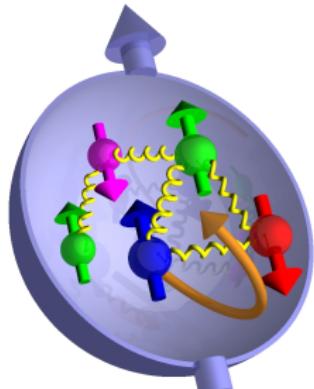


Transverse Force on Quarks in DIS

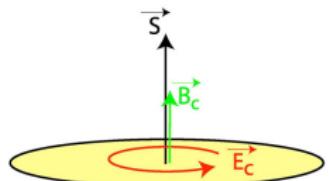
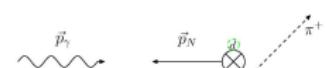
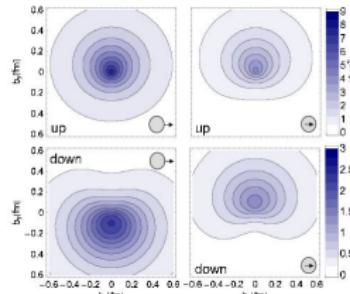
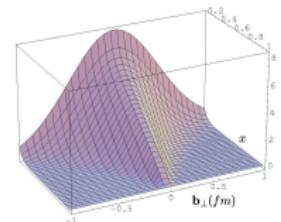
Matthias Burkardt

NMSU

April 30, 2015

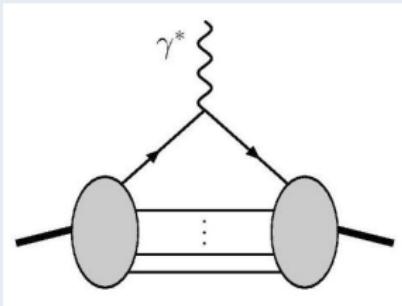


- 3D imaging of the nucleon
 - Single-Spin Asymmetries (SSAs)
 - Quark orbital angular momentum
- angular momentum decompositions (Jaffe v. Ji)
- quark-gluon correlations
- Summary



form factor

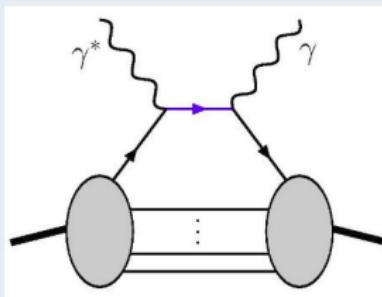
- electron hits nucleon, nucleon remains intact



- study amplitude that nucleon remains intact as function of momentum transfer $\rightarrow F(q^2)$
- $F(q^2) = \int dx GPD(x, q^2)$
- \hookrightarrow GPDs provide momentum dissected form factors

Compton scattering

- electron hits nucleon, nucleon remains intact & photon gets emitted



- study both energy & q^2 dependence
 - \hookrightarrow additional information about momentum fraction x of active quark
 - \hookrightarrow generalized parton distributions $GPD(x, q^2)$

MB, PRD62, 071503 (2000)

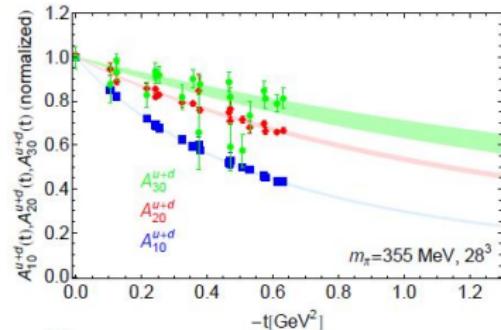
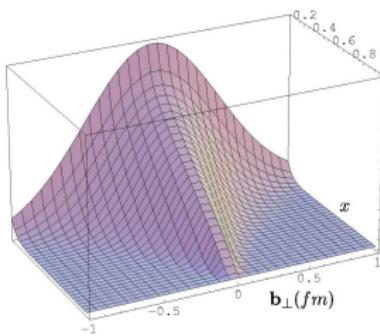
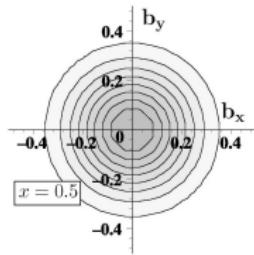
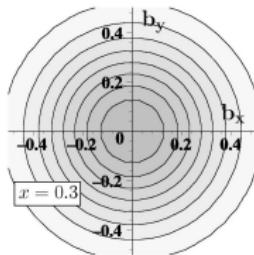
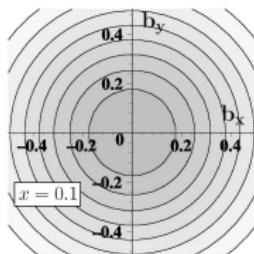
- form factors: $\xleftarrow{FT} \rho(\vec{r})$
- $GPDs(x, \vec{\Delta})$: form factor for quarks with momentum fraction x
 - ↪ suitable FT of $GPDs$ should provide spatial distribution of quarks with momentum fraction x

Impact Parameter Dependent Quark Distributions

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} GPD(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

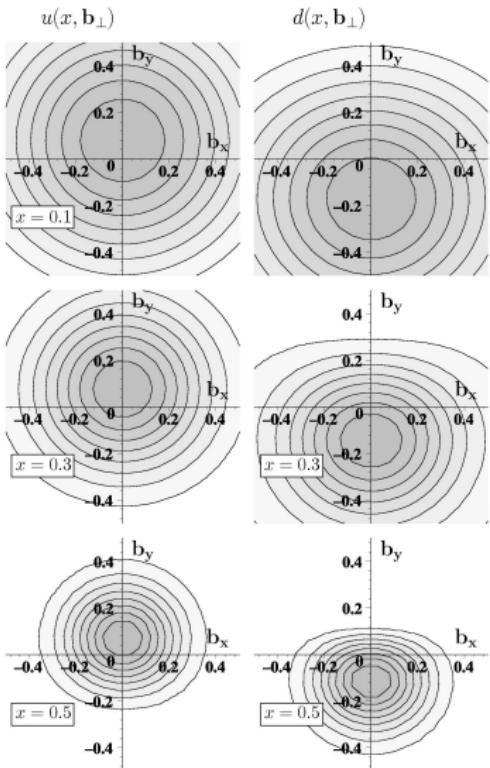
$q(x, \mathbf{b}_\perp)$ = parton distribution as a function of the separation \mathbf{b}_\perp from the transverse center of momentum $\mathbf{R}_\perp \equiv \sum_{i \in q,g} \mathbf{r}_{\perp,i} x_i$

- probabilistic interpretation!
- no relativistic corrections: Galilean subgroup! (MB,2000)
 - ↪ corollary: interpretation of 2d-FT of $F_1(Q^2)$ as charge density in transverse plane also free from relativistic corrections (MB,2003;G.A.Miller, 2007)

$q(x, \mathbf{b}_\perp)$ for unpol. p

unpolarized proton

- $q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$
 - $F_1(-\Delta_\perp^2) = \int dx H(x, 0, -\Delta_\perp^2)$
 - x = momentum fraction of the quark
 - \mathbf{b}_\perp relative to \perp center of momentum
 - small x : large 'meson cloud'
 - larger x : compact 'valence core'
 - $x \rightarrow 1$: active quark becomes center of momentum
- ↪ $\vec{b}_\perp \rightarrow 0$ (narrow distribution) for $x \rightarrow 1$



proton polarized in $+\hat{x}$ direction
no axial symmetry!

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

$$-\frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

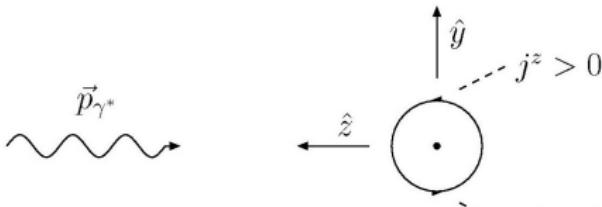
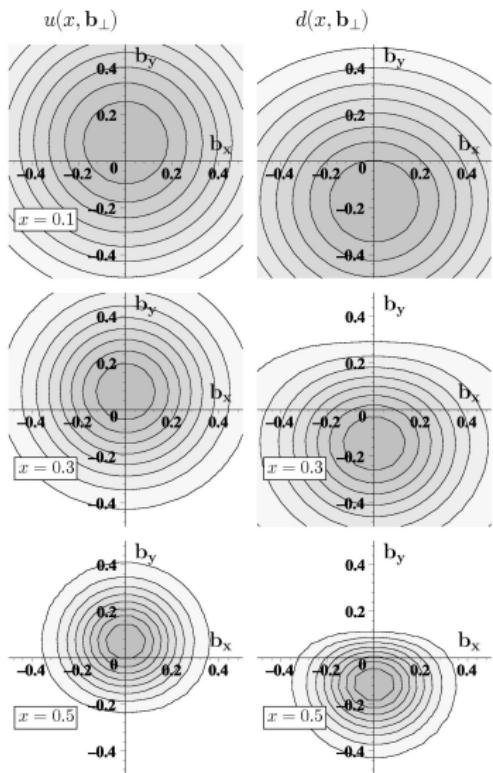
Physics: relevant density in DIS is
 $j^+ \equiv j^0 + j^3$ and left-right asymmetry
from j^3

intuitive explanation

- moving Dirac particle with anomalous magnetic moment has electric dipole moment \perp to \vec{p} and \perp magnetic moment
- γ^* 'sees' flavor dipole moment of oncoming nucleon

Impact parameter dependent quark distributions

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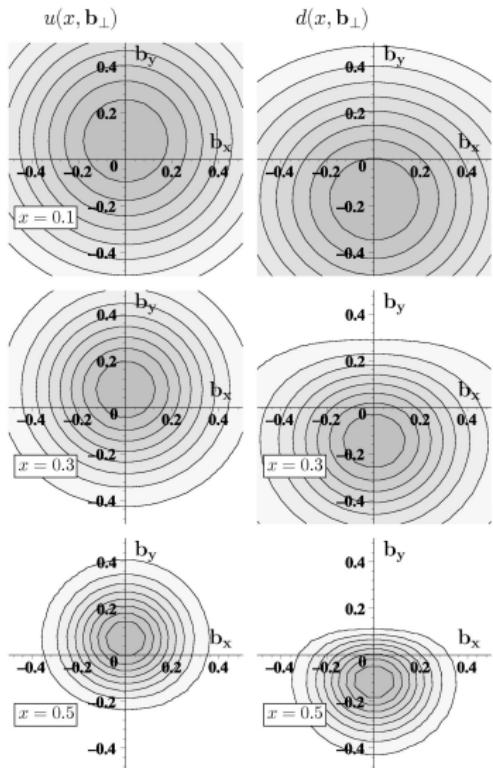


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Physics: relevant density in DIS is $j^+ \equiv j^0 + j^3$ and left-right asymmetry from j^3



proton polarized in $+\hat{x}$ direction

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} H_q(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

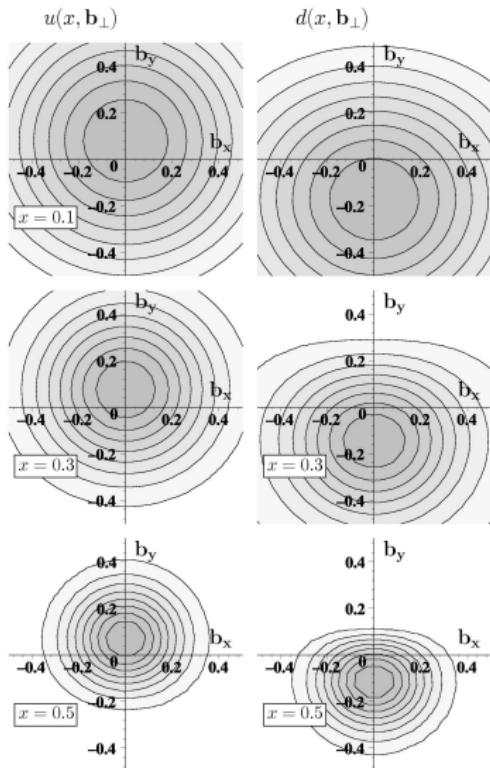
$$- \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} E_q(x, -\Delta_\perp^2) e^{-i \mathbf{b}_\perp \cdot \Delta_\perp}$$

sign & magnitude of the average shift

model-independently related to p/n
anomalous magnetic moments:

$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y$$

$$= \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_q}{2M}$$



sign & magnitude of the average shift
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$$= \frac{1}{2M} \int dx E_q(x, 0) = \frac{\kappa_q}{2M}$$

$$\kappa^p = 1.913 = \frac{2}{3} \kappa_u^p - \frac{1}{3} \kappa_d^p + \dots$$

- u -quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

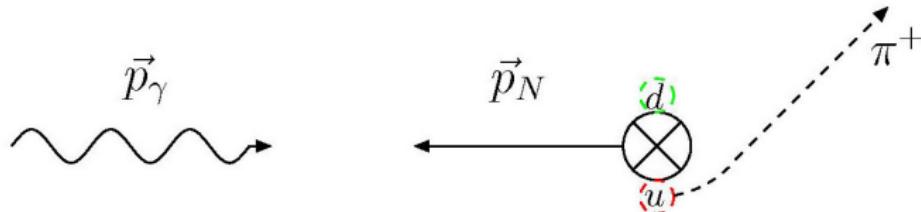
↪ shift in $+\hat{y}$ direction

- d -quarks: $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$

↪ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 \text{ fm})$!!!!

example: $\gamma p \rightarrow \pi X$



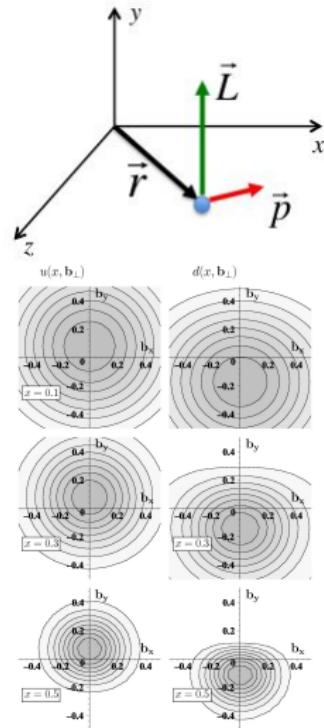
- u, d distributions in \perp polarized proton have left-right asymmetry in \perp position space (T-even!); sign “determined” by κ_u & κ_d
- attractive final state interaction (FSI) deflects active quark towards the center of momentum
- ↗ FSI translates position space distortion (before the quark is knocked out) in $+\hat{y}$ -direction into momentum asymmetry that favors $-\hat{y}$ direction → **chromodynamic lensing**

\Rightarrow

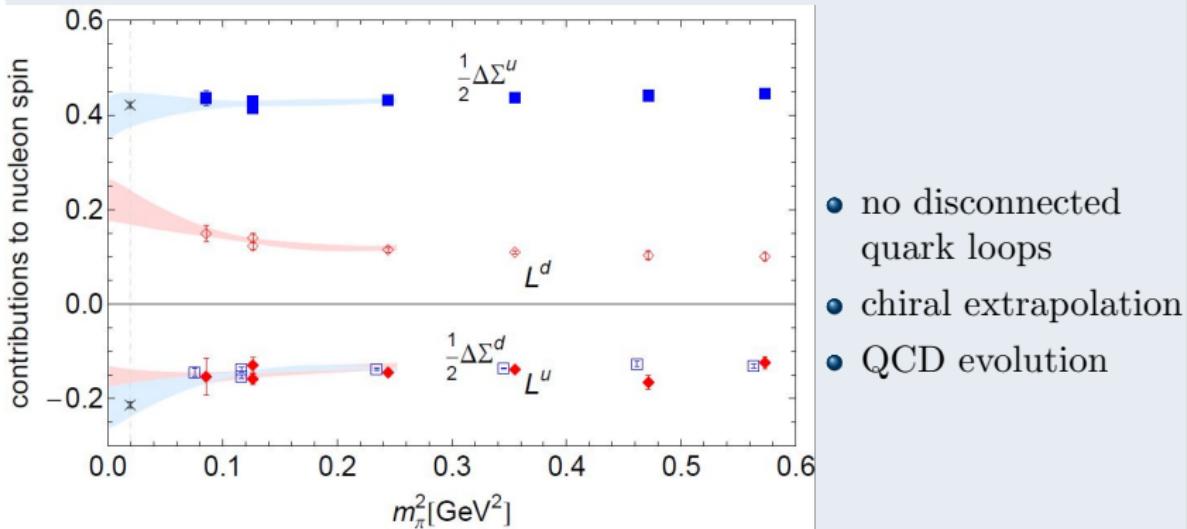
$\kappa_p, \kappa_n \longleftrightarrow$ sign of SSA!!!!!!! (MB,2004)

- confirmed by HERMES & COMPASS data

- $L_x = yp_z - zp_y$
- if state invariant under rotations about \hat{x} axis then $\langle yp_z \rangle = -\langle zp_y \rangle$
- $\langle L_x \rangle = 2\langle yp_z \rangle$
- GPDs provide simultaneous information about p_z & \mathbf{b}_{\perp}
- use quark GPDs to determine angular momentum carried by quarks
- $J_q^x = \frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$ (X.Ji, 1996)
- interpretation in terms of 3D distribution (MB,2001,2005)



lattice: (lattice hadron physics collaboration - LHPC)



$$J^q = \frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]$$

$$L^q = J^q - \frac{1}{2} \Delta\Sigma^q$$

QED with electrons

$$\begin{aligned}
 \vec{J}_\gamma &= \int d^3r \vec{r} \times (\vec{E} \times \vec{B}) = \int d^3r \vec{r} \times [\vec{E} \times (\vec{\nabla} \times \vec{A})] \\
 &= \int d^3r [E^j (\vec{r} \times \vec{\nabla}) A^j - \vec{r} \times (\vec{E} \cdot \vec{\nabla}) \vec{A}] \\
 &= \int d^3r [E^j (\vec{r} \times \vec{\nabla}) A^j + (\vec{r} \times \vec{A}) \vec{\nabla} \cdot \vec{E} + \vec{E} \times \vec{A}]
 \end{aligned}$$

- replace 2nd term (eq. of motion $\vec{\nabla} \cdot \vec{E} = ej^0 = e\psi^\dagger\psi$), yielding

$$\vec{J}_\gamma = \int d^3r [\psi^\dagger \vec{r} \times e\vec{A}\psi + E^j (\vec{r} \times \vec{\nabla}) A^j + \vec{E} \times \vec{A}]$$

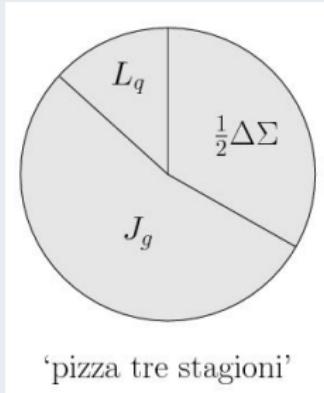
- $\psi^\dagger \vec{r} \times e\vec{A}\psi$ cancels similar term in electron OAM $\psi^\dagger \vec{r} \times (\vec{p} - e\vec{A})\psi$

↪ decomposing \vec{J}_γ into spin and orbital also shuffles angular momentum from photons to electrons!

The Nucleon Spin Pizzas

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Ji decomposition



$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + J_g$$

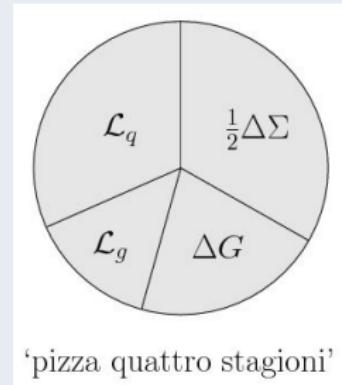
$$\frac{1}{2} \Delta q = \frac{1}{2} \int d^3x \langle P, S | q^\dagger(\vec{x}) \Sigma^3 q(\vec{x}) | P, S \rangle$$

$$\mathcal{L}_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) (\vec{x} \times i\vec{D})^3 q(\vec{x}) | P, S \rangle$$

$$J_g = \int d^3x \langle P, S | [\vec{x} \times (\vec{E} \times \vec{B})]^3 | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe-Manohar decomposition



light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P, S | \bar{q}(\vec{r}) \gamma^+(\vec{r} \times i\vec{\partial})^z \bar{q}(\vec{r}) | P, S \rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \langle P, S | \text{Tr} F^{+i} A^j | P, S \rangle$$

$$\mathcal{L}_g = 2 \int d^3r \langle P, S | \text{Tr} F^{+j} (\vec{x} \times i\vec{\partial})^z A^j | P, S \rangle$$

manifestly gauge invariant definition
for each term exists (\rightarrow Hatta)

Ji decomposition

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + L_q + J_g$$

$$\frac{1}{2} \Delta q = \frac{1}{2} \int d^3x \langle P, S | q^\dagger(\vec{x}) \Sigma^3 q(\vec{x}) | P, S \rangle$$

$$L_q = \int d^3x \langle P, S | q^\dagger(\vec{x}) (\vec{x} \times i\vec{D})^3 q(\vec{x}) | P, S \rangle$$

$$J_g = \int d^3x \langle P, S | [\vec{x} \times (\vec{E} \times \vec{B})]^3 | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

Jaffe-Manohar decomposition

light-cone framework & gauge $A^+ = 0$

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

$$\mathcal{L}_q = \int d^3r \langle P, S | \bar{q}(\vec{r}) \gamma^+ (\vec{r} \times i\vec{\partial})^z q(\vec{r}) | P, S \rangle$$

$$\Delta G = \varepsilon^{+-ij} \int d^3r \langle P, S | \text{Tr} F^{+i} A^j | P, S \rangle$$

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manifestly gauge invariant definitions
for each term exist (\rightarrow Hatta)

- GPDs $\longrightarrow L^q$
- $\overleftrightarrow{p \cdot p} \longrightarrow \Delta G \longrightarrow \mathcal{L} \equiv \sum_{i \in q,g} \mathcal{L}^i$
- QED: $\mathcal{L}^e \neq L^e$ [M.B. + Hikmat BC, PRD **79**, 071501 (2009)]
- $\mathcal{L}^q - L^q = ?$
 - can we calculate/predict/measure the difference?
 - what does it represent?

Wigner Functions (Belitsky, Ji, Yuan; Metz et al.)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ q(\xi) | P S \rangle.$$

- TMDs: $f(x, \mathbf{k}_\perp) = \int d^2 \mathbf{b}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$
- GPDs: $q(x, \mathbf{b}_\perp) = \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$
- $L_z = \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) (b_x k_y - b_y k_x)$
- need to include Wilson-line gauge link $\mathcal{U}_{t\xi} \sim \exp \left(i \frac{g}{\hbar} \int_0^\xi \vec{A} \cdot d\vec{r} \right)$ to connect 0 and ξ (Ji, Yuan; Hatta; Lorcé;...)
- ↪ crucial for SSAs in SIDIS et al.

Light-Cone Staple for $\mathcal{U}_{0\xi}^{\pm LC}$ (Hatta)

straight line (Ji et al.)

straigth Wilson line from 0 to ξ yields
Ji-OAM:

$$L^q = \int d^3 x \langle P, S | q^\dagger(\vec{x}) (\vec{x} \times i \vec{D})^z q(\vec{x}) | P, S \rangle$$



'light-cone staple' yields $\mathcal{L}_{Jaffe-Manohar}$

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ \mathcal{U}_{0\xi} q(\xi) | P S \rangle.$$

$\langle \vec{k}_\perp \rangle \equiv \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) \vec{k}_\perp$ depends on choice of path!

straight-line gauge link

$$\langle \vec{k}_\perp \rangle = \langle P, S | \bar{q}(\vec{x}) \gamma^+ i \vec{D} q(\vec{x}) | P, S \rangle$$

- $i \vec{D} = i \vec{\partial} - g \vec{A}(\vec{x})$

- $\langle \vec{k}_\perp \rangle = 0$ (T-odd !)

light-cone staple



- correct choice for \mathbf{k}_\perp distributions relevant for SIDIS

$$\langle \vec{\mathcal{K}}_\perp \rangle = \langle P, S | \bar{q}(\vec{x}) \gamma^+ i \vec{\mathcal{D}} q(\vec{x}) | P, S \rangle$$

- $i \vec{\mathcal{D}} = i \vec{\partial} - g \vec{A}(x^- = \infty, \mathbf{x}_\perp)$ $A^+ = 0$

- $\langle \vec{\mathcal{K}}_\perp \rangle \neq 0$ (FSI! Brodsky, Hwang, Schmidt)

difference $\langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle$

$$\langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_{\perp}) q(\vec{x}) | P, S \rangle$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

straight-line gauge link

$$\langle \vec{k}_{\perp} \rangle = \langle P, S | \bar{q}(\vec{x}) \gamma^+ i \vec{D} q(\vec{x}) | P, S \rangle$$

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light-cone staple



- correct choice for \mathbf{k}_{\perp} distributions relevant for SIDIS

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- $i \vec{\mathcal{D}} = i \vec{\partial} - g \vec{A}(x^- = \infty, \mathbf{x}_{\perp}) + g \int_{x^-}^{\infty} dr^- \vec{\partial} A^+$
- $i \mathcal{D}^j = i \partial^j - g A^j(x^-, \mathbf{x}_{\perp}) - g \int_{x^-}^{\infty} dr^- F^{+j}$

difference $\langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle$

$$\langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \int_{x^-}^{\infty} dr^- F^{+\perp}(r^-, \mathbf{x}_{\perp}) q(\vec{x}) | P, S \rangle$$

color Lorentz Force acting on ejected quark (Qiu, Sterman)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B} \right)^y \text{ for } \vec{v} = (0, 0, -1)$$

Impulse due to FSI

$\Delta \vec{k}_{\perp}^q \equiv \langle \vec{\mathcal{K}}_{\perp}^q \rangle - \langle \vec{k}_{\perp}^q \rangle$
 = (average) change in \perp momentum due to FSI!

straight-line gauge link

$$\langle \vec{k}_{\perp} \rangle = \langle P, S | \bar{q}(\vec{x}) \gamma^+ i \vec{D} q(\vec{x}) | P, S \rangle$$

- $i \vec{D} = i \vec{\partial} - g \vec{A}(\vec{x})$

- $\langle \vec{k}_{\perp} \rangle = 0$ (T-odd !)

light-cone staple



- correct choice for \mathbf{k}_{\perp} distributions relevant for SIDIS

$$\langle \vec{\mathcal{K}}_{\perp} \rangle = \langle P, S | \bar{q}(\vec{x}) \gamma^+ i \vec{D} q(\vec{x}) | P, S \rangle$$

- $i \vec{D} = i \vec{\partial} - g \vec{A}(x^- = \infty, \mathbf{x}_{\perp}) + g \int_{x^-}^{\infty} dr^- \vec{\partial} A^+$

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difference $\langle \vec{K}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle$

$$\langle \vec{K}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \int_{x^-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x}) | P, S \rangle$$

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Corollary: $d_2 \leftrightarrow$ average \perp force on quark in DIS from \perp pol target
polarized DIS: MB, PRD 88 (2013) 114502

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$ • $\sigma_{LT} \propto g_T \equiv g_1 + g_2$
- 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1
- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \langle P, S | \bar{q}(0) \gamma^+ g F^{+y}(0) q(0) | P, S \rangle$$

matrix element defining d_2

\leftrightarrow 1st integration point in QS-integral

difference $\langle \vec{K}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle$

$$\langle \vec{K}_\perp^q \rangle - \langle \vec{k}_\perp^q \rangle = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \int_{x_-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) q(\vec{x}) | P, S \rangle$$

color Lorentz Force acting on ejected quark (Qiu, Sterman)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B} \right)^y \text{ for } \vec{v} = (0, 0, -1)$$

Corollary: $d_2 \leftrightarrow$ average \perp force on quark in DIS from \perp pol target
polarized DIS: MB, PRD 88 (2013) 114502

- $\sigma_{LL} \propto g_1 - \frac{2Mx}{\nu} g_2$ • $\sigma_{LT} \propto g_T \equiv g_1 + g_2$
- 'clean' separation between g_2 and $\frac{1}{Q^2}$ corrections to g_1
- $g_2 = g_2^{WW} + \bar{g}_2$ with $g_2^{WW}(x) \equiv -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$

$$d_2 \equiv 3 \int dx x^2 \bar{g}_2(x) = \frac{1}{2MP^{+2}S^x} \langle P, S | \bar{q}(0) \gamma^+ gF^{+y}(0) q(0) | P, S \rangle$$

sign of d_2^q opposite Sivers $f_{1T}^{\perp q}$ \leftrightarrow \perp deformation of quark distributions

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ \mathcal{U}_{0\xi} q(\xi) | P S \rangle.$$

W and thus $L_z = \int dx \int d^2 \mathbf{b}_\perp \int d^2 \mathbf{k}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp) (b_x k_y - b_y k_x)$ may depend on choice of path!

straight line (Ji et al.)

straigth Wilson line from 0 to ξ yields

$$L^q = \int d^3 x \langle P, S | q^\dagger(\vec{x}) \left(\vec{x} \times i \vec{D} \right) \overset{z}{\vec{q}}(\vec{x}) | P, S \rangle$$

- $i \vec{D} = i \vec{\partial} - g \vec{A}(\vec{x})$
- same as Ji-OAM
- $f(x, \mathbf{k}_\perp) = \int d^2 \mathbf{b}_\perp W(x, \vec{b}_\perp, \vec{k}_\perp)$
not the TMDs relevant for SIDIS
(missing FSI!)

Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

$$W(x, \vec{b}_\perp, \vec{k}_\perp) \equiv \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^3} e^{ik \cdot \xi} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \langle P' S' | \bar{q}(0) \gamma^+ \mathcal{U}_{0\xi} q(\xi) | PS \rangle.$$

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Light-Cone Staple for $\mathcal{U}_{0\xi}^{\pm LC}$ (Hatta)

- want Wigner function that yields TMDs relevant for SIDIS when integrated $d^2 \mathbf{b}_\perp$
- ↪ path for gauge link → 'light-cone staple' → $\mathcal{U}_{0\xi}^{+LC}$

$$\mathcal{L}_+^q = \int d^3 x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{\mathcal{D}})^z q(\vec{x}) | P, S \rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp) \quad (A^+ = 0)$$



Wigner Functions with gauge link $\mathcal{U}_{0\xi}$ (Ji, Yuan; Hatta)

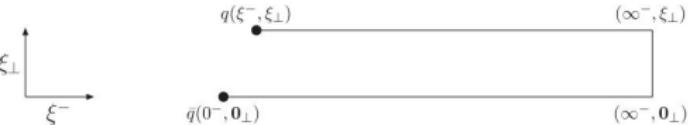
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$$\begin{aligned} \mathcal{L}_+^q &= \int d^3 x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{\mathcal{D}})^z q(\vec{x}) | P, S \rangle \\ i\vec{\mathcal{D}} &= i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp) + g \int_{x_-^-}^\infty dr^- \vec{\partial} A^+ \\ i\mathcal{D}^j &= i\partial^j - g A^j(x^-, \mathbf{x}_\perp) - g \int_{x_-^-}^\infty dr^- F^{+j} \end{aligned}$$



straight line (\rightarrow Ji)

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \textcolor{red}{L}_q + J_g$$

$$\textcolor{red}{L}_q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \left(\vec{x} \times i\vec{D} \right)^z q(\vec{x}) | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}$

light-cone staple (\rightarrow Jaffe-Manohar)

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \textcolor{red}{\mathcal{L}}_q + \Delta G + \mathcal{L}_g$$

$$\textcolor{red}{\mathcal{L}}^q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \left(\vec{x} \times i\vec{\mathcal{D}} \right)^z q(\vec{x}) | P, S \rangle$$

$$i\vec{\mathcal{D}} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$$

difference $\mathcal{L}^q - L^q$

$$\mathcal{L}^q - L^q = -g \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ \left[\vec{x} \times \int_{x^-}^\infty dr^- F^{+\perp}(r^-, \mathbf{x}_\perp) \right]^z q(\vec{x}) | P, S \rangle$$

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x$$

straight line ($\rightarrow \text{Ji}$)

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \textcolor{red}{L}_q + J_g$$

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color Lorentz Force acting on ejected quark (MB: arXiv:08103589)

$$\sqrt{2}F^{+y} = F^{0y} + F^{zy} = -E^y + B^x = -\left(\vec{E} + \vec{v} \times \vec{B}\right)^y \text{ for } \vec{v} = (0, 0, -1)$$

straight line ($\rightarrow J_i$)

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \textcolor{red}{L}_q + J_g$$

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$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \textcolor{red}{\mathcal{L}}_q + \Delta G + \mathcal{L}_g$$

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Torque along the trajectory of q

$$T^z = \left[\vec{x} \times \left(\vec{E} - \hat{\vec{z}} \times \vec{B} \right) \right]^z$$

Change in OAM

$$\Delta L^z = \int_{x^-}^\infty dr^- \left[\vec{x} \times \left(\vec{E} - \hat{\vec{z}} \times \vec{B} \right) \right]^z$$

straight line ($\rightarrow J_i$)

$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + J_g$$

$$\mathcal{L}_q = \int d^3x \langle P, S | \bar{q}(\vec{x}) \gamma^+ (\vec{x} \times i\vec{D})^z q(\vec{x}) | P, S \rangle$$

- $i\vec{D} = i\vec{\partial} - g\vec{A}(\vec{x})$

light-cone staple (\rightarrow Jaffe-Manohar)

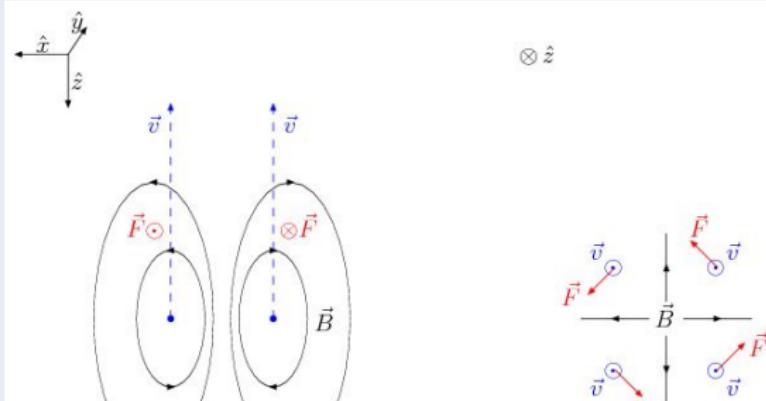
$$\frac{1}{2} = \sum_q \frac{1}{2} \Delta q + \mathcal{L}_q + \Delta G + \mathcal{L}_g$$

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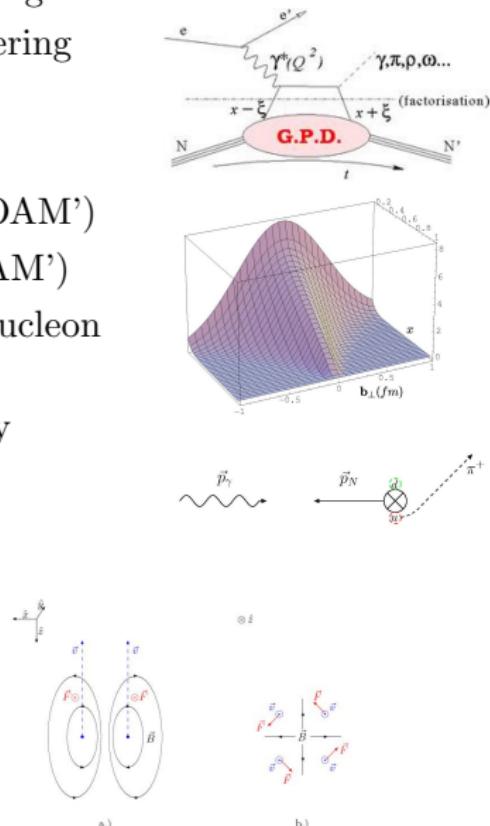
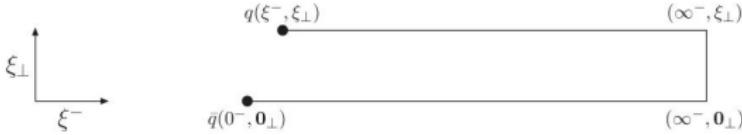
- $i\vec{D} = i\vec{\partial} - g\vec{A}(x^- = \infty, \mathbf{x}_\perp)$

difference $\mathcal{L}^q - L^q$ (\rightarrow Wakamatsu: L_{pot}^q) $\mathcal{L}^q - L^q = \Delta L_{FSI}^q =$ change in OAM as quark leaves nucleon

example: torque in magnetic dipole field



- generalized parton distributions and 3d imaging
- final state interactions in deep-inelastic scattering
- GPDs and OAM
- TMDs and OAM from Wigner distributions
- straight-(Wilson)line gauge link $\rightarrow L^q$ ('Ji-OAM')
- light-cone staple- gauge link $\rightarrow \mathcal{L}_+^q$ ('JM-OAM')
- $\mathcal{L}_+^q - L^q =$ change in OAM as quark leaves nucleon (torque due to FSI)
- $A^+ = 0$ gauge (with anti-symmetric boundary condition) $\mathcal{L}_+^q \rightarrow$ canonical OAM (Jaffe-Manohar-OAM)
- JM-OAM from lattice QCD



antisymm. boundary condition

- $A^+ = 0$
- fix residual gauge inv. $A^\mu \rightarrow A^\mu + \partial^\mu \phi(\vec{x}_\perp)$
by imposing $\vec{A}_\perp(\infty, \vec{x}_\perp) = -\vec{A}_\perp(-\infty, \vec{x}_\perp)$
- $\vec{A}_\perp(\infty, \vec{x}_\perp) - \vec{A}_\perp(-\infty, \vec{x}_\perp) = \int dx^- F^{+\perp}$ gauge inv.
- \mathcal{L}_+ involves $i\vec{\mathcal{D}}_+ = i\vec{\partial} - g\vec{A}(\infty, \mathbf{x}_\perp)$
- \mathcal{L}_- involves $i\vec{\mathcal{D}}_- = i\vec{\partial} - g\vec{A}(-\infty, \mathbf{x}_\perp)$
- $\mathcal{L}_+ = \mathcal{L}_- \rightarrow$ no contribution from $\vec{A}(\infty, \mathbf{x}_\perp)$
 \hookrightarrow 'naive' JM OAM $\mathcal{L}_{JM} = \mathcal{L}_+ = \mathcal{L}_-$

alternative: Bashinsky-Jaffe

- $A^+ = 0$
- $\vec{x} \times i\vec{\partial} \rightarrow \vec{x} \times [i\vec{\partial} - g\vec{A}(\vec{x}_\perp)]$
- $\vec{A}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-}$

antisymm. boundary condition

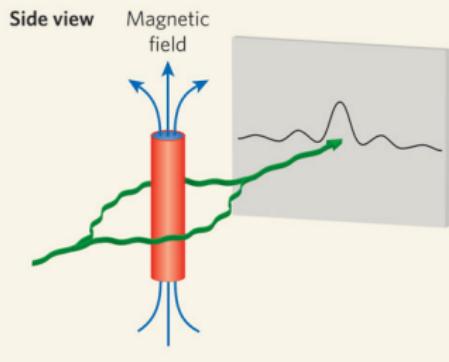
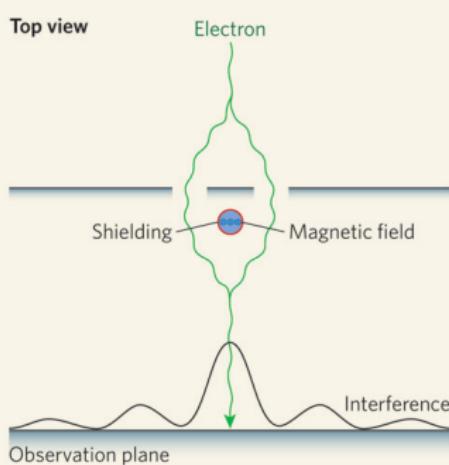
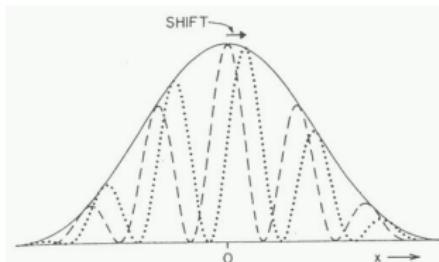
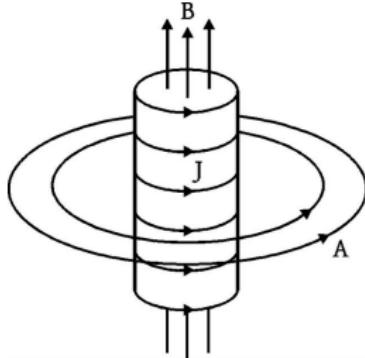
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- ↪ 'naive' JM OAM $\mathcal{L}_{JM} = \mathcal{L}_+ = \mathcal{L}_-$

alternative: Bashinsky-Jaffe

- $A^+ = 0$
 - $\vec{x} \times i\vec{\partial} \rightarrow \vec{x} \times [i\vec{\partial} - g\vec{\mathcal{A}}(\vec{x}_\perp)]$
 - $\vec{\mathcal{A}}_\perp(\vec{x}_\perp) = \frac{\int dx^- \vec{A}_\perp(x^-, \vec{x}_\perp)}{\int dx^-} = \frac{1}{2} (\vec{A}_\perp(\infty, \vec{x}_\perp) + \vec{A}_\perp(-\infty, \vec{x}_\perp))$
- ↪ $\mathcal{L}_{JB} = \frac{1}{2} (\mathcal{L}_+ + \mathcal{L}_-) = \mathcal{L}_+ = \mathcal{L}_-$

Aharonov Bohm effect

- double slit experiment with solenoid between beams
- no magnetic field at beam
- vector potential \vec{A} modifies QM phase
 $\sim \exp\left(i\frac{e}{\hbar} \oint \vec{A} \cdot d\vec{r}\right) = \exp\left(i\frac{e}{\hbar} \iint \vec{B} \cdot d\vec{S}\right)$
- $B \neq 0 \Rightarrow$ shifts interference pattern

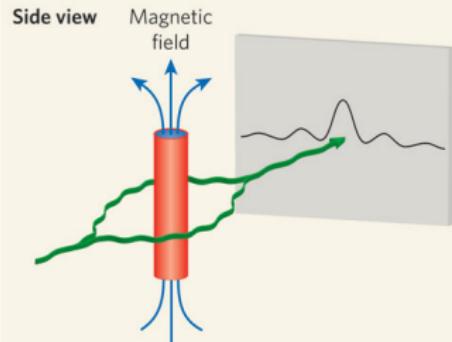
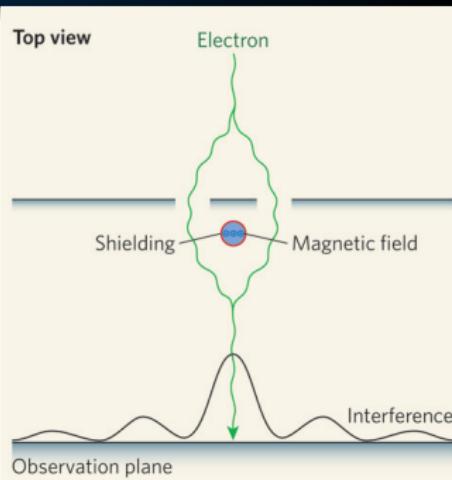


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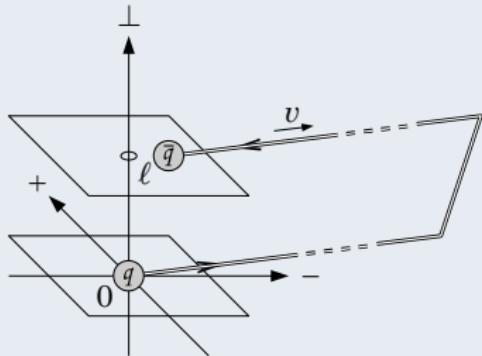
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transverse SSA

- Wilson line phase factor
 $\sim \exp\left(i\frac{g}{\hbar} \int \vec{A} \cdot d\vec{r}\right)$ crucial for SSAs in SIDIS et al.
- However, contrary to SSAs in SIDIS, AB-effect does NOT generate a net (average), k_{\perp}
 ↪ example for Ehrenfest's theorem



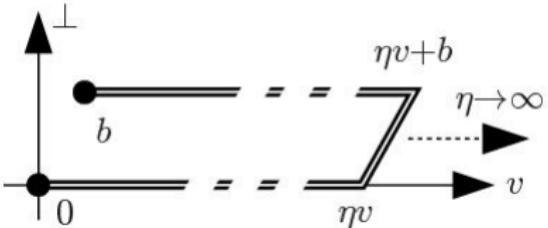
challenge



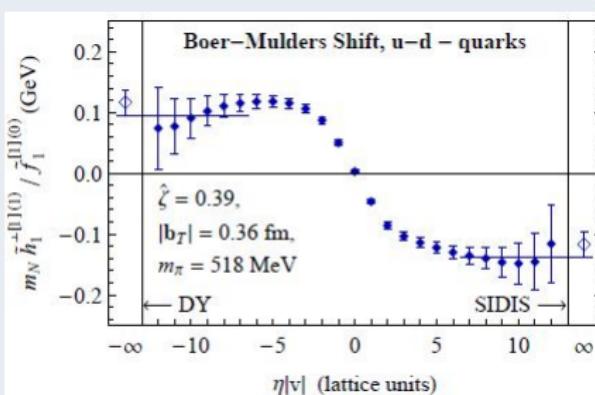
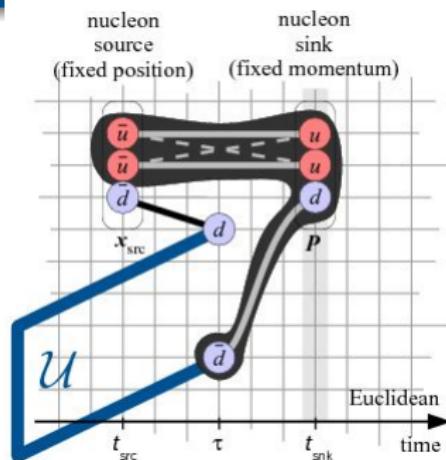
- TMDs/Wigner functions relevant for SIDIS require (near) light-like Wilson lines
- on Euclidean lattice, all distances are space-like

TMDs in lattice QCD

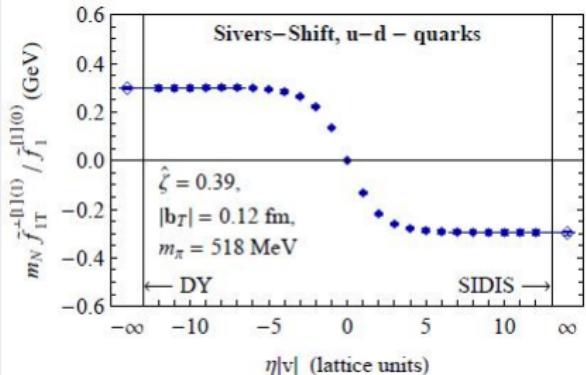
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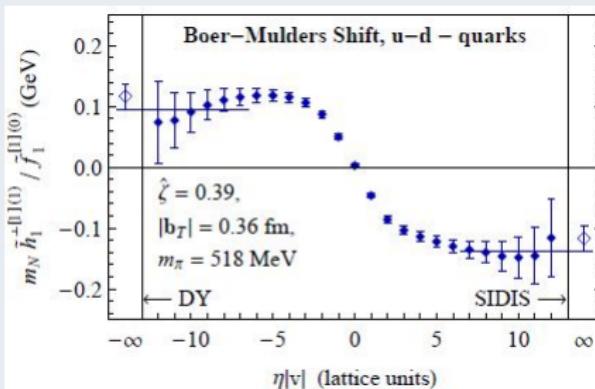
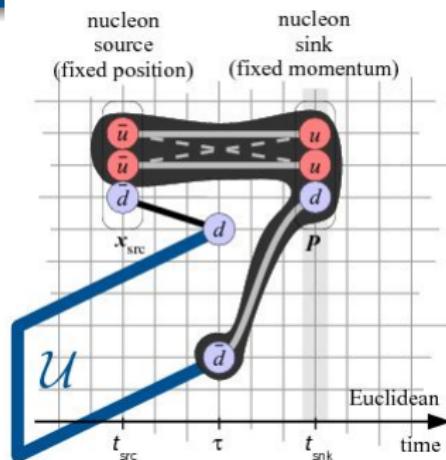
- calculate space-like staple-shaped Wilson line pointing in \hat{z} direction; length $L \rightarrow \infty$
- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios
- extrapolate/evolve to $P_z \rightarrow \infty$



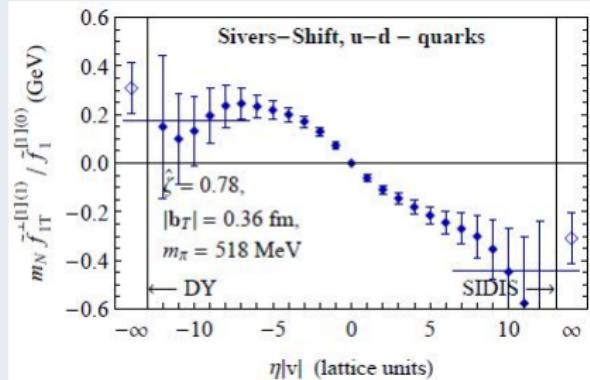
$$f_{1T,SIDIS}^\perp = -f_{1T,DY}^\perp \text{ (Collins)}$$



$f_{1T}^\perp(x, \mathbf{k}_\perp)$ is \mathbf{k}_\perp -odd term in quark-spin averaged momentum distribution in \perp polarized target



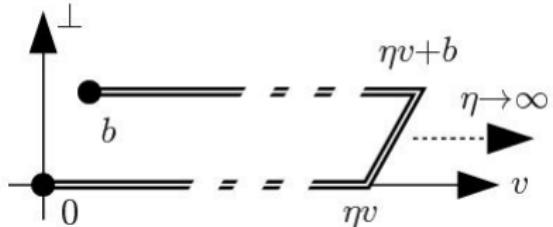
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TMDs in lattice QCD

B. Musch, P. Hägler, M. Engelhardt



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- momentum projected nucleon sources/sinks
- remove IR divergences by considering appropriate ratios
- extrapolate/evolve to $P_z \rightarrow \infty$

next: Orbital Angular Momentum

- same operator as for TMDs, only nonforward matrix elements:
 - momentum transfer provides position space information ($\rightarrow \mathbf{r}_\perp \times \mathbf{k}_\perp$)
 - staple with long side in \hat{z} direction
 - (large) nucleon momentum in \hat{z} direction
 - small momentum transfer in \hat{y} direction
- generalized TMD F_{14} (Metz et al.)
- quark OAM
- renormalization same as f_{1T}^\perp
- study ratios...