NLO weighted Sivers asymmetry in SIDIS: three-gluon correlator

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Based on the work done with Kang, Prokudin, Vitev

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Outlines

1. Introduction

2. Transverse spin-dependent cross section: matching and coefficient function

3. NLO $p_{\perp}$ weighted spin dependent cross section
   QCD evolution of Qiu-Sterman function

4. Summary
Transverse spin physics

- Single transverse spin asymmetry (SSA) is a very interesting observable, and has received great attention from both experimental and theoretical sides recently
  - Experimental measurements have been performed for both p+p and e+p processes

\[
A_N \equiv \frac{\Delta \sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}
\]
Theory: SSAs

- On the theory side, it is now understood that these SSAs are directly related to parton transverse motion.
- Two QCD formalisms have been developed to describe them:
  - Transverse momentum dependent (TMD) factorization
  - Collinear twist-3 factorization
- They are consistent with each other. 
  
\[ f(x, k_T) \]  
\[ T_{q,F}(x_1, x_2) \]  

\[ xP + k_T \]  
\[ (x_2 - x_1)P \]  
\[ x_2P \]  

Ji, Qiu, Vogelsang, Yuan, 06, …
Two processes to study spin asymmetry

- Low pt particle at SIDIS: sensitive to TMDs
  \[ e + p^\uparrow \rightarrow e + h + X \]
- High pt inclusive particle in pp: sensitive to collinear twist-3 functions
  \[ p + p^\uparrow \rightarrow (h, \gamma, jet, \cdots) + X \]

Two important questions to address in theory

- What are the relations between TMDs and collinear twist-3 functions?
  Lead to a unified description and also successful phenomenology for both ep and pp data
- What are the energy evolutions of these functions?
  Especially the experimental data are measured at different energies, any precise/consistent QCD formalisms should take into account evolution for these functions
Meaning: QCD collinear evolution

- Schematic meaning: parton distribution function depends on the resolution scale
  \[ Q_0^2 \quad Q^2 > Q_0^2 \]

- Physical meaning: evolution = include important perturbative corrections
  - DGLAP evolution of collinear PDFs: what it does is to resum the so-called single logarithms in the higher order perturbative calculations

\[ P = \text{\small\textcircled{\textbullet}} \quad xP \quad (\alpha_s \ln \frac{Q^2}{\mu^2})^n \]
TMD evolution

- TMD evolution naturally links TMDs and collinear functions
  - QCD evolution of TMDs in Fourier space (solution of TMD evolution equation)

\[ F(x, b; Q) \approx \left( \frac{d\mu}{\mu} \right) \exp \left\{ - \int_{c/b^*}^{Q} \left( A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \times \exp \left( -S_{\text{non-pert}}(b, Q) \right) \]

- TMD evolution needs
  - Coefficient function C: expand TMD in terms of collinear function
  - Need QCD evolution of collinear function \( F(x, c/b) \)

The goal of this talk

- Present our recent result on the QCD evolution for Qiu-Sterman function
- Present the C function for quark Sivers function
- Study the matching between TMD and collinear twist-3 formalism at the cross section level
A complete NLO calculation for spin-dependent observable should help us achieve these goals: SIDIS and three-gluon contributions.

- Collinear twist-3 factorization, spin-dependent cross section, three-gluon contributions
- Derive cross section in the limit of $p_{h\perp} \ll Q$, compare to TMD formalism in $\Lambda_{QCD} \ll p_{h\perp} \ll Q$
- Coefficient functions
- NLO results lead to QCD evolution equation of Qiu-Sterman
Three-gluon correlation functions

- Three-gluon correlators: two color structures

\[
M_{F,abc}^{\alpha \beta \gamma}(x_1, x_2) = g_s \int \frac{dy_1^- dy_2^-}{2\pi} e^{ix_1 p^+ y_1^-} e^{i(x_2-x_1) p^+ y_2^-} \frac{1}{p^+} \langle PS|F_b^{\beta+}(0)F_c^{\gamma+}(y_1^-)F_a^{\alpha+}(y_2^-)|PS\rangle
\]

\[
= \frac{N_c}{(N_c^2 - 1)(N_c^2 - 4)} d^{abc} O^{\alpha \beta \gamma}(x_1, x_2) - \frac{i}{N_c(N_c^2 - 1)} f^{abc} N^{\alpha \beta \gamma}(x_1, x_2)
\]

- Three-gluon correlators: three Lorentz structures

\[
O^{\alpha \beta \gamma}(x_1, x_2) = \frac{1}{2} \left[ O(x_1, x_2) g_\perp^{\alpha \beta} \epsilon^{\gamma n \bar{n} s} + O(x_2, x_2 - x_1) g_\perp^{\beta \gamma} \epsilon^{\alpha n \bar{n} s} + O(x_1, x_1 - x_2) g_\perp^{\gamma \alpha} \epsilon^{\beta n \bar{n} s} \right]
\]

\[
N^{\alpha \beta \gamma}(x_1, x_2) = \frac{1}{2} \left[ N(x_1, x_2) g_\perp^{\alpha \beta} \epsilon^{\gamma n \bar{n} s} - N(x_2, x_2 - x_1) g_\perp^{\beta \gamma} \epsilon^{\alpha n \bar{n} s} - N(x_1, x_1 - x_2) g_\perp^{\gamma \alpha} \epsilon^{\beta n \bar{n} s} \right]
\]

SIDIS: photon-gluon fusion channel

- Photon-gluon channel is sensitive to gluon dynamics

- Collinear expansion (kt-expansion) to extract twist-3 contribution

\[
w(p, q, pc) = (v_1 - v_2) \frac{1}{x^2} \left( \frac{dF_{NO}^{\rho \sigma \lambda}(x, x)}{dx} - \frac{2F_{NO}^{\rho \sigma \lambda}(x, x)}{x} \right) H_{\rho \sigma}^L(x, x, 0) + \frac{F_{NO}^{\rho \sigma \lambda}(x, x)}{x^2} \\
\times \lim_{k_\perp \to 0} \frac{\partial}{\partial k_\perp} \left[ H_{\rho \sigma}^L(x + (v_2 - v_1) \cdot k_\perp, x + v_2 \cdot k_\perp, k_\perp) - H_{\rho \sigma}^L(x, x + v_1 \cdot k_\perp, k_\perp) \right]
\]

- Three-gluon correlation functions $N(x_1, x_2)$ and $O(x_1, x_2)$

arXiv:1409.5851
LO Feynman diagram

- LO Feynman diagrams for three-gluon contribution

+ mirror diagrams
Spin-dependent cross section

- Transverse spin-dependent differential cross section

\[
\frac{d\Delta\sigma}{dx_Bdydz_hd^2p_{h\perp}} = \sigma_0 \left( e^{\alpha_\beta} s_{\perp} p_{h\perp} \right) \sum_q e_q^2 \left( \frac{1}{4} \right) \frac{\alpha_s}{2\pi^2} \int \frac{dx}{x} \frac{dz}{z} D_{h/q}(z) \frac{1}{zQ^2} \delta \left( p_{h\perp}^2 - z_h Q^2 \left( \frac{1}{\hat{x}} - 1 \right) \left( \frac{1}{\hat{z}} - 1 \right) \right) \\
\times \left\{ \left[ \left( \frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right) H_1 + \left( \frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right) H_2 + \frac{O(x,x)}{x} H_3 + \frac{O(x,0)}{x} H_4 \right] \\
+ \left[ \left( \frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right) H_1 - \left( \frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right) H_2 + \frac{N(x,x)}{x} H_3 - \frac{N(x,0)}{x} H_4 \right] \right\} 
\]

- Results are consistent with those from Koike, Tanaka, et.al. 2010

- More interesting here let us explore the matching to the TMD factorization formalism: thus study \( \Lambda_{QCD} \ll p_{h\perp} \ll Q \) region
Matching: TMD and collinear twist-3

- TMD factorization formalism for SIDIS

\[
\frac{d\Delta \sigma}{dx_B dy dz_H d^2 p_{h\perp}} = \sigma_0 \sum_q e_q^2 \int d^2 k_{\perp} d^2 p_{\perp} d^2 \lambda_{\perp} \delta^2 \left( z_h \vec{k}_{\perp} + \vec{p}_{\perp} + \lambda_{\perp} - \vec{p}_{h\perp} \right) \\
\times \frac{\epsilon^{\alpha\beta} s_\perp^{\alpha} k_\perp^{\beta}}{M} f_{1T}^{-q}(x_B, k_{\perp}^2) D_{h/q}(z_h, p_{\perp}^2) S(\lambda_{\perp}) H(Q^2),
\]

- Study the perturbative expansion/tail of the quark Sivers function

\[
\frac{1}{M} f_{1T}^{-q}(x_B, k_{\perp}^2) = -\frac{\alpha_s}{2\pi^2} \frac{1}{(k_{\perp}^2)^2} \int_{x_B}^1 dx \frac{1}{x^2} P_{q\rightarrow g}(\hat{x}) \left( \frac{1}{2} \right) [O(x, x) + O(x, 0) + N(x, x) - N(x, 0)].
\]

Plug in the TMD formalism, and use LO results for soft factor, hard factor, and fragmentation function, we arrive at the exactly same expression as we obtained from collinear twist-3 formalism.

We thus demonstrate the matching between two formalisms for twist-3 three-gluon correlator.
At this point, we could also obtain the so-called coefficient functions in the TMD evolution formalism

- We have the expansion of quark Sivers function in momentum space, now we need to convert to Fourier transformed coordinate $b$-space where TMD evolution equations are typically derived.
- For Sivers function, the relevant quantity is the $k_t$-weighted expression.

\[ f_{1T}^q(\alpha)(x_B, b) = \frac{1}{M} \int d^2k_\perp e^{-ik_\perp \cdot b} k_\perp^\alpha f_{1T}^q(x_B, k_\perp^2) \]

- Fourier transform contains divergence, we thus have to re-perform the calculations using dimensional regularization $\eta = 4 - 2\epsilon$

\[
\frac{1}{M} f_{1T}^q(x_B, k_\perp^2) = -\frac{\alpha_s}{2\pi^2} \frac{(4\pi^2 \mu^2)^\epsilon}{(k_\perp^2)^2} \int_{x_B}^1 dx \frac{dx}{x^2} \left\{ P_{q\rightarrow g}(\hat{x}) \left( \frac{1}{2} \right) [O(x, x) + O(x, 0) + N(x, x) - N(x, 0)] - \frac{\epsilon}{4} [O(x, x) + N(x, x)] - \epsilon \hat{x} (1 - \hat{x}) [O(x, 0) - N(x, 0)] \right\},
\]
Coefficient function II

- Fourier transform

\[ f_{1T}^{q}(x_B, b) = \frac{1}{M} \int d^{2-2\epsilon}k_{\perp}e^{-ik_{\perp}b}k_{\perp}^{\alpha}f_{1T}^{q}(x_B, k_{\perp}^{2}), \]

\[ f_{1T}^{q}(x_B, b) = \left( \frac{ib^{\alpha}}{2} \right) \left\{ \frac{\alpha_{s}}{2\pi} \left( -\frac{1}{\hat{c}} \right) \int \frac{dx}{x^{2}} P_{q\rightarrow g}(\hat{x}) \left( \frac{1}{2} \right) \left[ O(x, x) + O(x, 0) + N(x, x) - N(x, 0) \right] \right\} + \frac{\alpha_{s}}{4\pi} \int \frac{dx}{x^{2}} \left[ P_{q\rightarrow g}(\hat{x}) \ln \left( \frac{c^{2}}{b^{2}\mu^{2}} \right) + \hat{x}(1 - \hat{x}) \right] \left[ O(x, x) + N(x, x) \right] + \frac{\alpha_{s}}{4\pi} \int \frac{dx}{x^{2}} \left[ P_{q\rightarrow g}(\hat{x}) \ln \left( \frac{c^{2}}{b^{2}\mu^{2}} \right) - \frac{1}{2} \left( 1 - 6\hat{x} + 6\hat{x}^{2} \right) \right] \left[ O(x, 0) - N(x, 0) \right] \}

- The divergence in the 1st term simply reflects the collinear divergence of Qiu-Sterman function

- The 2nd term is the usual coefficient functions at \( O(\alpha_{s}) \)

\[ f_{1T, SIDIS}(x, b; \mu) = \left( \frac{ib^{\alpha}}{2} \right) T_{q,F}(x, x, \mu) + \ldots \]
Let us now study the NLO corrections to $p_{h\perp}$-weighted spin-dependent cross section for SIDIS

- From such a study we can identify QCD evolution equation for the Qiu-Sterman function, especially the off-diagonal term = contribution from the three-gluon correlation function

Redo our computation in dimensional regularization

- Definition of $p_{h\perp}$-weighted spin-dependent cross section

\[ \frac{d\langle p_{h\perp}\Delta\sigma \rangle}{dx_Bdydz_h} = \int d^2p_{h\perp}\epsilon^{\alpha\beta}s_{h\perp}^{\alpha}p_{h\perp}^{\beta} \frac{d\Delta\sigma}{dx_Bdydz_hd^2p_{h\perp}}. \]

- Leading order (LO) result depends on Qiu-Sterman function $T_{q,F}(x, x)$

\[ \frac{d\langle p_{h\perp}\Delta\sigma \rangle}{dx_Bdydz_h} = \frac{z_h\sigma_0}{2} \sum_q e_q^2 \int \frac{dx}{x} \frac{dz}{z} T_{q,F}(x, x)D_{h/q}(z)\delta(1 - \hat{x})\delta(1 - \hat{z}). \]
NLO contribution

- The $p_{h\perp}$-weighted cross section contains the divergence, which exactly reflects the collinear divergence of the Qiu-Sterman function
  - Divergence piece

\[
\frac{d\langle p_{h\perp} \Delta \sigma \rangle}{dx_B dy_d z_h} = -\frac{z_h \sigma_0}{2} \sum_q e_q^2 \int \frac{dz}{z} D_{h/q}(z) \delta(1 - \hat{\tau}) \left(-\frac{1}{\hat{\tau}} + \ln \left(\frac{Q^2}{\mu^2}\right)\right)
\]

\[
\times \frac{\alpha_s}{2\pi} \int \frac{dx}{x^2} P_{q\rightarrow g}(\hat{x}) \left(\frac{1}{2}\right) \left[O(x, x) + O(x, 0) + N(x, x) - N(x, 0)\right] + \ldots,
\]

- QCD evolution for Qiu-Sterman function: off-diagonal piece

\[
\frac{\partial}{\partial \ln \mu_f^2} T_{q,F}(x_B, x_B, \mu_f^2) = \frac{\alpha_s}{2\pi} \int_{x_B}^{1} \frac{dx}{x^2} P_{q\rightarrow g}(\hat{x}) \left(\frac{1}{2}\right) \left[O(x, x, \mu_f^2) + O(x, 0, \mu_f^2) + N(x, x, \mu_f^2) - N(x, 0, \mu_f^2)\right]
\]
Full NLO results: finite part

- NLO corrections for the three-gluon correlation functions to the $p_{h\perp}$-weighted cross section:

\[
\frac{d\langle p_{h\perp}\Delta\sigma}\rangle}{dx_Bdydz_h} = -\frac{z_h\sigma_0}{2\pi} \sum_q e_q^2 \int_0^1 \frac{dx}{x^2} \int_{z_h}^1 \frac{dz}{z} D_{h/q}(z) \left\{ \delta(1-z) \ln \left( \frac{Q^2}{\mu_f^2} \right) P_{q\leftarrow g}(\hat{x}) \right. \\
\times \left( \frac{1}{2} \right) \left[ O(x,x,\mu_f^2) + O(x,0,\mu_f^2) + N(x,x,\mu_f^2) - N(x,0,\mu_f^2) \right] \\
+ \left( \frac{1}{4} \right) \left[ \left( \frac{dO(x,x,\mu_f^2)}{dx} - \frac{2O(x,x,\mu_f^2)}{x} \right) \hat{H}_1 + \left( \frac{dO(x,0,\mu_f^2)}{dx} - \frac{2O(x,0,\mu_f^2)}{x} \right) \hat{H}_2 \right] \\
+ \frac{O(x,x,\mu_f^2)}{x} \hat{H}_3 + \frac{O(x,0,\mu_f^2)}{x} \hat{H}_4 \right] + \left( \frac{1}{4} \right) \left[ \left( \frac{dN(x,x,\mu_f^2)}{dx} - \frac{2N(x,x,\mu_f^2)}{x} \right) \hat{H}_1 \right. \\
- \left. \left( \frac{dN(x,0,\mu_f^2)}{dx} - \frac{2N(x,0,\mu_f^2)}{x} \right) \hat{H}_2 + \frac{N(x,x,\mu_f^2)}{x} \hat{H}_3 - \frac{N(x,0,\mu_f^2)}{x} \hat{H}_4 \right\},
\]

- If such type of $p_{h\perp}$-weighted cross section can be measured in the future experiments, it will be very useful to study the phenomenological consequences.
4. Summary

| Matching | We demonstrated the matching between TMD and twist-3 formalisms for three-gluon correlation contribution in the intermediate $p_{h\perp}$ region |
| C-function | Derived the so-called coefficient functions when one expands quark Sivers function in terms of three-gluon correlator, which is a very useful piece for TMD evolution |
| NLO | NLO corrections for the three-gluon correlation functions to the $p_{h\perp}$-weighted transverse spin dependent differential cross section |
| Evolution | Obtained the three-gluon contribution to the DGLAP type evolution equation of Qiu-Sterman function |
Thank You!
How to derive Evolution of Qiu-Sterman function

- Renormalized Qiu-Sterman function

\[ T_{q,F}(x_B, x_B, \mu_f^2) = T_{q,F}^{(0)}(x_B, x_B) + \left( -\frac{1}{\hat{\epsilon}} + \ln \left( \frac{\mu_f^2}{\mu^2} \right) \right) \frac{\alpha_s}{2\pi} \int_{x_B}^{1} \frac{dx}{x^2} \times P_{q\rightarrow g}(\hat{x}) \left( \frac{1}{2} \right) \left[ O(x, x) + O(x, 0) + N(x, x) - N(x, 0) \right] \]

“Bare” Qiu-Sterman function
The hard functions are given here:

\[ \hat{H}_1 = \delta(1 - \hat{z})(1 - \hat{x}) \left[ (2\hat{x}^2 - 2\hat{x} + 1) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 2 \right) - 1 \right] - \frac{(1 - \hat{x})(2\hat{x}^2 - 2\hat{x} + 1 - 2\hat{z} + 2\hat{z}^2)}{\hat{z}^2(1 - \hat{z})_+}, \]

\[ \hat{H}_2 = \delta(1 - \hat{z})(1 - \hat{x})(1 - 2\hat{x})^2 \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 3 \right) - \frac{(1 - \hat{x})(4\hat{x}^2 - 4\hat{x} + 1 - 2\hat{z} + 2\hat{z}^2)}{\hat{z}^2(1 - \hat{z})_+}, \]

\[ \hat{H}_3 = \delta(1 - \hat{z})(1 - \hat{x})2\hat{x}(1 - 2\hat{x}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 2 \right) - \frac{(1 - \hat{x)2\hat{x}(1 - 2\hat{x})}{\hat{z}^2(1 - \hat{z})_+}, \]

\[ \hat{H}_4 = \delta(1 - \hat{z})(1 - \hat{x})2\hat{x} \left[ (1 - 4\hat{x}) \left( \ln \frac{\hat{x}}{1 - \hat{x}} + 2 \right) + 2 \right] - \frac{(1 - \hat{x)2\hat{x}(1 - 4\hat{x})}{\hat{z}^2(1 - \hat{z})_+}. \]

Plus function:

\[ \int_0^1 dy \frac{f(y)}{(1-y)_+} = \int_0^1 dy \frac{f(y) - f(1)}{1-y} \]