Particle Production in Hybrid Formalizm: Dotting the i's

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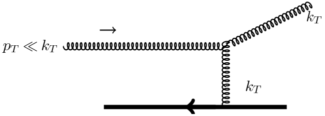
[T. Altinoluk, N. Armesto, G. Beuf, A.K. and M. Lublinsky, Phys. Rev. D in press]

Introduction

Desperately searching for saturation. Where to look for it?
Forward particle production on a dense target: projectile is partonic and perturbatively under control; target is dense and therefore likely saturated.
This observable has been studied in the last several years within the "Hybrid" formalism

"Hybrid" Formalism (Dimitru+ Hayashigaki+ Jalilian-Marian):

- Projectile wave function is treated perturbatively (collinear factorization including DGLAP evolution)
- Target is assumed to be dense: distribution of strong color fields which transfers transverse momentum to the propagating partonic configuration
  The original DHJM LO expression for inclusive hadron production:



$$\frac{dN}{d^2k d\eta} = \frac{1}{(2\pi)^2}\int_{x_F}^{1}\frac{dz}{z^2}\Big[x_1 f_g(x_1,Q^2)N_A(x_2,\frac{k}{z},b=0)D_{h/g}(z,Q)$$
$$+\Sigma_q x_1 f_q(x_1,Q^2)N_F(x_2,\frac{k}{z},b=0)D_{h/q}(z,Q)\Big]$$

- where longitudinal momentum fractions: $x_F = \frac{k}{\sqrt{s}}e^{\eta}$; $x_1 = \frac{x_F}{z}$; $x_2 = x_1 e^{-2\eta}$

NLO bites.

Initial Hybrid fits of RHIC data were reasonable.

And then we got greedy (or honest).

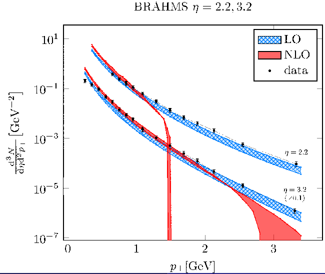
NLO corrections:

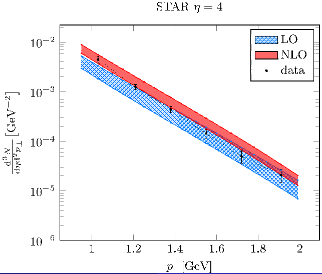
T.Altinoluk and A. K. - 2011 Elastic and Inelastic contributions (part of NLO); G.A. Chirilli, B.W. Xiao, F. Yuan - 2012 Full NLO calculation...

Numerical resuts:

J. Jalilian Marian and A. Rezaeian- 2011; A.M.Stasto, B.W.Xiao, D. Zaslavsky, - 2013 Numerical analysis...

Trouble: effect of NLO corrections very large, and disturbingly negative





Why is that?

- Maybe perturbation theory requires resummation? After all BFKL at NLO is problematic.
- But maybe NLO calculation is not quite NLO. There are some subtleties. Let's check it out.

First Things First - The Setup

Choose the right frame:

- The projectile wave function is perturbative.
- The scattering on the target is eikonal.

The projectile needs to move fast, but not too fast
Energy is large, so most of the energy is carried by the target.
PROJ frame:

$$P^{+}_{P,\ PROJ} = \frac{s}{2P^{-}_{T,\ PROJ}}$$

Scaling with energy: $P^{-}_{T,\ PROJ} \propto s$; $P^{+}_{P,\ PROJ} = \mathrm{const}$.
The target is evolved to s from an initial s_0 via BK evolution.
s_0 is arbitrary within the limits (has to be large enough for eikonal approximation to hold).
To get to s_0 we boost the projectile from its rest frame to rapidity Y_P, and the target from its rest frame by rapidity Y^0_T

$$s_0 = 2P^{+}_{P,PROJ}P^{-0}_{T}; \quad P^{+}_{P,PROJ} = \frac{M_P}{\sqrt{2}}\, e^{Y_P}; \quad P^{-0}_{T} = \frac{M_T}{\sqrt{2}}\, e^{Y^0_T}; \quad P^{-}_{T} = \frac{M_T}{\sqrt{2}}\, e^{Y^0_T + Y_T}$$

The projectile at any energy (x_p - Bjorken x of the produced hadron; $Y_0 = O(1)$)

$$Y_p = \ln\frac{1}{x_p} + Y_0$$

The target is evolved from s_0 by $Y_T = \ln\frac{s}{s_0}$,

Rapidity Balance

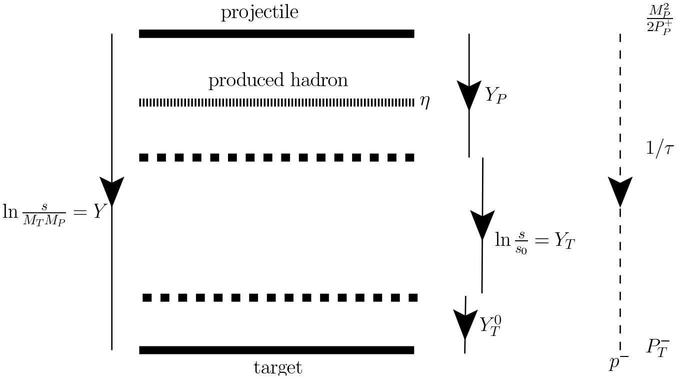


Figure: The rapidity balance.

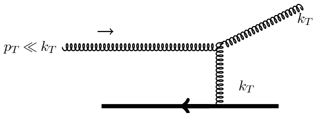
Y_T vs Y_g

Common practice: evolve the target to rapidity (as in DHJM)

$$Y_g = \ln \frac{1}{x_g}; \qquad x_g = \frac{p_\perp}{\sqrt{s}} e^{-\eta}$$

Y_T is rather different: does not depend on $p_\perp$.

Kinematic argument



- Incoming projectile parton carries momentum $(p^+, 0, 0)$;
- Outgoing parton carries momentum $(p^+, p^-, p_\perp)$;
- Is on shell $\rightarrow p^- = p_\perp^2 / 2p^+$.
- During scattering $p^- = \frac{p_\perp^2}{2p^+} = e^{-\eta} \frac{p_\perp}{\sqrt{2}}$ is transferred from the target.
- *If* the momentum has been transferred by a single gluon of the target, the gluon must carry at least this p^-,
- Longitudinal momentum fraction of the target $x_g = \frac{p^-}{P^-} = e^{-\eta} \frac{p_\perp}{\sqrt{s}}$
- A hadronic wave function is dominated by the softest gluons.
- Thus x_g is the longitudinal momentum fraction of the softest gluons in the target, and the target has to be evolved to Y_g.

Not Y_g.

But the target is dense. Mulitple scatterings dominate!

x_g is the upper limit on x_{Bj} of the target gluons, *sic.* there must be more evolution than Y_g.

Should Y_T depend on $p_\perp$?

- In the dense scattering regime, $k_\perp$ "'random walks"' as the parton propagates through the target.
- Thus, $p_\perp^2 \propto N_g$, where N_g is the number of gluons exchanges.
- But k^- does not random walk - all the gluons in the target have same sign k^-.
- Thus $p^- \propto N_g$ - consistent with the on shell relation between p^- and $p_\perp$.
- So $p^- \approx N_g x_{Bj}$.
- Increasing $p_\perp$ of the on shell observed parton, does not change x_{Bj} of individual target gluons that participate in the scattering.
- *Ergo* Y_T should not depend on $p_\perp$.

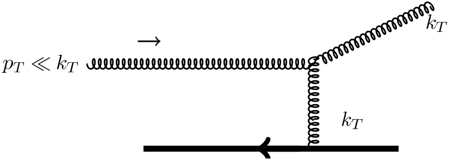
In the LO calculation it does not matter what is the evolution interval, as long as it is logarithmically $\ln s$ - at least in principle Y_T and Y_g are equally valid.

But we are doing NLO, and here it matters.

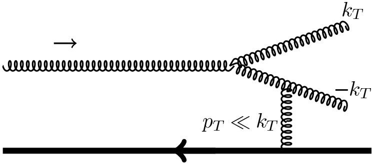
This is the first point of departure of our calculation: different evolution interval.

What scatters?

The basic LO processes that lead to particle production:



New NLO process: parton splits in the projectile wave function, the pair subsequently scatters on the target.



In the high energy approximation partons scatter eikonally:

$$|g,x\rangle \to S(x)|g,x\rangle; \qquad |g,x;g,y\rangle \to S(x)S(y)|g,x;g,y\rangle$$

What really scatters?

Of course, this is not quite true.

If all partonic configurations scattered, the cross section would be divergent - the ubiquituos rapidity divergence. This divergence is cutoff in the calculations and is "resummed" in the rapidity evolution of the target.

How do we cut off this divergence? Again, in leading order it does not matter, but in NLO one has to come up with something better than just " a cutoff".

There is a clear physical reason why not all configurations scatter eikonally.

The target has a finite longitudinal size τ. Partonic fluctuations which do not exist long enough to propagate through the target, cannot scatter.

Thus the physical parameter that cuts off part of the phase space is the Ioffe time.

Ioffe time

For example, a quark of the projectile fluctuates into a quark-gluon state

$$
\begin{aligned}
&|(q)\, x_B P^+, \alpha, s\rangle_D = |(q)\, x_B P^+, \alpha, s\rangle \\
&+g \int_{\xi, k_\perp} F_{(qg)}(x_B P^+, \xi, k_\perp)_{s\bar{s};j}\, t^a_{\alpha\beta} \\
&|(q)\ k_\perp, p^+ = (1-\xi) x_B P^+, \beta, \bar{s}; (g)\ -k_\perp, q^+ = \xi x_B P^+, a, j\rangle
\end{aligned}
$$

F- a perturbatively calculable amplitude.

The Ioffe (coherence) time for a $q-g$ configuration is

$$t_c = \frac{2(1-\xi)\xi x_B P^+}{k_\perp^2}$$

The $q-g$ pair scatters coherently only if $t_c > \tau$.

If $t_c < \tau$, the fluctuation is not resolved, and the scattering amplitude is equal to that of the parent quark.

What is τ ?

At initial energy s_0 we simply have $\tau = 1/P_T^- = 2P^+/s_0$
It turns out τ does not depend on energy.

Intuitively: the longitudinal size of the target does not depend on energy due to the cloud of soft gluons.

Mathematically: independence of the cross section on s_0.

Ioffe time restriction:

$$\frac{(1-\xi)\xi x_B}{k_\perp^2} > s_0^{-1}$$

Cuts off emissions of very soft gluons (ξ too small) and very large transverse momentum pairs (k_T too large).

The Final Result

The final expressions are long (multiple production channels) but all have the same structure. E.g.: hadron production from a fragmentation of the final state quark, which originates from the quark in the projectile wave function:

$$
\begin{aligned}
\frac{d\sigma^{q\to H}}{d^2p_\perp d\eta} &= \frac{1}{(2\pi)^2}\int\frac{d\zeta}{\zeta^2}D_H^q(\zeta)\frac{x_p}{\zeta}f_{p_\perp}^q\left(\frac{x_p}{\zeta}\right)\int_{y\bar{y}}e^{i\frac{p_\perp}{\zeta}(y-\bar{y})}s_{Y_T}[y,\bar{y}] \\
&+ \int\frac{d\zeta}{\zeta^2}D_H^q(\zeta)\frac{d\bar{\sigma}^q}{d^2p_\perp d\eta}\left(\frac{p_\perp}{\zeta},\frac{x_p}{\zeta}\right)
\end{aligned}
$$

$s_{Y_T}[y,\bar{y}]$ - dipole cross section evolved with BK equation from initial rapidity Y_T^0 by the rapidity interval $Y_T = \ln\frac{s}{s_0}$.

$$
\frac{\partial}{\partial Y}s(y,\bar{y}) = -\frac{\alpha_s N_c}{\pi}\int_{y,\bar{y},z}\frac{(y-\bar{y})^2}{(y-z)^2(\bar{y}-z)^2}\Big[s(y,\bar{y})-s(y,z)s(z,\bar{y})\Big] \qquad (1)
$$

The NLO piece:

$$
\frac{d\bar{\sigma}^q}{d^2p_\perp d\eta}(p_\perp,x) = \left[\frac{d\bar{\sigma}^q}{d^2p_\perp d\eta}(p_\perp,x)\right]_{Chirilli,Xiao,Yuan,Y_g\to Y_T} + \delta\sigma^q \qquad (2)
$$

The extra piece

$$\delta\sigma^q = \frac{g^2}{(2\pi)^3} N_c x_p f^q_{\mu^2}(x_p) \int_0^1 \frac{d\xi}{\xi} \int_{y\bar{y}z} e^{ip_\perp(y-\bar{y})} \left[A^i_\xi(y-z) - A^i_\xi(\bar{y}-z)\right]^2 \times \left[s(y,z)s(z,\bar{y}) - s(y,\bar{y})\right]$$

with the Ioffe time cutoff modified Weizsacker-Williams field

$$\begin{aligned} A^i_\xi(y-z) &= -i \int_{l^2_\perp < \xi x_p s_0} \frac{d^2 l_\perp}{(2\pi)^2} \frac{l^i_\perp}{l^2_\perp} e^{il_\perp(y-z)} \\ &= -\frac{1}{2\pi} \frac{(y-z)^i}{(y-z)^2} \left[1 - \mathrm{J}_0\left(|y-z|\sqrt{\xi x_p s_0}\right)\right] \end{aligned}$$

Almost like extra evolution of the leading order term, but not really!

Change the order of integration: ξ and $l_\perp$ in WW field:

$$\int d^2 l_\perp \int d^2 m_\perp \ln\left(\frac{1}{\xi_{min}}\right) \frac{d}{dY} s(l_\perp + p_\perp, m_\perp - p_\perp) \qquad (3)$$

with

$$\xi_{min} = \max\left\{\frac{l_\perp^2}{x_p s_0}, \frac{m_\perp^2}{x_p s_0}\right\}$$

Together with leading order, this is like effective evolution by

$$Y_T + \ln\frac{1}{\xi_{min}} = \ln\frac{1}{x_g} + \ln\frac{p_\perp^2}{l_\perp^2}$$

$l_\perp$ - momentum of the splitting, so does not exist at LO.
If splittings were dominated by $l_\perp = p_\perp$, this would effectively reproduce CXY.
But we do not expect this to be the case.
E.g. $l_\perp \sim Q_s$ should be very significant for $p_\perp > Q_s$.

Summary

We found corrections to previous results due to careful treatment of the physical Ioffe time cutoff, and a different treatment of the evolution scale at NLO.

These corrections are strictly NLO and do not involve resummations of higher orders.

Whether these differences are significant numerically remains to be seen.