NNLL Resummation of Event Shapes in e^+e^- Annihilation

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Resummation in e^+e^-

When calculating high-energy processes in QCD, certain regions are not calculable using the standard perturbative series in α_s.

Looking at the dijet region implicitly puts constraints on the allowed real radiation, which introduces large logarithms.

These ruin the perturbation expansion and must be resummed to all orders in the strong coupling to regain calculability.

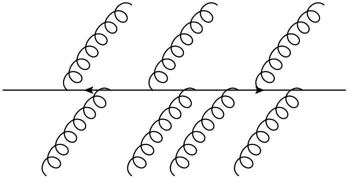
Start in the simplest and cleanest environment: e^+e^- annihilation.

Event Shapes

Parametrise the geometry of events by observables, so-called 'event-shapes'.
(Let's call the value of the observable in a given event V.)

The dijet region, $V = 0$

Close to the dijet region, soft and collinear gluon emission dominates, generating large logarithms.



The quasi-dijet region: a pair of hard quarks emit soft/collinear gluons, $V \approx 0$

In the soft-collinear region of phase space for real emissions,

$$\sigma \propto \alpha_s \int\limits_v \frac{dk_t}{k_t} \int\limits_v \frac{d\theta}{\theta} = \alpha_s \ln^2\left(\frac{1}{v}\right)$$

When the value of the observable becomes small, the logarithms become large.

Each power of α_s is accompanied by up to two of these kinematic logarithms.

When this is the case, $\alpha_s \ln(\frac{1}{v}) \approx 1$, and ‘leading’ and ‘higher-order’ corrections become comparable.

If leading logarithms exponentiate
$1 + \alpha_s + \alpha_s^2 + \alpha_s^3 + \cdots \to$

$$1+(\alpha_s L+\alpha_s L^2)+(\alpha_s^2 L+\alpha_s^2 L^2+\alpha_s^2 L^3+\cdots)+(\alpha_s^3 L+\alpha_s^3 L^2+\alpha_s^3 L^3 \cdots)+\cdots$$

$$= e^{Lg_1(\alpha_s L)}(1 + G_2(\alpha_s L) + \alpha_s G_3(\alpha_s L) + \alpha_s^2 G_4(\alpha_s L) + \cdots)$$

$(L \equiv \ln(\frac{1}{v}))$

g_1 resums all of the leading logarithmic terms (LL) of form $\alpha_s^n L^{n+1}$ in the exponent; G_2 the next-to-leading terms (NLL) of form $\alpha_s^n L^n$, and so on.

Our Observables

We calculate the thrust, τ, and its relatives the C parameter and the heavy jet mass m_H.

$$\tau \equiv 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{Q} \;,$$

where the vector $\vec{n}$ maximised by the sum defines the thrust axis.

$$\rho_H \equiv \max_{i=1,2} \frac{M_i^2}{Q^2}\,, \qquad M_i^2 \equiv \left(\sum_{j \in \mathcal{H}^{(i)}} p_j \right)^2$$

$$C \equiv 3 \left(1 - \frac{1}{2} \sum_{i,j} \frac{(p_i \cdot p_j)^2}{(p_i \cdot Q)(p_j \cdot Q)} \right)$$

The indices i and j, where applicable, run over all final-state partons in the event.

As well as the total and wide broadenings, B_T and B_W, and their relatives oblateness, O, and the thrust major, T_M.

$$B_T \equiv B_L + B_R, \qquad B_W \equiv \max\{B_L, B_R\}$$

where

$$B_L \equiv \sum_{i\in\mathcal{H}^{(1)}} \frac{|\vec{p_i} \times \vec{n_T}|}{2Q}, \quad B_R \equiv \sum_{i\in\mathcal{H}^{(2)}} \frac{|\vec{p_i} \times \vec{n_T}|}{2Q}$$

$$T_M \equiv \max_{\vec{n}\cdot\vec{n_T}=0} \frac{\sum_i |\vec{p_i} \cdot \vec{n}|}{Q}$$

$$O \equiv T_M - T_{\mathrm{m}}, \qquad T_{\mathrm{m}} \equiv \frac{\sum_i |p_{i,x}|}{Q}$$

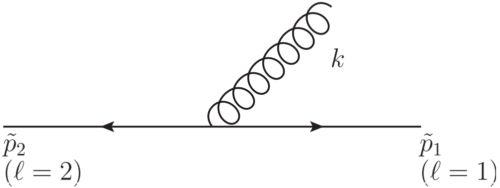
and where x is the direction perpendicular to both the thrust and the thrust-major axes.

Resummation to NLL

Our starting point is the generic observable used by the CAESAR (Computer Automated Semi-Analytical Resummer) [1].

$$V(\{\tilde{p}\}, k) = d_\ell \left(\frac{k_t^{(\ell)}}{Q} \right)^{a_\ell} e^{-b_\ell \eta^{(\ell)}} g_\ell(\phi^{(\ell)})$$

($a = b_\ell = 1$ for thrust-type observables, and $a = 1, b_\ell = 0$ for broadening-type.)



[1] A. Banfi, G. P. Salam and G. Zanderighi, JHEP 0503 (2005) 073 [hep-ph/0407286]

The only requirements: observable must be **continuously global** and **rIRC safe**.

continuously global : the observable parametrisation $V(\{\tilde{p}\}, k)$ holds everywhere in phase space.

IRC safety : independence of the observable to an extra soft or collinear emission.

rIRC safety [1] : the observable's scaling properties should be the same in the presence of any number of extra soft and/or collinear emissions.

[1] A. Banfi, G. P. Salam and G. Zanderighi, JHEP 0503 (2005) 073 [hep-ph/0407286]

For CAESAR observables, the resummed cumulative distribution has the form (for $V < v$)

$$\Sigma(v) = \frac{1}{\sigma}\int_0^v dv' \frac{d\sigma(v')}{dv'}$$

$$= e^{Lg_1(\lambda)+g_2(\lambda)}\mathcal{F}_{\mathrm{NLL}}(\lambda) = \exp\left(-\int_v [dk]|M^2(k)|\right)\mathcal{F}_{NLL}(\lambda)$$

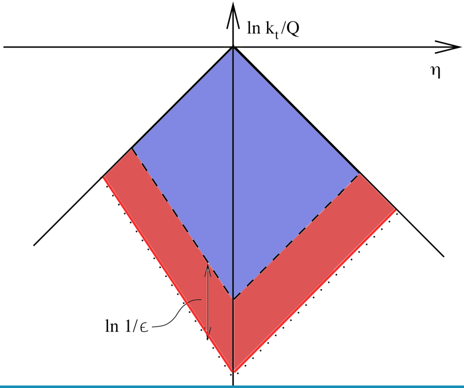
The Sudakov form factor, containing all the virtual corrections and unresolved real emissions as double logs (a LL contribution).

The $\mathcal{F}$ function, containing single logs coming from soft-collinear real emissions that are widely separated in rapidity and independent from one another (an NLL contribution).

$(L = \ln(1/v)$, $\lambda = \alpha_s(Q)\beta_0 L)$

$$\Sigma(v) = \frac{1}{\sigma}\int_0^v dv' \frac{d\sigma(v')}{dv'}$$

$$= e^{Lg_1(\lambda)+g_2(\lambda)}\mathcal{F}_{\rm NLL}(\lambda) = \exp\left(-\int_v [dk]|M^2(k)|\right)\mathcal{F}_{NLL}(\lambda)$$



To NLL order, emissions are:

- soft and collinear
- widely separated in rapidity
- emitted independently (QCD coherence effect)
- event shapes: emissions have arbitrary rapidity fraction

rIRC safety confines real emissions to live between a resolution cut-off, ϵv, and the upper value of the observable, v.
This region is size $\sim \ln\left(\frac{1}{v}\right)$ (NLL).

Resummation to NNLL

In e^+e^- processes - to gain a **measurement of** α_s at ‰ level accuracy.

Contains all the basic elements also required for hadron-hadron collisions.

Start with the NLL formulation and relax any one of the NLL assumptions, one emission at a time.
This will result in contributions at NNLL accuracy (and beyond).

Manipulate the result to attain a pure NNLL correction.

Going to NNLL: corrections to the Sudakov exponent (virtual corrections) and the multiple emissions function (real emissions).

The resummed cumulative distribution is now

$$\Sigma(v) = \frac{1}{\sigma}\int_0^v dv' \frac{d\sigma(v')}{dv'}$$
$$= e^{Lg_1(\lambda)+g_2(\lambda)+\frac{\alpha_s(Q)}{\pi}g_3(\lambda)}\left[\mathcal{F}_{\mathrm{NLL}}(\lambda)+\frac{\alpha_s(Q)}{\pi}\delta\mathcal{F}_{\mathrm{NNLL}}(\lambda)\right]$$

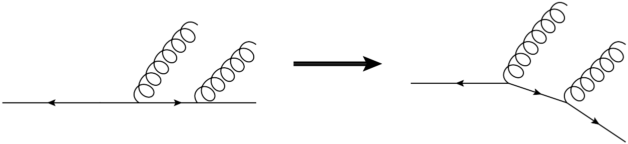
We determine the Sudakov factor using dimensional regularisation, cancelling the real and virtual IR divergences.
On the other hand, the multiple emissions function is finite in 4D, and we use a Monte Carlo to determine its value.

(Recall that $\lambda = \alpha_s(Q)\beta_0 L$)

Sources of $\delta\mathcal{F}_{\mathrm{NNLL}}(\lambda)$

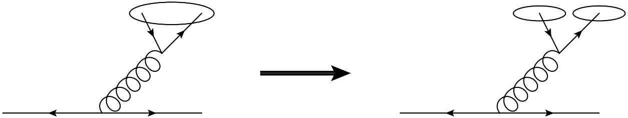
Emissions at the edges of phase space:
One emission is soft but not collinear: $\delta\mathcal{F}_{wa}$
One emission is collinear but not soft: $\delta\mathcal{F}_{hc}$

Also $\delta\mathcal{F}_{rec}$, correctly treating the recoil of one particle.



Sources of $\delta\mathcal{F}_{\mathrm{NNLL}}(\lambda)$

Subsequent gluon splitting can happen non-inclusively: $\delta\mathcal{F}_{correl}$



One emission has its exact kinematic rapidity bounds: $\delta\mathcal{F}_{sc}$

N.B: *all other emissions* obey the assumptions in $\mathcal{F}_{NLL}$.

The function g_3

We have not yet derived the g_3 function in terms of the generic parameters $(a, b_\ell, d_\ell, g_\ell(\phi))$.

Since the Sudakov factor depends only on the scaling of the observable in the presence of a single soft-collinear emission (a and b_ℓ parameters), two observables with the same soft-collinear behaviour will have the same Sudakov exponent: we use g_3 functions from the literature. [2,3,4]

The Sudakov factor is the emission probability for a single inclusive-branching gluon, whilst all configurations associated with real emissions are contained in the multiple emissions functions.

[2] T. Becher and M. D. Schwartz, JHEP 0807 (2008) 034 [arXiv:0803.0342 [hep-ph]]
[3] P. F. Monni, T. Gehrmann and G. Luisoni, JHEP 1108, 010 (2011) [arXiv:1105.4560 [hep-ph]]
[4] A. Banfi, P. F. Monni, G. P. Salam and G. Zanderighi, Phys. Rev. Lett. 109 (2012) 202001 [arXiv:1206.4998 [hep-ph]]

Semi-numerical Resummation

For additive observables, a fully analytic resummation is possible.

Our Monte Carlo program computes the real corrections to each event-shape using a generic procedure for each correction.

Additive observables (here, the thrust-type observables that we consider) satisfy

$$V(\{\tilde{p}\}, k_1, \ldots, k_n) = \sum_{i=1}^{n} V(\{\tilde{p}\}, k_i) + \mathcal{O}(V^2)\,,$$

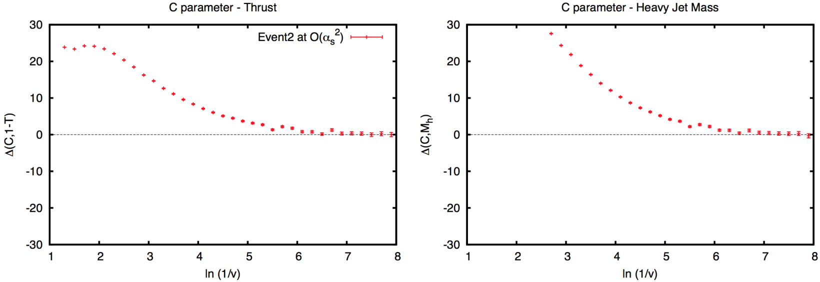
Such event shapes' NNLL contributions can be factorised by $\mathcal{F}_{\mathrm{NLL}}$.

Our results for τ, B_T, B_W and m_H agree to NNLL@NNLO (i.e. up to terms $\alpha_s^3 L^2$) with those already analytically resummed to NNLL. [2,3,5,6]

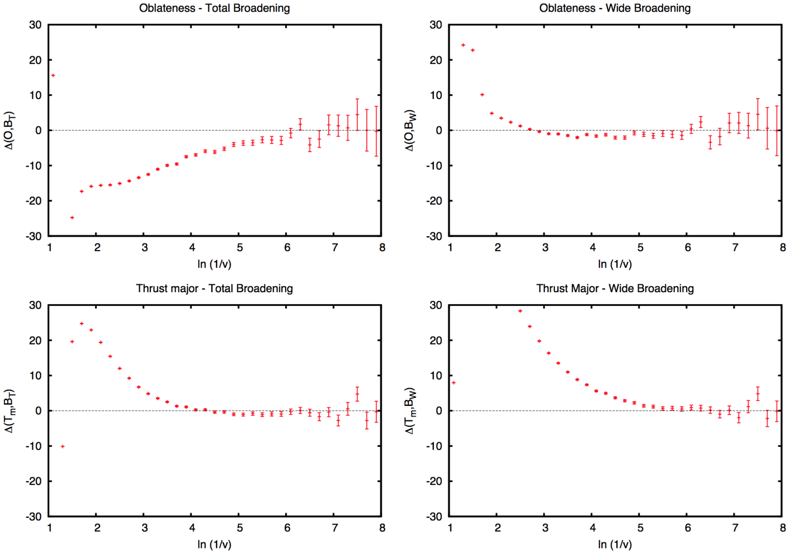
The three new results (C-parameter**, O, T_M) are checked by expanding our result to α_s^2 and comparing with exact fixed order predictions (Event2 [7]).

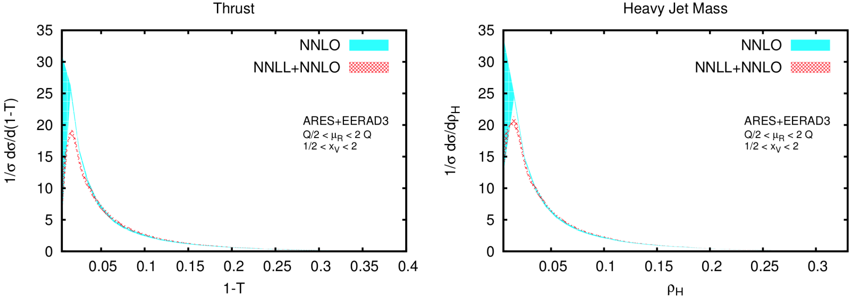
$$\Delta(v_1, v_2) = \left(\frac{1}{\sigma_0} \frac{d\sigma^{\mathrm{NLO}}}{d \ln \frac{1}{v_1}} - \frac{1}{\sigma_0} \frac{d\sigma^{\mathrm{NNLL}}\big|_{\mathrm{expanded}}}{d \ln \frac{1}{v_1}} \right) - \{v_1 \to v_2\} \, .$$

[2] T. Becher and M. D. Schwartz, JHEP 0807 (2008) 034 [arXiv:0803.0342 [hep-ph]]
[3] P. F. Monni, T. Gehrmann and G. Luisoni, JHEP 1108 (2011) 010 [arXiv:1105.4560 [hep-ph]]
[5] Y. T. Chien and M. D. Schwartz, JHEP 1008 (2010) 058 [arXiv:1005.1644 [hep-ph]]
[6] T. Becher and G. Bell, JHEP 1211 (2012) 126 [arXiv:1210.0580 [hep-ph]]
[7] S. Catani and M. H. Seymour, Nucl. Phys. B 485 (1997) 291 [Erratum-ibid. B 510 (1998) 503] [hep-ph/9605323]
[**] A. H. Hoang, D. W. Kolodrubetz, V. Mateu and I. W. Stewart, arXiv:1411.6633 [hep-ph]

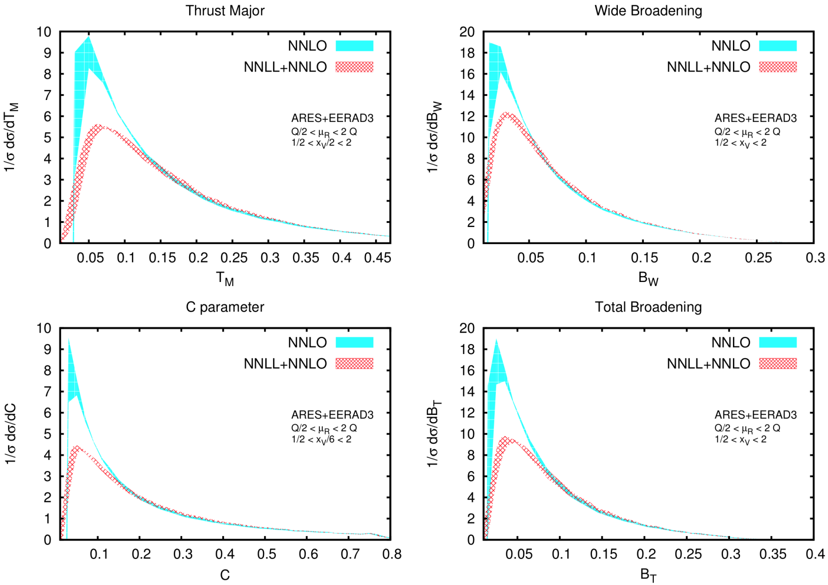


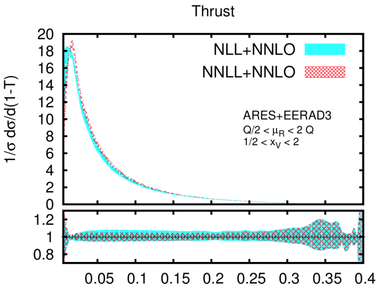
In the region $v \to 0$ the distribution is dominated by the large logs which require resumming, so the difference between the expansion of our results and the full result must go to zero as $\ln(1/v) \to$ large positive values.

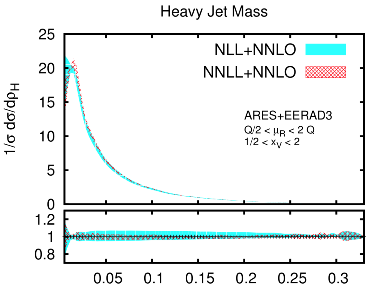

Oblateness - Total Broadening
Δ(O,B_T)
ln (1/v)
Oblateness - Wide Broadening
Δ(O,B_W)
ln (1/v)
Thrust major - Total Broadening
Δ(T_m,B_T)
ln (1/v)
Thrust Major - Wide Broadening
Δ(T_m,B_W)
ln (1/v)



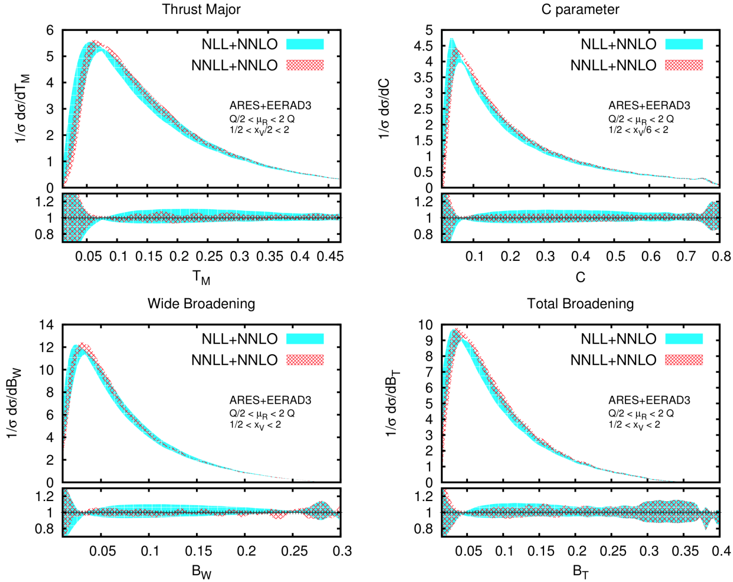
Comparison of the pure fixed order to $\mathcal{O}(\alpha_s^2)$ and the same fixed order plus our resummation. The convergence of the perturbative prediction is restored in the dijet region.


Thrust Major
NNLO
NNLL+NNLO
ARES+EERAD3
Q/2 < μR < 2 Q
1/2 < xV/2 < 2
1/σ dσ/dTM
TM
Wide Broadening
NNLO
NNLL+NNLO
ARES+EERAD3
Q/2 < μR < 2 Q
1/2 < xV < 2
1/σ dσ/dBW
BW
C parameter
NNLO
NNLL+NNLO
ARES+EERAD3
Q/2 < μR < 2 Q
1/2 < xV/6 < 2
1/σ dσ/dC
C
Total Broadening
NNLO
NNLL+NNLO
ARES+EERAD3
Q/2 < μR < 2 Q
1/2 < xV < 2
1/σ dσ/dBT
BT





Reduction of scale uncertainties going from NLL to NNLL.


Thrust Major
NLL+NNLO
NNLL+NNLO
ARES+EERAD3
Q/2 < μR < 2 Q
1/2 < xV/2 < 2
1/σ dσ/dTM
TM
C parameter
NLL+NNLO
NNLL+NNLO
ARES+EERAD3
Q/2 < μR < 2 Q
1/2 < xV/6 < 2
1/σ dσ/dC
C
Wide Broadening
NLL+NNLO
NNLL+NNLO
ARES+EERAD3
Q/2 < μR < 2 Q
1/2 < xV < 2
1/σ dσ/dBW
BW
Total Broadening
NLL+NNLO
NNLL+NNLO
ARES+EERAD3
Q/2 < μR < 2 Q
1/2 < xV < 2
1/σ dσ/dBT
BT

Summary

We have carried out calculations of event shapes in e+e- to NNLL accuracy using a novel and general method.

We are working to extend this procedure to include further variables such as jet-rates.

This framework will allow us to calculate observables in a range of processes relevant for probing physics at colliders.

Outlook

Obtaining the general expression for the full NNLL Sudakov factor will give us control over any well-behaved (rIRC safe and global) observable in e^+e^-.

The extension of this framework to include hadron-hadron collisions will be addressed in the near future.