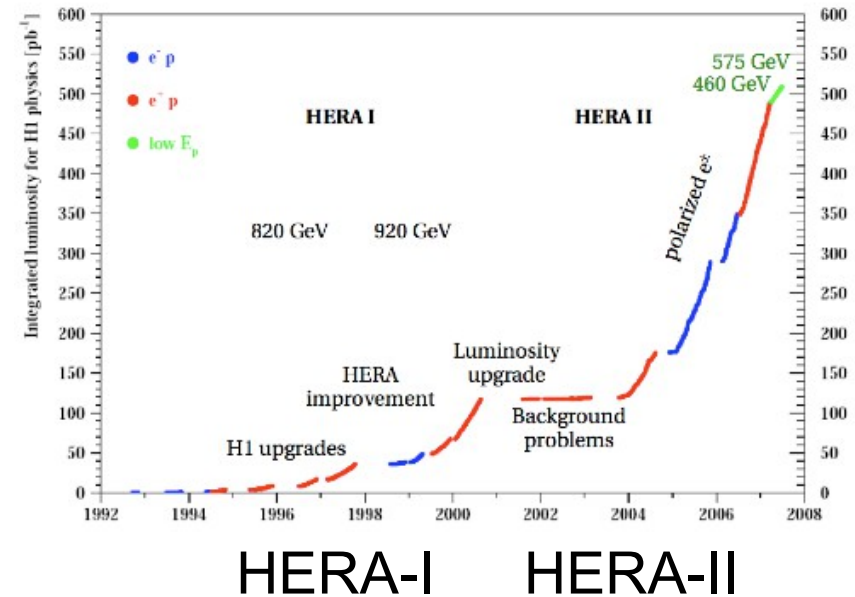
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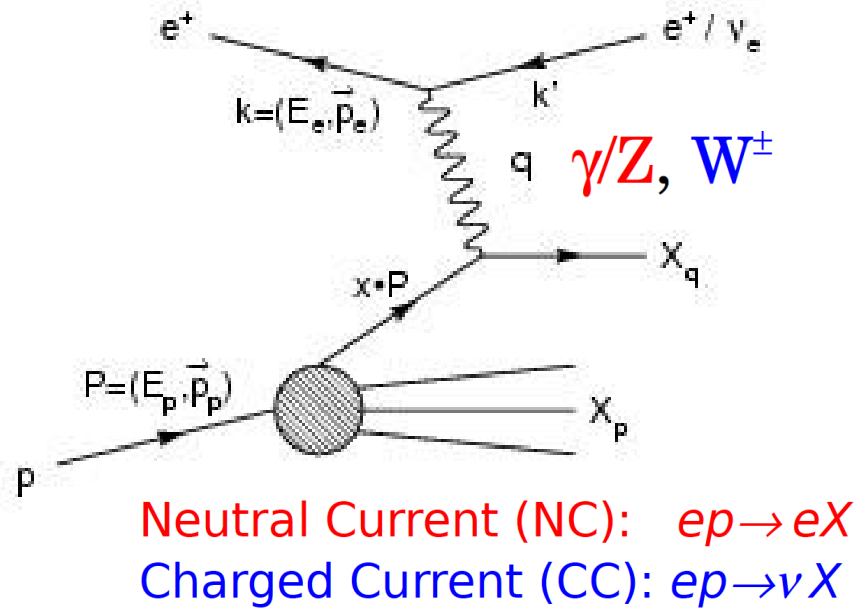
$e p$ Collisions: HERA, H1 and ZEUS

- HERA world's only $e p$ collider, 1992 - 2007
- Centre-of-mass energies 225 -- 318 GeV,
- $\sim 1 \text{ fb}^{-1}$ of total physics data recorded
- Two all-purpose detectors: H1 and ZEUS
- Most HERA measurements now final, in particular inclusive, charm, and jet cross sections used here
 - Combinations between experiments being finalized



Deep-Inelastic Scattering

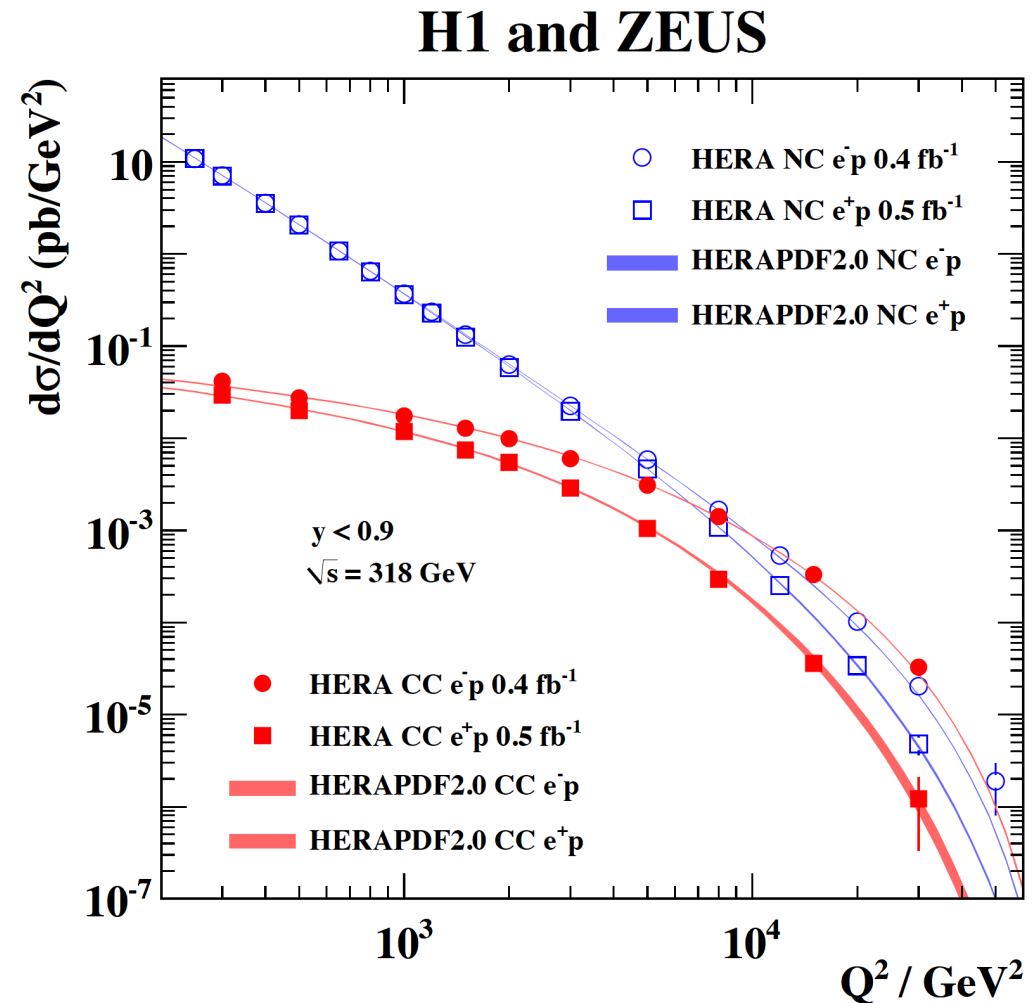
- Inclusive DIS cross sections form the backbone of all modern global QCD analyses and PDF fits
- Cover wide range of $6 \cdot 10^{-7} < x < 0.65$ and $0.045 < Q^2 < 50000 \text{ GeV}^2$



$$Q^2 = -q^2 = -(k - k')^2 \quad \text{Photon virtuality}$$

$$x = \frac{Q^2}{2p \cdot q} \quad \text{Bjorken variable}$$

$$y = \frac{p \cdot q}{p \cdot k} \quad \text{Inelasticity}$$



Overview HERAPDF2.0

- The HERAPDF2.0 fits are based on the latest and most comprehensive combinations of HERA data
- Several flavors released
 - Different assumptions on Q_{\min}^2 , flavour scheme, NLO, NNLO
 - with and without charm and jets

Will focus here on HERAPDF2.0Jets including charm and jets data

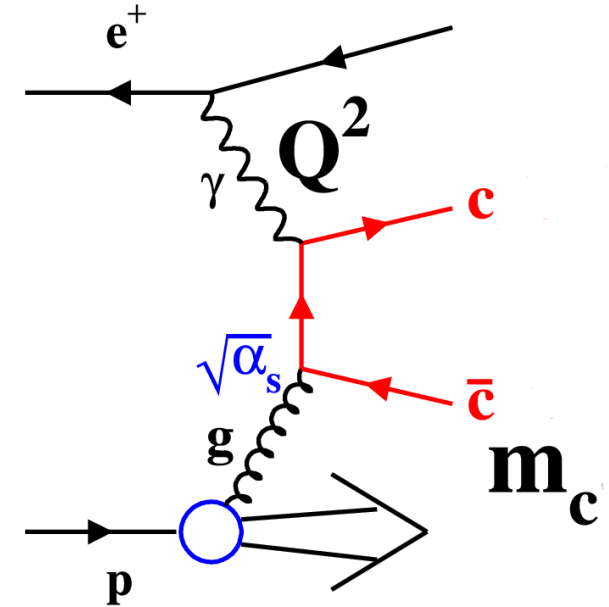


HERAPDF	$Q_{\min}^2 [\text{GeV}^2]$	χ^2	d.o.f.	$\chi^2/\text{d.o.f.}$
2.0 NLO	3.5	1357	1131	1.200
2.0HiQ2 NLO	10.0	1156	1002	1.154
2.0 NNLO	3.5	1363	1131	1.205
2.0HiQ2 NNLO	10.0	1146	1002	1.144
2.0 AG NLO	3.5	1359	1132	1.201
2.0HiQ2 AG NLO	10.0	1161	1003	1.158
2.0 AG NNLO	3.5	1385	1132	1.223
2.0HiQ2 AG NNLO	10.0	1175	1003	1.171
2.0 NLO FF3A	3.5	1351	1131	1.195
2.0 NLO FF3B	3.5	1315	1131	1.163
2.0Jets $\alpha_s(M_Z^2)$ fixed	3.5	1568	1340	1.170
2.0Jets $\alpha_s(M_Z^2)$ free	3.5	1568	1339	1.171

- More talks on HERAPDF2.0 already given here:
 - HERA Collider Results (A. Cooper-Saarkar, Plenary)
 - Inclusive combination (K. Wichmann, WG1)
 - QCD Analysis and variants (V. Myronenko, WG1)

Charm Production at HERA

- Charm is produced in virtual photon-gluon fusion
- M_c important scale in the pQCD calculations
- Sensitive to gluon PDF
 - Charm contributes up to 30% to PDFs at high Q^2
- Consequences for electroweak precision measurements at LHC



Cross section in terms of structure functions F_2^{cc} , F_L^{cc}

$$\frac{d^2\sigma^{c\bar{c}}}{dx dQ^2} = \frac{2\pi\alpha^2(Q^2)}{xQ^4} ([1 + (1 - y)^2] F_2^{c\bar{c}}(x, Q^2) - y^2 F_L^{c\bar{c}}(x, Q^2))$$

Reduced cross section:

$$\begin{aligned} \sigma_{\text{red}}^{c\bar{c}} &= \frac{d^2\sigma^{c\bar{c}}}{dx dQ^2} \cdot \frac{xQ^4}{2\pi\alpha^2(Q^2) (1 + (1 - y)^2)} \\ &= F_2^{c\bar{c}} - \frac{y^2}{1 + (1 - y)^2} F_L^{c\bar{c}}. \end{aligned}$$

Quark Mass Treatment Schemes in QCD

- Several large scales complicate QCD analysis of charm production
- Various *ansätze* starting from basic factorization theorem

Fixed Flavour Number Scheme (FFNS)

- Heavy quarks are massive
- Treated like final-state particles (not as partons)
- Expected to be valid at scale $\sim M_c$
- Cross section calculations: HVQDIS, FMNR

Zero Mass Flavour Number Scheme (ZM-VFNS)

- Neglects heavy quark masses
- Uses resummation for terms $\sim \log(Q^2/m_c^2)$
- Expected to be valid at scales $\gg m_c$
- Cannot describe charm data at HERA

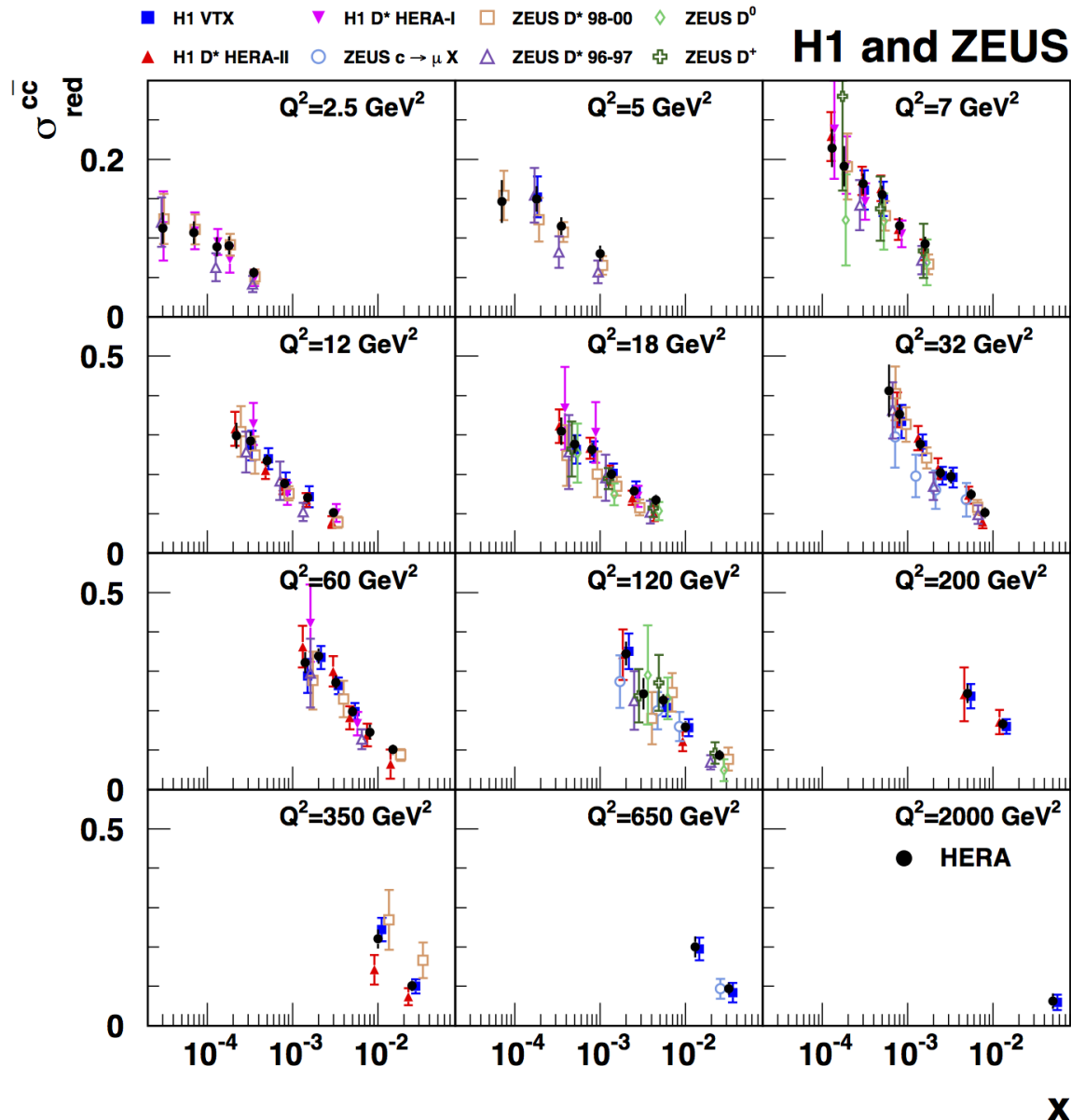
General Mass Variable Flavour Number Scheme (GM-VFNS)

- Interpolates between FFNS and ZM-VFNS
- Various approaches possible and in existence (RT, ACOT, ...)

Schemes can be tested with HERA charm data

Combined HERA Charm Data

Eur. Phys. J. C73 (2013) 2311



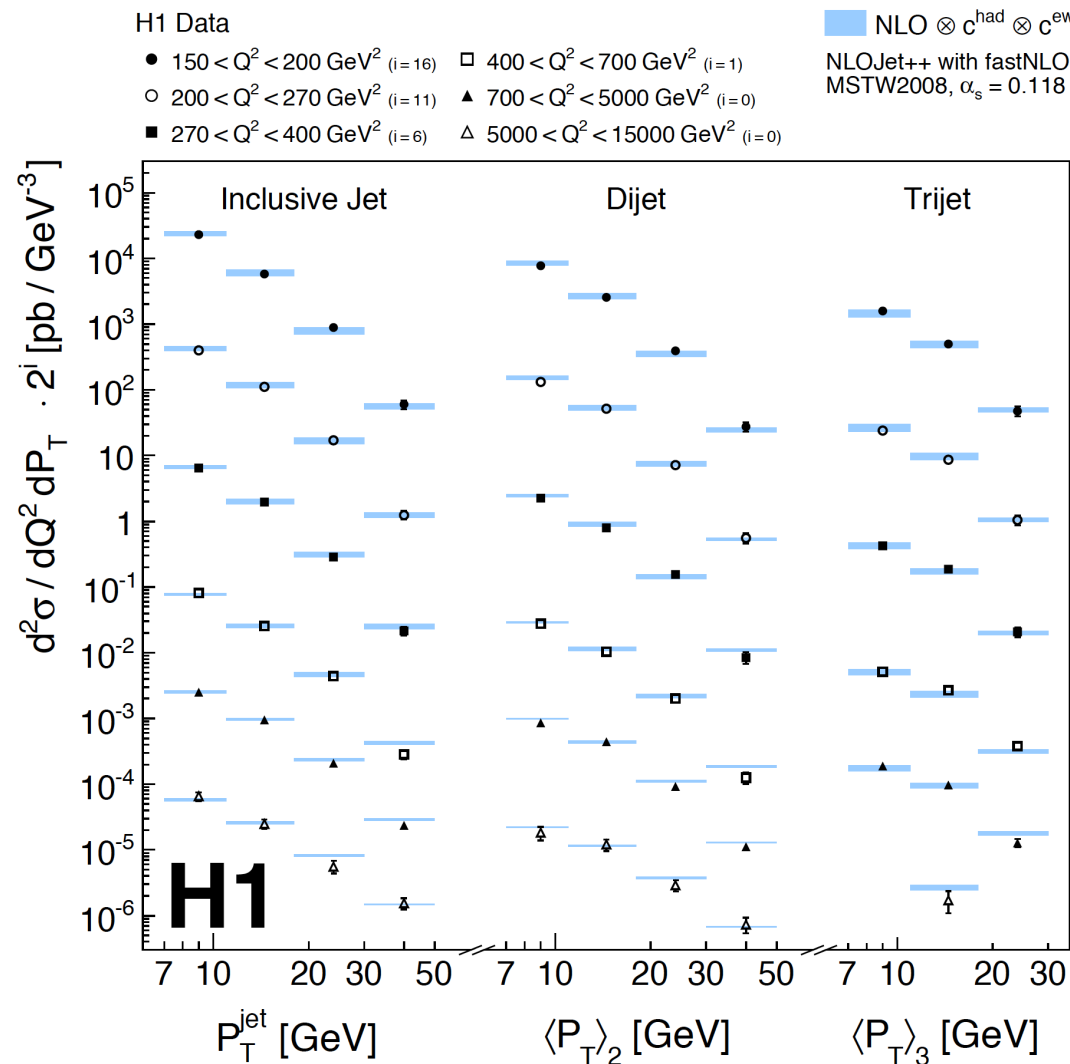
- Combined reduced cross sections together with input data
- 155 measurements combined to 52 cross section measurements
- 48 sources of correlated uncertainty taken into account
- Good consistency of combination with data
 $\chi^2 / \text{ndof} = 62 / 103$



Jets Data

- Select HERA jet data used in HERAPDF2.0Jets PDF fit
- ZEUS Inclusive jets (HERA I)
[Phys. Lett. B 547, 164 \(2002\) \[hep-ex/0208037\]](#)
- ZEUS Dijets (HERA II)
[Eur. Phys. J. C 70, 965 \(2010\) \[arXiv:1010.6167\]](#)
- H1 Inclusive jets (HERA I)
 - Low Q²
[Eur. Phys. J. C 67, 1 \(2010\), \[arXiv:0911.5678\]](#)
 - High Q²
[Phys. Lett. B 653, 134 \(2007\), \[arXiv:0706.3722\]](#)
- H1 Multijet Production (HERA II)
[Eur. Phys. J. C 75 \(2015\) 2, 65 \[arxiv:1406.4709\]](#)

--> More more details in talk by
R. Placakyte, WG4

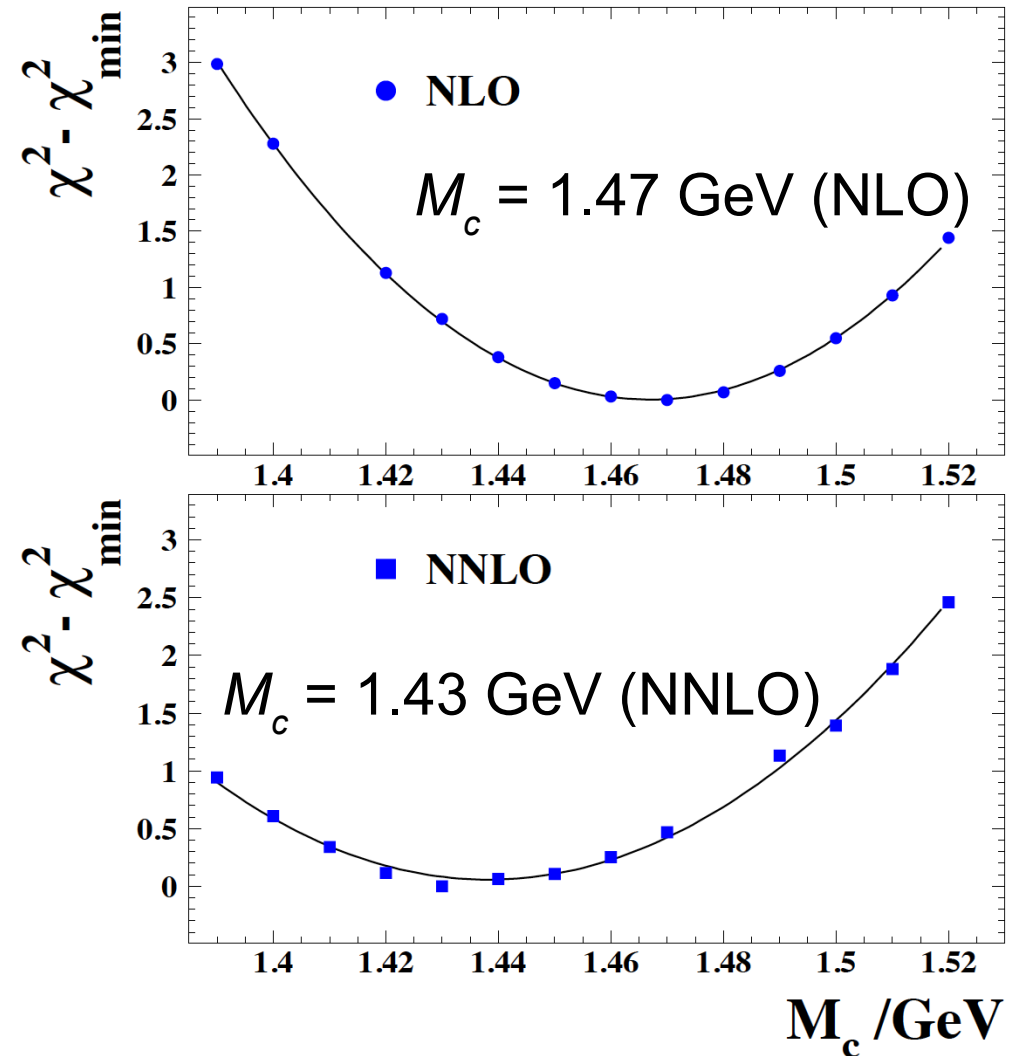


Charm Mass Parameter in HERAPDF 2.0

- Reminder: In VFNS the charm mass parameter is a scale for the calculation – charm quark *not* treated as free particle
- Using GM-VFNS RTOPT scheme
- Optimal M_c found by repeating PDF fits scanning the range $1.2 \text{ GeV} < M_c < 1.6 \text{ GeV}$ and finding the minimum

$$\chi^2(M_c) = \chi_{\min}^2 + \left(\frac{M_c - M_c^{\text{opt}}}{\sigma(M_c^{\text{opt}})} \right)^2$$

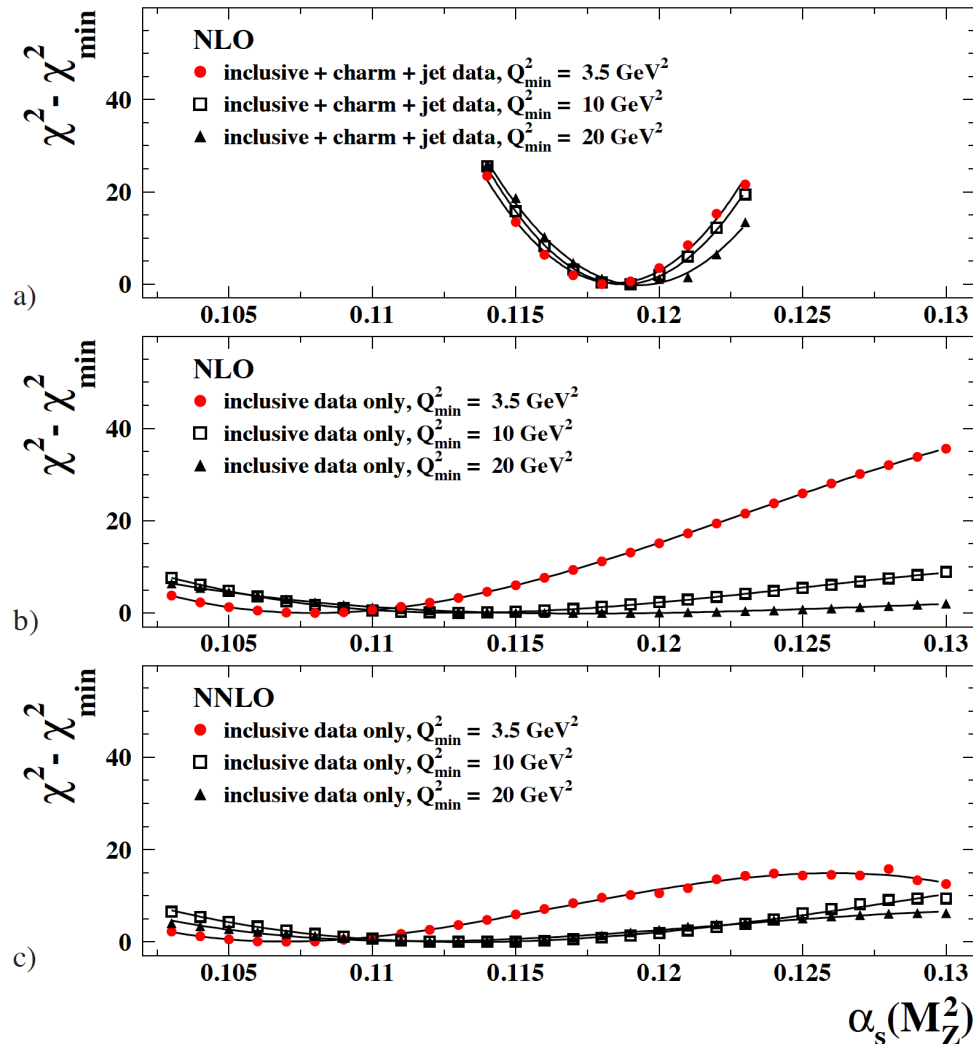
H1 and ZEUS



α_s Choice in Fit and Validation

- For fixed fits the strong coupling constant was chosen to be $\alpha_s(M_Z) = 0.118$ at (N)NLO and $\alpha_s(M_Z) = 0.130$ at LO
- Confirm by leaving it free and fitting in a scan

H1 and ZEUS

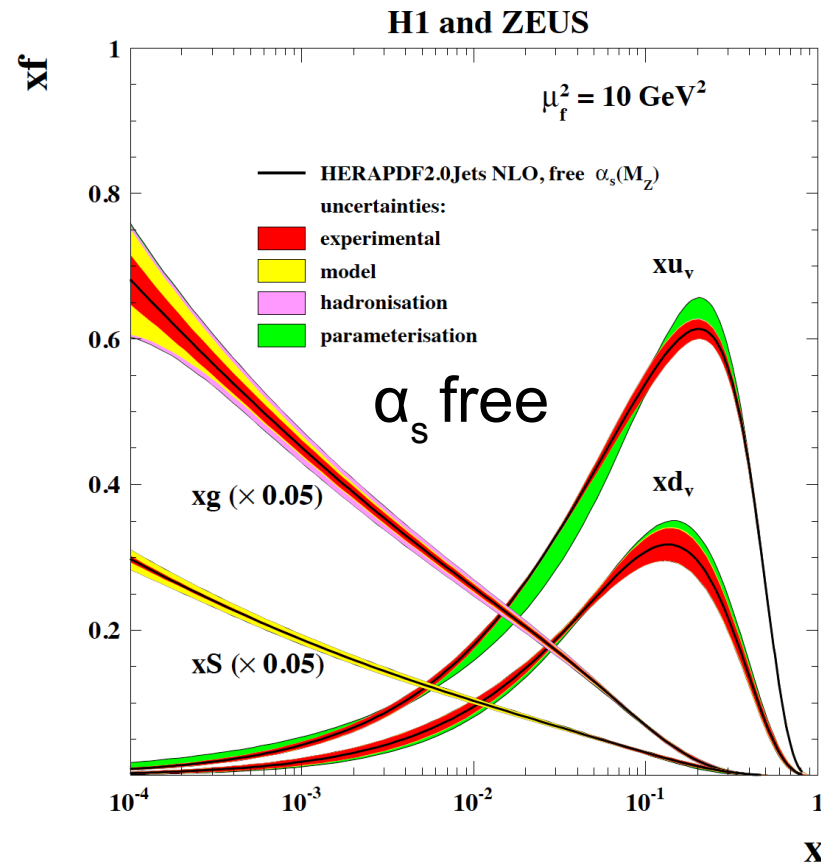
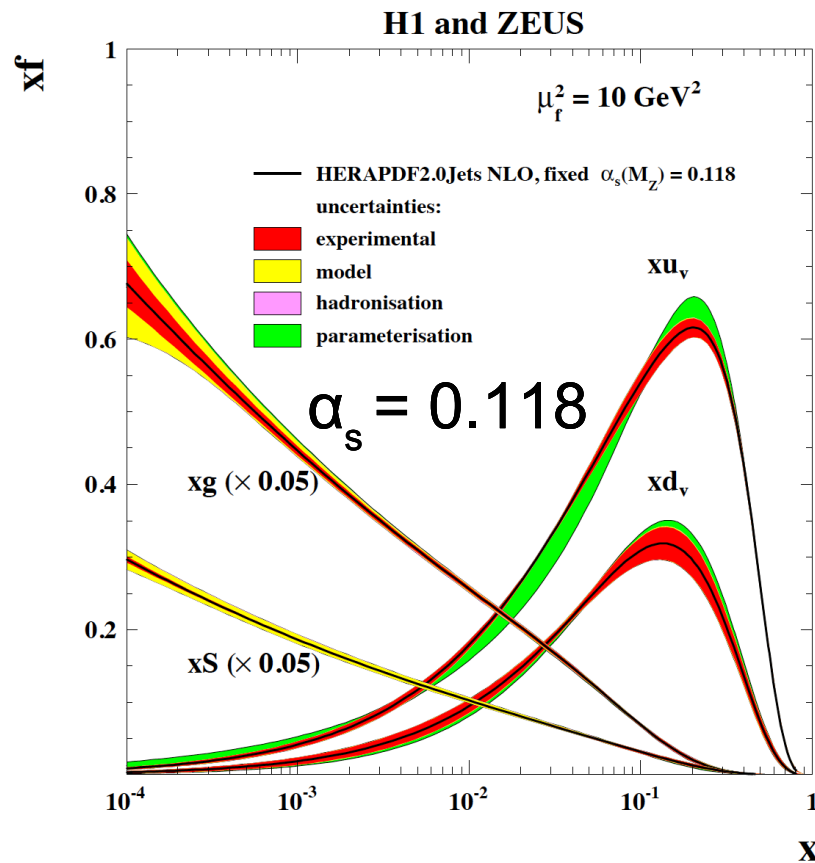


Inclusive, charm and jet data at NLO

- Fit arrives at $\alpha_s(M_Z) = 0.118$
- Largely independent of Q_{min}^2
- Due to inclusion of jet data in fit

- Inclusive only fits at NLO and NNLO cannot constrain $\alpha_s(M_Z)$ very well
- Strong dependence on Q_{min}^2

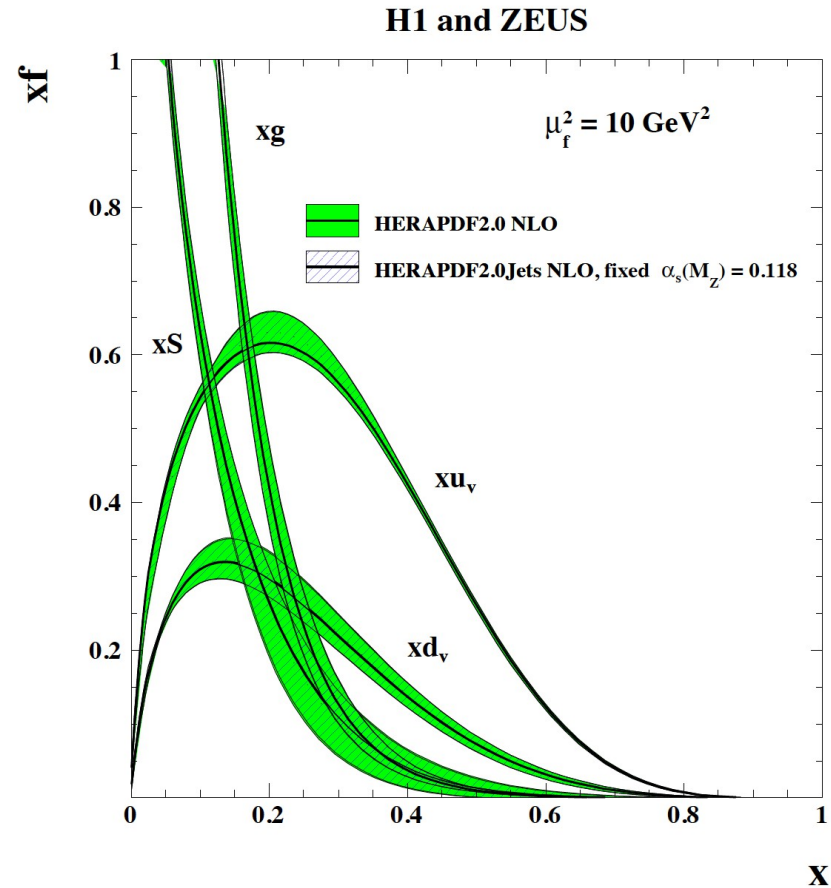
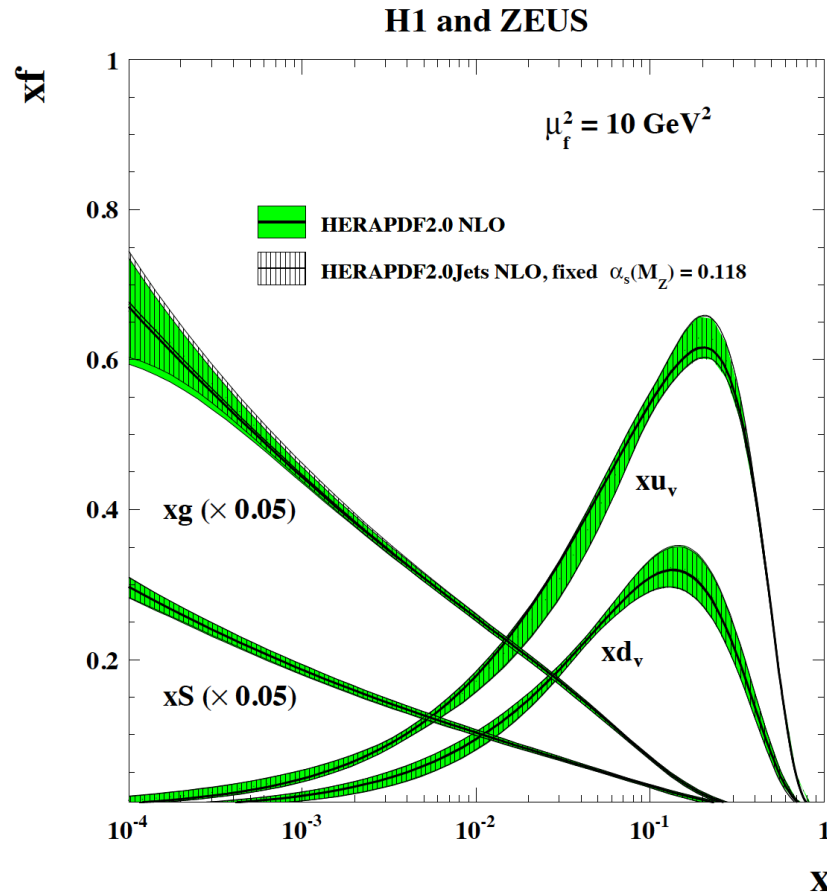
HERAPDF2.0Jets NLO at $\mu_f = 10 \text{ GeV}^2$



- Fits very similar in both cases (assumed as corresponds to fit result)
 - Confirms choice of $\alpha_s = 0.118$ in fixed fit
- Full treatment of uncertainties in both cases
 - Gluon uncertainties in free fit only slightly larger – constrained by data
- Fit with free $\alpha_s(M_Z)$ results in

$$\alpha_s(M_Z^2) = 0.1183 \pm 0.0009(\text{exp}) \pm 0.0005(\text{model/parameterisation}) \\ \pm 0.0012(\text{hadronisation}) {}^{+0.0037}_{-0.0030}(\text{scale}) .$$

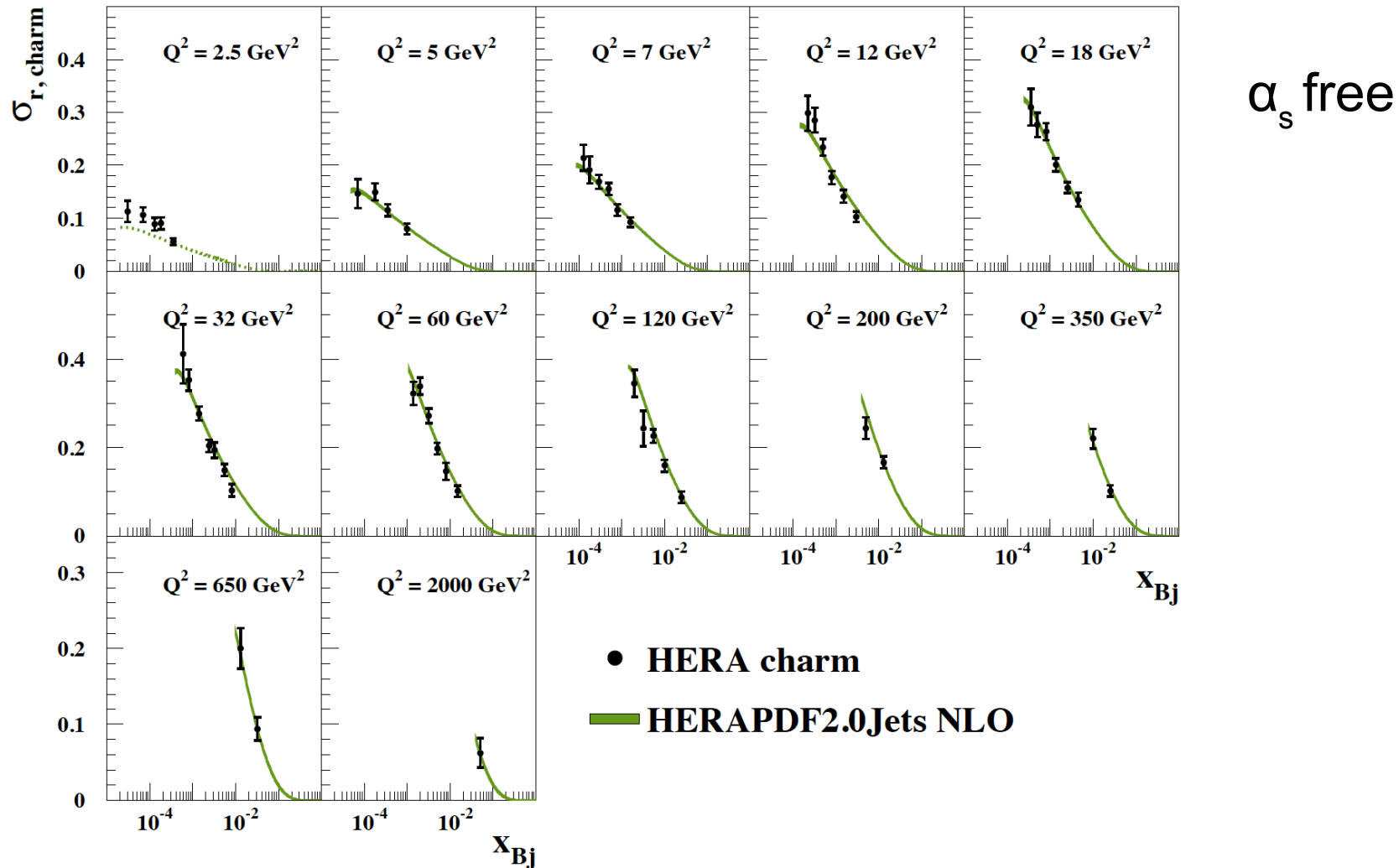
HERAPDF2.0 compared to HERAPDF2.0Jets NLO



- Fits almost identical
- Main difference is small reduction in uncertainty on gluon distribution

HERAPDF2.0Jets vs. red. Charm Cross Section

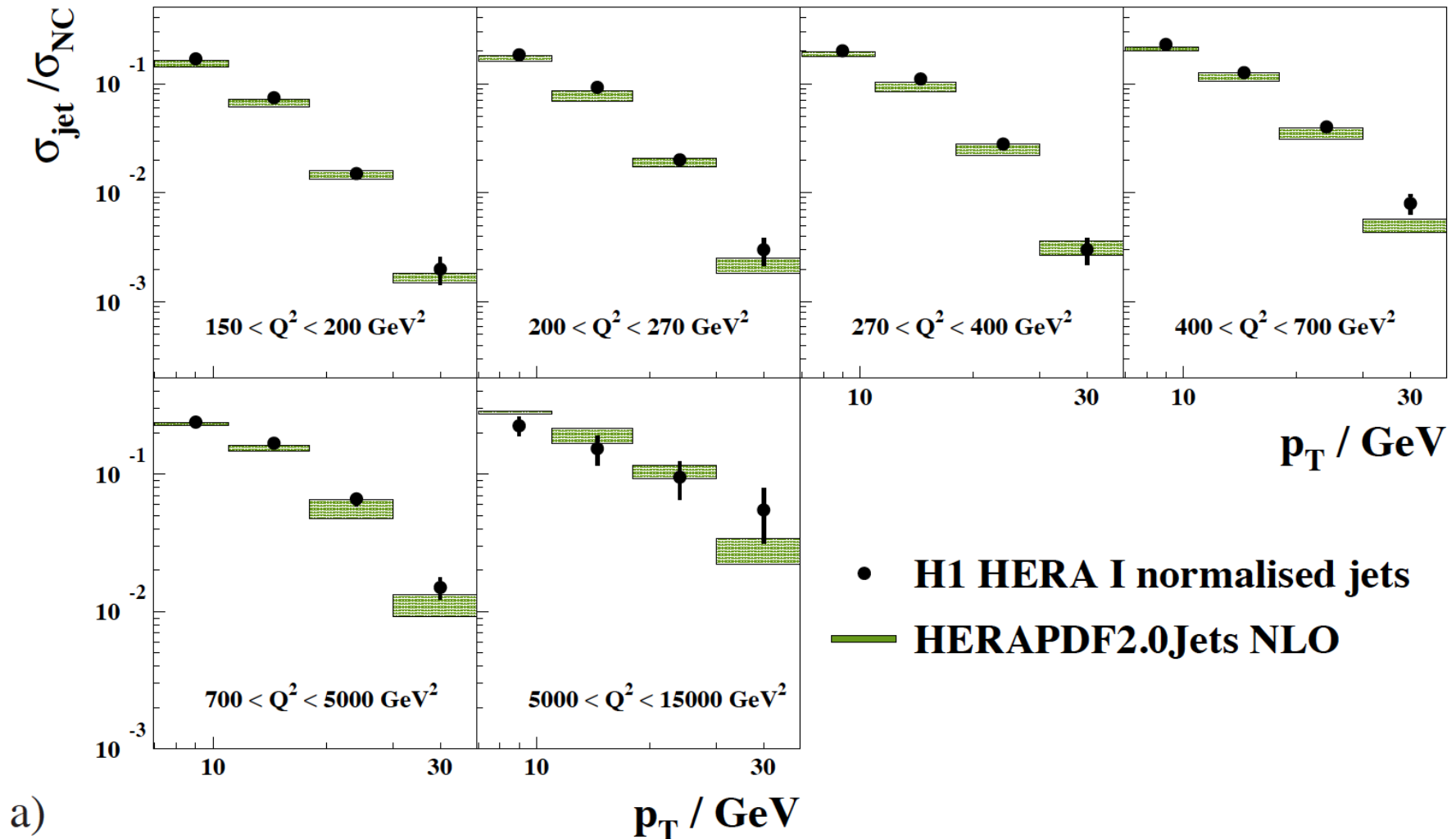
H1 and ZEUS



- Combined HERA charm data together with the HERAPDF2.0Jets NLO fit
- Excellent description

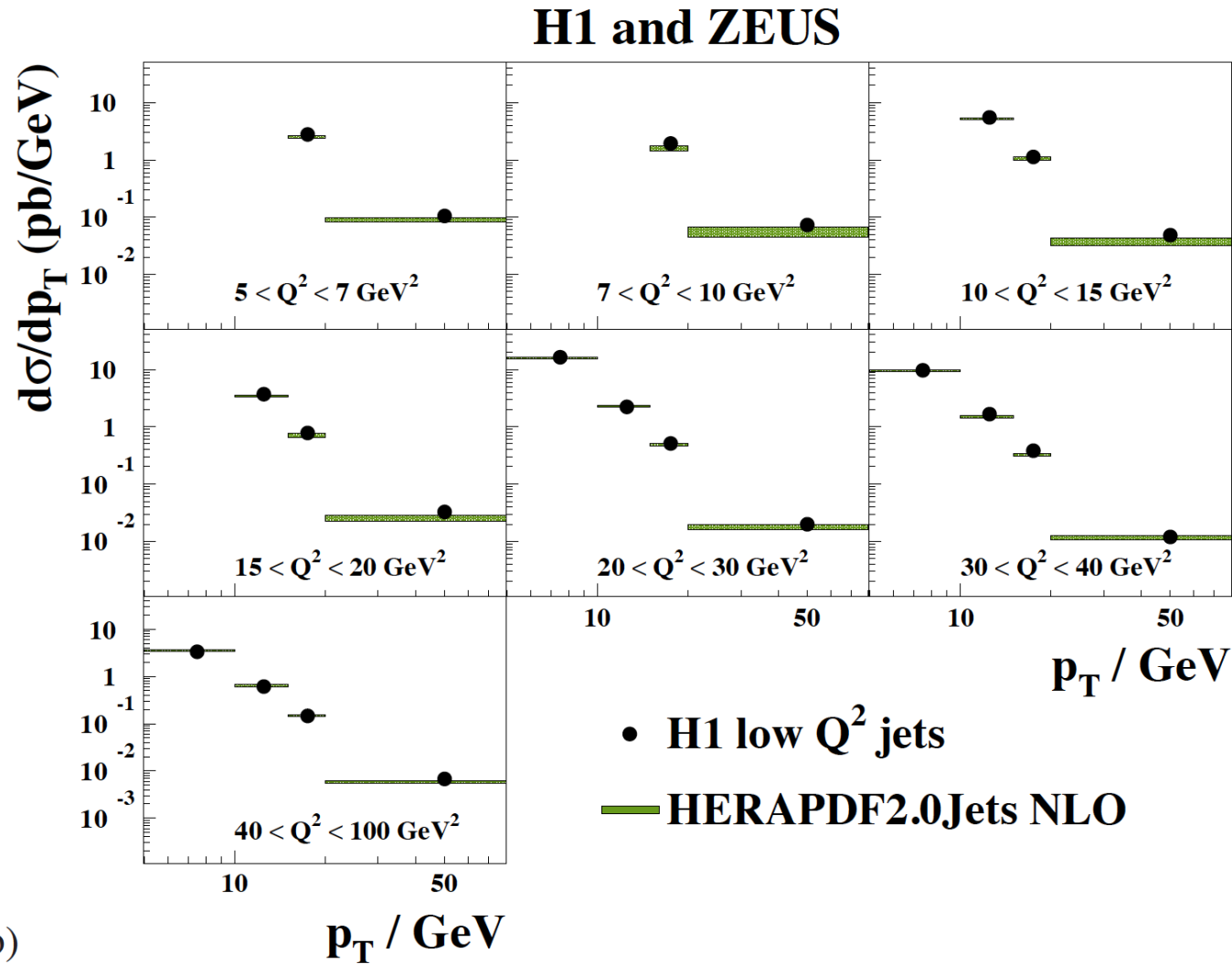
HERAPDF2.0Jets vs. H1 HERA-I Jet Cross Sections

H1 and ZEUS



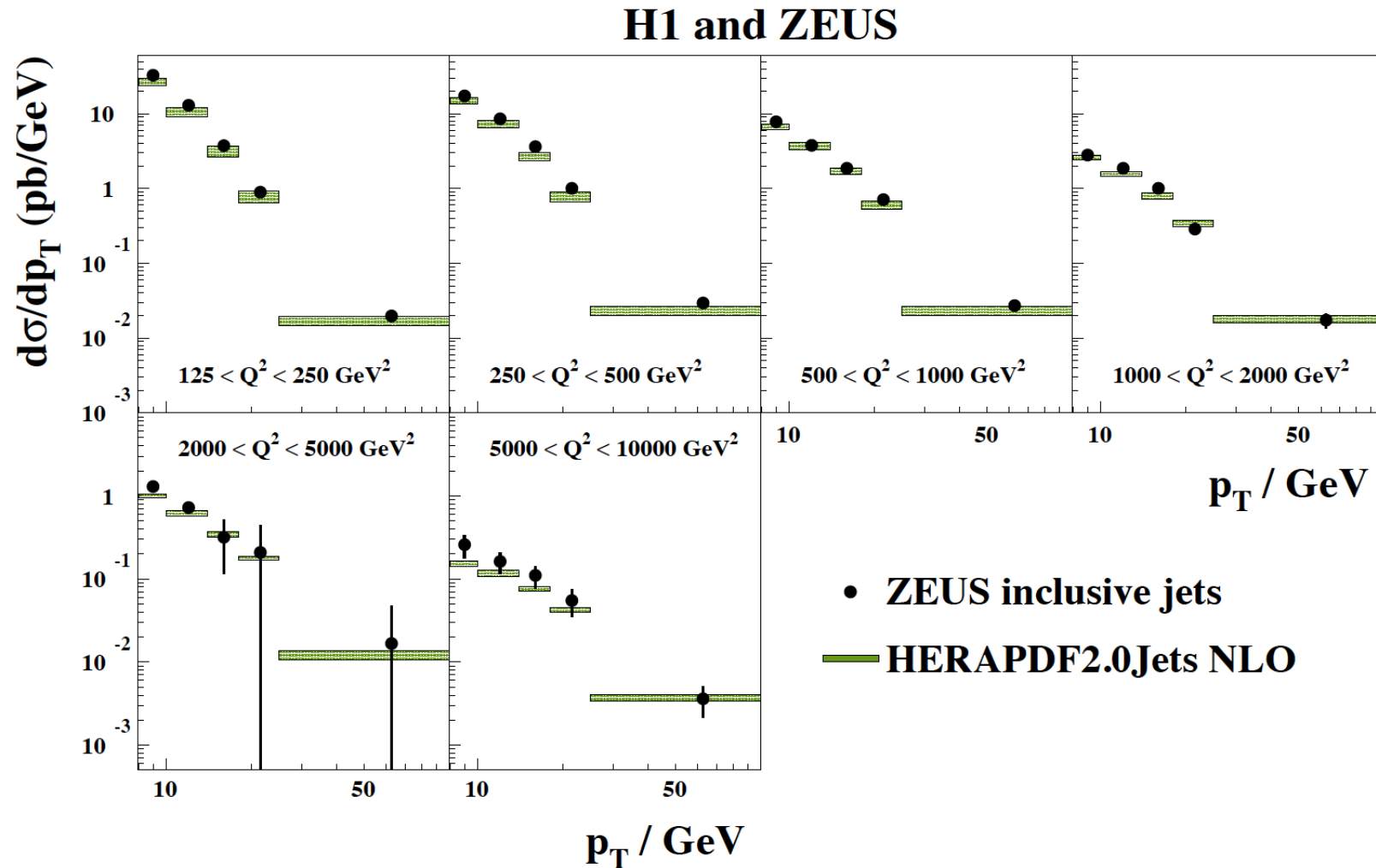
- Differential H1 HERA-I jet cross sections $d\sigma/dp_T$ normalized to NC inclusive cross sections compared to HERAPDF2.0Jets
- Good description of data

HERAPDF2.0Jets vs. H1 low Q^2 Jet Cross Sections



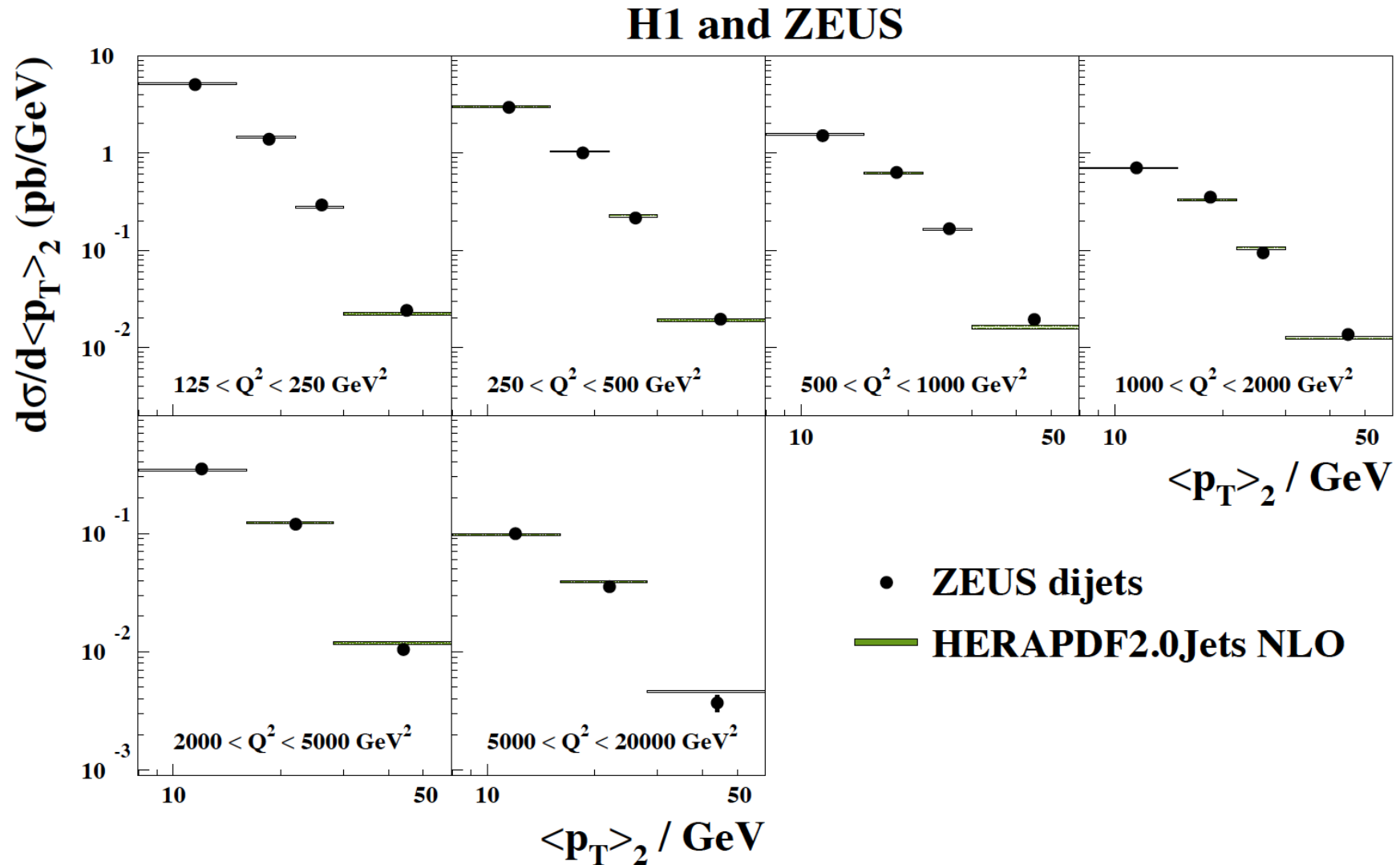
- Differential H1 low Q^2 jet cross sections $d\sigma/dp_T$ normalized to NC inclusive cross sections compared to HERAPDF2.0Jets
- Excellent description of data

HERAPDF2.0Jets vs. ZEUS Inclusive Jet Cross Sections



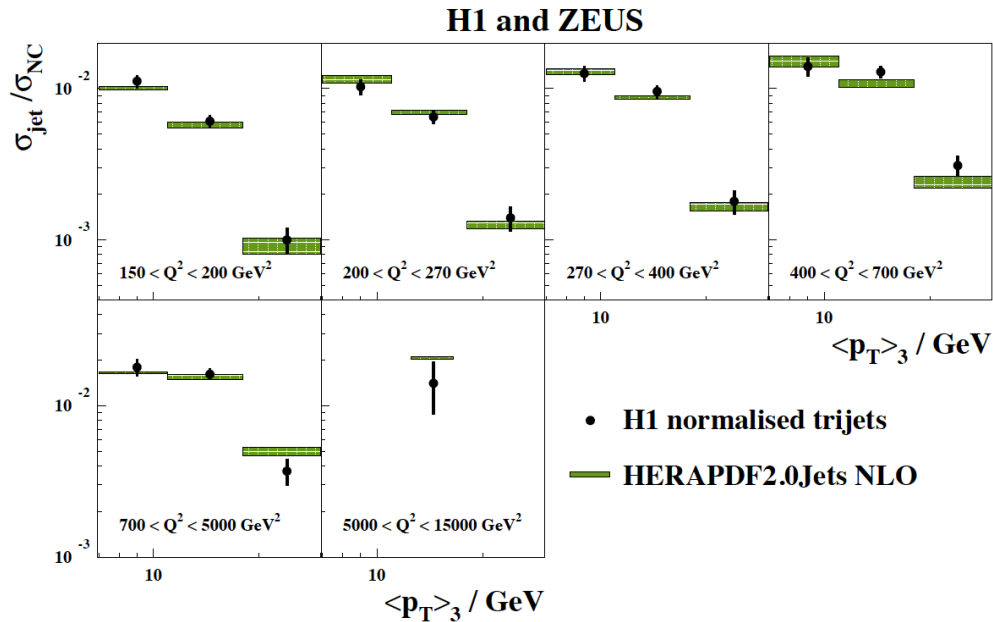
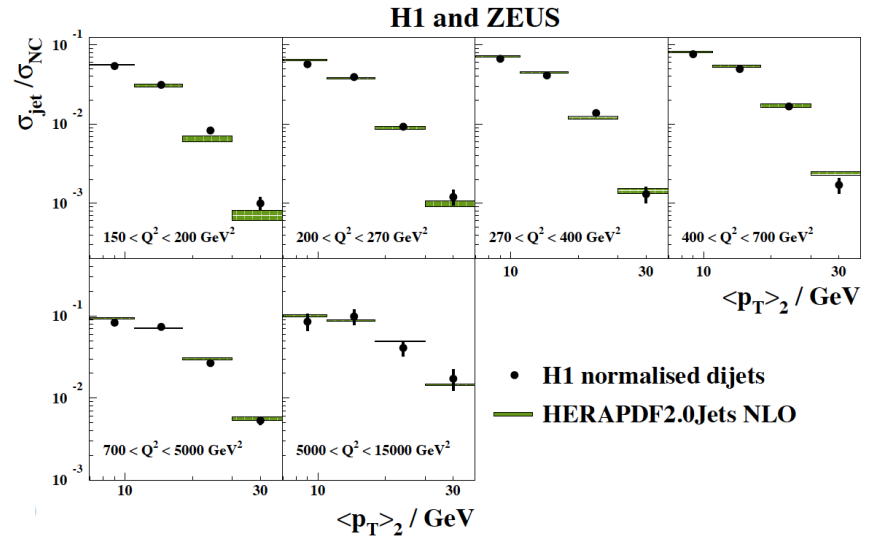
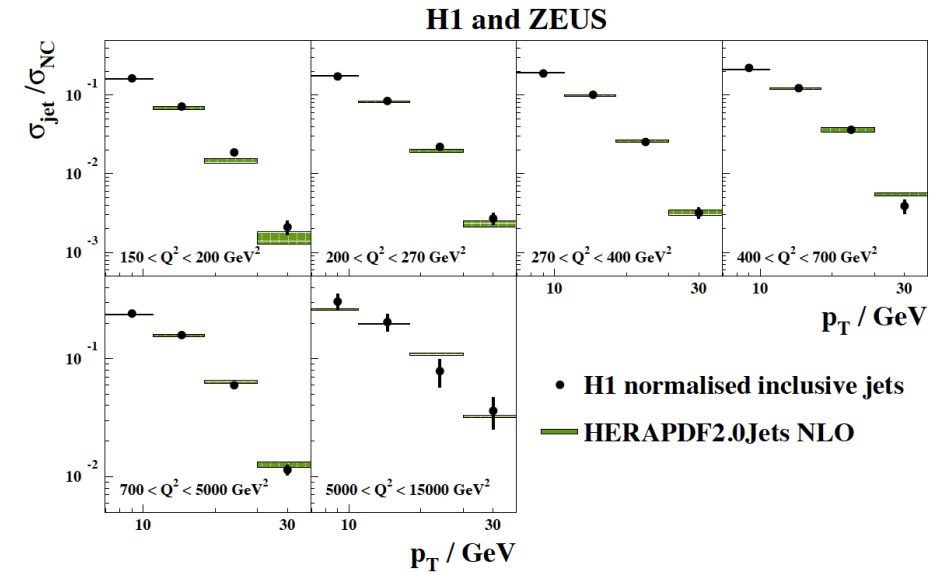
- Differential ZEUS jet cross sections $d\sigma/dp_T$ compared to HERAPDF2.0Jets
- Good description of data

HERAPDF2.0Jets vs. ZEUS Dijet Cross Sections



- Differential ZEUS dijet cross sections $d\sigma/d\langle p_T \rangle_2$ compared to HERAPDF2.0Jets
- Good description of data

HERAPDF2.0Jets vs. H1 Multijet Cross Sections



- Differential H1 inclusive, dijet and trijet cross sections normalised to inclusive NC cross sections
- Good description of data

Summary and Conclusions

- Measurements of HERA inclusive, charm and jets cross sections have been performed over six orders of magnitude
- Most precise set of data of ep scattering ever published
 - One of the legacies of HERA
- Extensive QCD analysis of data performed resulting in the HERAPDF2.0 set of PDFs along with derivations
- Adding charm and jets data: **HERAPDF2.0Jets**
 - Does not change PDFs significantly
 - But significant reduction of uncertainty on gluon PDF
 - Allows *simultaneous* extraction of PDFs and of the strong coupling constant

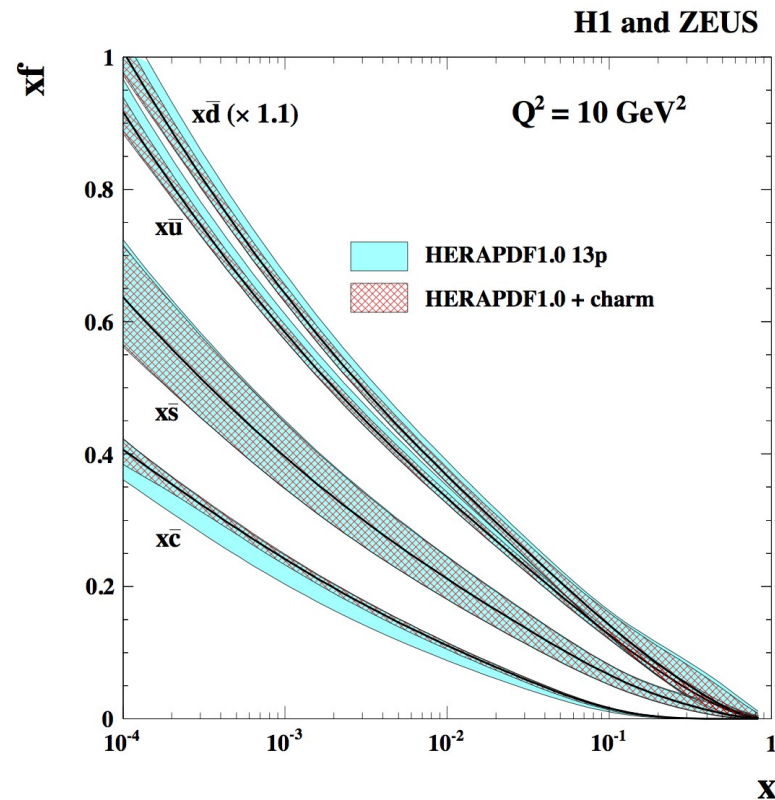
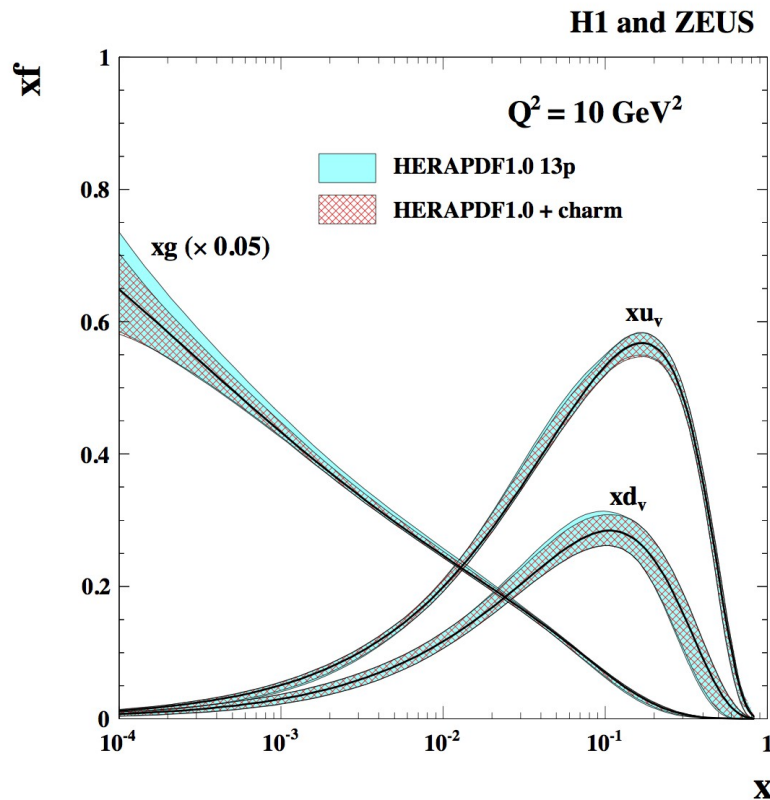
$$\alpha_s(M_Z^2) = 0.1183 \pm 0.0009(\text{exp}) \pm 0.0005(\text{model/parameterisation}) \\ \pm 0.0012(\text{hadronisation}) {}^{+0.0037}_{-0.0030}(\text{scale}) .$$

Excellent agreement with world average

$$\alpha_s(M_Z^2) = 0.1185$$

BACKUP

Reminder: Impact of Charm Data on PDF Fits



- 13 parameter fit using RT optimised VFNS based on HERAPDF1.0 fit
- Inclusion of charm data does not impact PDF fit significantly (neither central values nor uncertainties)
- Main effect:
Uncertainty on gluon pdf is marginally reduced due to inclusion of data from $\gamma g \rightarrow c\bar{c}$ process
- For HERAPDF2.0 no extra fit with only adding charm was released

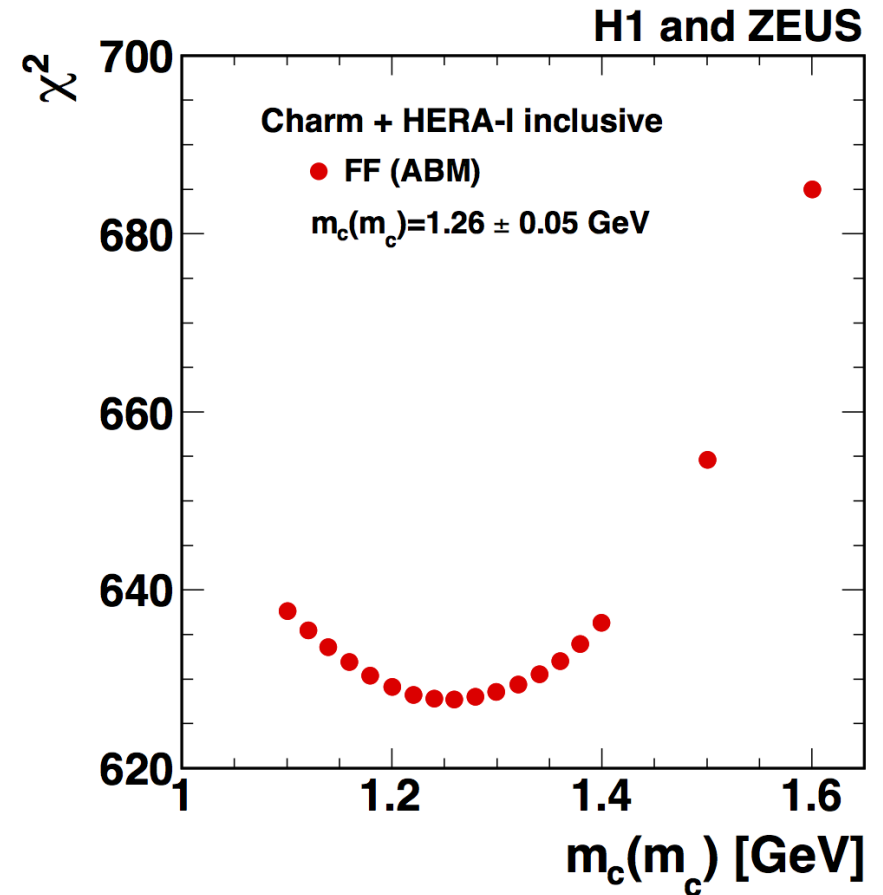
Measurement of the Charm Quark Mass

- Reminder: In fixed flavour schemes we interpret the charm quark like a free, final state particle with a mass
- Fit m_c in NLO QCD Analysis based on ABM FFNS scheme using HERA inclusive and charm data
- 3 active flavors, $\alpha_s^{\text{nf}=3}(M_Z)=0.105$
- Fit result using OPENQCDRAD:

$$m_c(m_c) = 1.26 \pm 0.05_{\text{exp}} \pm 0.03_{\text{mod}} \pm 0.02_{\text{param}} \pm 0.02_{\alpha_s} \text{ GeV}$$

- Compatible with PDG value

$$m_c(m_c) = 1.275 \pm 0.025 \text{ GeV}$$



Theory Calculations Compared to Charm Data

- Can compare data to theoretical predictions before QCD analysis

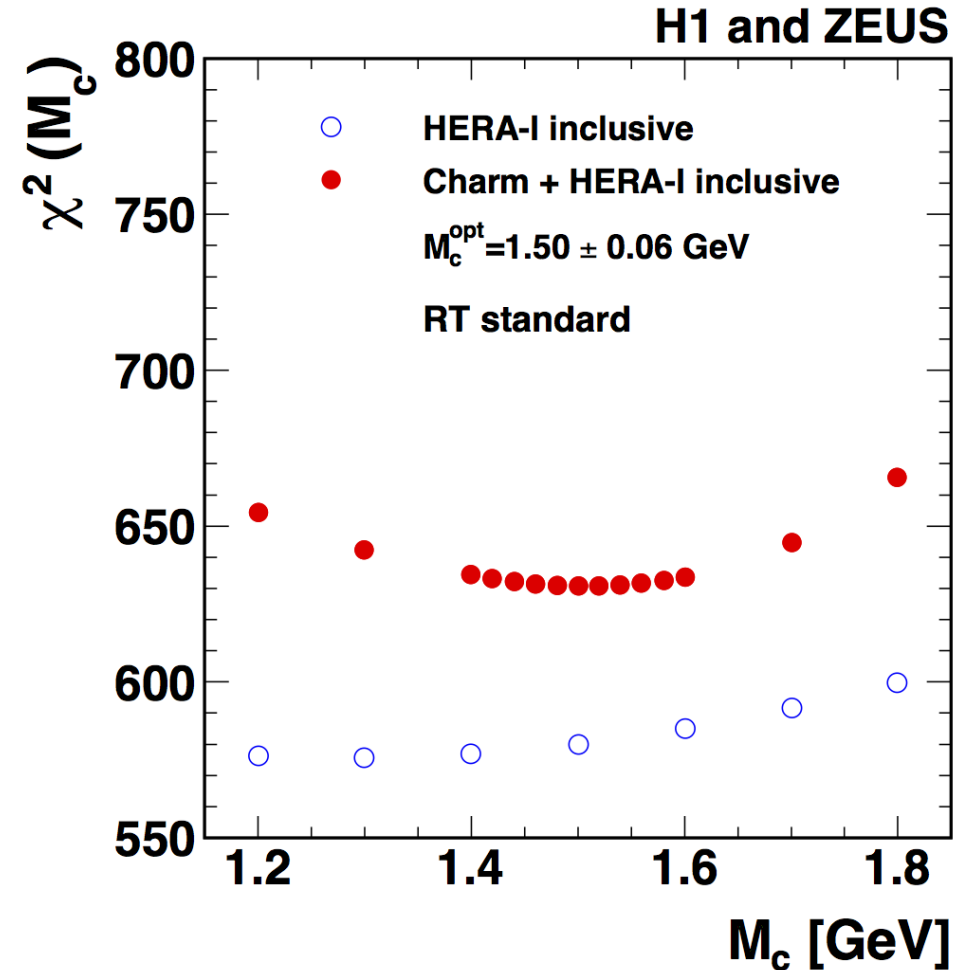
Theory	Scheme	Ref.	$F_{2(L)}$ def.	m_c [GeV]	Massive ($Q^2 \lesssim m_c^2$)	Massless ($Q^2 \gg m_c^2$)	$\alpha_s(m_Z)$ ($n_f = 5$)	Scale	Included charm data
MSTW08 NLO	RT standard	[28]	$F_{2(L)}^c$	1.4 (pole)	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	0.12108	Q	[1,4–6,8,9,11]
MSTW08 NNLO					approx.- $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^2)$	0.11707		
MSTW08 NLO (opt.)	RT optimised	[31]			$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	0.12108		
MSTW08 NNLO (opt.)					approx.- $\mathcal{O}(\alpha_s^3)$	$\mathcal{O}(\alpha_s^2)$	0.11707		
HERAPDF1.5 NLO	RT standard	[55]	$F_{2(L)}^c$	1.4 (pole)	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s)$	0.1176	Q	HERA inclusive DIS only
NNPDF2.1 FONLL A	FONLL A	[30]	n.a.	$\sqrt{2}$	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s)$	0.119	Q	[4–6,12,13,15,18]
NNPDF2.1 FONLL B	FONLL B		$F_{2(L)}^c$	$\sqrt{2}$ (pole)	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s)$			
NNPDF2.1 FONLL C	FONLL C		$F_{2(L)}^c$	$\sqrt{2}$ (pole)	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^2)$			
CT10 NLO	S-ACOT- χ	[22]	n.a.	1.3	$\mathcal{O}(\alpha_s)$	$\mathcal{O}(\alpha_s)$	0.118	$\sqrt{Q^2 + m_c^2}$	[4–6,8,9]
CT10 NNLO (prel.)		[56]	$F_{2(L)}^{c\bar{c}}$	1.3 (pole)	$\mathcal{O}(\alpha_s^2)$	$\mathcal{O}(\alpha_s^2)$			
ABKM09 NLO	FFNS	[57]	$F_{2(L)}^{c\bar{c}}$	1.18 ($\overline{\text{MS}}$)	$\mathcal{O}(\alpha_s^2)$	-	0.1135	$\sqrt{Q^2 + 4m_c^2}$	for mass optimisation only
ABKM09 NNLO					approx.- $\mathcal{O}(\alpha_s^3)$	-			

Fitting the charm mass scale M_c in VFNS

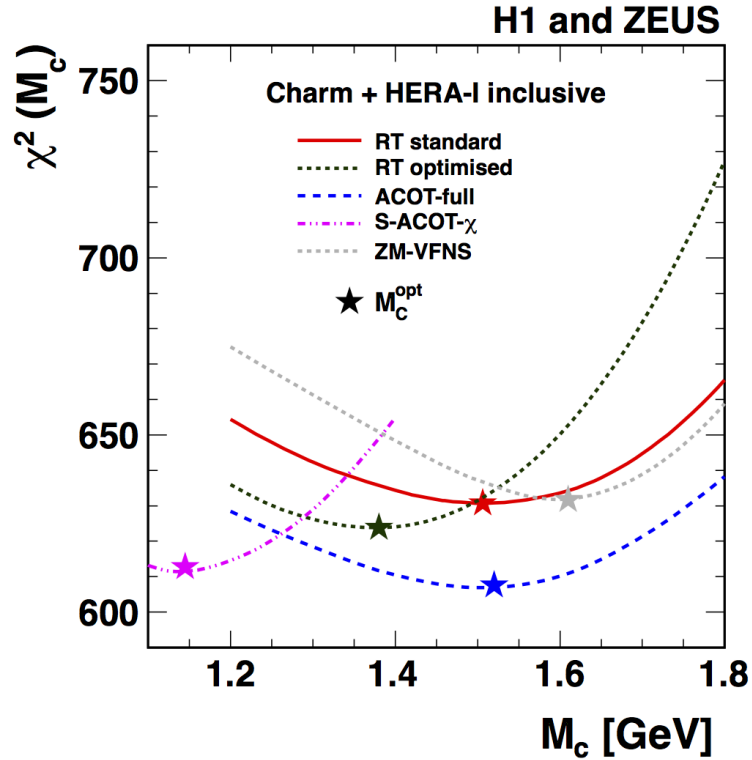
- Optimal M_c^{opt} found by repeating PDF fits scanning the range $1.2 \text{ GeV} < M_c < 1.8 \text{ GeV}$ and finding the minimum

$$\chi^2(M_c) = \chi_{\text{min}}^2 + \left(\frac{M_c - M_c^{\text{opt}}}{\sigma(M_c^{\text{opt}})} \right)^2$$

- Charm + HERA-I data allow to constrain M_c much better than HERA-I data alone

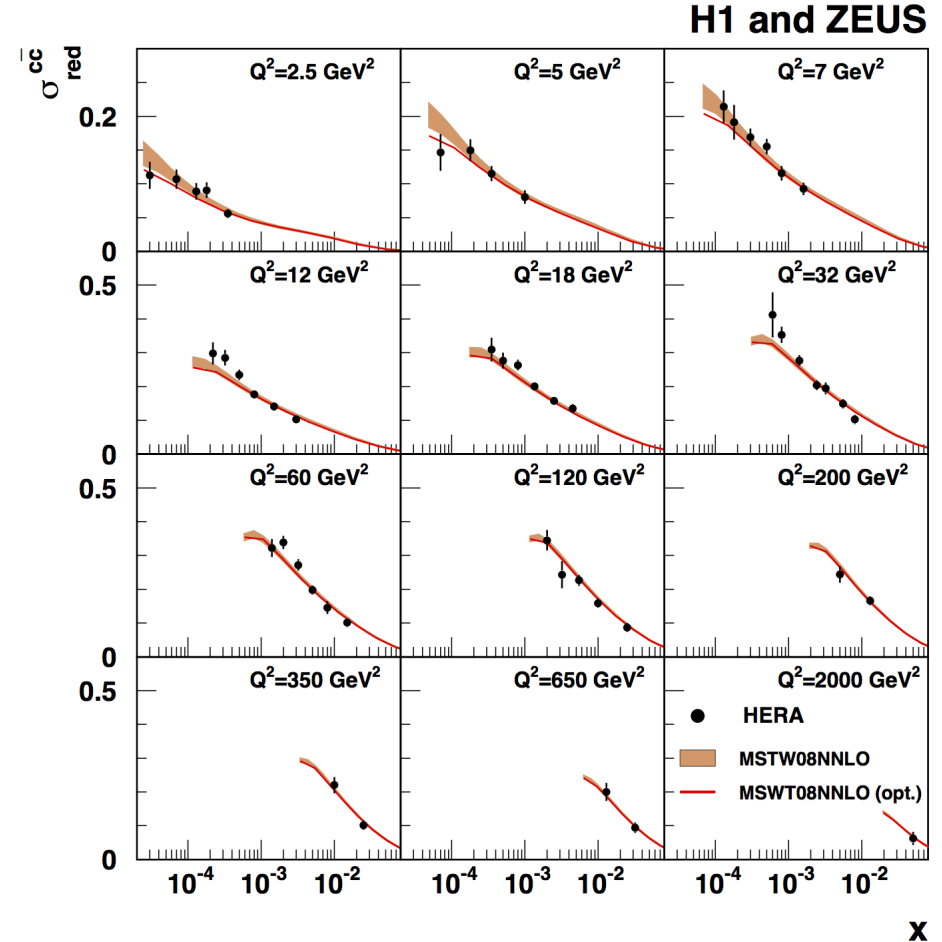
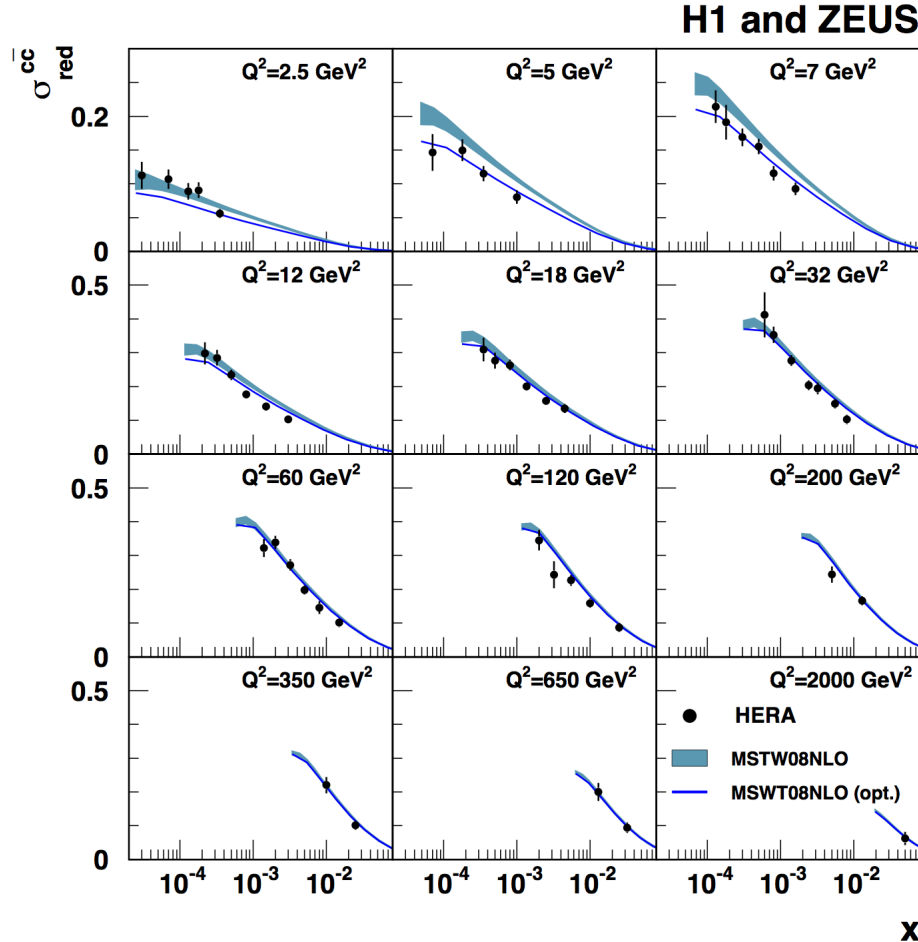


Extraction of M_c^{opt} in various flavour schemes



scheme	M_c^{opt} [GeV]	χ^2/n_{dof} $\sigma_{\text{red}}^{NC,CC} + \sigma_{\text{red}}^{c\bar{c}}$	χ^2/n_{dp} $\sigma_{\text{red}}^{c\bar{c}}$
RT standard	$1.50 \pm 0.06_{\text{exp}} \pm 0.06_{\text{mod}} \pm 0.01_{\text{param}} \pm 0.003_{\alpha_s}$	630.7/626	49.0/47
RT optimised	$1.38 \pm 0.05_{\text{exp}} \pm 0.03_{\text{mod}} \pm 0.01_{\text{param}} \pm 0.01_{\alpha_s}$	623.8/626	45.8/47
ACOT-full	$1.52 \pm 0.05_{\text{exp}} \pm 0.12_{\text{mod}} \pm 0.01_{\text{param}} \pm 0.06_{\alpha_s}$	607.3/626	53.3/47
S-ACOT- χ	$1.15 \pm 0.04_{\text{exp}} \pm 0.01_{\text{mod}} \pm 0.01_{\text{param}} \pm 0.02_{\alpha_s}$	613.3/626	50.3/47
ZM-VFNS	$1.60 \pm 0.05_{\text{exp}} \pm 0.03_{\text{mod}} \pm 0.05_{\text{param}} \pm 0.01_{\alpha_s}$	631.7/626	55.3/47

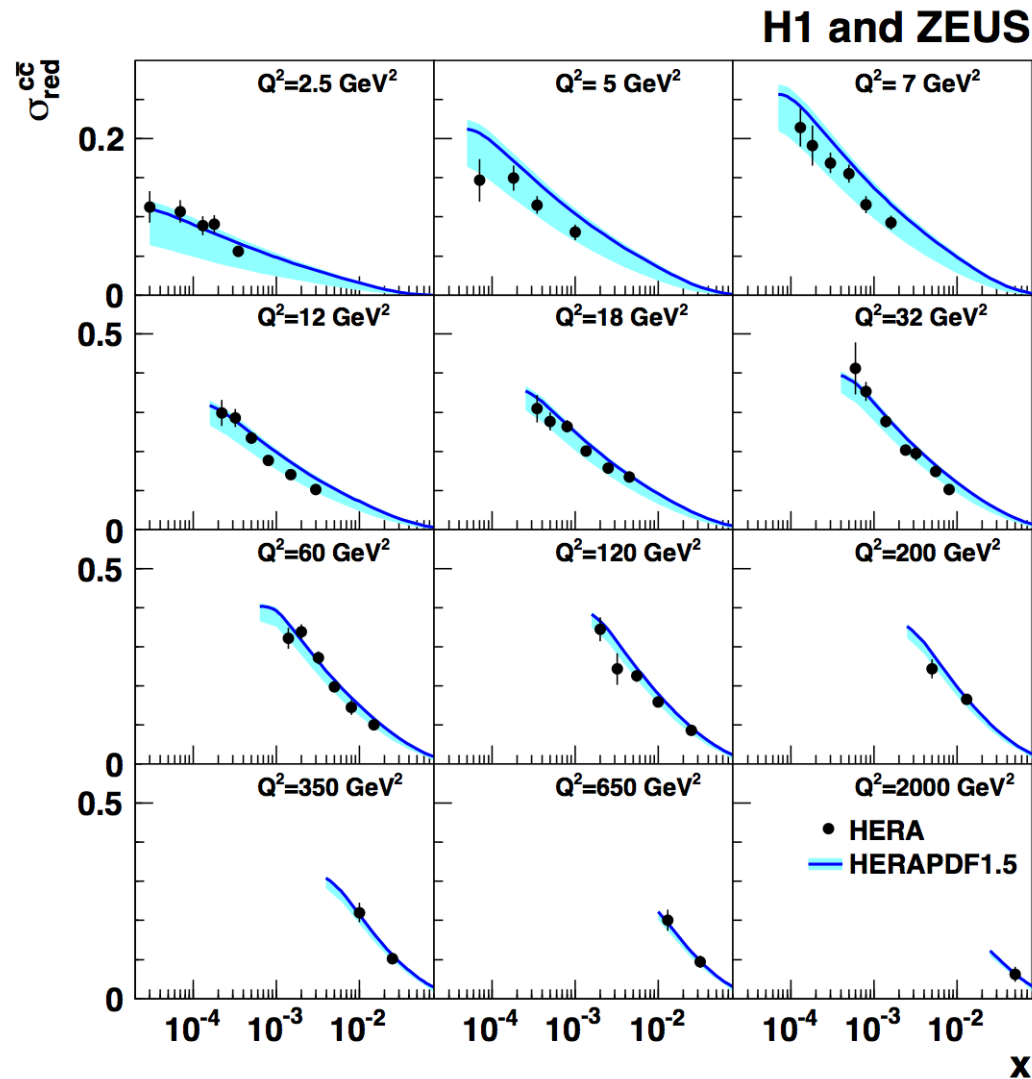
Comparison of $\sigma_{\text{red}}^{\text{cc}}$ to MSTW at NLO and NNLO



- MSTW at NLO with RT standard and optimised interpolation procedure
- RT optimised works better at low Q^2

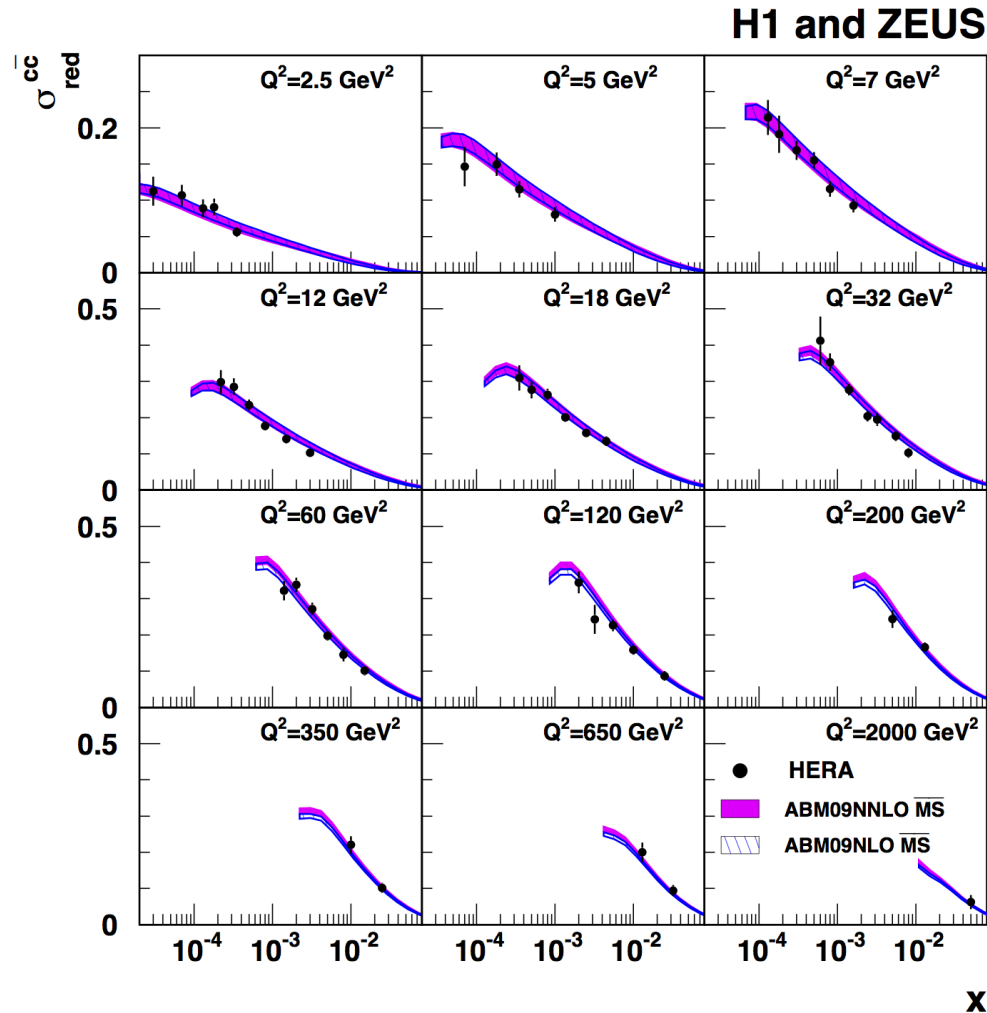
- MSTW at NNLO describes data better than NLO

Comparison of $\sigma_{\text{red}}^{\text{cc}}$ to HERAPDF1.5



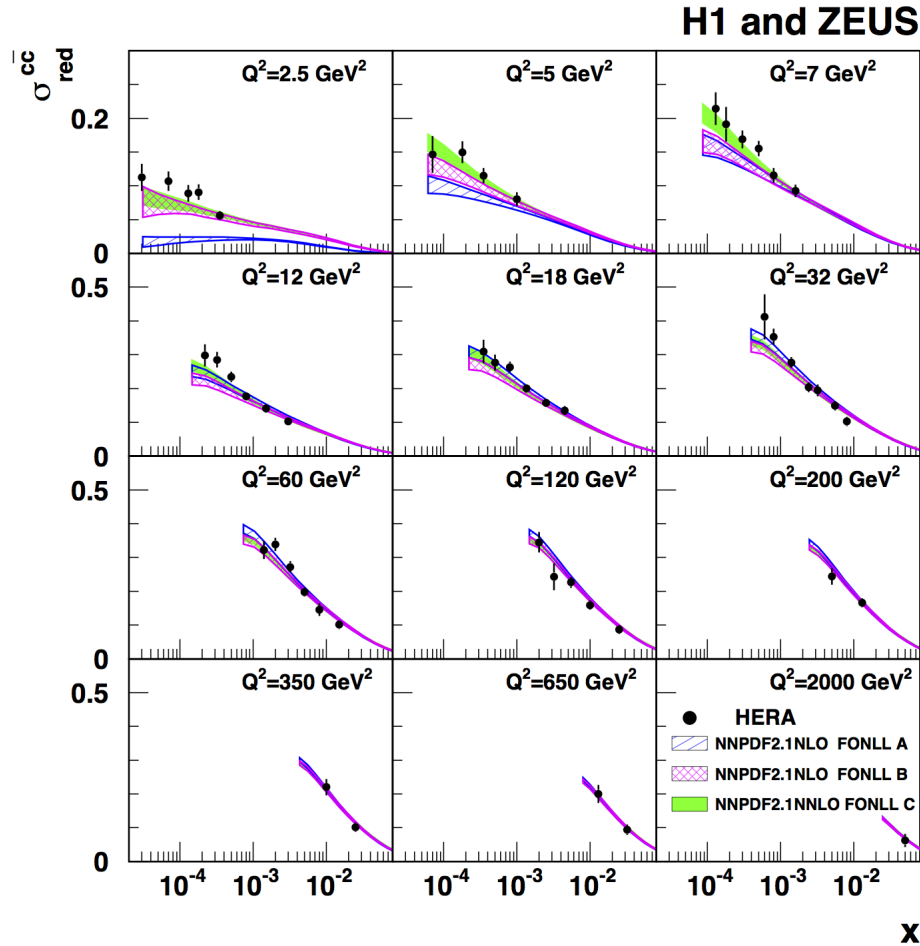
- Using $M_c = 1.4 \text{ GeV}$
($1.35 \text{ GeV} < M_c < 1.65 \text{ GeV}$)
- Data described well within full HERAPDF1.5 uncertainties

Comparison of $\sigma_{\text{red}}^{\text{cc}}$ to ABM (FFNS)

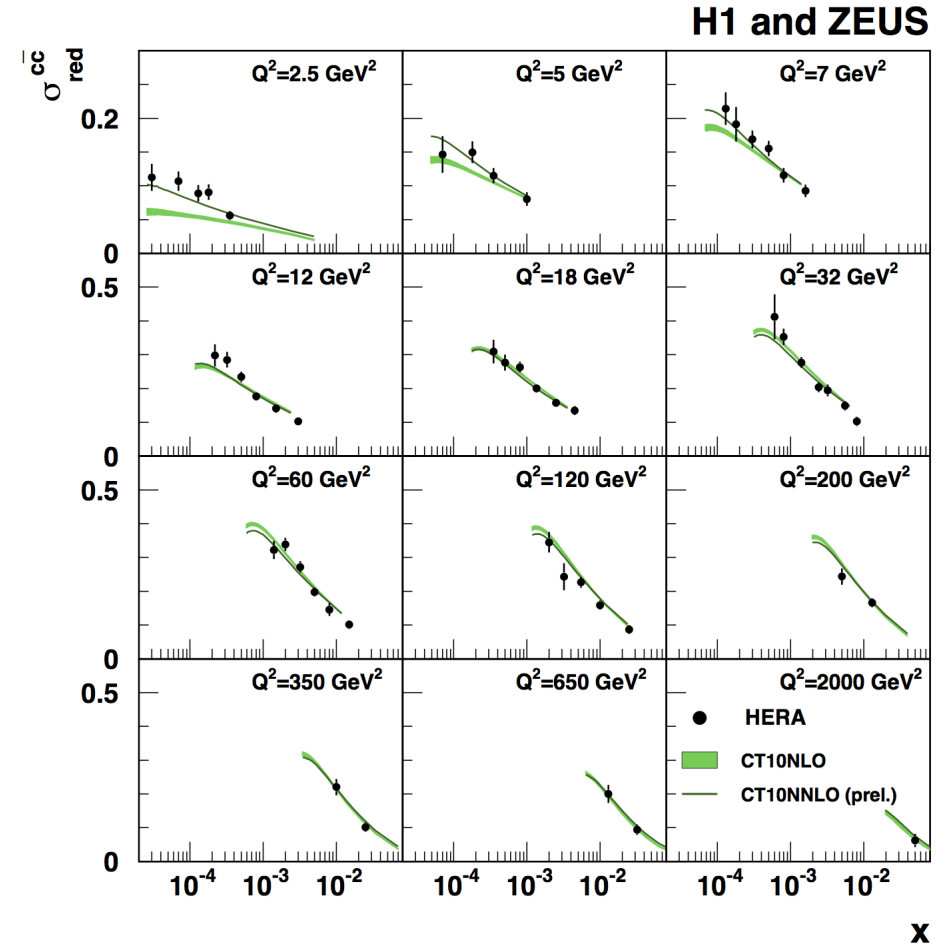


- Data described well over whole Q^2 range at NLO and NNLO

Comparison of $\sigma_{\text{red}}^{\text{cc}}$ to NNPDF and CT (GM-VFNS)

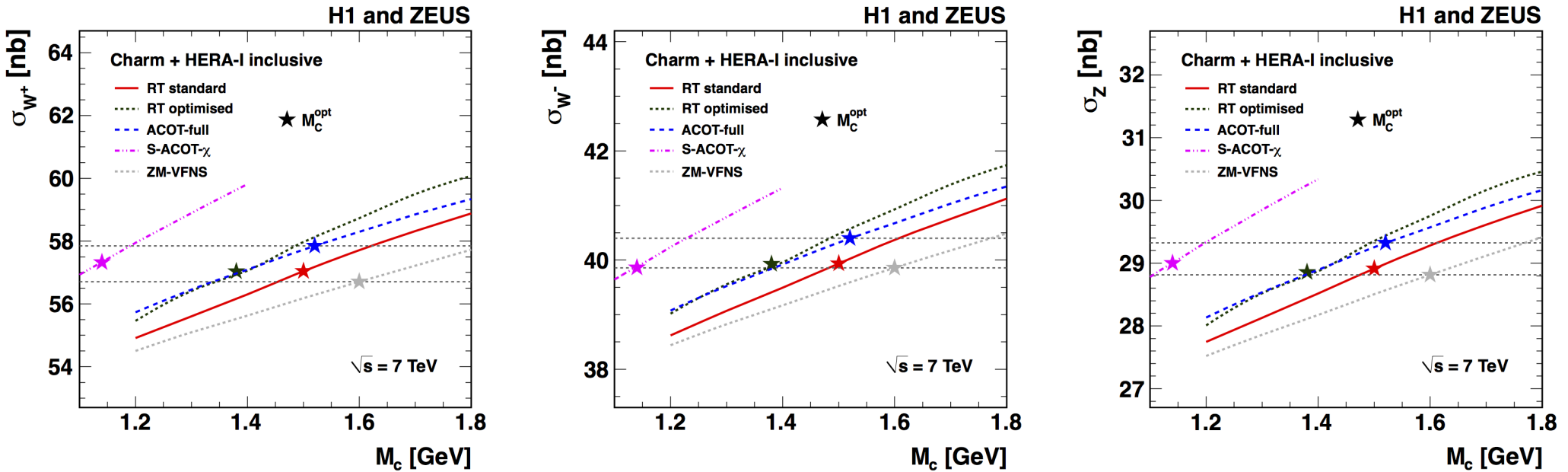


- NNPDF FONLL-A and FONLL-B fail to describe data at $Q^2 < 100 \text{ GeV}^2$
- Improved in FONLL-C



- CT10 (NLO) with S-ACOT-X scheme fails to describe data at $Q^2 < 5 \text{ GeV}^2$
- Improved at NNLO

Impact of HERA charm data on LHC predictions



- Predictions for W^\pm and Z production at LHC calculated using MCFM interfaced to APPLGRID for various VFNS schemes
- Monotonic rise in all schemes
 - Suppression of charm leads to more light quarks and gluons
 - More light sea quarks and gluon splitting lead to larger cross section
- Significant spread of 6% over whole M_c fit range
- Spread reduced to $<2\%$ at M_c^{opt} values
 - Can reduce uncertainty on LHC predictions when using HERA charm data at optimal M_c determined by HERA

Inputs to charm combination

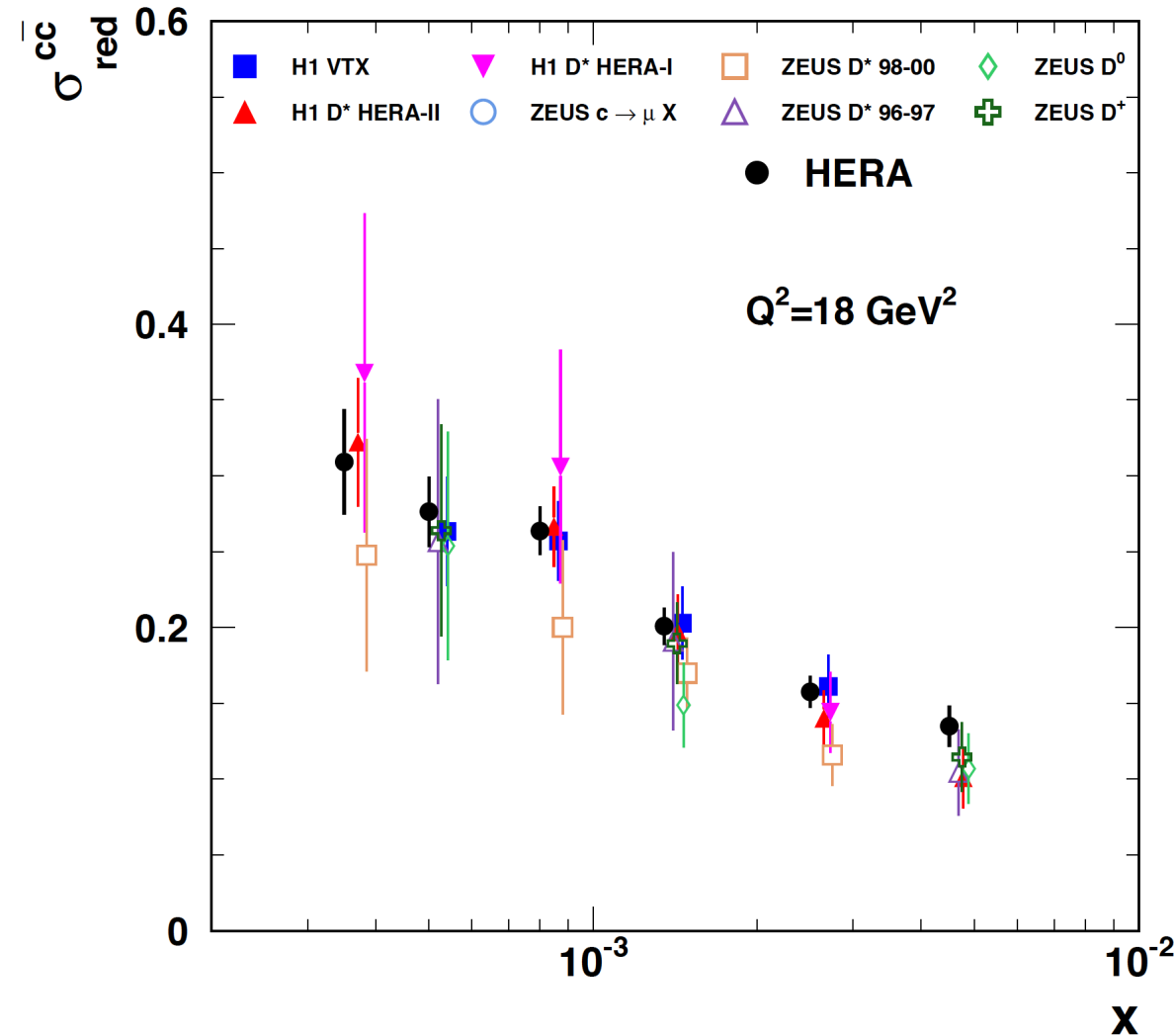
- This talk does not discuss the original charm measurements, but their combination and QCD analysis

Data set	Tagging method	Q^2 range [GeV ²]	N	\mathcal{L} [pb ⁻¹]
1 H1 VTX [14]	Inclusive track lifetime	5 – 2000	29	245
2 H1 D^* HERA-I [10]	D^{*+}	2 – 100	17	47
3 H1 D^* HERA-II [18]	D^{*+}	5 – 100	25	348
4 H1 D^* HERA-II [15]	D^{*+}	100 – 1000	6	351
5 ZEUS D^* (96-97) [4]	D^{*+}	1 – 200	21	37
6 ZEUS D^* (98-00) [6]	D^{*+}	1.5 – 1000	31	82
7 ZEUS D^0 [12]	$D^{0,\text{no}D^{*+}}$	5 – 1000	9	134
8 ZEUS D^+ [12]	D^+	5 – 1000	9	134
9 ZEUS μ [13]	μ	20 – 10000	8	126

Data sets used in combination

Details of $\sigma_{\text{red}}^{\text{cc}}$ at $Q^2 = 18 \text{ GeV}^2$

H1 and ZEUS



- Total uncertainty of combination much smaller than those of input data
- At $Q^2 = 18 \text{ GeV}^2$ improvement of factor ~ 2 with 6% - 10% resulting uncertainty
- Inputs shifted in x for presentation

Charm Quark Mass Definitions

- Pole mass
- Treat charm quark as if it was a free particle (not confined)

$$m_c(Q) = m_{c,\text{pole}} \left[1 - \frac{\alpha_s}{\pi} - \frac{3\alpha_s}{4\pi} \ln \left(\frac{Q^2}{m_c(m_c)^2} \right) \right]$$

- Running quark mass
 - Used in the $\overline{\text{MS}}$ renormalization scheme
 - $m(\mu_R)$ implements a scale dependent, running mass

Combination Method for Charm Data

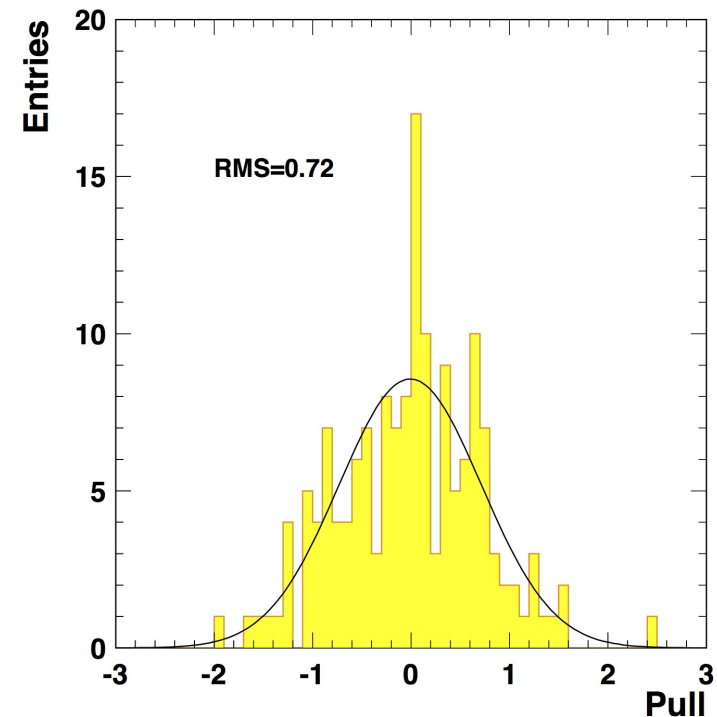
- Based on Chi2 combination procedure

$$\chi_{\text{exp},e}^2(\mathbf{m}, \mathbf{b}) = \sum_i \frac{\left(m^i - \sum_j \gamma_j^{i,e} m^i b_j - \mu^{i,e}\right)^2}{(\delta_{i,e,\text{stat}} \mu^{i,e})^2 + (\delta_{i,e,\text{uncor}} m^i)^2} + \sum_j b_j^2$$

$$\chi_{\text{tot}}^2 = \sum_e \chi_{\text{exp},e}^2$$

- Allowed to shift in fit
(max. shift < 1.2 sigma)
- Pulls show gaussian behavior,
fit works

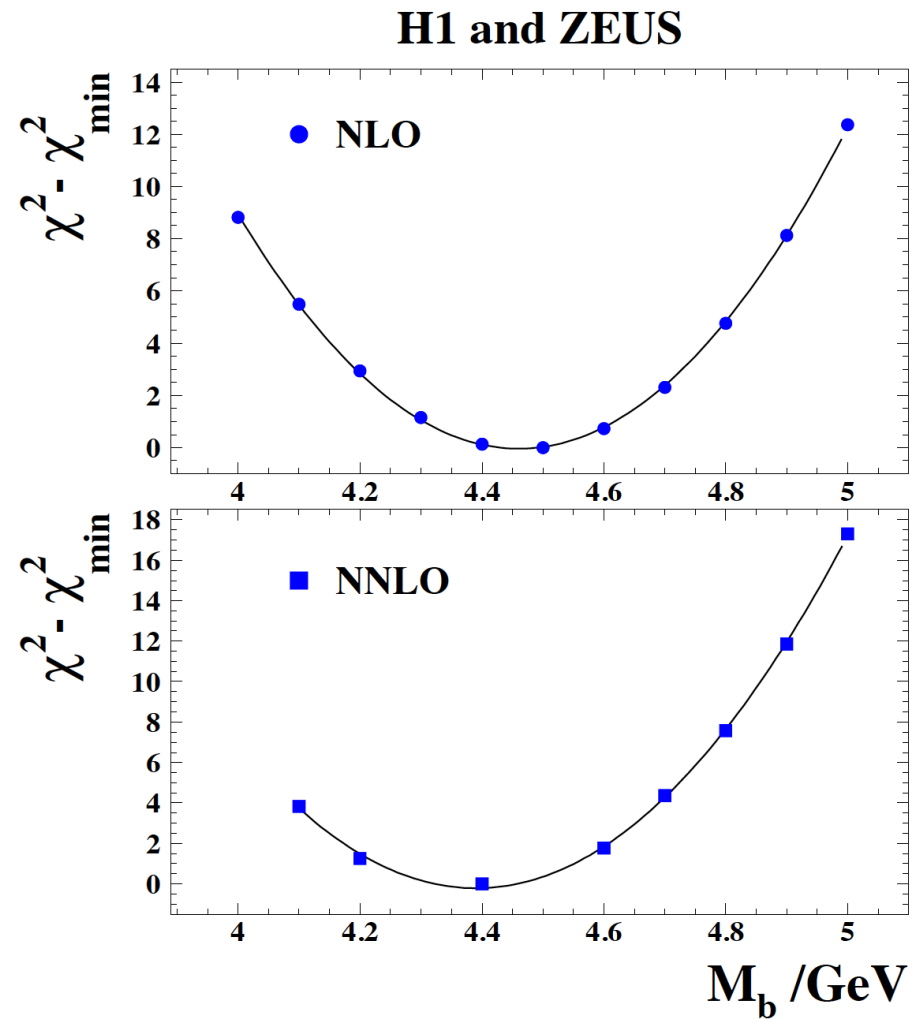
HERAverager



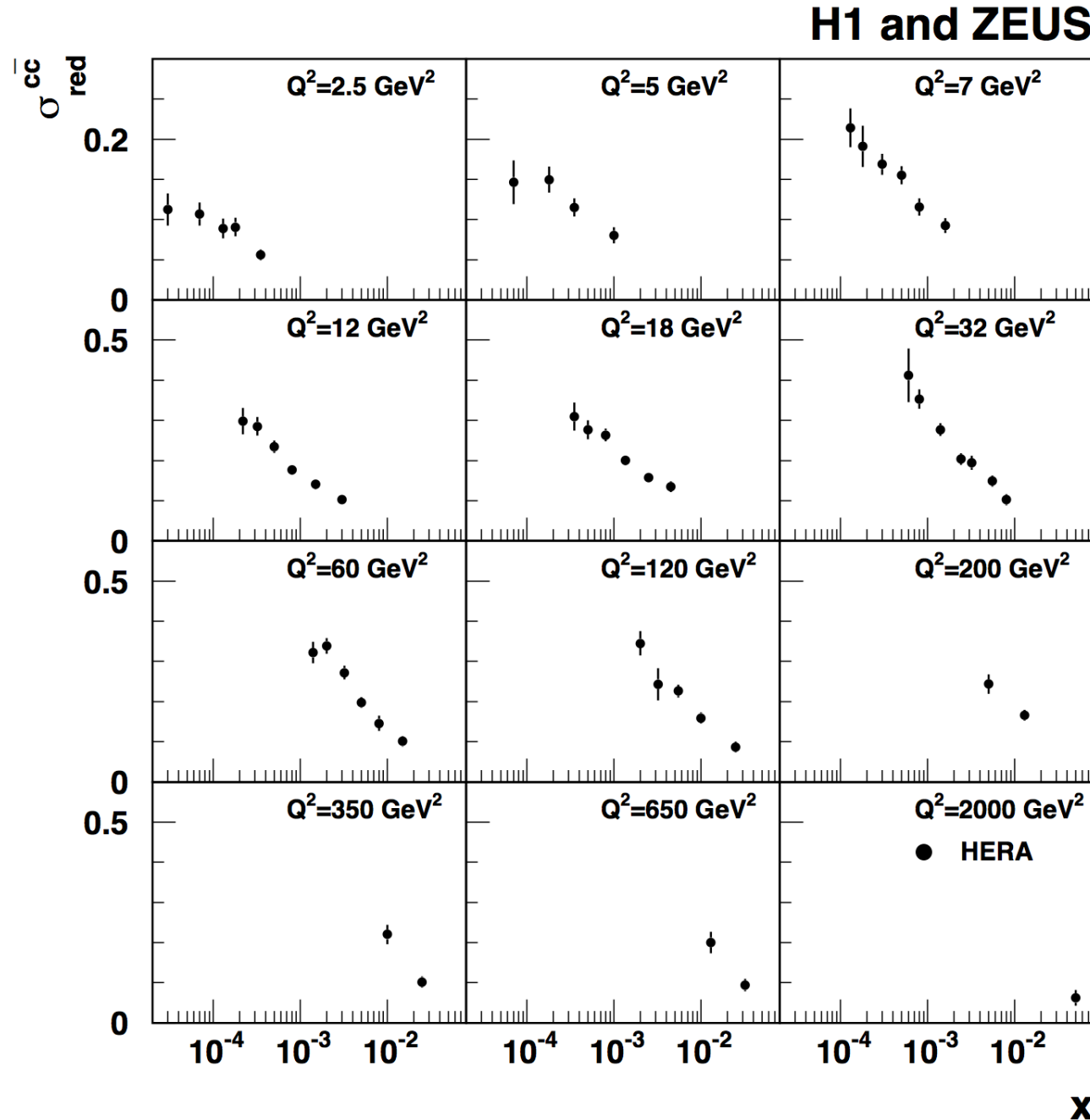
Differences HERAPDF1.5 to HERAPDF2.0

	HERAPDF2.0		HERAPDF1.5	
	NNLO	NLO	NNLO	NLO
Data as in Table 1 Uncertainties: Experimental Procedural	combination Hessian 7		preliminary combination Hessian 3	
Parameterisation	as in Equations 27 to 31		as in Equations 27 to 31	
Number of Parameters	14	14	14**	10 *
– Variations	15 [D_{u_v}]	15 [D_{u_v}]	none	11 [D_{u_v}], 12 [$D_{\bar{U}}$]
$\mu_{f_0}^2$ [GeV ²]	1.9	1.9	1.9	1.9
– Variations	1.6, 2.2	1.6, 2.2	1.5, 2.5	1.5* ^f , 2.5*
M_c [GeV]	1.43	1.47	1.4	1.4*
– Variations	1.37* ^c , 1.49	1.41, 1.53	1.35* ^c , 1.65	1.35* ^c , 1.65*
M_b [GeV]	4.5	4.5	4.75	4.75*
– Variations	4.25, 4.75	4.25, 4.75	4.30, 5.00	4.30, 5.00*
f_s [GeV]	0.40	0.40	0.31	0.31*
– Variations	0.30, 0.50	0.30, 0.50	0.23, 0.38	0.23, 0.38*
Q_{\min}^2 [GeV ²] of Data	3.5	3.5	3.5	3.5*
– Variations	2.5, 5.0	2.5, 5.0	2.5, 5.0	2.5, 5.0*
Fixed α_s	0.118	0.118	0.1176	0.1176*

Beauty Mass Parameter Fit in HERAPDF 2.0



HERA Combined Reduced Charm Cross Sections



Combined Charm Cross Section Results

Averaged reduced charm cross sections at HERA

$Q^2[\text{GeV}^2]$	x	y	$\sigma_{\text{red}}^{c\bar{c}}$	$\delta_{\text{unc}}[\%]$	$\delta_{\text{cor}}[\%]$	$\delta_{\text{proced}}[\%]$	$\delta_{\text{tot}}[\%]$
2.5	0.00003	0.824	0.1126	14.0	10.9	0.3	17.8
2.5	0.00007	0.353	0.1068	9.0	9.9	0.2	13.4
2.5	0.00013	0.190	0.0889	10.0	9.1	2.2	13.7
2.5	0.00018	0.137	0.0907	9.5	8.3	1.4	12.7
2.5	0.00035	0.071	0.0560	8.7	8.2	0.0	11.9
5.	0.00007	0.706	0.1466	15.6	10.0	0.2	18.5
5	0.00018	0.274	0.1495	8.4	6.8	1.1	10.8
5	0.00035	0.141	0.1151	7.1	6.7	0.6	9.8
5	0.00100	0.049	0.0803	9.2	8.2	0.6	12.4
7	0.00013	0.532	0.2142	8.1	8.0	0.2	11.4
7	0.00018	0.384	0.1909	10.2	8.5	2.1	13.4
7	0.00030	0.231	0.1689	4.6	6.3	0.4	7.8
7	0.00050	0.138	0.1553	4.3	5.9	0.6	7.3
7	0.00080	0.086	0.1156	7.2	6.0	0.7	9.4
7	0.00160	0.043	0.0925	6.4	7.6	0.6	9.9
12	0.00022	0.539	0.2983	8.4	7.2	0.1	11.1
12	0.00032	0.371	0.2852	4.7	6.5	0.6	8.1
12	0.00050	0.237	0.2342	4.3	5.1	0.5	6.6
12	0.00080	0.148	0.1771	3.8	5.7	0.1	6.9
12	0.00150	0.079	0.1413	5.5	6.8	0.1	8.7
12	0.00300	0.040	0.1028	6.1	8.0	0.2	10.1
18	0.00035	0.508	0.3093	9.2	6.5	1.0	11.3
18	0.00050	0.356	0.2766	4.7	7.0	0.5	8.4
18	0.00080	0.222	0.2637	3.8	4.6	0.6	6.1
18	0.00135	0.132	0.2009	3.3	5.2	0.0	6.2
18	0.00250	0.071	0.1576	3.5	5.7	0.1	6.7
18	0.00450	0.040	0.1349	5.8	8.0	1.4	10.0

$Q^2[\text{GeV}^2]$	x	y	$\sigma_{\text{red}}^{c\bar{c}}$	$\delta_{\text{unc}}[\%]$	$\delta_{\text{cor}}[\%]$	$\delta_{\text{proced}}[\%]$	$\delta_{\text{tot}}[\%]$
32	0.00060	0.527	0.4119	15.1	5.7	0.1	16.2
32	0.00080	0.395	0.3527	4.3	5.3	0.3	6.9
32	0.00140	0.226	0.2767	3.9	4.2	0.4	5.8
32	0.00240	0.132	0.2035	4.8	4.9	0.3	6.9
32	0.00320	0.099	0.1942	7.1	5.6	0.3	9.0
32	0.00550	0.058	0.1487	6.9	6.0	0.4	9.1
32	0.00800	0.040	0.1027	10.7	8.3	0.4	13.5
60	0.00140	0.424	0.3218	6.1	5.4	1.4	8.3
60	0.00200	0.296	0.3387	4.3	3.7	0.4	5.7
60	0.00320	0.185	0.2721	4.7	3.9	0.4	6.1
60	0.00500	0.119	0.1975	4.7	4.9	0.3	6.8
60	0.00800	0.074	0.1456	12.0	5.2	0.6	13.1
60	0.01500	0.040	0.1008	10.6	6.4	0.8	12.4
120	0.00200	0.593	0.3450	7.1	5.2	0.6	8.8
120	0.00320	0.371	0.2432	15.9	4.0	2.1	16.5
120	0.00550	0.216	0.2260	5.2	4.5	0.6	6.9
120	0.01000	0.119	0.1590	6.6	5.4	0.8	8.6
120	0.02500	0.047	0.0866	13.7	6.8	1.2	15.3
200	0.00500	0.395	0.2439	8.1	5.7	0.7	9.9
200	0.01300	0.152	0.1659	6.7	4.8	0.4	8.3
350	0.01000	0.346	0.2250	8.8	5.0	4.1	10.9
350	0.02500	0.138	0.1016	11.2	5.8	5.1	13.6
650	0.01300	0.494	0.2004	11.1	7.2	1.1	13.3
650	0.03200	0.201	0.0939	12.4	10.6	0.9	16.4
2000	0.05000	0.395	0.0622	27.7	14.4	1.7	31.2

Z and W cross section predictions for LHC

scheme	σ_Z [nb]	σ_{W^+} [nb]	σ_{W^-} [nb]
RT standard	28.91 ± 0.30	57.04 ± 0.55	39.94 ± 0.35
RT optimised	28.85 ± 0.24	57.03 ± 0.45	39.93 ± 0.27
ACOT-full	29.32 ± 0.42	57.84 ± 0.74	40.39 ± 0.47
S-ACOT- χ	29.00 ± 0.22	57.32 ± 0.42	39.86 ± 0.24
ZM-VFNS	28.81 ± 0.24	56.71 ± 0.40	39.86 ± 0.25

NLO VFNS predictions for Z/W+- cross sections at the LHC for 7 TeV using MCFM

