NLO weighted Sivers asymmetry in SIDIS: three-gluon correlator

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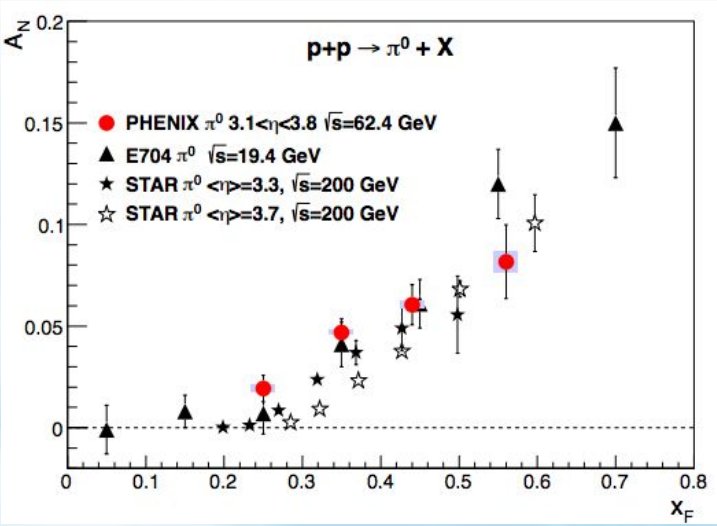
Based on the work done with Kang, Prokudin, Vitev
arXiv:1409.5851

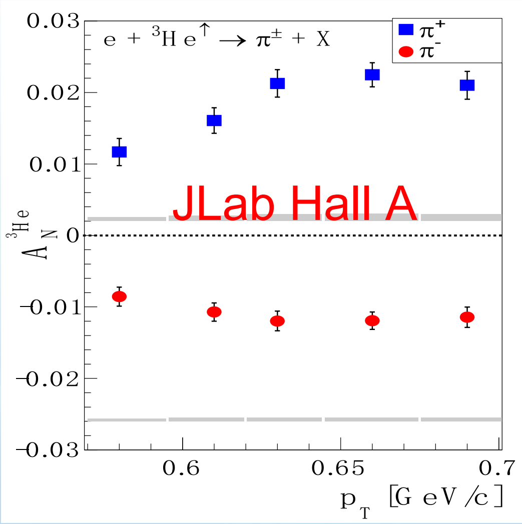
Outlines

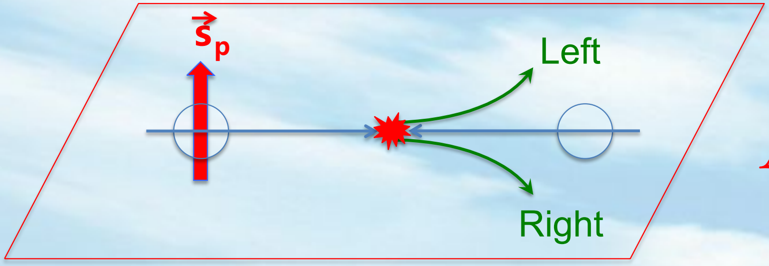
1. Introduction
2. transverse spin-dependent cross section: matching and coefficient function
3. NLO $p_\perp$ weighted spin dependent cross section
   QCD evolution of Qiu-Sterman function
4. Summary

Transverse spin physics

- Single transverse spin asymmetry (SSA) is a very interesting observable, and has received great attention from both experimental and theoretical sides recently
  - Experimental measurements have been performed for both p+p and e+p processes







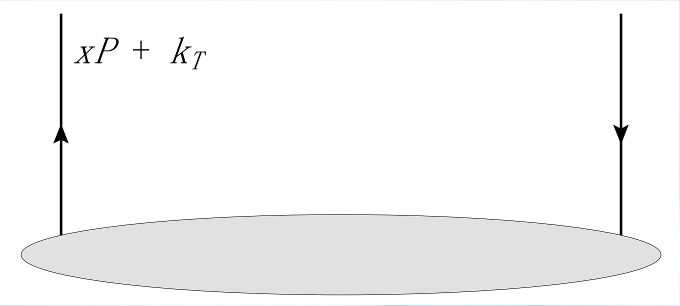
$$A_N \equiv \frac{\Delta\sigma(\ell,\vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell,\vec{s})-\sigma(\ell,-\vec{s})}{\sigma(\ell,\vec{s})+\sigma(\ell,-\vec{s})}$$

Theory: SSAs

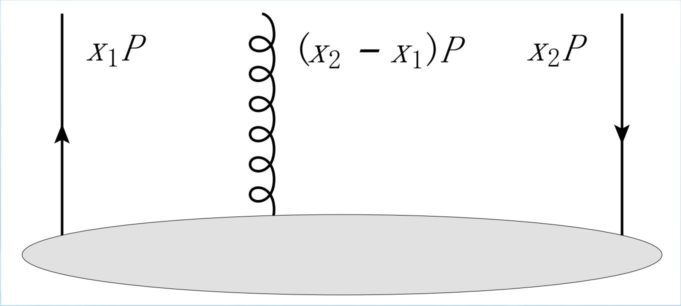
- On the theory side, it is now understood that these SSAs are directly related to parton transverse motion
- Two QCD formalisms have been developed to describe them
  - Transverse momentum dependent (TMD) factorization
  - Collinear twist-3 factorization
  - They are consistent with each other

Ji,Qiu,Vogelsang,Yuan, 06, …

$$f(x, k_T)$$



$$T_{q,F}(x_1, x_2)$$



Two processes and two important questions

- Two processes to study spin asymmetry
  - Low pt particle at SIDIS: sensitive to TMDs

$$e + p^{\uparrow} \to e + h + X$$

  - High pt inclusive particle in pp: sensitive to collinear twist-3 functions

$$p + p^{\uparrow} \to (h, \gamma, jet, \cdots) + X$$

- Two important questions to address in theory
  - What are the relations between TMDs and collinear twist-3 functions?

    Lead to a unified description and also successful phenomenology for both ep and pp data

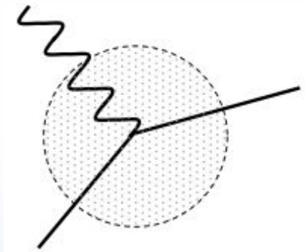
  - What are the energy evolutions of these functions?

    Especially the experimental data are measured at different energies, any precise/consistent QCD formalisms should take into account evolution for these functions

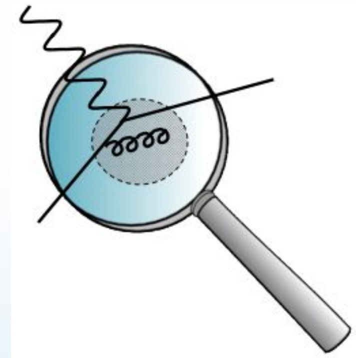
Meaning: QCD collinear evolution

- Schematic meaning: parton distribution function depends on the resolution scale

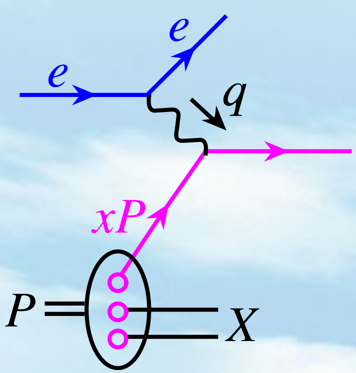
Q_0^2

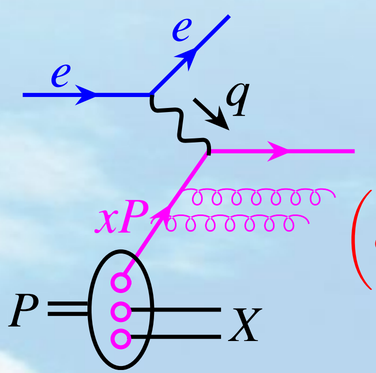


$Q^2 > Q_0^2$



- Physical meaning: evolution = include important perturbative corrections
  - DGLAP evolution of collinear PDFs: what it does is to resum the so-called single logarithms in the higher order perturbative calculations





$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$

TMD evolution

- TMD evolution naturally links TMDs and collinear functions
  - QCD evolution of TMDs in Fourier space (solution of TMD evolution equation) Collins, Rogers, Prokudin, Kang, Qiu, Yuan, …

$$F(x,b;Q) \approx \boxed{C \otimes F(x,c/b^*)} \times \exp\left\{-\int_{c/b^*}^{Q} \frac{d\mu}{\mu}\left(A \ln\frac{Q^2}{\mu^2} + B\right)\right\} \times \exp\left(-S_{\text{non-pert}}(b,Q)\right)$$

  - TMD evolution needs

    Coefficient function C: expand TMD in terms of collinear function

    Need QCD evolution of collinear function F(x, c/b)
- The goal of this talk
  - Present our recent result on the QCD evolution for Qiu-Sterman function
  - Present the C function for quark Sivers function
  - Study the matching between TMD and collinear twist-3 formalism at the cross section level

NLO corrections: SIDIS

- A complete NLO calculation for spin-dependent observable should help us achieve these goals: SIDIS and three-gluon

cross section → **matching** → **evolution**

cross section	matching	evolution
Collinear twist-3 factorization, spin-dependent cross section, *three-gluon contributions*	Derive cross section in the limit of $p_{h\perp} \ll Q$, compare to TMD formalism in $\Lambda_{QCD} \ll p_{h\perp} \ll Q$	▪ Coefficient functions ▪ NLO results lead to QCD evolution equation of Qiu-Sterman

Three-gluon correlation functions

- Three-gluon correlators: two color structures

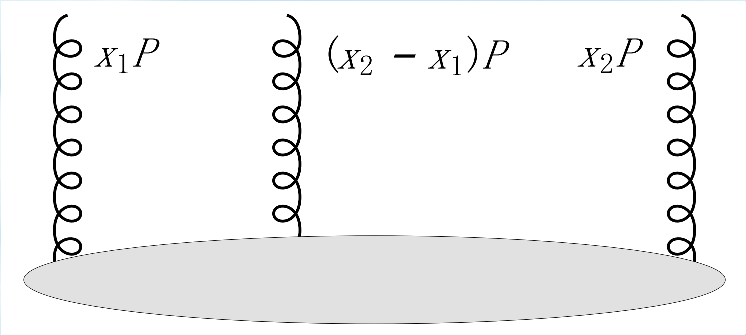
$$M_{F,abc}^{\alpha\beta\gamma}(x_1,x_2) = g_s \int \frac{dy_1^- dy_2^-}{2\pi} e^{ix_1 p^+ y_1^-} e^{i(x_2-x_1)p^+ y_2^-} \frac{1}{p^+} \langle PS|F_b^{\beta+}(0)F_c^{\gamma+}(y_1^-)F_a^{\alpha+}(y_2^-)|PS\rangle$$

$$= \frac{N_c}{(N_c^2-1)(N_c^2-4)} d^{abc} O^{\alpha\beta\gamma}(x_1,x_2) - \frac{i}{N_c(N_c^2-1)} f^{abc} N^{\alpha\beta\gamma}(x_1,x_2)$$

- Three-gluon correlators: three Lorentz structures

$$O^{\alpha\beta\gamma}(x_1,x_2) = \frac{1}{2}\left[O(x_1,x_2)g_\perp^{\alpha\beta}\epsilon^{\gamma n\bar{n}s} + O(x_2,x_2-x_1)g_\perp^{\beta\gamma}\epsilon^{\alpha n\bar{n}s} + O(x_1,x_1-x_2)g_\perp^{\gamma\alpha}\epsilon^{\beta n\bar{n}s}\right]$$

$$N^{\alpha\beta\gamma}(x_1,x_2) = \frac{1}{2}\left[N(x_1,x_2)g_\perp^{\alpha\beta}\epsilon^{\gamma n\bar{n}s} - N(x_2,x_2-x_1)g_\perp^{\beta\gamma}\epsilon^{\alpha n\bar{n}s} - N(x_1,x_1-x_2)g_\perp^{\gamma\alpha}\epsilon^{\beta n\bar{n}s}\right]$$

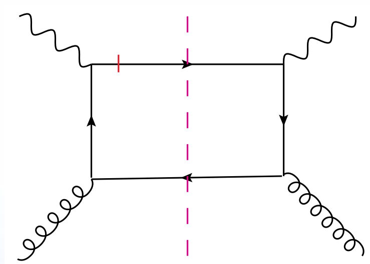


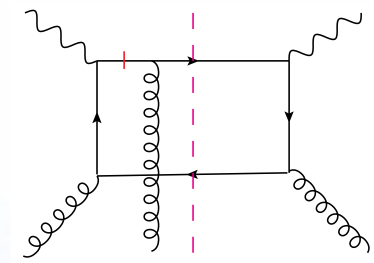
Beppu, Koike, Tanaka, Yoshida, 2010, et.al.

SIDIS: photon-gluon fusion channel

- Photon-gluon channel is sensitive to gluon dynamics

Unpolarized

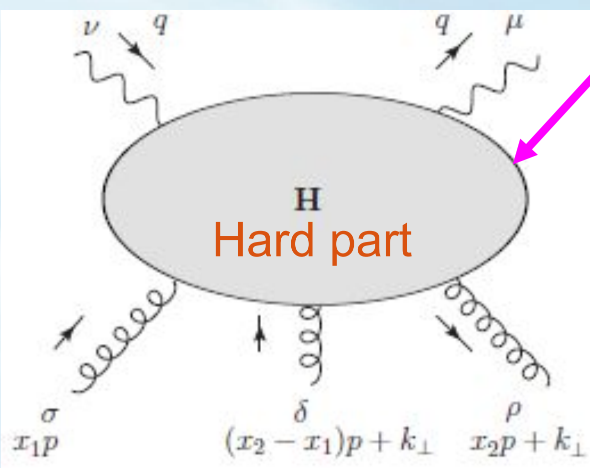




Spin-dependent

- Collinear expansion (kt-expansion) to extract twist-3 contribution

$$w(p,q,p_c) = (v_1 - v_2)_\lambda \frac{1}{x^2}\left(\frac{dF_{NO}^{\rho\sigma\lambda}(x,x)}{dx} - \frac{2F_{NO}^{\rho\sigma\lambda}(x,x)}{x}\right) H^L_{\rho\sigma}(x,x,0) + \frac{F_{NO}^{\rho\sigma\lambda}(x,x)}{x^2}$$
$$\times \lim_{k_\perp \to 0} \frac{\partial}{\partial k_\perp^\lambda}\left[H^L_{\rho\sigma}(x + (v_2 - v_1)\cdot k_\perp, x + v_2 \cdot k_\perp, k_\perp) - H^R_{\rho\sigma}(x, x + v_1 \cdot k_\perp, k_\perp)\right]$$

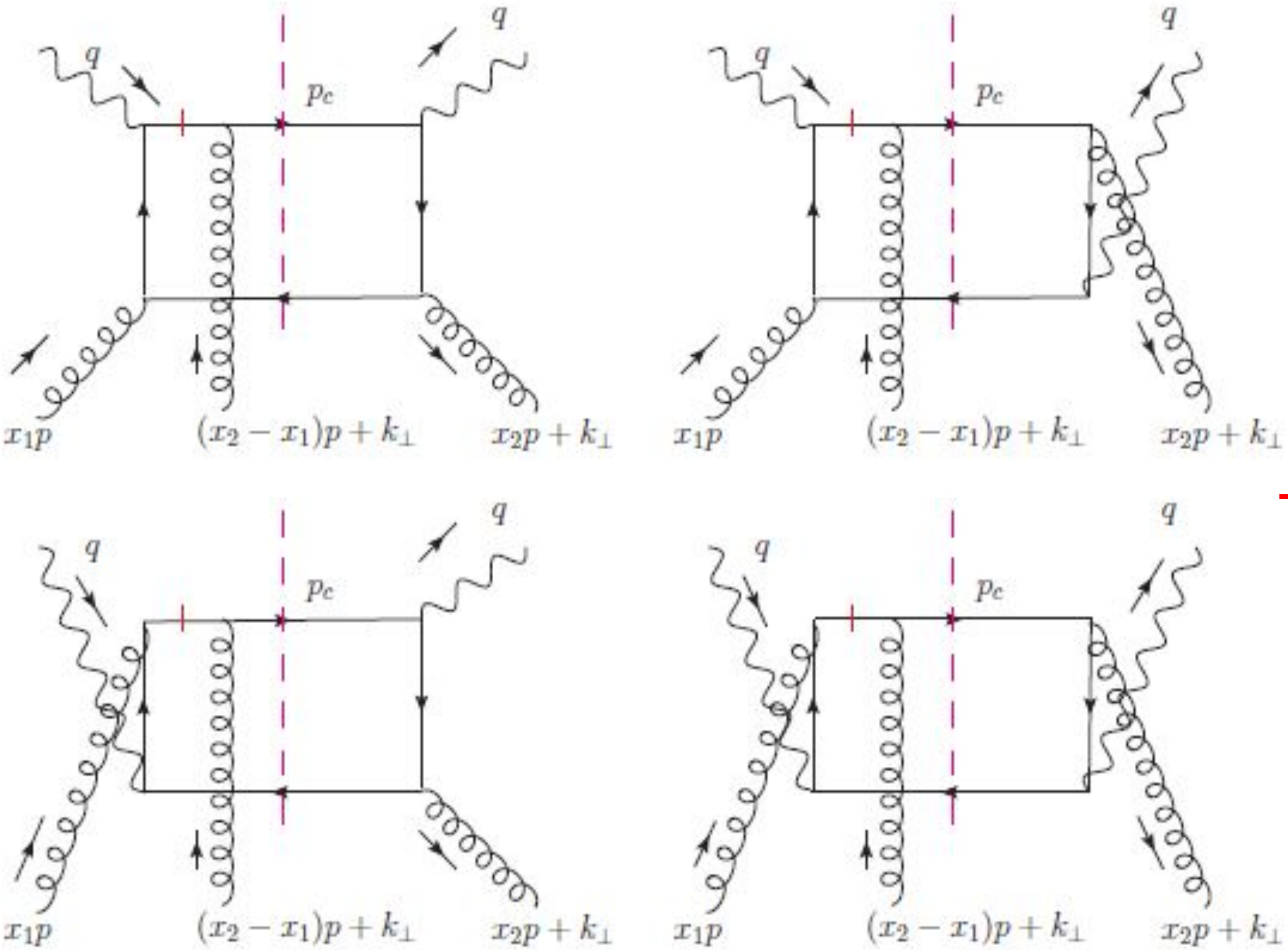


arXiv:1409.5851

Three-gluon correlation functions
N(x1, x2) and O(x1,x2)

LO Feynman diagram

- LO Feynman diagrams for three-gluon contribution



+ mirror diagrams

Spin-dependent cross section

- Transverse spin-dependent differential cross section

$$\frac{d\Delta\sigma}{dx_B dy dz_h d^2p_{h\perp}} = \sigma_0 \left(\epsilon^{\alpha\beta} s_\perp^\alpha p_{h\perp}^\beta\right) \sum_q e_q^2 \left(\frac{1}{4}\right) \frac{\alpha_s}{2\pi^2} \int \frac{dx}{x}\frac{dz}{z} D_{h/q}(z) \frac{1}{zQ^2} \delta\left(p_{h\perp}^2 - z_h^2 Q^2 \left(\frac{1}{\hat{x}} - 1\right)\left(\frac{1}{\hat{z}} - 1\right)\right)$$
$$\times \left\{\left[\left(\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x}\right) H_1 + \left(\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x}\right) H_2 + \frac{O(x,x)}{x} H_3 + \frac{O(x,0)}{x} H_4\right]\right.$$
$$\left. + \left[\left(\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x}\right) H_1 - \left(\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x}\right) H_2 + \frac{N(x,x)}{x} H_3 - \frac{N(x,0)}{x} H_4\right]\right\}$$

- Results are consistent with those from Koike, Tanaka, et.al. 2010
- More interesting here let us explore the matching to the TMD factorization formalism: thus study $\Lambda_{QCD} \ll p_{h\perp} \ll Q$ region

$$\left.\frac{d\Delta\sigma}{dx_B dy dz_h d^2p_{h\perp}}\right|_{p_{h\perp} \ll Q} = -z_h \sigma_0 \left(\epsilon^{\alpha\beta} s_\perp^\alpha p_{h\perp}^\beta\right) \frac{1}{(p_{h\perp}^2)^2} \sum_q e_q^2 \frac{\alpha_s}{2\pi^2} \int \frac{dz}{z} D_{h/q}(z) \delta(1-\hat{z})$$
$$\times \int \frac{dx}{x^2} P_{q\leftarrow g}(\hat{x}) \left(\frac{1}{2}\right) [O(x,x) + O(x,0) + N(x,x) - N(x,0)]$$

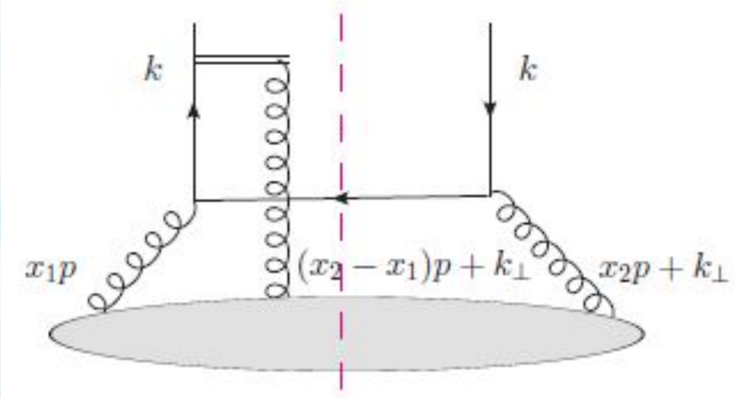
Matching: TMD and collinear twist-3

- TMD factorization formalism for SIDIS

$$\frac{d\Delta\sigma}{dx_B dy dz_h d^2 p_{h\perp}} = \sigma_0 \sum_q e_q^2 \int d^2k_\perp d^2p_\perp d^2\lambda_\perp \delta^2\left(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\lambda}_\perp - \vec{p}_{h\perp}\right) \times \frac{\epsilon^{\alpha\beta} s_\perp^\alpha k_\perp^\beta}{M} f_{1T}^{\perp q}(x_B, k_\perp^2) D_{h/q}(z_h, p_\perp^2) S(\lambda_\perp) H(Q^2),$$

- Study the perturbative expansion/tail of the quark Sivers function

$$\frac{1}{M} f_{1T}^{\perp q}(x_B, k_\perp^2) = -\frac{\alpha_s}{2\pi^2}\frac{1}{(k_\perp^2)^2}\int_{x_B}^1 \frac{dx}{x^2} P_{q\leftarrow g}(\hat{x})\left(\frac{1}{2}\right)\left[O(x,x) + O(x,0) + N(x,x) - N(x,0)\right]$$



Plug in the TMD formalism, and use LO results for soft factor, hard factor, and fragmentation function, we arrive at the exactly same expression as we obtained from collinear twist-3 formalism
We thus demonstrate the matching between two formalisms for twist-3 three-gluon correlator

Coefficient function I

- At this point, we could also obtain the so-called coefficient functions in the TMD evolution formalism
  - We have the expansion of quark Sivers function in momentum space, now we need to convert to Fourier transformed coordinate b-space where TMD evolution equations are typically derived
  - For Sivers function, the relevant quantity is the kt-weighted expression

$$f_{1T}^{\perp q(\alpha)}(x_B, b) = \frac{1}{M} \int d^2k_\perp e^{-ik_\perp \cdot b} k_\perp^\alpha f_{1T}^{\perp q}(x_B, k_\perp^2)$$

  - Fourier transform contains divergence, we thus have to re-perform the calculations using dimensional regularization $n = 4 - 2\epsilon$

$$\frac{1}{M} f_{1T}^{\perp q}(x_B, k_\perp^2) = -\frac{\alpha_s}{2\pi^2} \frac{(4\pi^2\mu^2)^\epsilon}{1-\epsilon} \frac{1}{(k_\perp^2)^2} \int_{x_B}^1 \frac{dx}{x^2} \Big\{ P_{q\leftarrow g}(\hat{x}) \left(\frac{1}{2}\right) [O(x,x) + O(x,0) + N(x,x) - N(x,0)]$$
$$- \frac{\epsilon}{4} [O(x,x) + N(x,x)] - \epsilon \hat{x}(1-\hat{x}) [O(x,0) - N(x,0)] \Big\},$$

Coefficient function II

- Fourier transform $f_{1T}^{\perp q(\alpha)}(x_B,b) = \frac{1}{M}\int d^{2-2\epsilon}k_\perp e^{-ik_\perp\cdot b}k_\perp^\alpha f_{1T}^{\perp q}(x_B,k_\perp^2),$

$$
\begin{aligned}
f_{1T}^{\perp q(\alpha)}(x_B,b) = \left(\frac{ib^\alpha}{2}\right)\Bigg\{&\frac{\alpha_s}{2\pi}\left(-\frac{1}{\hat\epsilon}\right)\int\frac{dx}{x^2}P_{q\leftarrow g}(\hat x)\left(\frac{1}{2}\right)[O(x,x)+O(x,0)+N(x,x)-N(x,0)]\\
&+\frac{\alpha_s}{4\pi}\int\frac{dx}{x^2}\left[P_{q\leftarrow g}(\hat x)\ln\left(\frac{c^2}{b^2\mu^2}\right)+\hat x(1-\hat x)\right][O(x,x)+N(x,x)]\\
&+\frac{\alpha_s}{4\pi}\int\frac{dx}{x^2}\left[P_{q\leftarrow g}(\hat x)\ln\left(\frac{c^2}{b^2\mu^2}\right)-\frac{1}{2}\left(1-6\hat x+6\hat x^2\right)\right][O(x,0)-N(x,0)]\Bigg\},
\end{aligned}
$$

  - The divergence in the 1st term simply reflects the collinear divergence of Qiu-Sterman function $f_{1T,\mathrm{SIDIS}}^{\perp q(\alpha)}(x,b;\mu) = \boxed{\left(\frac{ib^\alpha}{2}\right)T_{q,F}(x,x,\mu)} + \cdots$

LO

  - The 2nd term is the usual coefficient functions at $O(\alpha_s)$

$$
f_{1T}^{\perp q(\alpha)}(x_B,b) = \left(\frac{ib^\alpha}{2}\right)\int_{x_B}^1\frac{dx}{x^2}\left\{C_{q\leftarrow g,1}(\hat x)[O(x,x)+N(x,x)]+C_{q\leftarrow g,2}(\hat x)[O(x,0)-N(x,0)]\right\}
$$

$$
\begin{aligned}
C_{q\leftarrow g,1}(\hat x) &= \frac{\alpha_s}{4\pi}\left[P_{q\leftarrow g}(\hat x)\ln\left(\frac{c^2}{b^2\mu^2}\right)+\hat x(1-\hat x)\right],\\
C_{q\leftarrow g,2}(\hat x) &= \frac{\alpha_s}{4\pi}\left[P_{q\leftarrow g}(\hat x)\ln\left(\frac{c^2}{b^2\mu^2}\right)-\frac{1}{2}\left(1-6\hat x+6\hat x^2\right)\right].
\end{aligned}
$$

NLO corrections to pt-weighted spin-dependent cross section

- Let us now study the NLO corrections to $p_{h\perp}$-weighted spin-dependent cross section for SIDIS
  - From such a study we can identify QCD evolution equation for the Qiu-Sterman function, especially the off-diagonal term = contribution from the three-gluon correlation function
- Redo our computation in dimensional regularization
  - Definition of pt-weighted spin-dependent cross section

$$\frac{d\langle p_{h\perp}\Delta\sigma\rangle}{dx_B dy dz_h} \equiv \int d^2p_{h\perp}\epsilon^{\alpha\beta}s_\perp^\alpha p_{h\perp}^\beta \frac{d\Delta\sigma}{dx_B dy dz_h d^2p_{h\perp}}.$$

  - Leading order (LO) result depends on Qiu-Sterman function $T_{q,F}(x,x)$

$$\frac{d\langle p_{h\perp}\Delta\sigma\rangle}{dx_B dy dz_h} = -\frac{z_h\sigma_0}{2}\sum_q e_q^2 \int \frac{dx}{x}\frac{dz}{z} T_{q,F}(x,x) D_{h/q}(z)\delta(1-\hat{x})\delta(1-\hat{z}).$$

Kang, Vitev, Xing, 2013

NLO contribution

- The $p_{h\perp}$-weighted cross section contains the divergence, which exactly reflects the collinear divergence of the Qiu-Sterman function
  - Divergence piece

Finite pieces

$$\frac{d\langle p_{h\perp}\Delta\sigma\rangle}{dx_B dy dz_h} = -\frac{z_h\sigma_0}{2}\sum_q e_q^2 \int \frac{dz}{z} D_{h/q}(z)\delta(1-\hat{z})\left(-\frac{1}{\hat{\epsilon}} + \ln\left(\frac{Q^2}{\mu^2}\right)\right)$$
$$\times \frac{\alpha_s}{2\pi}\int \frac{dx}{x^2} P_{q\leftarrow g}(\hat{x})\left(\frac{1}{2}\right)[O(x,x) + O(x,0) + N(x,x) - N(x,0)] + \cdots ,$$

  - QCD evolution for Qiu-Sterman function: off-diagonal piece

$$\frac{\partial}{\partial \ln \mu_f^2} T_{q,F}(x_B, x_B, \mu_f^2) = \frac{\alpha_s}{2\pi}\int_{x_B}^1 \frac{dx}{x^2} P_{q\leftarrow g}(\hat{x})\left(\frac{1}{2}\right)\left[O(x,x,\mu_f^2) + O(x,0,\mu_f^2) + N(x,x,\mu_f^2) - N(x,0,\mu_f^2)\right]$$

Full NLO results: finite part

- NLO corrections for the three-gluon correlation functions to the $p_{h\perp}$-weighted cross section:

$$\frac{d\langle p_{h\perp}\Delta\sigma\rangle}{dx_B dy dz_h} = -\frac{z_h\sigma_0}{2}\frac{\alpha_s}{2\pi}\sum_q e_q^2\int_{x_B}^1\frac{dx}{x^2}\int_{z_h}^1\frac{dz}{z}D_{h/q}(z)\Bigg\{\delta(1-\hat{z})\ln\left(\frac{Q^2}{\mu_f^2}\right)P_{q\leftarrow g}(\hat{x})$$
$$\times\left(\frac{1}{2}\right)\left[O(x,x,\mu_f^2)+O(x,0,\mu_f^2)+N(x,x,\mu_f^2)-N(x,0,\mu_f^2)\right]$$
$$+\left(\frac{1}{4}\right)\Bigg[\left(\frac{dO(x,x,\mu_f^2)}{dx}-\frac{2O(x,x,\mu_f^2)}{x}\right)\hat{H}_1+\left(\frac{dO(x,0,\mu_f^2)}{dx}-\frac{2O(x,0,\mu_f^2)}{x}\right)\hat{H}_2$$
$$+\frac{O(x,x,\mu_f^2)}{x}\hat{H}_3+\frac{O(x,0,\mu_f^2)}{x}\hat{H}_4\Bigg]+\left(\frac{1}{4}\right)\Bigg[\left(\frac{dN(x,x,\mu_f^2)}{dx}-\frac{2N(x,x,\mu_f^2)}{x}\right)\hat{H}_1$$
$$-\left(\frac{dN(x,0,\mu_f^2)}{dx}-\frac{2N(x,0,\mu_f^2)}{x}\right)\hat{H}_2+\frac{N(x,x,\mu_f^2)}{x}\hat{H}_3-\frac{N(x,0,\mu_f^2)}{x}\hat{H}_4\Bigg]\Bigg\},$$

- If such type of $p_{h\perp}$-weighted cross section can be measured in the future experiments, it will be very useful to study the phenomenological consequences

4. Summary

Matching

We demonstrated the matching between TMD and twist-3 formalisms for three-gluon correlation contribution in the intermediate $p_{h\perp}$ region

C-function

Derived the so-called coefficient functions when one expands quark Sivers function in terms of three-gluon correlator, which is a very useful piece for TMD evolution

NLO

NLO corrections for the three-gluon correlation functions to the $p_{h\perp}$-weighted transverse spin dependent differential cross section

Evolution

Obtained the three-gluon contribution to the DGLAP type evolution equation of Qiu-Sterman function

Thank You!

How to derive Evolution of Qiu-Sterman function

- Renormalized Qiu-Sterman function

"Bare" Qiu-Sterman function

$$T_{q,F}(x_B,x_B,\mu_f^2) = T_{q,F}^{(0)}(x_B,x_B) + \left(-\frac{1}{\hat{\epsilon}} + \ln\left(\frac{\mu_f^2}{\mu^2}\right)\right)\frac{\alpha_s}{2\pi}\int_{x_B}^{1}\frac{dx}{x^2}$$
$$\times P_{q\leftarrow g}(\hat{x})\left(\frac{1}{2}\right)\left[O(x,x)+O(x,0)+N(x,x)-N(x,0)\right]$$

Hard function

- The hard functions are given here

$$
\begin{aligned}
\hat{H}_1 =& \delta(1-\hat{z})(1-\hat{x})\left[(2\hat{x}^2-2\hat{x}+1)\left(\ln\frac{\hat{x}}{1-\hat{x}}+2\right)-1\right]-\frac{(1-\hat{x})(2\hat{x}^2-2\hat{x}+1-2\hat{z}+2\hat{z}^2)}{\hat{z}^2(1-\hat{z})_+},\\
\hat{H}_2 =& \delta(1-\hat{z})(1-\hat{x})(1-2\hat{x})^2\left(\ln\frac{\hat{x}}{1-\hat{x}}+3\right)-\frac{(1-\hat{x})(4\hat{x}^2-4\hat{x}+1-2\hat{z}+2\hat{z}^2)}{\hat{z}^2(1-\hat{z})_+},\\
\hat{H}_3 =& \delta(1-\hat{z})(1-\hat{x})2\hat{x}(1-2\hat{x})\left(\ln\frac{\hat{x}}{1-\hat{x}}+2\right)-\frac{(1-\hat{x})2\hat{x}(1-2\hat{x})}{\hat{z}^2(1-\hat{z})_+},\\
\hat{H}_4 =& \delta(1-\hat{z})(1-\hat{x})2\hat{x}\left[(1-4\hat{x})\left(\ln\frac{\hat{x}}{1-\hat{x}}+2\right)+2\right]-\frac{(1-\hat{x})2\hat{x}(1-4\hat{x})}{\hat{z}^2(1-\hat{z})_+}.
\end{aligned}
$$

- Plus function

$$\int_0^1 dy\,\frac{f(y)}{(1-y)_+} = \int_0^1 dy\,\frac{f(y)-f(1)}{1-y}$$