Towards a Direct Measurement of the Quark Orbital Angular Momentum Distribution

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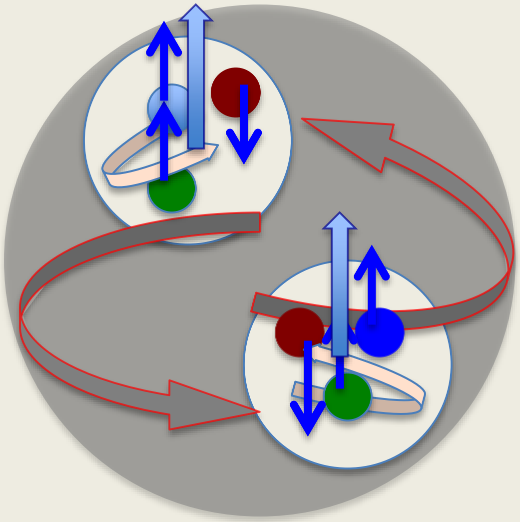

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In collaboration with: Aurore Courtoy, Michael Engelhardt, Abha Rajan

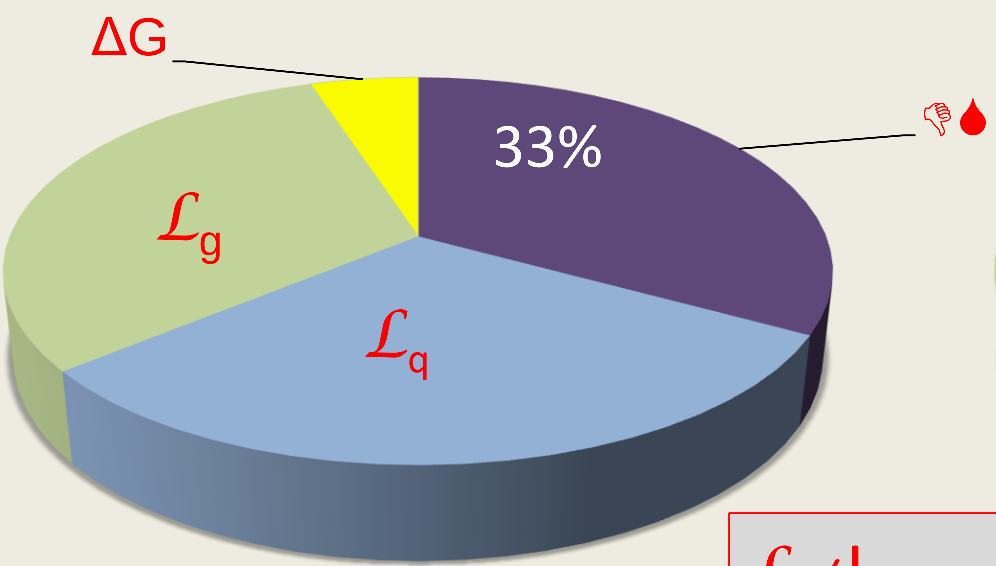
The spin crisis in a cartoon

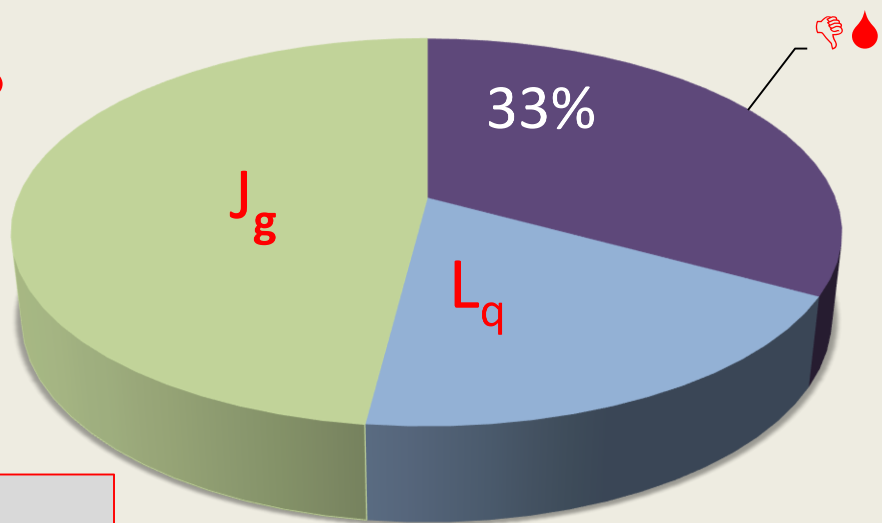
Jaffe Manohar

$$\frac{1}{2} = \frac{1}{2}\mathrm{DS} + \mathcal{L}_q + \mathrm{D}G + \mathcal{L}_g$$

Ji

$$\frac{1}{2} = \frac{1}{2}\mathrm{DS} + L_q + J_g$$

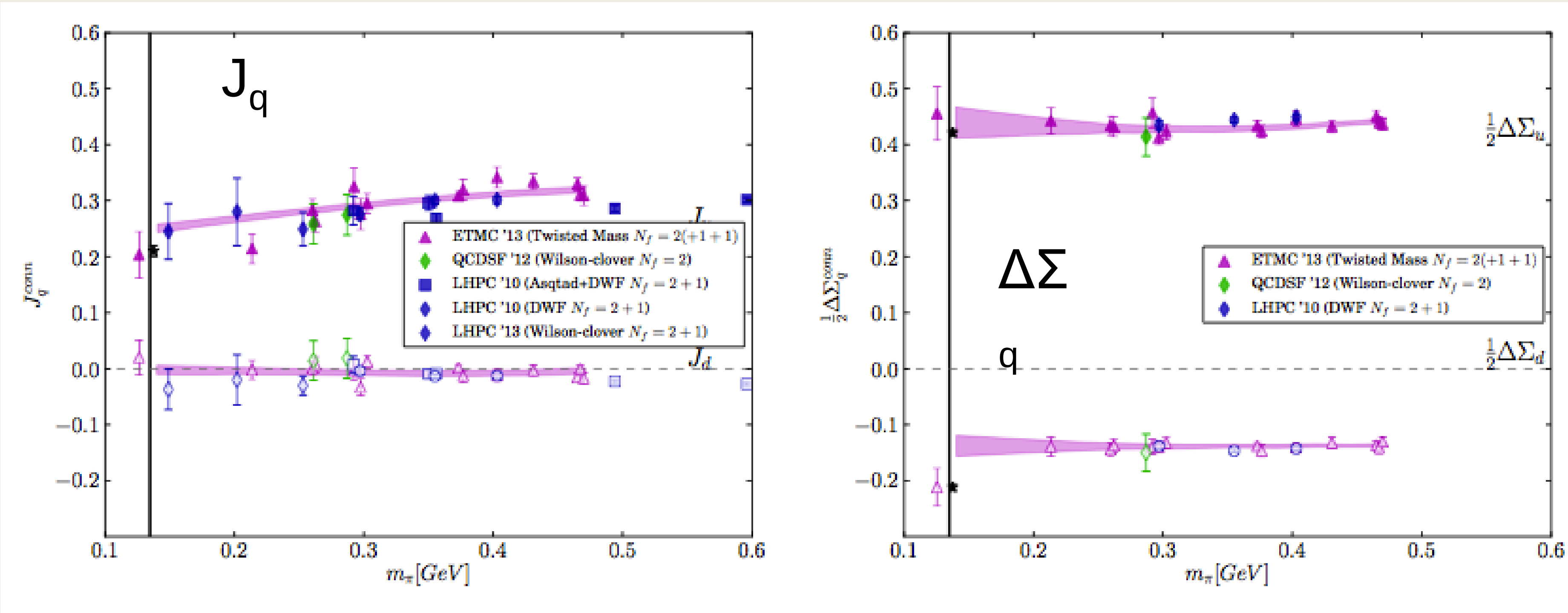




$\mathcal{L}_q \neq L_q$

$J_g \neq \mathcal{L}_g + \Delta G$

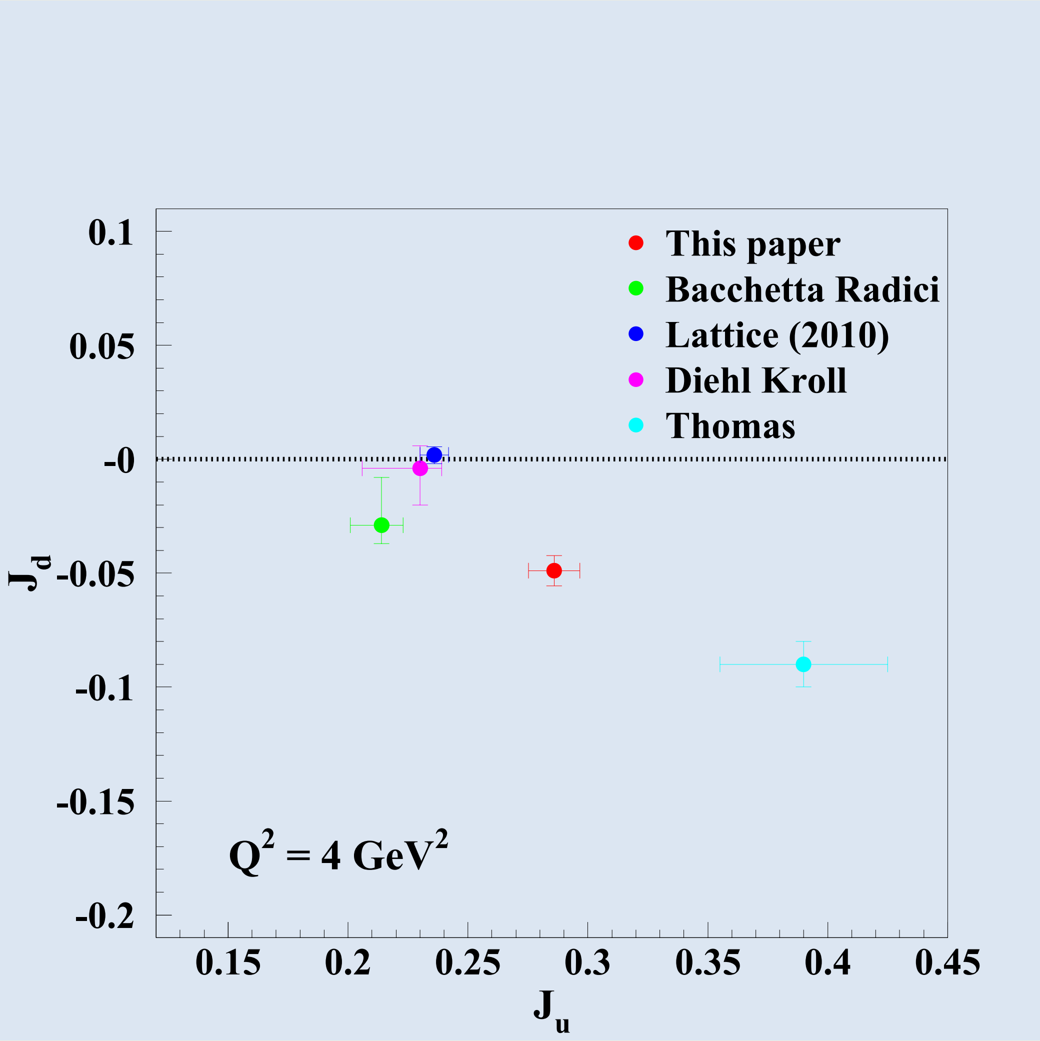
Lattice QCD



S. Syritsyn, PoS Lattice 2013

$\mathcal{L}_q$ L_q are also derived

Model dependent extractions of J_u and J_d



O. Gonzalez Hernandez et al.,
Phys. Rev. C88, 065206; arXiv:1206.1876

Angular Momentum Sum Rules

Express the various components, $\Delta\Sigma$, ΔG, L_q, J_q, J_g, $\mathcal{L}_q$, $\mathcal{L}_g$ in terms of non-local light-cone operators of twist 2 and twist 3.

Jaffe Manohar: in LC gauge rewrite AM using Dirac eqn. to isolate spin terms

$$M^{+12} = \psi^\dagger \sigma^{12}\psi + \psi^\dagger\left[\vec{x}\times(-i\partial)\right]^3\psi + Tr(\varepsilon^{+-ij}F^{+j}A^j) + 2i\,TrF^{+j}(\vec{x}\times\partial)A^j$$

$\Delta\Sigma$ $\mathcal{L}_q$ ΔG $\mathcal{L}_g$

Ji:

$$M^{+12} = \psi^\dagger \sigma^{12}\psi + \psi^\dagger\left[\vec{x}\times\left(-i\vec{D}\right)\right]^3\psi + \left[\vec{x}\times\left(\vec{E}\times\vec{B}\right)\right]^3$$

$\Delta\Sigma$ L_q J_g

The two sum rules give equivalently valid representations:

Different mechanisms for generating OAM in the proton can coexist

The ones that will make physical sense are the ones that can be measured

OAM represents the correlation between the position and momentum of the quarks and gluons

$$\mathcal{L}_q, L_q = \int dx\, d^2b\, d^2k_T \left(\vec{b} \times \vec{k}_T\right)_3 \mathcal{W}\left(x, \vec{b}, \vec{k}_T\right)$$

Hatta (2011)
Lorce, Pasquini (2011)

GTMD

OAM Wigner Distribution

$$\mathcal{W}\left(x, \vec{b}, \vec{k}_T\right) = \int \frac{d^2\Delta_T}{(2\pi)^2} e^{i\Delta_T \cdot b} \int \frac{d^2 z_T d^2 z^-}{(2\pi)^3} e^{i(xP^+ z^- - k_T \cdot z_T)} \langle P', \Lambda' | \bar{\psi}(0) \gamma^+ U(0, \infty | n) \psi(z) | P, \Lambda \rangle \Big|_{z^+=0}$$

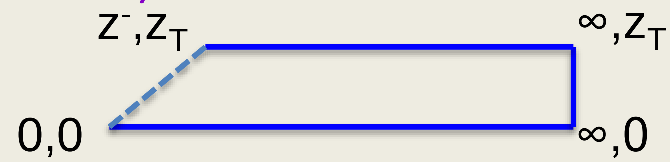
Hatta (2011)
Burkardt(2013)

Wilson-line gauge link, “n” defines the path along which we evaluate the vector potential

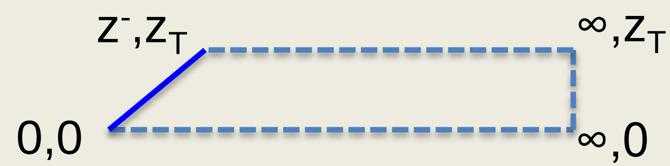
Burkardt's torque (2013)

Evaluate gauge links (Hatta Yoshida, 2012; Burkardt, 2013)

$\mathcal{L}_q$ ➔ "staple" gauge link,



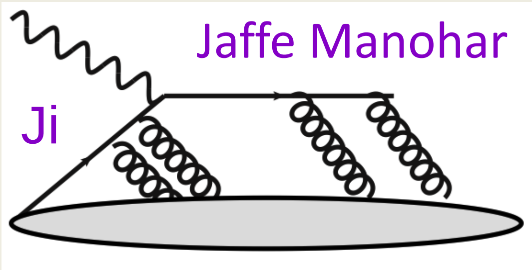
L_q ➔ "straight" gauge link



The difference of the two gives a <u>torque,</u>

$$\mathcal{L}_q - L_q = \int \frac{d^2 z_T\, dz^-}{(2\pi)^3} \langle P', \Lambda' | \bar{\psi}(z)\gamma^+ (-g) \int_{z^-}^{\infty} dy^- U \left[z_1 G^{+1}(y^-) - z_2 G^{+2}(y^-) \right] U \psi(z) | P, \Lambda \rangle \Big|_{z^+=0}$$

$\vec{\tau} = \vec{r} \times \vec{F}$ ➔ "Chromodynamic torque"*



*a Qiu-Sterman term type term analogous to f_{1T}^{perp}

Polyakov's relation (2000)

Polyakov et al. (2000), Hatta, Yoshida (2012): Define twist three GPDs

$$W^{\gamma^i}_{\Lambda'\Lambda} = \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\frac{\Delta^i_T}{M}G_1 + \frac{i\sigma^{ji}\Delta_j}{M}G_2 + \frac{Mi\sigma^{i+}}{P^+}G_4 + \frac{\Delta^i_T}{P^+}\gamma^+ G_3\right]U(p,\Lambda),$$

- ✓ Derive Wandzura Wilczeck relation using OPE tw 2 and tw 3 operators.
- ✓ Take off-forward matrix elements

Similarly to the forward case,

$$G_2(x,0,t,Q^2) = \left[\int_x^1 \frac{dy}{y}(H+E) - \int_x^1 \frac{dy}{y^2}\tilde{H}\right] + \bar{G}_2$$

genuine twist three

In the hypothesis that the genuine twist three integrates to 0, Ji's OAM distribution is identified with a twist 3 GPD

$$L_q(x) = -\,xG_2(x)$$

Generalized Wandzura Wilczek relation

$$\underbrace{xG_2(x)}_{t=3} = \underbrace{-x\int_x^1 \frac{dy}{y}\left[H(x,0,0)+E(x,0,0)\right] + x\int_x^1 \frac{dy}{y^2}\,\tilde{H}(x,0,0)}_{G_2^{WW} \rightarrow \tau=2} + \underbrace{\left[\bar{G}_2^{tw3} - \int_x^1 \frac{dy}{y}\,\bar{G}_2^{tw3}\right]}_{\tau=3}$$

k_T substructure

Burkardt, Hatta (2011)

$$\mathcal{L}_q(x) = \int d^2k_T \frac{k_T^2}{M^2} F_{14}^{"staple"}\left(x, \xi = 0, \vec{k}_T, t = 0\right) = L_q(x) + \text{"Qiu - Sterman"}$$

F_{14} (Meissner, Metz, Schlegel, 2009)

Both Jaffe Manohar and Ji OAM are identified with a k_T moment of a twist 2 GTMD

Courtoy et al. (2014)

$$L_q(x) = \int d^2k_T \left[\frac{\vec{k}_T \cdot \vec{\Delta}_T}{\Delta_T^2} F_{27}\left(x, 0, \vec{k}_T, 0\right) + F_{28}\left(x, 0, \vec{k}_T, 0\right) \right] \equiv - G_2(x)$$

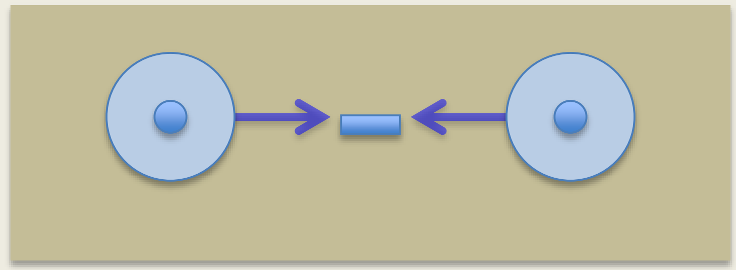
F_{27}, F_{28} (Meissner, Metz, Schlegel, 2009)

Ji's OAM is identified with the integral of twist 3 GTMDs

Spin correlations

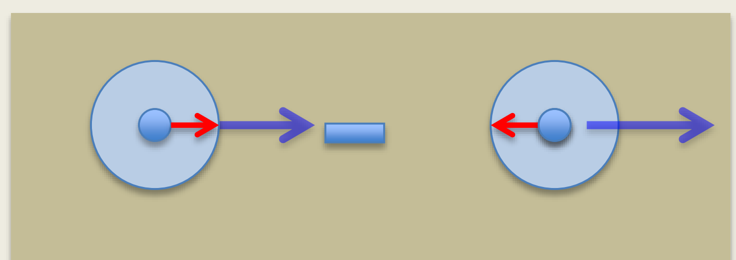
$$W^{[\gamma^+]}_{\Lambda'\Lambda} = \frac{1}{2M}\,\overline{u}(p',\Lambda')\left[F_{11} + \frac{i\sigma^{k_\perp +}}{P^+}F_{12} + \frac{i\sigma^{\Delta_\perp +}}{P^+}F_{13} + \frac{i\sigma^{k_\perp \Delta_\perp}}{M^2}F_{14}\right]u(p,\Lambda)$$

F_{14}



$$W^{\gamma^i}_{\Lambda'\Lambda} = \frac{1}{2P^+}\overline{U}(p',\Lambda')\left[\frac{\Delta_T^i}{M}G_1 + \frac{i\sigma^{ji}\Delta_j}{M}G_2 + \frac{Mi\sigma^{i+}}{P^+}G_4 + \frac{\Delta_T^i}{P^+}\gamma^+ G_3\right]U(p,\Lambda),$$

G_2



There exists a connection between F_{14} and G_2 that uncovers different types of quark-gluon interactions behind the Jaffe Manohar and Ji mechanisms for generating OAM
(A. Courtoy, M. Engelhardt, S. L., A. Rajan, 2015)

→ a unique probe of quark-gluon interactions

Using directly the unintegrated quark-quark correlator defining GTMDs,

$$W_{\Lambda\Lambda'}^{\sigma^{i+}\gamma_5} = \int \frac{d^2 z_T d^2 z^-}{(2\pi)^3} e^{ixP^+ z^- - i\vec{k}_T \cdot \vec{z}_T} \langle P', \Lambda' | \bar{\psi}(0) i\sigma^{i+}\gamma_5 \psi(z) | P, \Lambda \rangle \Big|_{z^+=0}$$

Insert the Equations of Motion,

$$\int \frac{d^2 z_T d^2 z^-}{(2\pi)^3} e^{ixP^+ z^- - i\vec{k}_T \cdot \vec{z}_T} \langle P', \Lambda' | \bar{\psi}(0) i\sigma^{i+} [i\not{D}(0) - m] \psi(z) | P, \Lambda \rangle \Big|_{z^+=0} = 0$$

$$iD_m = i\P_m + gA_m$$

$$i\not{D} = i\P_- g^+ - \P_T \times g_T + g\not{A}$$

$$k^+ W^{\gamma i \gamma_5}_{\Lambda\Lambda'} + ik^+ W^{\gamma i}_{\Lambda\Lambda'} = k_T^i W^{\gamma^+\gamma_5}_{\Lambda\Lambda'} + ik_T^i W^{\gamma^+}_{\Lambda\Lambda'}$$

$$+ \int \frac{d^2 z_T d^2 z^-}{(2\pi)^3} e^{ixP^+ z^- - i\vec{k}_T\cdot\vec{z}_T} \langle P',\Lambda' | \bar{\psi}(0)\gamma^+ g A_T(0)\psi(z) | P,\Lambda\rangle \Big|_{z^+=0}$$

Q-g-q correlator

For longitudinal polarization and Integrating over k_T

$$\underbrace{x\tilde{G}_2(x) + xG_2(x)}_{t=3} = \underbrace{\int d^2k_T \frac{k_T\cdot\Delta_T}{\Delta_T^2} G_{14}(x,0,\vec{k}_T) + \int d^2k_T \frac{k_T^2}{M^2} F_{14}(x,0,\vec{k}_T)}_{\tau=2} + \underbrace{\bar{G}_2^{tw3}}_{\tau=3}$$

$$x\tilde{G}_2(x) + xG_2(x) = G_{14}^{(1')} + F_{14}^{(1)} + \bar{G}_2^{tw3}$$

A sum rule relating Ji and JM OAM

$$\underbrace{xG_2(x)}_{t=3} = \underbrace{-x\int_x^1 \frac{dy}{y}\left[H(x,0,0) + E(x,0,0)\right] + x\int_x^1 \frac{dy}{y^2}\tilde{H}(x,0,0)}_{G_2^{WW} \rightarrow \tau=2} + \underbrace{\left[\bar{G}_2^{tw3} - \int_x^1 \frac{dy}{y}\bar{G}_2^{tw3}\right]}_{\tau=3}$$

Compare with gauge-links derivation

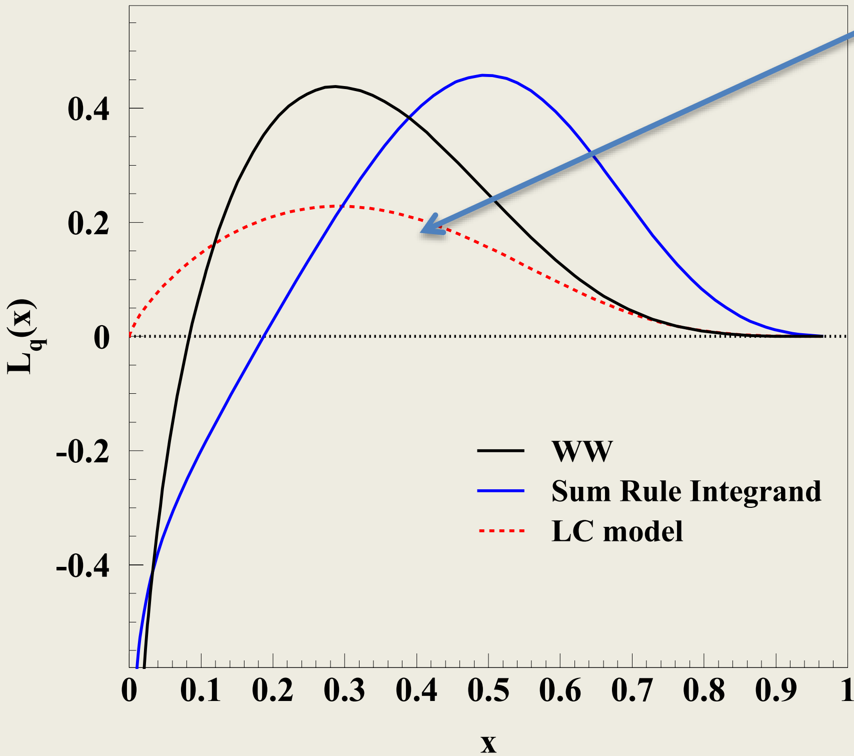
$$\int d^2k_T \frac{k_T^2}{M^2} F_{14}^{JM}\left(x, 0, \vec{k}_T\right) = \int d^2k_T \frac{k_T^2}{M^2} F_{14}^{Ji}\left(x, 0, \vec{k}_T\right) + \text{"Qiu - Sterman"}$$

$$\int d^2k_T \frac{k_T^2}{M^2} F_{14}^{JM}\left(x, 0, \vec{k}_T\right) = \int d^2k_T \frac{k_T^2}{M^2} F_{14}^{Ji}\left(x, 0, \vec{k}_T\right) + \text{"}tw3^{Ji}\text{"} - \text{"}tw3^{JM}\text{"}$$

We are investigating further this connection

$$L_q(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0)+E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0),$$

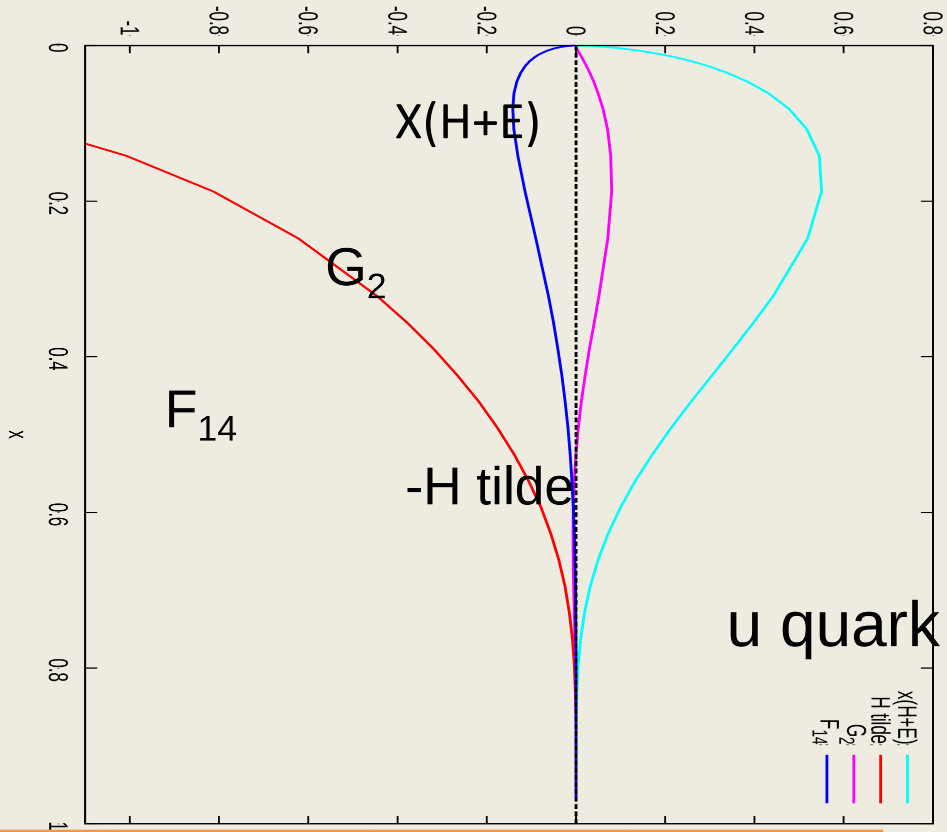
$\neq F_{14}$!



u quark

GPDs calculated in
Reggeized diquark model
GGL PRD (2010),
O. Gonzalez et al, PRC(2013)

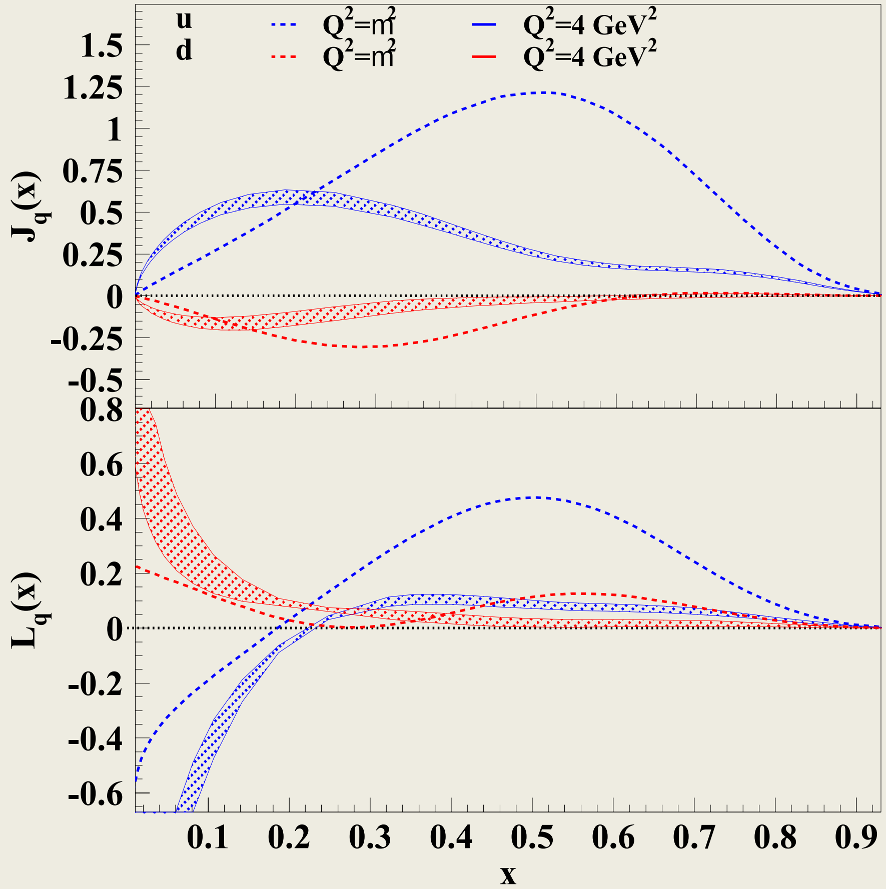
Preliminary results in reggeized diquark model (With Abha Rajan)



$Q^2=2\ GeV^2$

J_q	$\Delta\Sigma_q$	L_q	Int $-xG_2$	Int F_{14}
0.24	0.46	-0.22	-0.025	-0.045

Effect of evolution

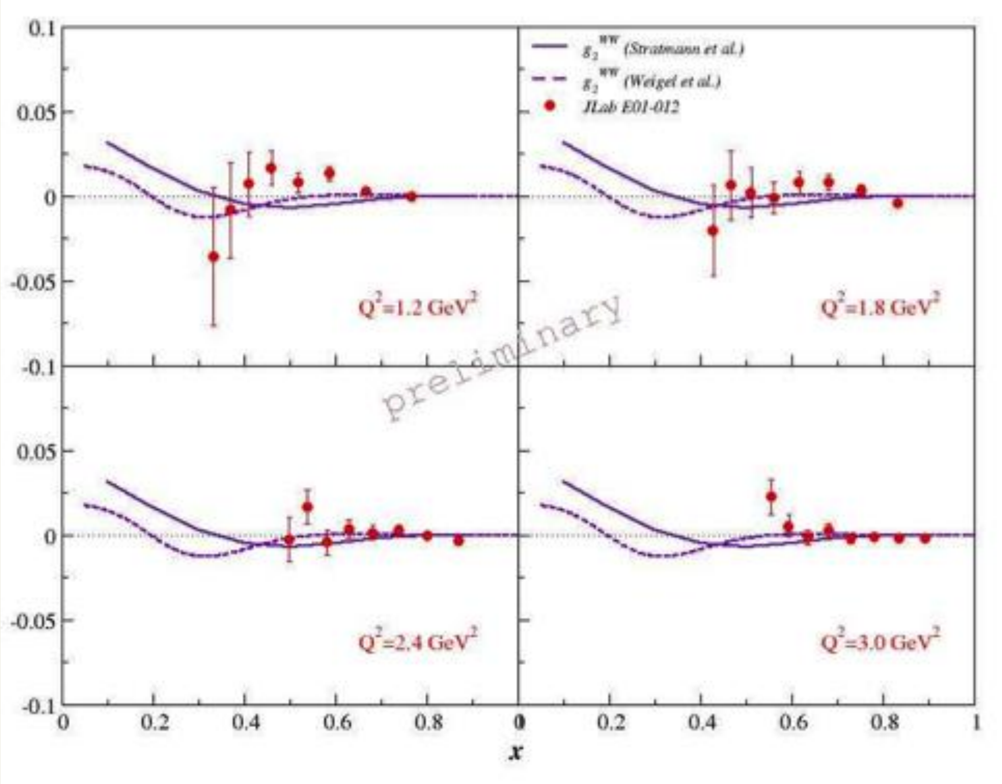


GPDs calculated in Reggeized diquark model

GGL PRD (2010), O. Gonzalez et al, PRC (2013)

$$\underbrace{g_2(x)}_{t=3} = \underbrace{-g_1(x) + \int_x^1 \frac{dy}{y} g_1(x)}_{g_2^{WW} \rightarrow \tau=2} + \underbrace{\left[\bar{g}_2^{tw3} - \int_x^1 \frac{dy}{y} \bar{g}_2^{tw3} \right]}_{\tau=3}$$

g_2^n Hall A



A few observations

- ✓ The new expression relates a GTMD (F_{14}) with a twist 3 GPD (G_2), intrinsic k_T enters even if integrated over ➔ it establishes a connection between transverse spatial and momentum dependences

- ✓ Similar to the Sivers effect but here the function vanishes for a straight link, only staple links are probed

- ✓ A unique setup to study/test transverse momentum dependence and related effects, factorization issues, renormalization issues…

- ✓ A unique handle on quark-gluon interactions through the explicit appearance of the quark-gluon-quark correlator in the sum rule

- ✓ The role of partonic k_T and off-shellness, k^2 is manifest.

- ✓ Similarities with g_2

Observability: Cross section and asymmetries

$$
\begin{aligned}
\frac{d^4\sigma}{dx_{Bj}\,dy\,d\phi\,dt} = \Gamma \Big\{ \Big[& F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{2\epsilon(\epsilon+1)} \cos\phi F_{UU}^{\cos\phi} + h \sqrt{2\epsilon(1-\epsilon)} \sin\phi F_{LU}^{\sin\phi} \\
+ & S_{||} \Big[\sqrt{2\epsilon(\epsilon+1)} \sin\phi F_{UL}^{\sin\phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \Big(\sqrt{1-\epsilon^2}\, F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos\phi\, F_{LL}^{\cos\phi} \Big) \Big] \\
+ & S_{\perp} \Big[\sin(\phi-\phi_S) \Big(F_{UT,T}^{\sin(\phi-\phi_S)} + \epsilon F_{UT,L}^{\sin(\phi-\phi_S)} \Big) + \epsilon \Big(\sin(\phi+\phi_S) F_{UT}^{\sin(\phi+\phi_S)} + \sin(3\phi-\phi_S) F_{UT}^{\sin(3\phi-\phi_S)} \Big) \\
+ & \sqrt{2\epsilon(1+\epsilon)} \Big(\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi-\phi_S) F_{UT}^{\sin(2\phi-\phi_S)} \Big) \Big] \\
+ & S_{\perp} h \Big[\sqrt{1-\epsilon^2} \cos(\phi-\phi_S) F_{LT}^{\cos(\phi-\phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \Big(\cos\phi_S F_{LT}^{\cos\phi_S} + \cos(2\phi-\phi_S) F_{LT}^{\cos(2\phi-\phi_S)} \Big) \Big] \Big\}
\end{aligned}
$$

$$
A_{LU} = \sqrt{\epsilon(1-\epsilon)}\, \frac{F_{LU}^{\sin\phi}}{F_{UU,T} + \epsilon F_{UU,L}}
$$

$$
A_{UL} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{\epsilon(\epsilon+1)} \sin\phi F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon\, F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon\, F_{UU,L}}
$$

$$
A_{LL} = \frac{N^{\rightarrow}_{s_z=+} - N^{\rightarrow}_{s_z=-} + N^{\leftarrow}_{s_z=+} - N^{\leftarrow}_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{1-\epsilon^2}\ F_{LL}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\sqrt{\epsilon(1-\epsilon)}\ \cos\phi\, F_{LL}^{\cos\phi}}{F_{UU,T} + \epsilon F_{UU,L}}
$$

We identified the quark-proton helicity amplitudes combinations

In terms of quark-proton helicity amplitudes,

$$i(\mathbf{k}\times\mathbf{\Delta})_3 F_{14} = A^{tw2}_{++,++} + A^{tw2}_{+-,+-} - A^{tw2}_{-+,-+} - A^{tw2}_{--,--}$$

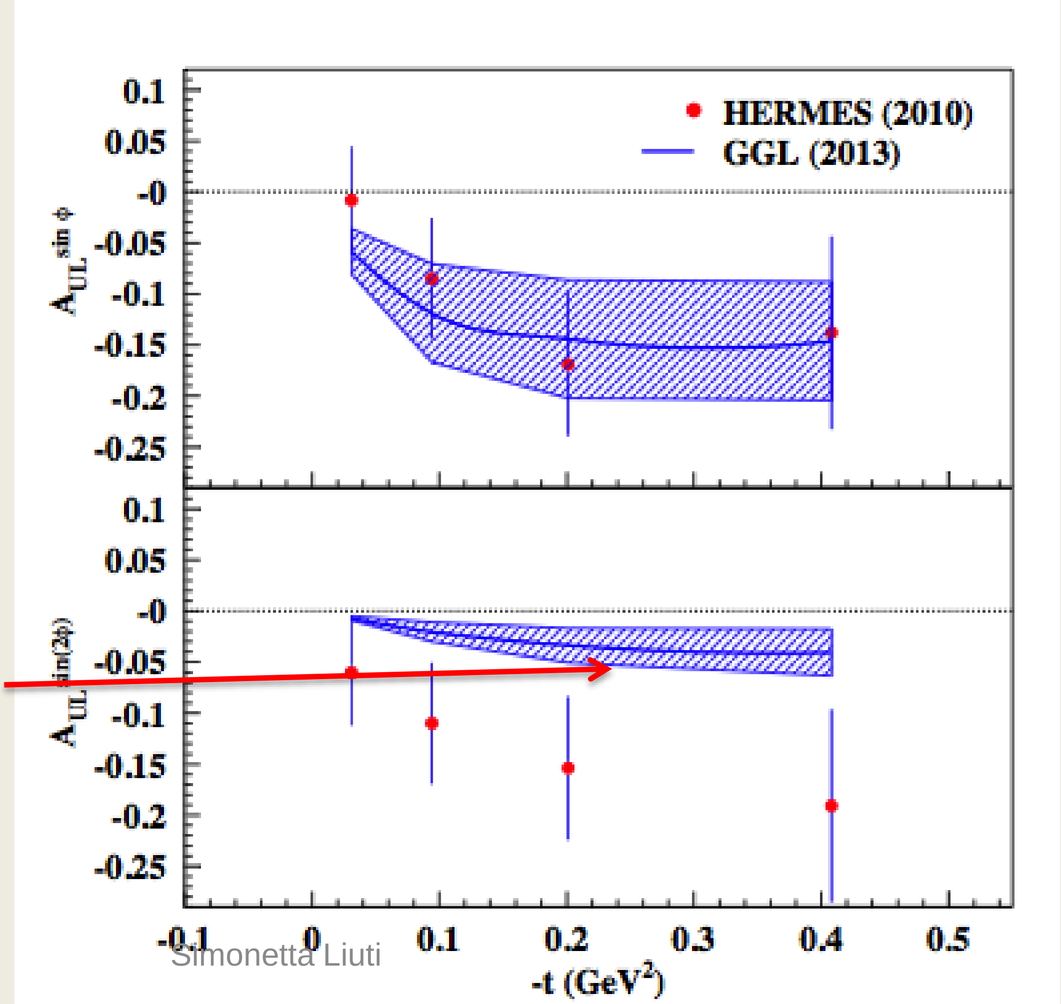
G_2

$$(k_1 - ik_2)F_{27} + (\Delta_1 - i\Delta_2)F_{28} = A^{tw3}_{+-^*,++} - A^{tw3}_{+-,++^*} - A^{tw3}_{--^*,-+} + A^{tw3}_{--,-+^*}$$

Direct measurement of OAM via G_2: DVCS on a longitudinally polarized target

$$A_{UL,L} = \frac{N_{s_z=+} - N_{s_z=-}}{N_{s_z=+} + N_{s_z=-}} = \frac{\sqrt{2\epsilon(\epsilon+1)}\sin\phi F_{UL}^{\sin\phi}}{F_{UU,T} + \epsilon F_{UU,L}} + \frac{\epsilon \sin 2\phi F_{UL}^{\sin 2\phi}}{F_{UU,T} + \epsilon F_{UU,L}}$$

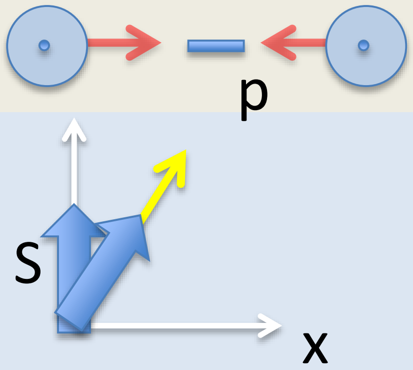
WW, small ξ



Hall B analysis in progress!
Avakian, Pisano

Direct measurement of (k_T moment of) F_{14}

UL correlation



F_{14} is Parity even: its matrix element transforms like:

$$S_z = p_z \mathrm{L}$$

it can therefore in principle represent OAM!

Most of the criticism in Kanazawa et al. (2014) is unfounded, it raised a controversy on an issue that is uncontroversial!

However, because F_{14} transforms opposite to helicity, we did raise an issue about its observability in terms of helicity amplitudes!!

Parity predicts a lack of a UL correlation at twist 2

TMDs

	U	T_x	T_y	L
U	f_1	$-i\frac{k_y}{M}\, h_1^\perp$	$i\frac{k_x}{M}\, h_1^\perp$	✗
T_x	$-i\frac{k_y}{M}\, f_{1T}$	$h_1 + \frac{k_x^2-k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$\frac{k_x}{M}\, g_{1T}$
T_y	$i\frac{k_x}{M}\, f_{1T}$	$\frac{k_x k_y}{M^2}\, h_{1T}^\perp$	$h_1 - \frac{k_x^2-k_y^2}{2M^2}\, h_{1T}^\perp$	$\frac{k_y}{M}\, g_{1T}$
L	✗	$\frac{k_x}{M}\, h_{1L}^\perp$	$\frac{k_y}{M}\, h_{1L}^\perp$	g_{1L}

GPDs

	U	T_x	T_y	L
U	H	$i\,\frac{\Delta_y}{2M}\,(2\tilde{H}_T + E_T)$	$-i\,\frac{\Delta_x}{2M}\,(2\tilde{H}_T + E_T)$	✗
T_x	$i\,\frac{\Delta_y}{2M}\,E$	$H_T - \frac{\Delta_x^2-\Delta_y^2}{4M^2}\,\tilde{H}_T$	$-\frac{\Delta_x\Delta_y}{2M^2}\,\tilde{H}_T$	
T_y	$-i\,\frac{\Delta_x}{2M}\,E$	$-\frac{\Delta_x\Delta_y}{2M^2}\,\tilde{H}_T$	$H_T + \frac{\Delta_x^2-\Delta_y^2}{4M^2}\,\tilde{H}_T$	
L	✗			$\tilde{H}$

✓The amps will cancel unless they are imaginary:

$$A_{++,++} = A^*_{--,--}\ ;\ A_{+-,+-} = A^*_{-+,-+}$$

✓But this cannot be when the scattering happens in one single hadronic plane. In this case there can be no relative phase between helicity amps (this is what we referred to as Parity Odd, not F_{14} itself!).

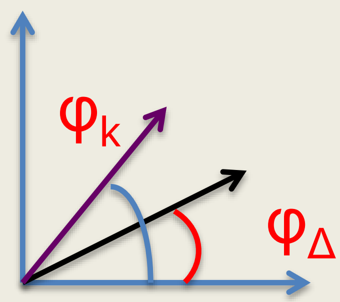
✓Different from GPD E where the amplitude's phase is a consequence of helicity flip and off-forward spinor rotation

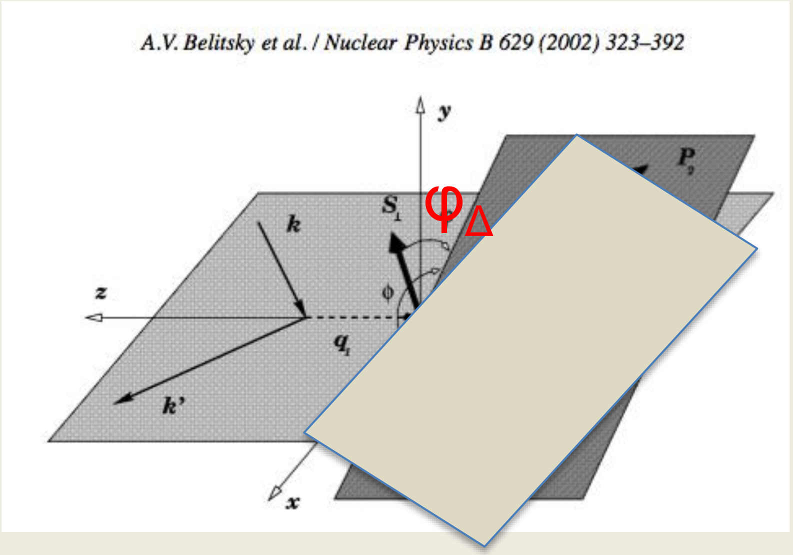
✓The phase in F_{14} is obtained by introducing two scattering planes

Off forward SIDIS

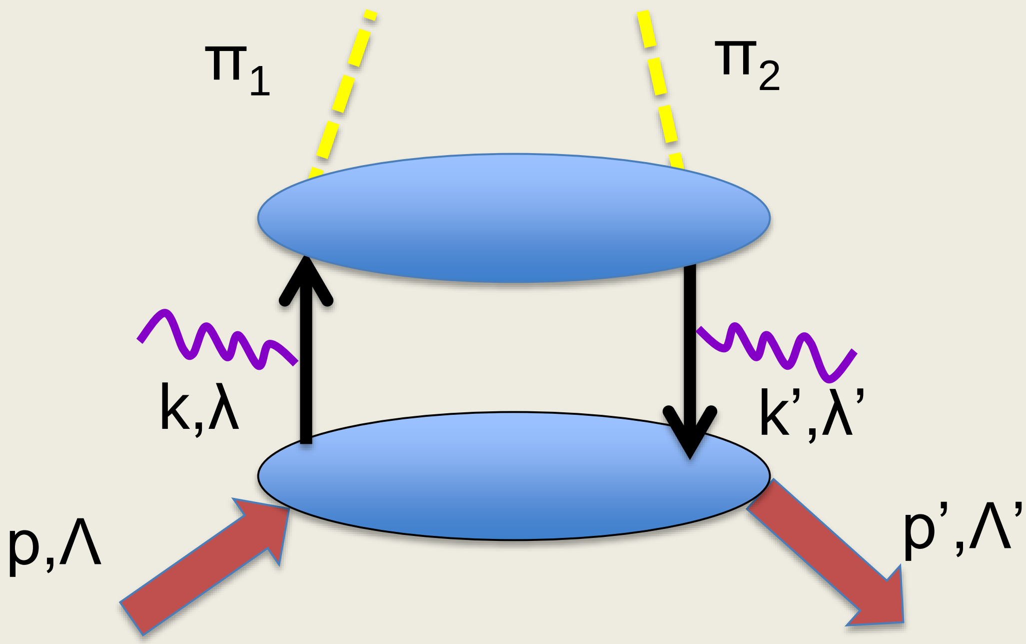
- To measure F_{14} one has to be in a frame where the reaction cannot be viewed as a two-body quark-proton scattering
- In the CoM the amplitudes are imaginary ➔UL connection goes to 0
- The way to accomplish this is to define two planes

virtual photon coming at you





… “off-forward SIDIS” allows us to introduce additional degrees of freedom:



$$g_{\Lambda'_\gamma,\Lambda'_N,0;\Lambda_\gamma,\Lambda_N,0} = \sum_{\lambda,\lambda'} \tilde{g}^{\lambda'\lambda}_{\Lambda'_\gamma\Lambda_\gamma} \otimes A_{\Lambda'_N,\lambda',\Lambda_N,\lambda}(x,\xi,t) \otimes F^{\pi_1}_{\lambda 0}(z) F^{\pi_2}_{\lambda' 0}(v)$$

$\Downarrow$

$$F_{14} \propto g_{1,+,0;1,+0} + g_{-1,+,0;-1,+0} - g_{1,-,0;1,-0} - g_{-1,-,0;-1,-0}$$

Conclusions and Outlook

With observables in hand we can now state that OAM acquires different meanings depending on the way we probe it

The difference between JM and Ji sheds light on the working of the quark-gluon correlations (twist analysis)

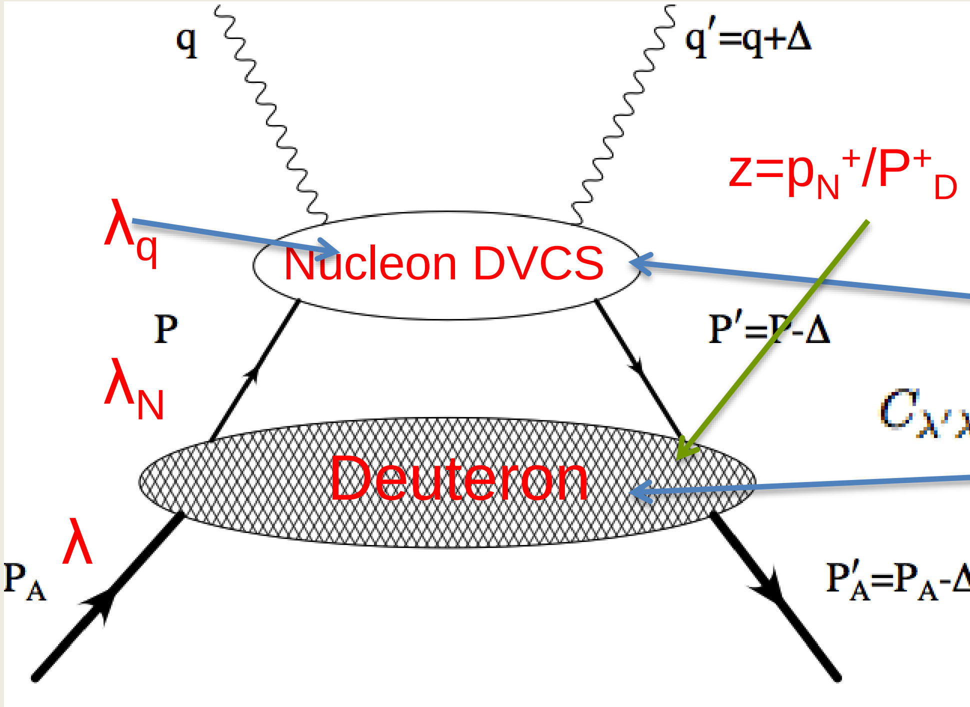
Unique connection between (G)TMDs and GPDs, allows us to study in detail the role of quark-gluon correlations, and of transverse momentum or off-shellness

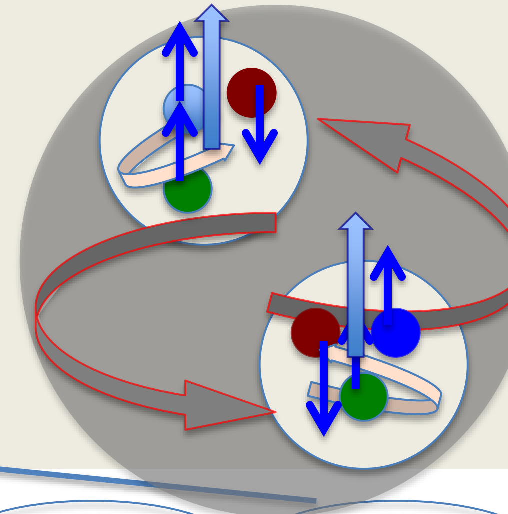
OAM was obtained so far by subtraction (also in lattice) We suggest an independent measurement of OAM

Many more interesting new connections: with transverse spin (Sivers effect, transverse spin) and axial vector sector (g_2)

Test renormalization issues, evolution etc…

ADVERTISEMENT: Sum Rules in Deuteron





$$C_{\lambda'\lambda'_q,\lambda\lambda_q} = \sum_{\lambda_N,\lambda'_N} B_{\lambda'\lambda'_N,\lambda\lambda_N} \otimes A_{\lambda'_N\lambda'_q,\lambda_N\lambda_q}$$

$$J_q = \frac{1}{2}\int dx\, x\,[H_q(x,0,0) + E_q(x,0,0)]\,,$$

Nucleon

→

$$J_q = \frac{1}{2}\int dx\, x\, H_2^q(x,0,0),$$

Deuteron

↓ $F_1+F_2= G_M$

↓ G_M

BackUp

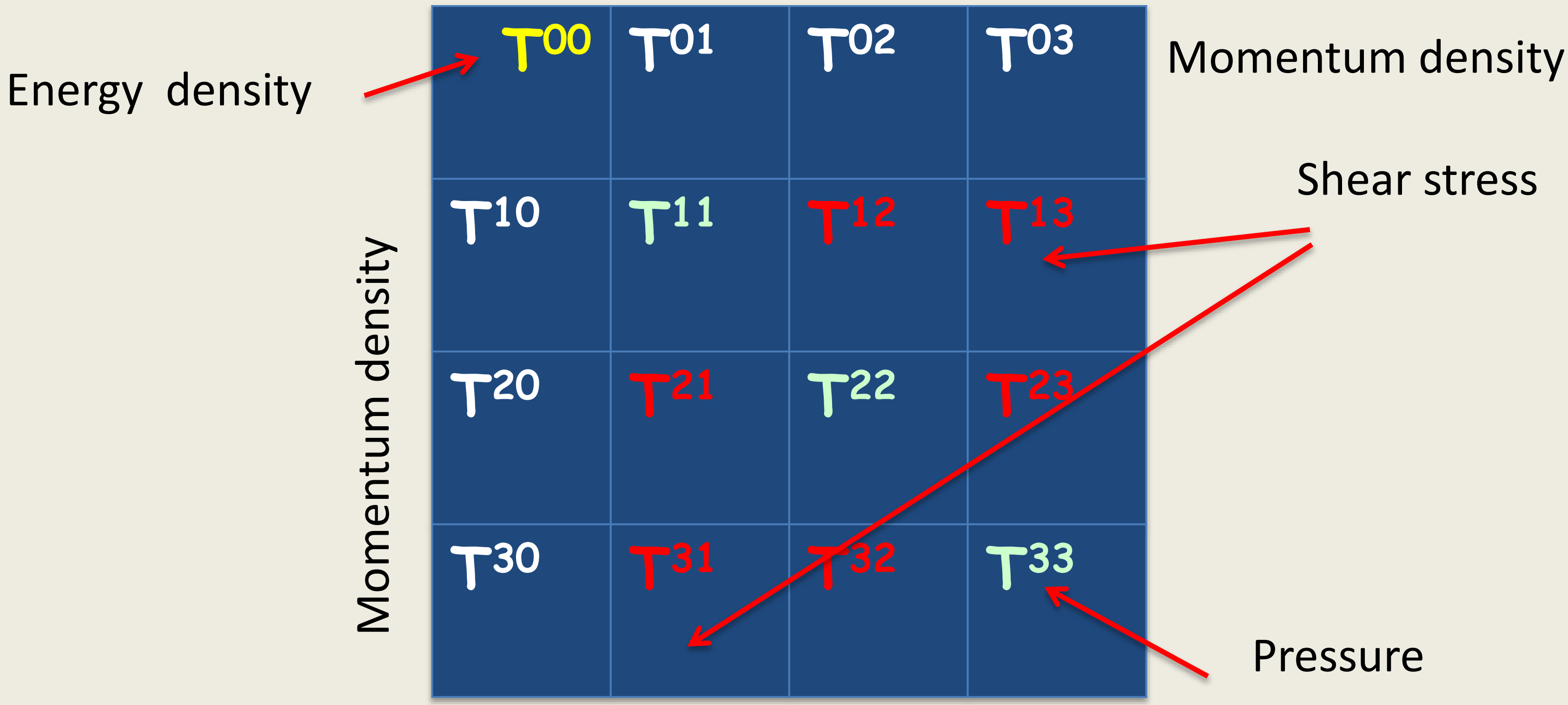
Twist 3 decomposition of hadronic tensor in various notations

Polyakov et al. [13]	$2G_1$	G_2	G_3	G_4
Meissner et al. [3]	$2\widetilde{H}_{2T}$	$\widetilde{E}_{2T}$	E_{2T}	H_{2T}
Belitsky et al. [16]	E_+^3	$\widetilde{H}_-^3$	$H_+^3 + E_+^3$	$\frac{1}{\xi}\widetilde{E}_-^3$

TABLE I: Comparison of notations for different twist 3 GPDs.

Courtoy et al, PLB (2014)arXiv:1310.5157

The QCD Energy Momentum Tensor



$$T^{\mu\nu} = \frac{1}{4} i\theta\bar{\psi}\left(\gamma^{\mu}\overleftrightarrow{D}^{\nu} + \gamma^{\nu}\overleftrightarrow{D}^{\mu}\right)\psi + Tr\left\{F^{\mu\alpha}F_{a}^{\nu} - \frac{1}{2}g^{\mu\nu}F^{2}\right\} \rightarrow M^{\mu\nu\lambda} = x^{\nu}T^{\mu\lambda} - x^{\lambda}T^{\mu\nu}$$

Angular Momentum density

Sum Rule: Part I

First define the angular momentum components

$$M^{\mu\nu\lambda} = x^{\nu} T^{\mu\lambda} - x^{\lambda} T^{\mu\nu} \quad \Rightarrow \quad J_q^i = \epsilon^{ijk} \int dz^- d^2 z \; M^{+jk}$$

then parametrize the EMT in terms of form factors A, B, C

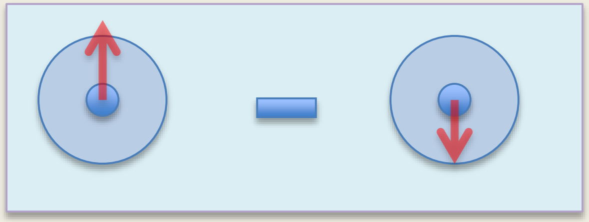
$$T^{\mu\nu} = A(\gamma^{\mu}\bar{P}^{\nu} + \gamma^{\nu}\bar{P}^{\mu}) + B\left(\frac{i\sigma^{\mu\alpha}\Delta_{\alpha}}{2M}\bar{P}^{\nu} + \frac{i\sigma^{\nu\alpha}\Delta_{\alpha}}{2M}\bar{P}^{\mu}\right) + C\,\frac{\Delta^{\mu}\Delta^{\nu} - \Delta^2 g^{\mu\nu}}{M}$$

Finally, connect the EMT matrix element with AM components

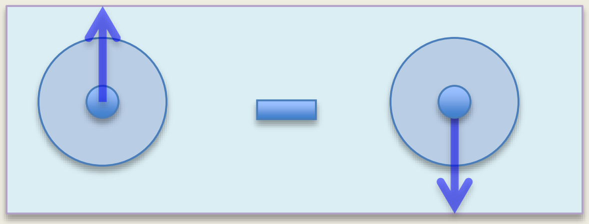
$$J_q = \frac{1}{2}\left(A_q + B_q\right) \Rightarrow \sum J_q + J_g = \frac{1}{2}$$

Jaffe Manohar (1990)
Ji (1997)

$h_1^{\wedge}$



$f_{1T}^{\wedge}$



Ah ha!
This is the same argument that allows us to observe the T-odd TMDs by understanding the role of the gauge links

$f_{1T}^{\wedge}$

S.J. Brodsky et al. / Physics Letters B 530 (2002) 99–107

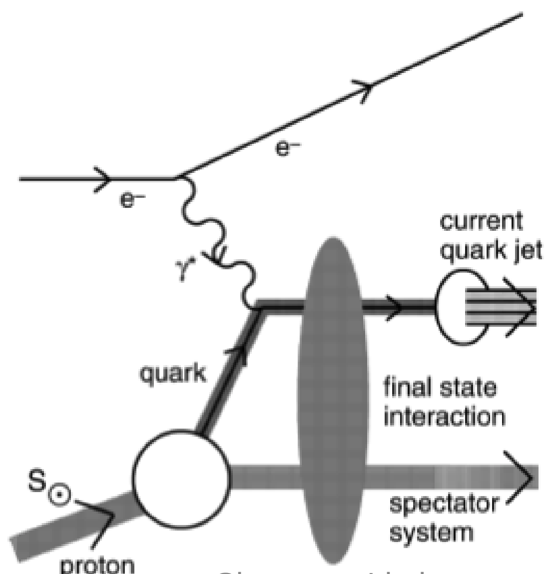


Fig. 1. The final-state interaction in the semi-inclusive deep inelastic lepton scattering $\ell p^{\uparrow} \to \ell' \pi X$.

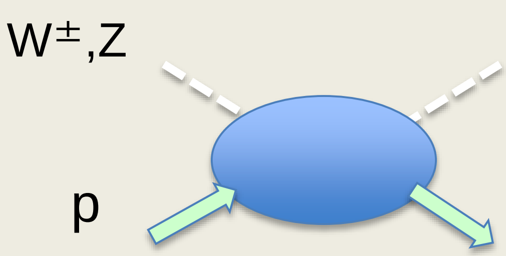
F_{14} appears in the unintegrated structure functions for deep inelastic scattering with electroweak currents

$$\tfrac{1}{4}\left(T_{1\,\frac{1}{2};1\,\frac{1}{2}} + T_{1\,-\frac{1}{2};1\,-\frac{1}{2}} + T_{-1\,\frac{1}{2};-1\,\frac{1}{2}} + T_{-1\,-\frac{1}{2};-1\,-\frac{1}{2}}\right) = T_1, \qquad F_1$$

$$\tfrac{1}{4}\left(T_{1\,\frac{1}{2};1\,\frac{1}{2}} - T_{1\,-\frac{1}{2};1\,-\frac{1}{2}} + T_{-1\,\frac{1}{2};-1\,\frac{1}{2}} - T_{-1\,-\frac{1}{2};-1\,-\frac{1}{2}}\right) = \frac{\nu}{M^2}\sqrt{1+\frac{M^2Q^2}{\nu^2}}\;A_1, \qquad A_1$$

$$\tfrac{1}{4}\left(T_{1\,\frac{1}{2};1\,\frac{1}{2}} - T_{1\,-\frac{1}{2};1\,-\frac{1}{2}} - T_{-1\,\frac{1}{2};-1\,\frac{1}{2}} + T_{-1\,-\frac{1}{2};-1\,-\frac{1}{2}}\right) = -\frac{\nu}{M^2}S_1 + \frac{Q^2}{M^2}S_2 + S_3, \qquad G_1$$

$$\tfrac{1}{4}\left(T_{1\,\frac{1}{2};1\,\frac{1}{2}} + T_{1\,-\frac{1}{2};1\,-\frac{1}{2}} - T_{-1\,\frac{1}{2};-1\,\frac{1}{2}} - T_{-1\,-\frac{1}{2};-1\,-\frac{1}{2}}\right) = \frac{\nu}{2M^2}\sqrt{1+\frac{Q^2M^2}{\nu^2}}\;T_3, \qquad F_3$$



X.Ji, NPB402 (1993)

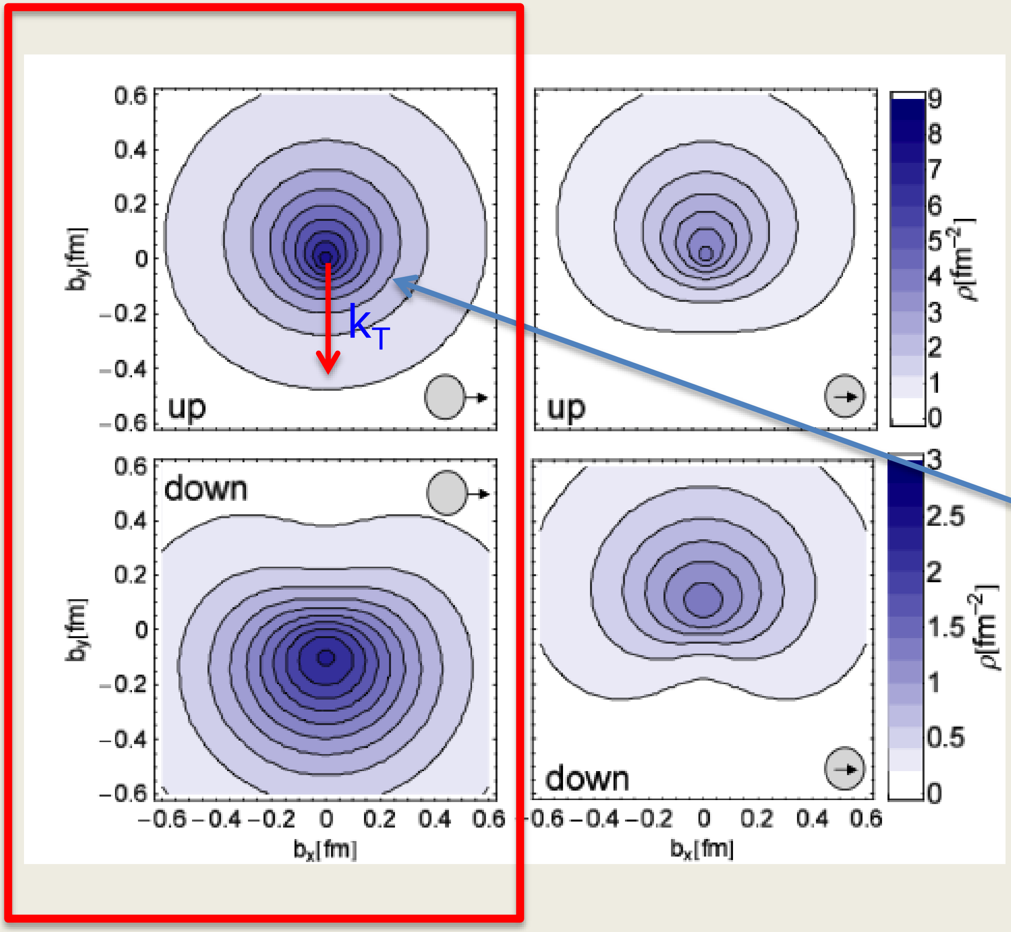
$$\begin{aligned} G_1 \propto\ & (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-}) \quad g_1 \\ & + (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-}) \quad F_{14} \end{aligned}$$

parity odd

$$\begin{aligned} A_1 \propto\ & (g'_V g_V + g'_A g_A) \otimes (A_{++,++} - A_{-+,-+} - A_{--,--} + A_{+-,+-}) \quad F_{14} \\ & + (g'_V g_A + g'_A g_V) \otimes (A_{++,++} - A_{-+,-+} + A_{--,--} - A_{+-,+-}), \quad g_1 \end{aligned}$$

F_{14} is the parity odd contribution to g_1 and the parity even contribution to A_1!

Analogous situation as for E wrt. transverse spin (M. Burkardt)



E $\rightarrow$ $s^{+j}\Delta_j \Rightarrow \vec{S}_T \times \vec{\Delta}$

The net b corresponds to net k_T in the opposite direction (attractive color force due to FSI)

$$\left(A^X_{++,++} + A^X_{+-,+-} + A^X_{-+,-+} + A^X_{--,--}\right) + \left(A^X_{++,++} + A^X_{+-,+-} - A^X_{-+,-+} - A^X_{--,--}\right)$$

$$\approx H - i\Delta_2 E$$