Twist-3 Spin Observables for Single-Hadron Production in DIS

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- Introduction and Motivation

- A related observable: double-spin asymmetry $A_{LT}$ for $\vec{\ell} N^\uparrow \rightarrow \ell X$

- Single-spin asymmetry $A_{UT} (A_N)$ for $\ell N^\uparrow \rightarrow h X$

- Single-spin asymmetry $A_{UT} (A_N)$ for $\ell N \rightarrow \Lambda^\uparrow X$

- Double-spin asymmetry $A_{LT}$ for $\vec{\ell} N^\uparrow \rightarrow h X$

- Summary

talk mainly based on
arXiv:1411.6459, Kanazawa, A.M., Pitonyak, Schlegel
Introduction and Motivation

1. Data exist for twist-3 spin asymmetries in $\ell N \rightarrow h X$
   - example: $A_N$ for $\ell N^{\uparrow} \rightarrow h X$ (HERMES, 2013 / JLab Hall A, 2013)
     \[
     A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}
     \]
     \[
     x_F = \frac{2P_{hL}}{\sqrt{s}}
     \]

   (HERMES, 2013)

   - more data: $A_N$ for $\ell N \rightarrow \Lambda^{\uparrow} X$ (HERMES, 2014)
     \[
     A_{LT} \quad \text{for} \quad \vec{\ell} N^{\uparrow} \rightarrow h X \quad \text{(JLab Hall A, 2015)}
     \]

   - can one understand those data, and what can one learn from them?
2. Related asymmetries in processes like $pp \to hX$

- example: $A_N$ for $pp^\uparrow \to \pi X$

(plot from Aidala, Bass, Hasch, Mallot, 2012)

- data exist for $A_N$ in $pp \to \Lambda^\uparrow X$ (Bunce et al, 1976 / ...)

- calculation available for $A_{LT}$ for $\bar{p}p^\uparrow \to (h, \gamma, jet) X$

- it is challenging to reveal the underlying physics of the available data

- maybe the asymmetries for $\ell N$ help to understand the asymmetries for $pp$
3. Playground to solidify and streamline theory tools
   (due to small number of Feynman graphs)
   • gauge-invariance of calculation
   • frame-independence of results
   • higher order corrections

4. Explore potential of those observables for future measurements
   • EIC should be ideal for future studies
     → measurements possible for large transverse momenta $P_{h\perp}$
Reminder: double-spin asymmetry $A_{LT}$ for $\vec{\ell} N^\uparrow \rightarrow \ell X$

- Re-scattering of struck quark matters at twist-3 (gluon with physical polarization)

- Contributing correlators after factorization
- collinear quark-quark correlator at twist-3 \( g_T(x) \)
- \( k_\perp \)-dependent quark-quark correlator \( \tilde{g}(x) = \int d^2 \vec{k}_\perp \frac{k_\perp^2}{2M^2} g_{1T}(x, \vec{k}_\perp) \)
- (collinear) quark-gluon-quark correlator \( F_{FT}(x, x_1) G_{FT}(x, x_1) \)

- Exploit relations between functions
  - relation due to QCD equation of motion

\[
x g_T(x) = \int dx_1 \left[ G_{DT}(x, x_1) - F_{DT}(x, x_1) \right]
\]

- Final result

\[
\frac{l'^0 d\sigma_{LT}}{d^3 \vec{l}'} = -8 \alpha_{em}^2 x_B^2 \sqrt{1 - y} M \lambda_\ell |s_\perp| \cos \phi_S \sum_q e_q^2 g_T^q(x_B)
\]

- twist-3 effect
- final result looks rather simple
- comparable twist-3 observables may have more complicated structure
Single-spin asymmetry $A_N$ for $\ell N^\uparrow \rightarrow h X$

(Gamberg, Kang, A.M., Pitonyak, Prokudin, 2014)

- Feynman diagrams for LO calculation

- twist-3 effects associated with nucleon, and with fragmentation process
- large scale for pQCD calculations provided by $P_{h\perp}$
- in LO formalism, hadron recoils against (undetected) final state lepton $\rightarrow Q^2$ large
Analytical result

\[
P_h^0 \frac{d\sigma_{UT}}{d^3P_h} = -\frac{8\alpha_{em}^2}{S} \varepsilon_{\mu\nu} S_{\mu \bot}^\nu \sum_q e_q^2 \int_{z_{min}}^1 dz \frac{1}{S + T/z} \frac{1}{x} \times \left\{ -\frac{\pi M}{\hat{u}} D_1^{h/q}(z) \left( F_{FT}^q(x, x) - x \frac{dF_{FT}^q(x, x)}{dx} \right) \left[ \frac{\hat{s}(\hat{s}^2 + \hat{u}^2)}{2\hat{t}^3} \right] \right. \\
+ \frac{M_h}{-x\hat{u} - \hat{t}} h_1^q(x) \left\{ \left( \hat{H}^{h/q}(z) - z \frac{d\hat{H}^{h/q}(z)}{dz} \right) \left[ \frac{(1 - x)\hat{s}\hat{u}}{\hat{t}^2} \right] \right. \\
+ \frac{1}{z} H^{h/q}(z) \left[ \frac{\hat{s}(\hat{s}^2 + (x - 1)\hat{u}^2)}{\hat{t}^3} \right] + 2z^2 \int_z^\infty dz_1 \frac{1}{z_1 - \frac{1}{z_1}} \hat{H}^{h/q,3}(z, z_1) \left[ \frac{x\hat{s}^2\hat{u}}{\xi_2 \hat{t}^3} \right] \right\} \right\}
\]

Ingredients for numerics
- twist-2 FF $D_1$ (de Florian, Sassot, Stratmann, 2007)
- Qiu-Sterman function $F_{FT}$, from Sivers function $f_{1T}^\perp$ (Anselmino et al, 2008)
- transversity $h_1$ (Anselmino et al, 2013)
- twist-3 FF $\hat{H}$, from Collins function $H_1^\perp$ (Anselmino et al, 2013)
- twist-3 FF $H$ and $\hat{H}^{S}_{FU}$ enter $A_N$ for $pp^\uparrow \rightarrow hX$ (A.M., Pitonyak, 2012)
use model-independent relation

$$H^{h/q}(z) = -2z\hat{H}^{h/q}(z) + 2z^3\int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{z-1} \hat{H}^{h/q,S}_{FU}(z, z_1)$$

and fit to RHIC data for $A_N$ (Kanazawa, Koike, A.M., Pitonyak, 2014)

* good fit can be obtained ($\chi^2$/d.o.f = 1.03)
* $A_N$ dominated by twist-3 FFs (beyond moment of Collins function)
* data on $A_N$ for $\ell N^\uparrow \rightarrow hX$ may allow cross check
Numerical results and discussion

- comparison with HERMES data (JLab data are at very low $P_{h\perp}$)

* error band based on uncertainties of $f_{1T}^\perp$, $h_1$, $H_1^\perp$ only

* relatively poor comparison with data, especially for $\pi^+$ production

* potential reasons for discrepancy:
  1. no error band for twist-3 FF $\hat{H}_{FU}^3$ and hence for FF $H$
  2. (significant) other source(s) for $A_N$ in $p p^\uparrow \rightarrow h X$
  3. leading order formalism not appropriate for rather low $P_{h\perp}$ of available data;
     HERMES: even data at highest $P_{h\perp}$ dominated by quasi-real photo-production
     → calculation of NLO correction needed
– better comparison with "DIS" sub-set of HERMES data \( Q^2 \geq 1 \text{ GeV}^2 \)

– prospects for measurement at an EIC \( \sqrt{S} = 63 \text{ GeV}, \ P_{h\perp} = 3 \text{ GeV} \)

with \( \hat{H}^{3}_{FU} \) 

without \( \hat{H}^{3}_{FU} \)

* description of \( A_N \) in \( p p^\uparrow \rightarrow h X \) through twist-3 fragmentation may be checked
Single-spin asymmetry $A_N$ for $\ell N \rightarrow \Lambda^\uparrow X$

(Kanazawa, A.M., Pitonyak, Schlegel, 2015)

- Same Feynman graphs as for $A_N$ in $\ell N^\uparrow \rightarrow h X$

- Analytical results
  - twist-3 distribution contribution

\[
\frac{P_h d\sigma_{\text{Dist}}}{d^3 P_h} = \frac{8\pi M \alpha_{\text{em}}^2}{S} \epsilon_{\perp} e^2 \int_{z_{\text{min}}}^{1} \frac{dz}{xz^3} \frac{1}{S+T/z} \frac{1}{\hat{u}} H_1^q(z) \left( x \frac{dH_{FU}^q(x,x)}{dx} \right) \left[ -\frac{s^2\hat{u}}{\hat{t}^3} \right]
\]

  * no non-derivative term (first observed by Zhou, Yuan, Liang, 2008)

- twist-3 fragmentation contribution

\[
\frac{P_h d\sigma_{\text{Frag}}}{d^3 P_h} = \frac{2M_h \alpha_{\text{em}}^2}{S} \epsilon_{\perp} e^2 \int_{z_{\text{min}}}^{1} \frac{dz}{xz^3} \frac{1}{S+T/z} \frac{1}{-\hat{t} - x\hat{u}} f_1^q(x) \\
\times \left[ z \frac{d\hat{D}_T^q(z)}{dz} \hat{\sigma}_D + \hat{D}_T^q(z) \hat{\sigma}_N + \frac{1}{z} D_T^q(z) \hat{\sigma}_2 + \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{1/z - 1/z_1} \frac{1}{\xi} \hat{D}_{FT}^{q,3}(z,z_1) \hat{\sigma}_3 \right]
\]

  * (again) contributions from three sources
• Discussion
  – identified all relevant parton correlation functions and their relations
  – first complete result in twist-3 collinear factorization
  – results obtained in both light-cone gauge and Feynman gauge
    (for the first time for twist-3 fragmentation contribution to transverse SSA)
  – checked fragmentation contribution to $A_N$ for $\ell N^\uparrow \rightarrow h X$ in Feynman gauge
  – numerical estimate needed
  – ingredients available for calculation of $A_N$ for process like $p p \rightarrow \Lambda^\uparrow X$
Double-spin asymmetry $A_{LT}$ for $\vec{\ell} N^\uparrow \rightarrow h X$

(Kanazawa, A.M., Pitonyak, Schlegel, 2014)

- Same Feynman graphs as before, but somewhat different treatment of kinematics

- Analytical results (calculation in lepton-nucleon cm frame)
  - results obtained in both light-cone gauge and Feynman gauge
  - twist-3 distribution contribution

\[
\frac{P_h^0 d\sigma^{\text{Dist}}_{LT}(\lambda_\ell, \vec{S}_\perp)}{d^3 P_h} = -\frac{8\alpha_{em}^2}{S} \frac{M}{\vec{P}_h \cdot \vec{S}_\perp} \lambda_\ell \sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3} \frac{1}{S + T/z} \frac{1}{x \hat{u}} D_{1/h/q}(z)
\]
\[
\times \left\{ \left( \tilde{g}^q(x) - x \frac{d\tilde{g}^q(x)}{dx} \right) \left[ \frac{\hat{s}(\hat{s} - \hat{u})}{2\hat{t}^2} \right] + x g_T^q(x) \left[ \frac{\hat{u}}{2\hat{t}} \right] + \int dx_1 G_{DT}^q(x, x_1) \left[ \frac{\hat{u}(\hat{s} - \hat{u})}{\xi \hat{t}^2} \right] \right\}
\]

* (again) contributions from three sources

- twist-3 fragmentation contribution

\[
\frac{P_h^0 d\sigma^{\text{Frag}}_{LT}(\lambda_\ell, \vec{S}_\perp)}{d^3 P_h} = -\frac{8\alpha_{em}^2}{S} M_h \frac{M}{\vec{P}_h \cdot \vec{S}_\perp} \lambda_\ell \sum_q e_q^2 \int_{z_{\min}}^1 \frac{dz}{z^3} \frac{1}{S + T/z} \frac{1}{z x \hat{t}} \tilde{h}_1^q(x) E_{h/q}(z) \left[ -\frac{\hat{s}}{\hat{t}} \right]
\]

* final result becomes rather simple
Comparison between lepton-nucleon cm-frame and nucleon-hadron cm-frame

- agreement after taking into account the following relation (LIR): (Bukhvostov, Kuraev, Lipatov, 1984 / ...)

\[ g_T(x) = g_1(x) - \frac{2}{x} \int dx_1 \frac{1}{\xi} G_{DT}(x, x_1) \]

- matters here only for twist-3 distribution contribution
- LIR also allows one to simplify the final result

\[ \frac{P_h^0 d\sigma_{LT}^{Dist}(\lambda, \vec{S}_\perp)}{d^3 \vec{P}_h} = -\frac{8\alpha_{em}^2}{S} M \vec{P}_h \cdot \vec{S}_\perp \lambda \sum_q e_q^2 \int_{z_{min}}^{1} \frac{dz}{z^3} \frac{1}{S + T/z} \frac{1}{x\hat{u}} D_1^{h/q}(z) \]

\[ \times \left\{ \left[ \bar{g}^q(x) - x \frac{d\bar{g}^q(x)}{dx} \right] \left[ \frac{\hat{s}(\hat{s} - \hat{u})}{2\hat{t}^2} \right] + x g_1^q(x) \left[ \frac{-\hat{s}\hat{u}}{\hat{t}^2} \right] + x g_1^q(x) \left[ \frac{\hat{u}(\hat{s} - \hat{u})}{2\hat{t}^2} \right] \right\} \]

* in particular, numerics becomes much easier \((g_1\) instead of \(G_{DT}\))
Numerical results and discussion (twist-3 distribution term only)

- use Wandzura-Wilceck approximation for $g_T$

- two models for $\tilde{g}$
  1. WW-type approximation: $\tilde{g}(x) = g_{1T}^{(1)}(x) \approx x \int_x^1 \frac{dy}{y} g_1(y)$
  2. Large $N_c$ analysis suggests: $\tilde{g}(x) \approx -f_{1T}^{(1)}(x)$ (Pobylitsa, 2003)

- comparison with data from JLab Hall A

* agreement in sign
* quantitative description only for $\pi^+$ for WW-type approximation
* but, keep in mind that $P_{h\perp}$ of data certainly too low
* in general, $A_{LT}$ may allow one to study the TMD $g_{1T}$

(JLab Hall A, 2015)
– prediction for COMPASS ($\sqrt{S} = 17.3$ GeV, $P_{h\perp} = 2$ GeV)

![COMPASS prediction graph]

– prediction for an EIC ($\sqrt{S} = 63$ GeV)

![EIC prediction graphs]

– $A_{LT}$ becomes very small at higher energies
Summary

• Twist-3 spin asymmetries for $\ell N \rightarrow h X$ are interesting "new" observables

• They may give new insight into non-perturbative parton correlation functions

• They may give new insight into corresponding asymmetries for hadronic collisions

• First data available for three such asymmetries

• LO calculations available for those three asymmetries
  – helped to solidify theory tools (gauge invariance, frame independence)
  – phenomenology is at exploratory stage

• Calculation of higher order corrections needed