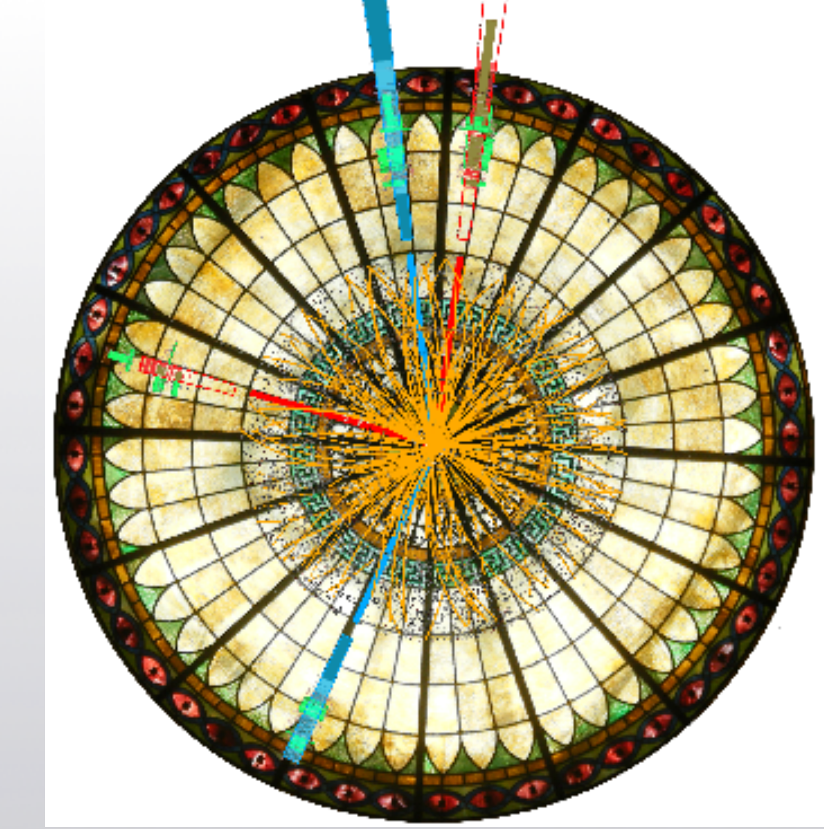
HOLOGRAPHIC APPROACH TO DEEP INELASTIC SCATTERING AT SMALL-X



Chung-I Tan, Brown University
SMU, Dallas, TX
April 27-May 1, 2015

DIS 2015

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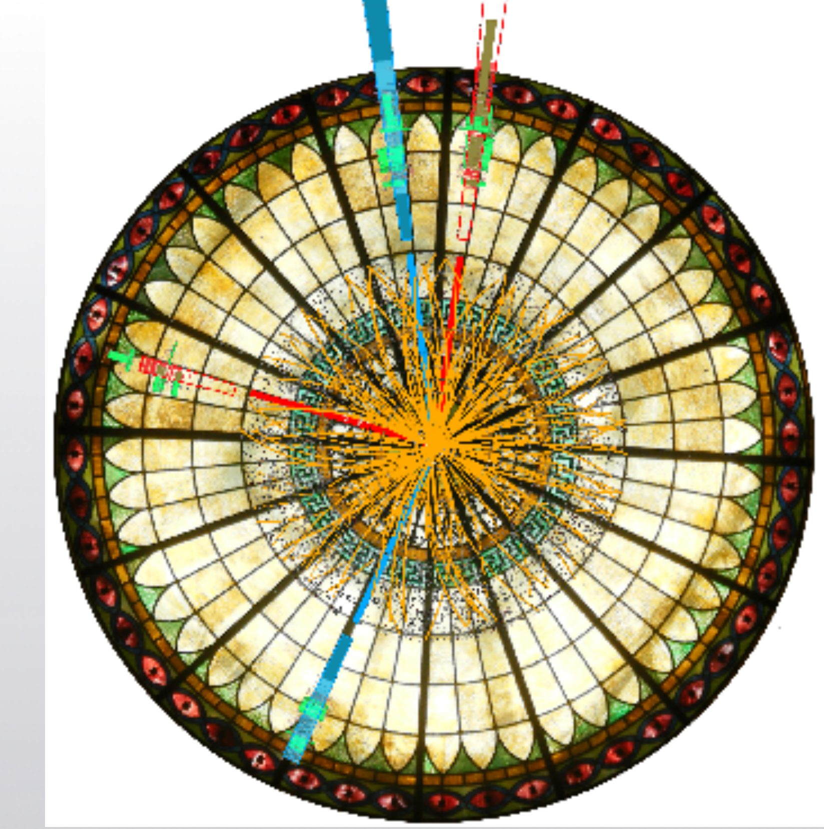

DIS 2015

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Richard Brower, Joseph Polchinski, Matthew Strassler and Chung-I Tan, "*The Pomeron and Gauge/String Duality*", JHEP 0712:005(2007), arXiv:hep-th/0603115

Richard Brower, Marko Djurić, Ina Sarcević and Chung-I Tan: *String-Gauge Dual Description of DIS and Small-x*, 10.1007/JHEP 11(2010)051, arXiv:1007.2259

Richard Brower, Miguel Costa, Marko Djuric, Timothy Raben and Chung-I Tan, "*Strong Coupling Expansion for the Conforaml Pomeorn/Odderon Trajectories*", 10.1007/JHEP02(2015)104, arXiv:1409.2730

R. Brower, M. Djuric, T. Raben and C-I Tan, "DIS, Confinement and Soft-Wall", (to appear)

Outline

- **AdS/CFT and Holographic QCD:**
- **DIS at small-x:**
  - Holographic treatment
  - Conformal and Anomalous Dimensions, DGLAP, etc.
  - Confinement, Softwall
  - Saturation
- **Pomeron/Odderon Intercepts in strong coupling:**
- **Summary and Outlook:**

Outline

- **AdS/CFT and Holographic QCD:** **Unification and Universality**
- **DIS at small-x:**
    - Holographic treatment
    - Conformal and Anomalous Dimensions, DGLAP, etc.
    - Confinement, Softwall
    - Saturation
- **Pomeron/Odderon Intercepts in strong coupling:**
- **Summary and Outlook:**

[redacted]

- AdS/CFT and Holographic QCD: **Unification and Universality**

[redacted]

Soft Pomeron trajectory [Donnachie, Landshoff]

- Trajectory selected by exchanged quantum numbers. For elastic scattering these are the vacuum quantum numbers.

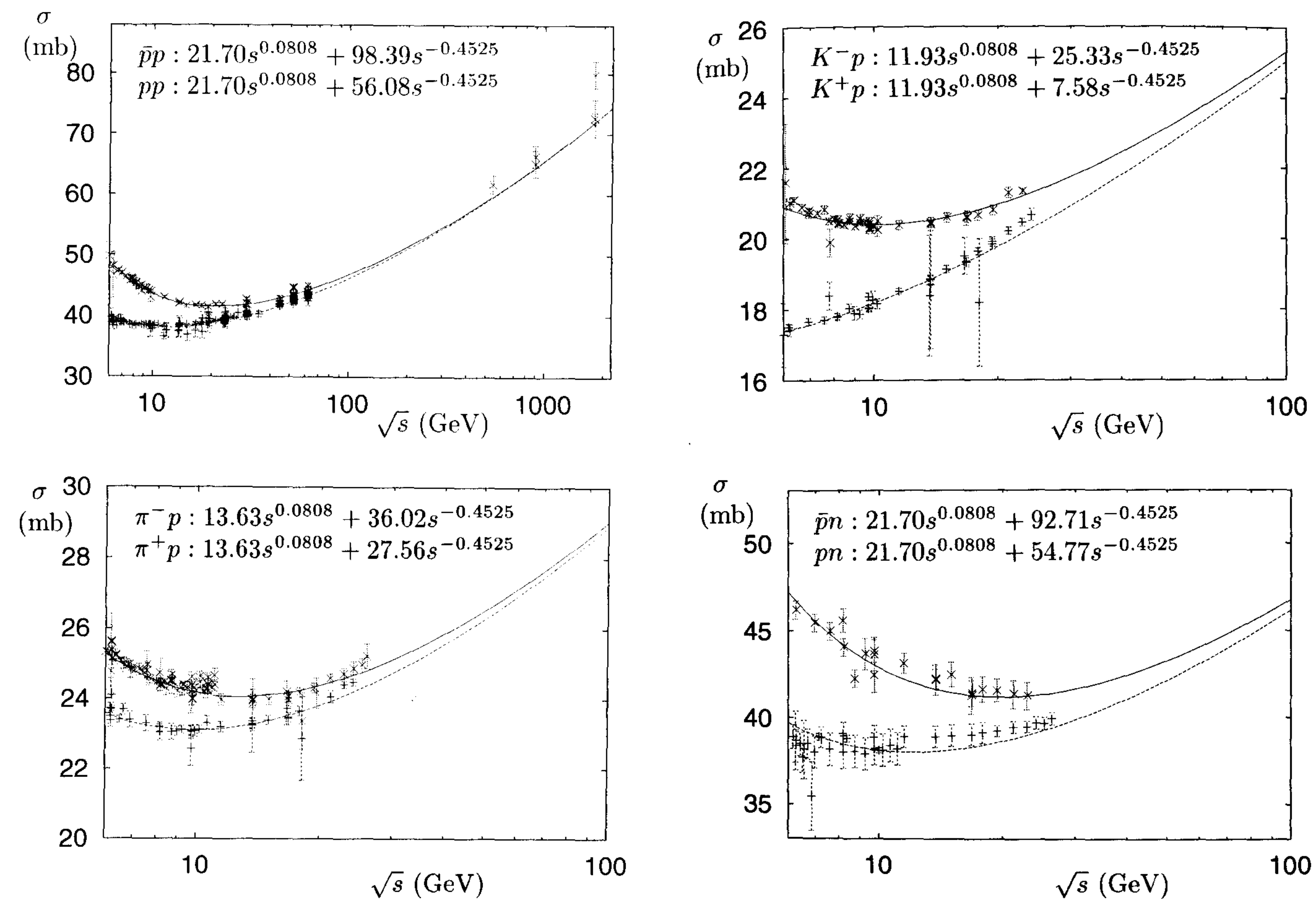
$$j_P(t) \approx 1.08 + 0.25t \quad \text{(GeV units)}$$

Elastic cross sections in QCD

$$\sigma \sim s^{j_P(0)-1} \sim s^{0.08}$$

Exchange of even spin glueballs $(J \geq 2)$

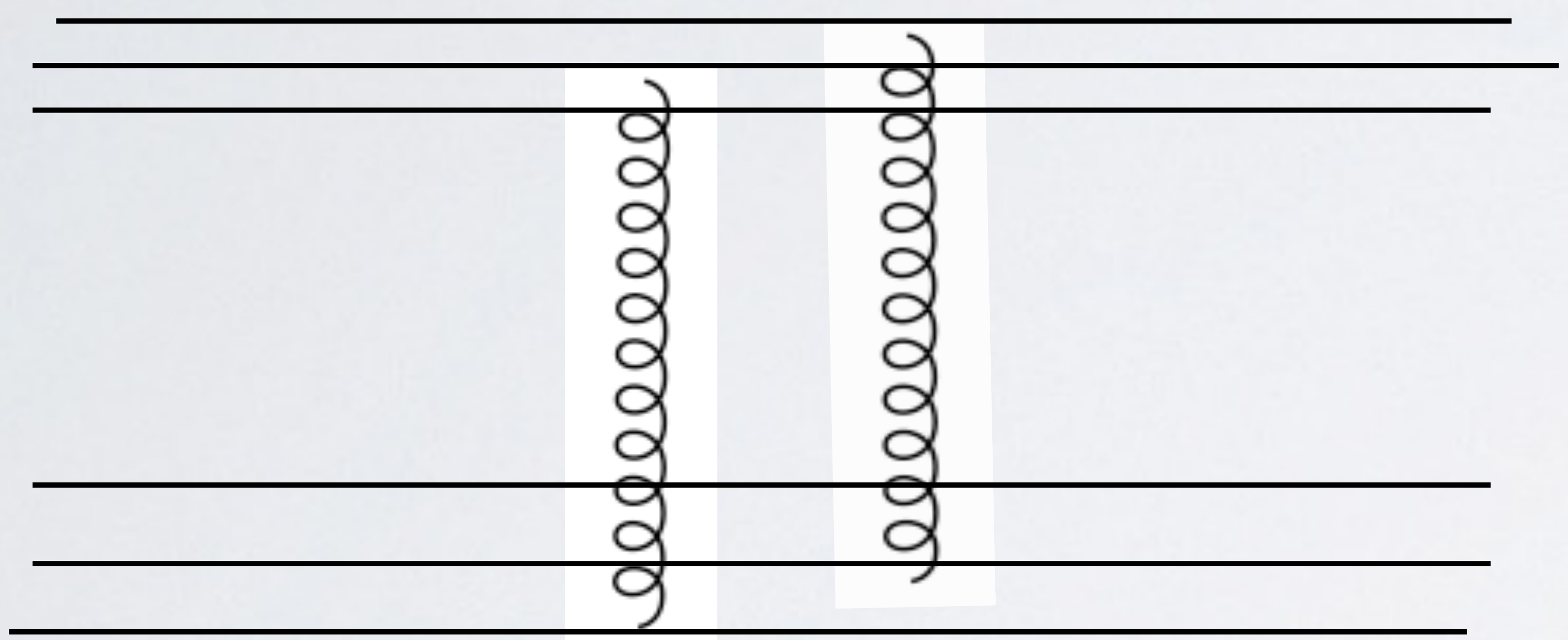
$$\mathcal{O}_J \sim F_{\alpha[\beta_1} D_{\beta_2} \ldots D_{\beta_{J-1}} F_{\beta_J]}{}^{\alpha}$$



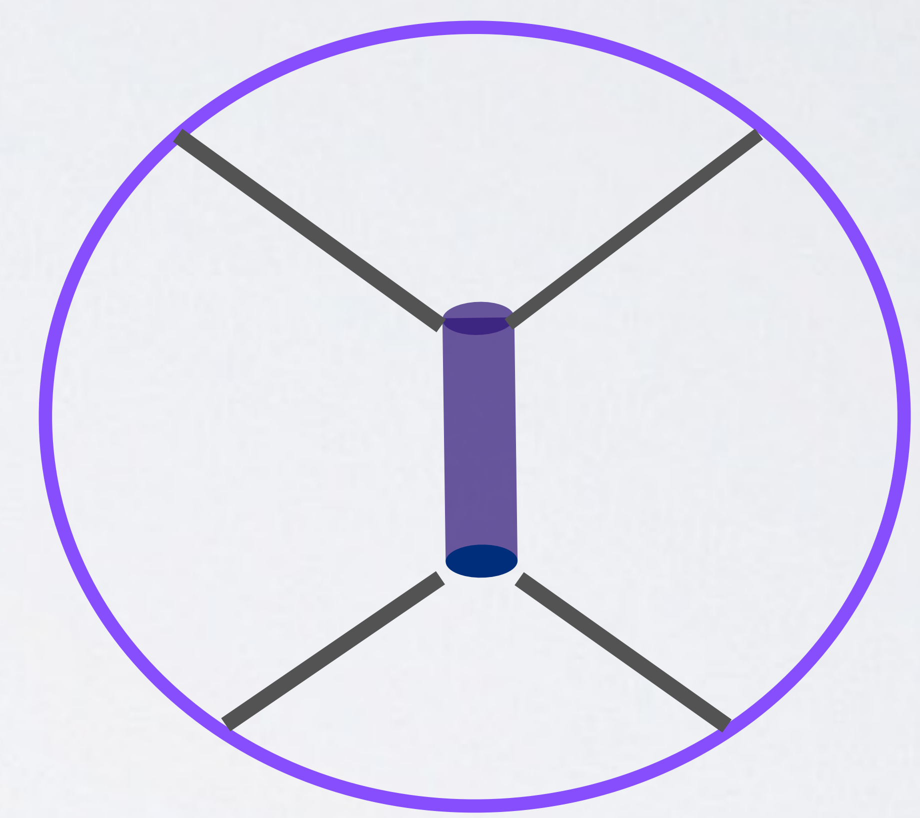
HIGH ENERGY SCATTERING <=> POMERON

WHAT IS THE POMERON ?

WEAK: TWO-GLUON <=> STRONG: ADS GRAVITON



$$J_{cut} = 1 + 1 - 1 = 1$$



$$J = 2$$

$$S = \frac{1}{2\kappa^2} \int d^4x dz \sqrt{-g(z)} \Big(- \mathcal{R} + \frac{12}{R^2} + \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi \Big)$$

F.E. Low. Phys. Rev. D 12 (1975), p. 163.
S. Nussinov. Phys. Rev. Lett. 34 (1975), p. 1286.

AdS Witten Diagram: Adv. Theor. Math. Physics 2 (1998)253

Unification and Universality:

Gauge/String Duality (AdS/CFT) $\longrightarrow$ 2-GLUONS $\simeq$ GRAVITON

- Unification of Soft and Hard Physics in High Energy Collision
- Gauge Dynamics — Geometry of Space-Time
- Weak-Strong Duality

- Improved phenomenology based on "Large Pomeron intercept", e.g., DIS at small-x: (DGLAP vs Pomeron), Elastic/Total Cross sections, DVCS, Central Diffractive Higgs Production, etc.

Unification and Universality:

Gauge/String Duality (AdS/CFT) $\longrightarrow$ 2-GLUONS $\simeq$ GRAVITON

- Gauge Dynamics — Geometry of Space-Time
- **Weak-Strong Duality**

II. Gauge-String Duality: AdS/CFT

Weak Coupling:

Gluons and Quarks: $A_\mu^{ab}(x), \psi_f^a(x)$

Gauge Invariant Operators: $\bar{\psi}(x)\psi(x), \;\; \bar{\psi}(x)D_\mu\psi(x)$

$S(x) = TrF_{\mu\nu}^2(x), \;\; O(x) = TrF^3(x)$

$T_{\mu\nu}(x) = TrF_{\mu\lambda}(x)F_{\lambda\nu}(x), \;\; etc.$

$$\mathcal{L}(x) = -TrF^2 + \bar{\psi}\not{D}\psi + \cdots$$

Strong Coupling:

Metric tensor: $G_{mn}(x) = g_{mn}^{(0)}(x) + h_{mn}(x)$

Anti-symmetric tensor (Kalb-Ramond fields): $b_{mn}(x)$

Dilaton, Axion, etc. $\phi(x), \; a(x), \; etc.$

Other differential forms: $C_{mn\cdots}(x)$

$$\mathcal{L}(x) = \mathcal{L}(G(x), b(x), C(x), \cdots)$$

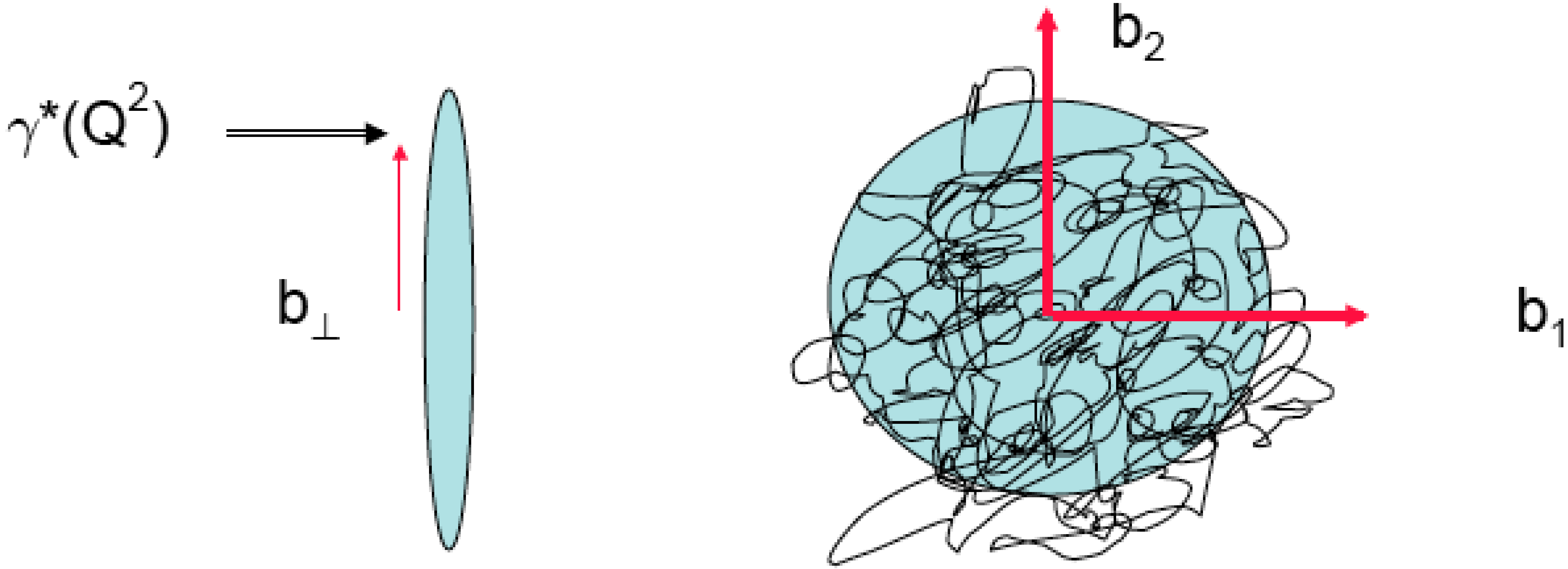
Unification and Universality:

Gauge/String Duality (AdS/CFT) $\longrightarrow$ 2-GLUONS $\simeq$ GRAVITON

- Gauge Dynamics — Geometry of Space-Time — **Extra Dimension**
- Weak-Strong Duality

QCD EMERGENCE OF 5-DIM ADS

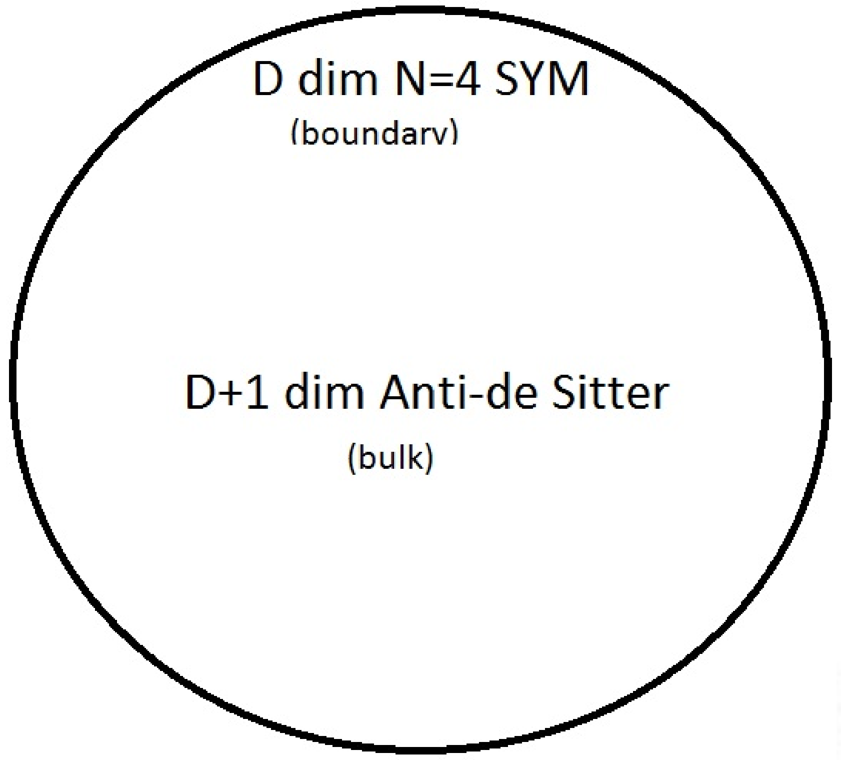
“Fifth” co-ordinate is size z / z’ of proj/target



5 kinematical Parameters:

2-d Longitudinal	$p^\pm = p^0 \pm p^3 \simeq \exp[\pm \log(s/\Lambda_{qcd})]$
2-d Transverse space:	$x'_\perp - x_\perp = b_\perp$
1-d Resolution:	$z = 1/Q$ (or $z' = 1/Q'$)

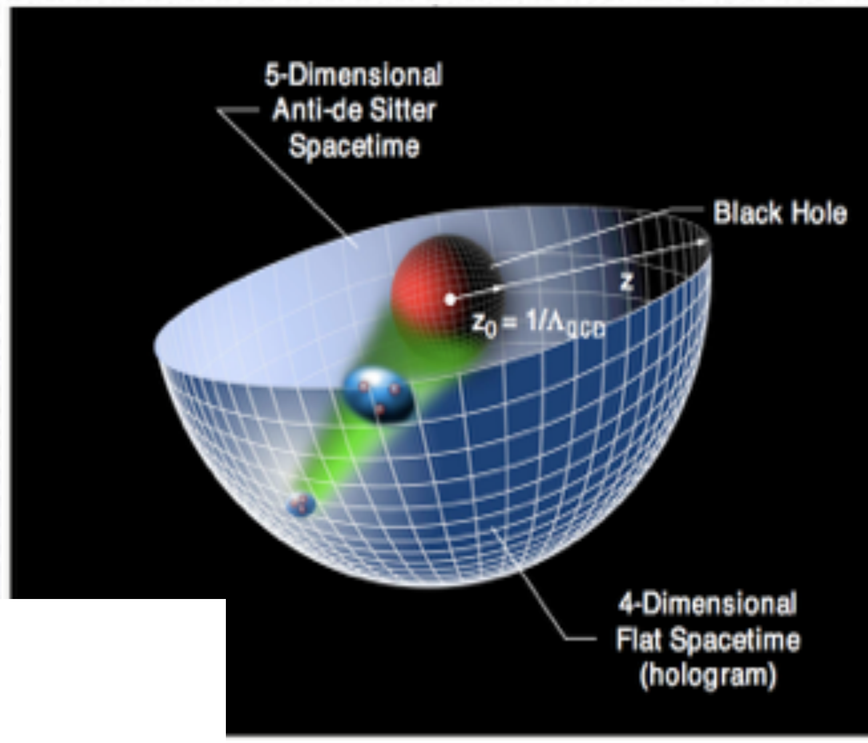
Background and Motivation



The AdS/CFT is a holographic duality
that equates a string theory (gravity) in high dimension
with a conformal field theory (gauge) in 4 dimensions.
Specifically, compactified 10 dimensional super string theory
is conjectured to correspond to $\mathcal{N} = 4$ Super Yang Mills theory
in 4 dimensions in the limit of large 't Hooft coupling:

$$\lambda = g_s N = g_{ym}^2 N_c = R^4/\alpha'^2 >> 1.$$

$$\boxed{ds^2 = \frac{R^2}{z^2}\left[dz^2 + dx\cdot dx\right] + R^2 d\Omega_5 \rightarrow e^{2A(z)}\left[dz^2 + dx\cdot dx\right] + R^2 d\Omega_5}$$



Unification and Universality:

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- Gauge Dynamics — Geometry of Space-Time
- Weak-Strong Duality

$\mathcal{N} = 4$ SYM Scattering at High Energy

$$\langle e^{\int d^4x \phi_i(x)\mathcal{O}_i(x)} \rangle_{CFT} = \mathcal{Z}_{string}\left[\phi_i(x,z)|_{z\sim 0} \rightarrow \phi_i(x)\right]$$

Bulk Degrees of Freedom from type-IIB Supergravity on AdS_5:

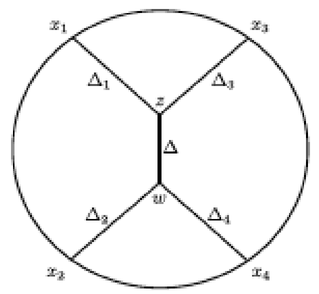
- metric tensor: G_{MN}
- Kalb-Ramond 2 Forms: B_{MN}, C_{MN}
- Dilaton and zero form: ϕ and C_0

$$\lambda = g^2 N_c \rightarrow \infty$$

Supergravity limit

- Strong coupling
- Conformal
- Pomeron as Graviton in AdS

Conformal Invariance and Pomeron Interaction from AdS/CFT



+ ....

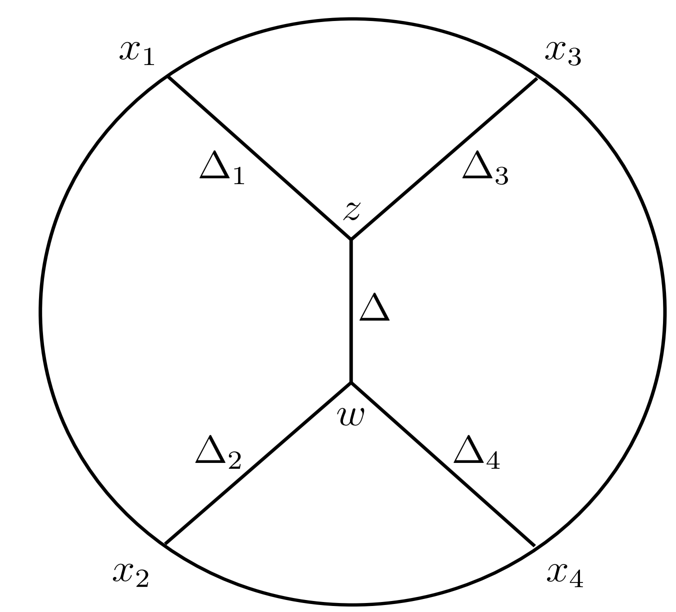
Technique: Summing generalized Witten Diagrams

Freedman et al., hep-th/9903196

Brower, Polchinski, Strassler, and Tan, hep-th/0003115

- Draw all "Witten-Feynman" Diagrams in AdS_5,
- High Energy Dominated by Spin-2 Exchanges:

$$p_1 + p_2 \to p_3 + p_4$$

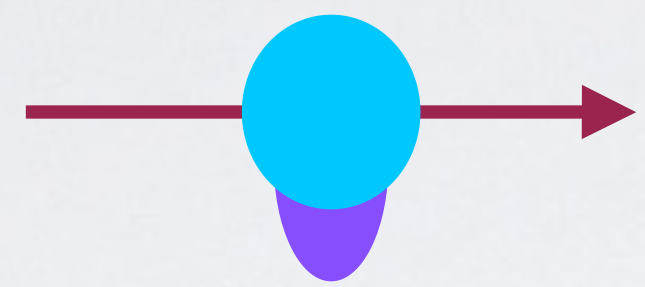


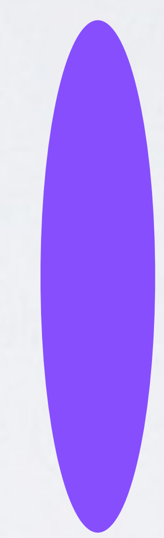
One Graviton Exchange at High Energy

$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5}\int \frac{dz'}{z'^5}\, \tilde\Phi_\Delta(p_1^2,z)\tilde\Phi_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde\Phi_\Delta(p_2^2,z')\tilde\Phi_\Delta(p_4^2,z')$$

$$\boxed{\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')}$$

BASIC BUILDING BLOCK

- Elastic Vertex: 

- Pomeron/Graviton Propagator: 

$$\mathcal{K}(s,b,z,z') = -\left(\frac{(zz')^2}{R^4}\right)\int \frac{dj}{2\pi i}\,\left(\frac{1+e^{-i\pi j}}{\sin \pi j}\right)\,\widehat{s}^j\,G_j(z,x^{\perp},z',x'^{\perp};j)$$

conformal:

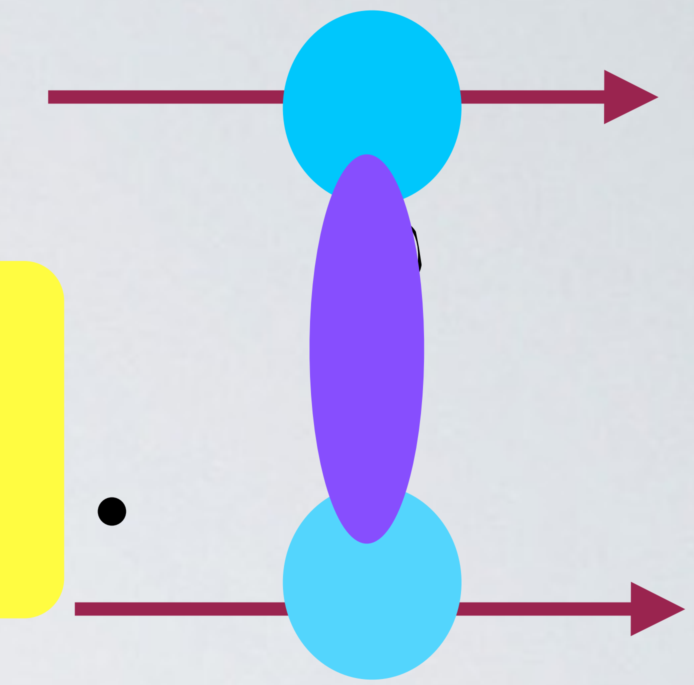
$$G_j(z,x^{\perp},z',x'^{\perp}) = \frac{1}{4\pi z z'}\frac{e^{(2-\Delta(j))\xi}}{\sinh \xi}\ ,$$

$$\Delta(j) = 2 + \sqrt{2}\,\lambda^{1/4}\sqrt{(j-j_0)}$$

ADS BUILDING BLOCKS BLOCKS

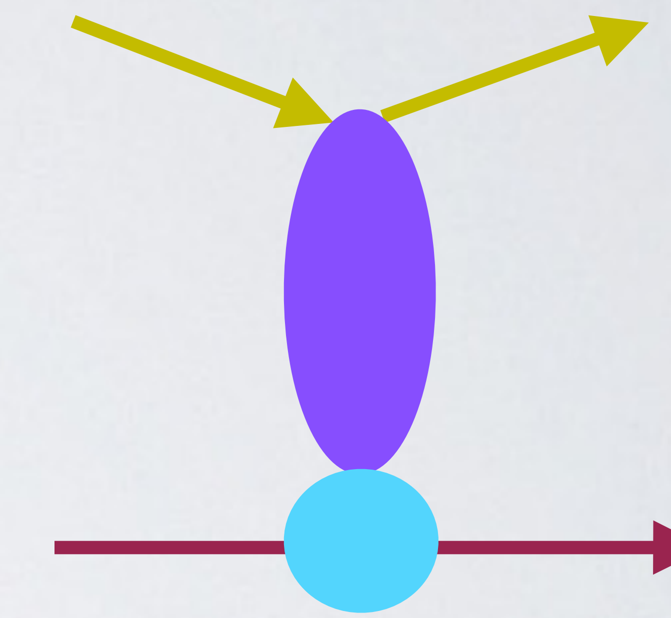
For 2-to-2

$$A(s,t) = \Phi_{13} * \widetilde{\mathcal{K}}_P * \Phi_{24}\,.$$



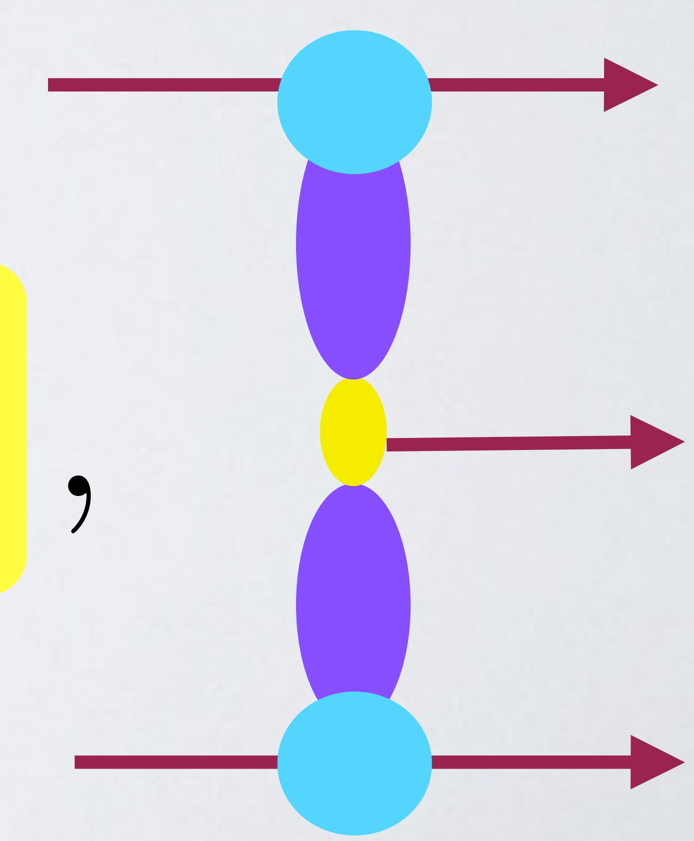
$$A(s,t) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}'\; e^{i\mathbf{q}_\perp\cdot(\mathbf{x}-\mathbf{x}')}\; \Phi_{13}(z)\; \mathcal{K}(s,\mathbf{x}-\mathbf{x}',z,z')\; \Phi_{24}(z')$$

$$d^3\mathbf{b} \equiv dz d^2x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$



For 2-to-3

$$A(s,s_1,s_2,t_1,t_2) = \Phi_{13} * \widetilde{\mathcal{K}}_P * V * \widetilde{\mathcal{K}}_P * \Phi_{24}\,,$$



Additional Steps for QCD:

- Conformal, therefore no scale and no particles, etc.
  - Confinement
  - Need to consider running coupling in weak coupling
- Spin-2 leads to too rapid an increase for cross sections.
  - Need to consider $\lambda = g^2 N$ finite.

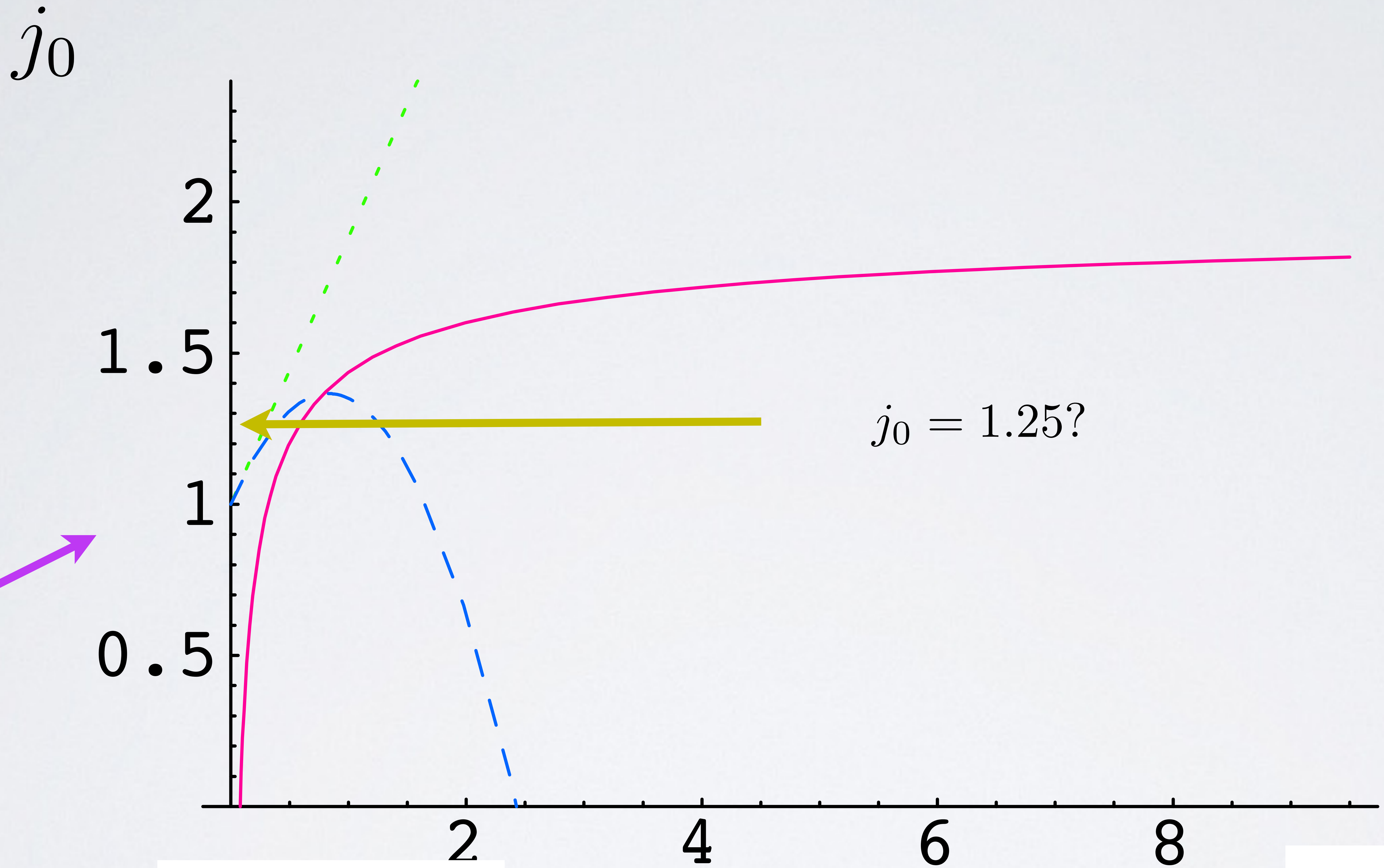
Holographic Approach to QCD

- Spin-2 leads to too fast a rise for cross sections
- Need to consider $\lambda \equiv g^2 N_c$ finite
- Graviton (Pomeron) becomes j-Plane singularity at

$$\boxed{j_0 : \; 2 \to 2 - 2/\sqrt{\lambda}}$$

- Brower, Polchinski, Strassler, and Tan: "The Pomeron and Gauge/String Duality," hep-th/063115

$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$



$j_0 = 2$

Graviton

$j_0 = 1$

Two Gluon

BFKL

QCD?

BPST

$j_0 = 1 + \ln(2) g^2 N_c / \pi^2$

$j_0 = 2 - 2/\sqrt{g^2 N_c}$

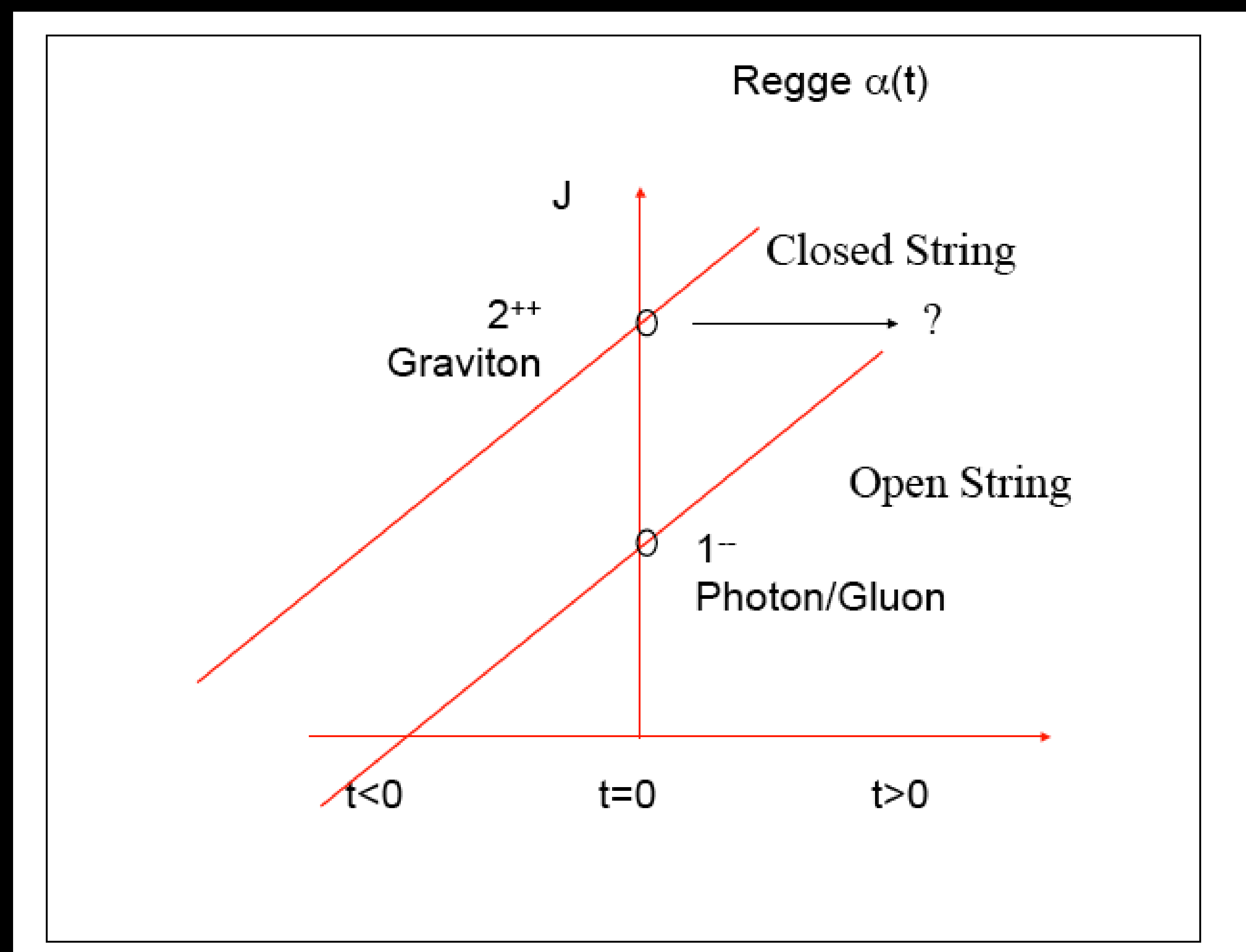
Holographic Approach to QCD

- <u>Comfinement: Particles and Regge trajectories</u>

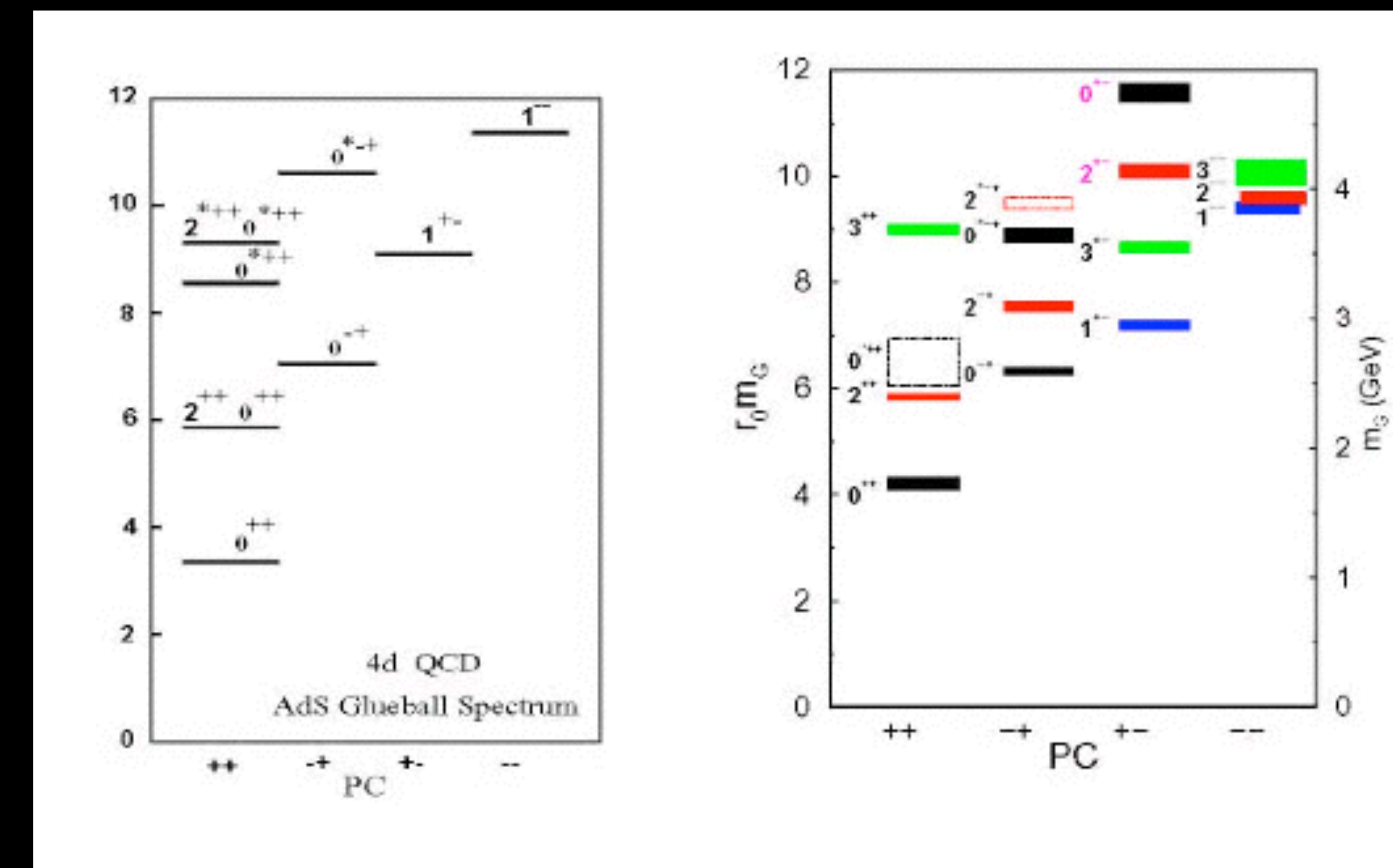
- Brower, Polchinski, Strassler, and Tan: “The Pomeron and Gauge/String Duality,” hep-th/063115

QCD Pomeron <===> Graviton (metric) in AdS

Flat-space String



Confinement



Conformal Invariance

Fixed cut in J-plane:

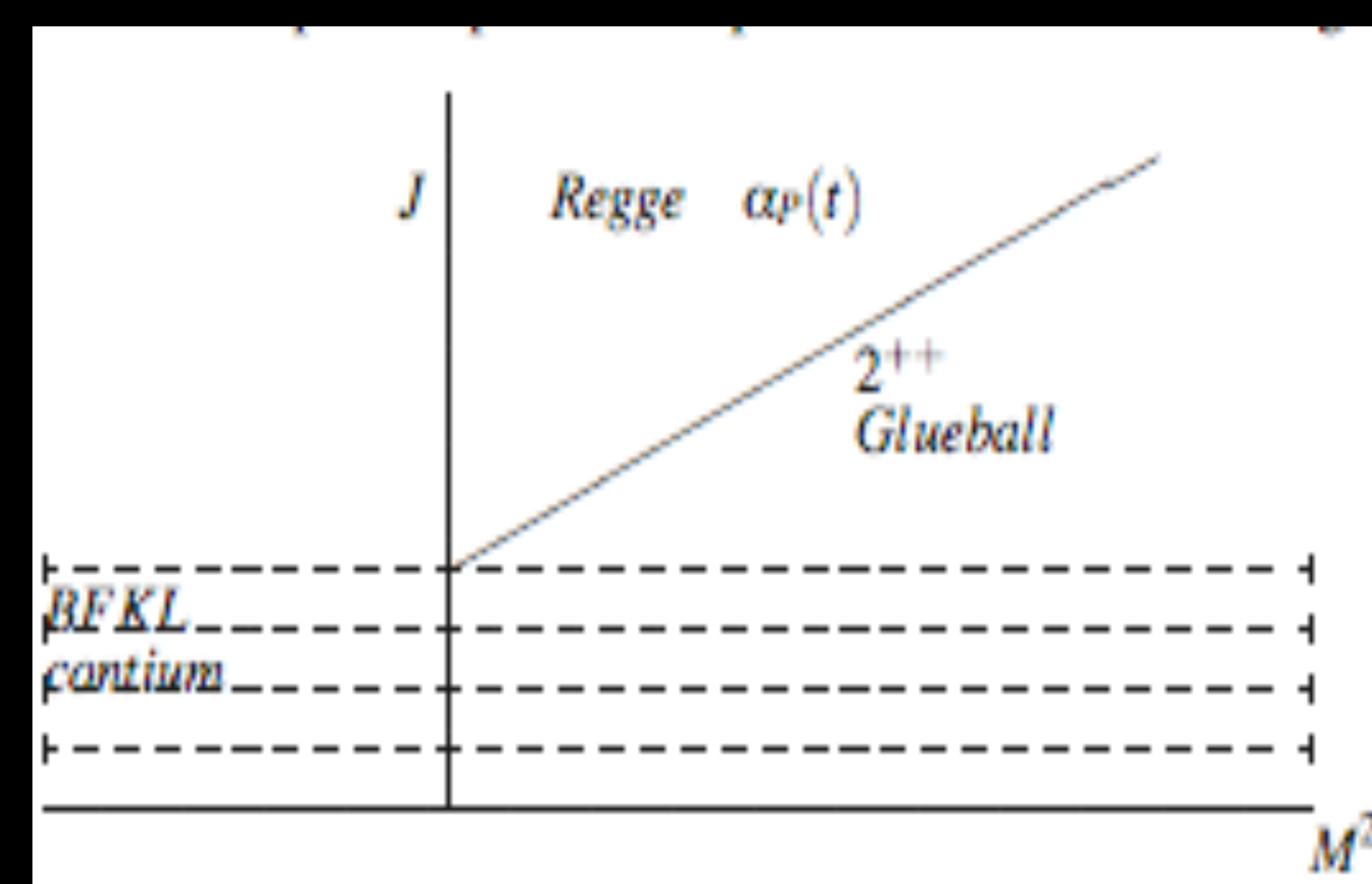
Weak coupling:
(BFKL)

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N$$

Strong coupling:

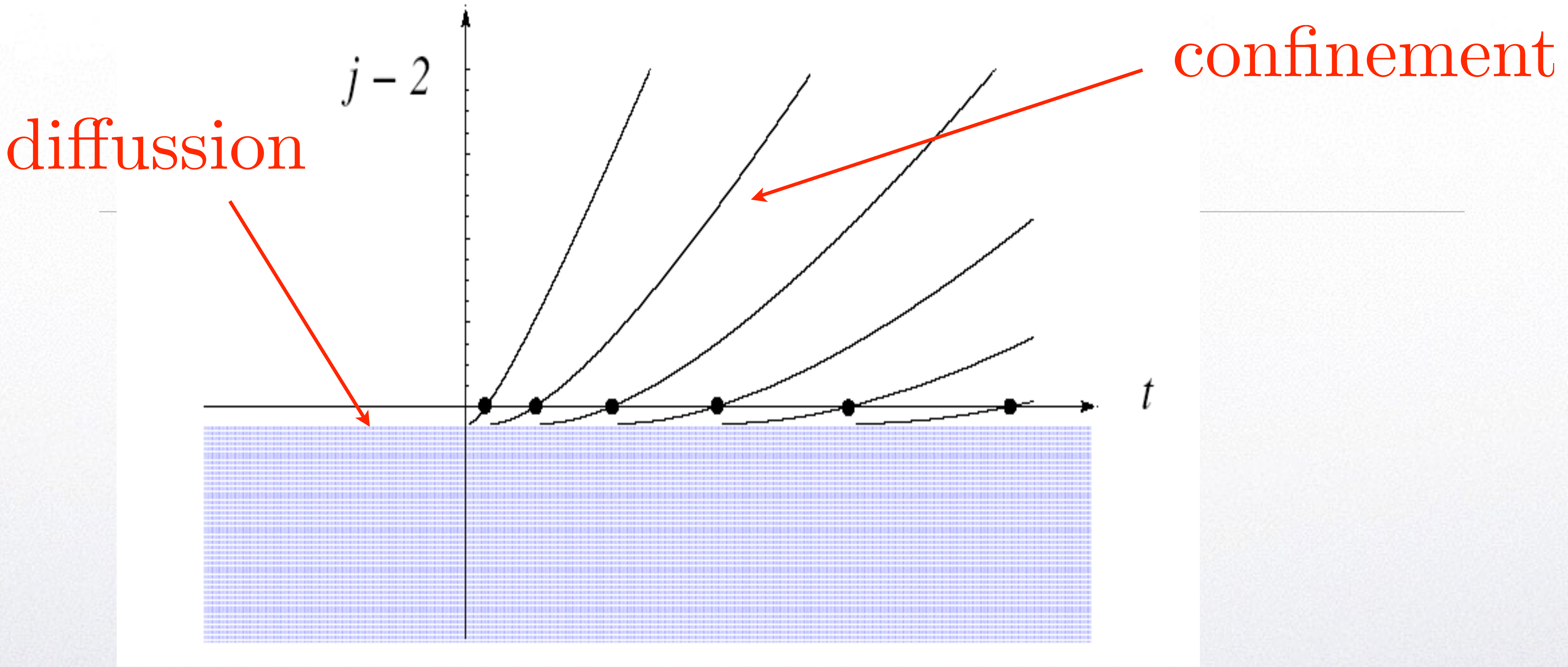
$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

Pomeron in AdS Geometry

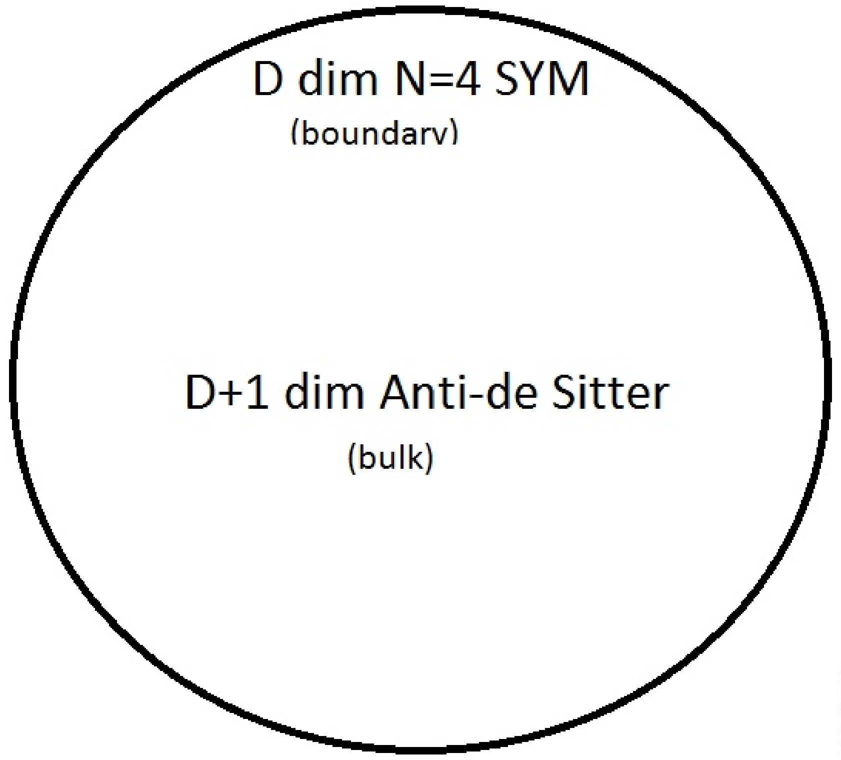


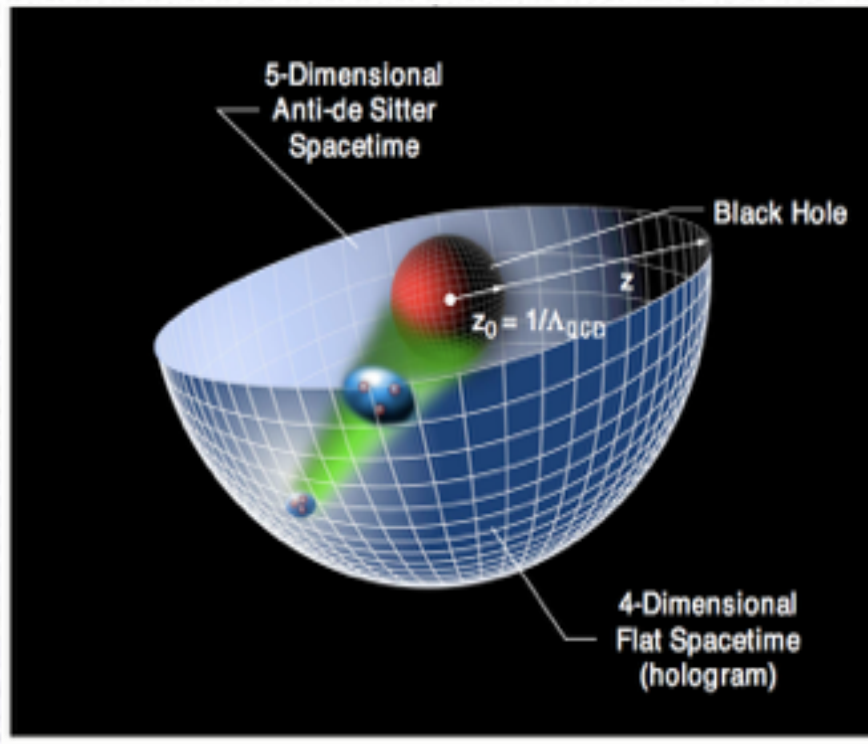
Confinement and Diffusion:

At finite λ, due to Confinement in AdS, *at* $t > 0$ aymptotical linear Regge trajectories



Deformed AdS and Confinement



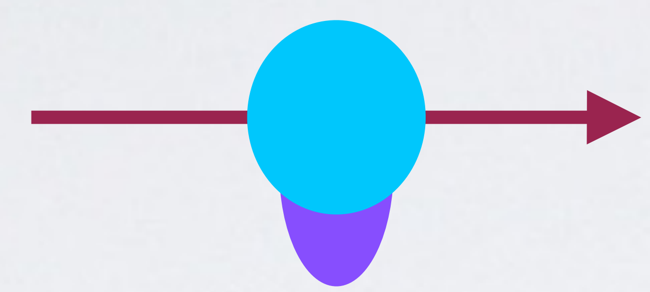


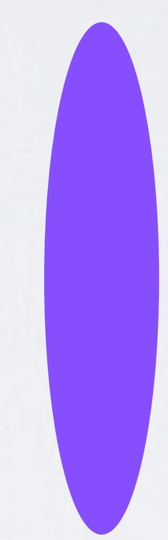
$$ds^2 = \frac{R^2}{z^2}\left[dz^2 + dx\cdot dx\right] + R^2 d\Omega_5 \rightarrow e^{2A(z)}\left[dz^2 + dx\cdot dx\right] + R^2 d\Omega_5$$

For AdS, $A = -\log(z/R)$. As The function $A(z)$ is changed for z large, the space is "deformed" away from pure AdS

$$\text{"Soft-Wall"}:\ A(z) \rightarrow -\log(z/R) + (\Lambda z)^2$$

BASIC BUILDING BLOCK

- Elastic Vertex: 

- Pomeron/Graviton Propagator: 

$$\mathcal{K}(s,b,z,z') = -\left(\frac{(zz')^2}{R^4}\right)\int \frac{dj}{2\pi i}\left(\frac{1+e^{-i\pi j}}{\sin\pi j}\right)\widehat{s}^j\, G_j(z,x^\perp,z',x'^\perp;j)$$

conformal: $$G_j(z,x^\perp,z',x'^\perp) = \frac{1}{4\pi zz'}\frac{e^{(2-\Delta(j))\xi}}{\sinh\xi}\;,$$

$$\Delta(j) = 2+\sqrt{2}\,\lambda^{1/4}\sqrt{(j-j_0)}$$

confinement: $G_j(z,x^\perp,z',x'^\perp;j) \longrightarrow$ discrete sum

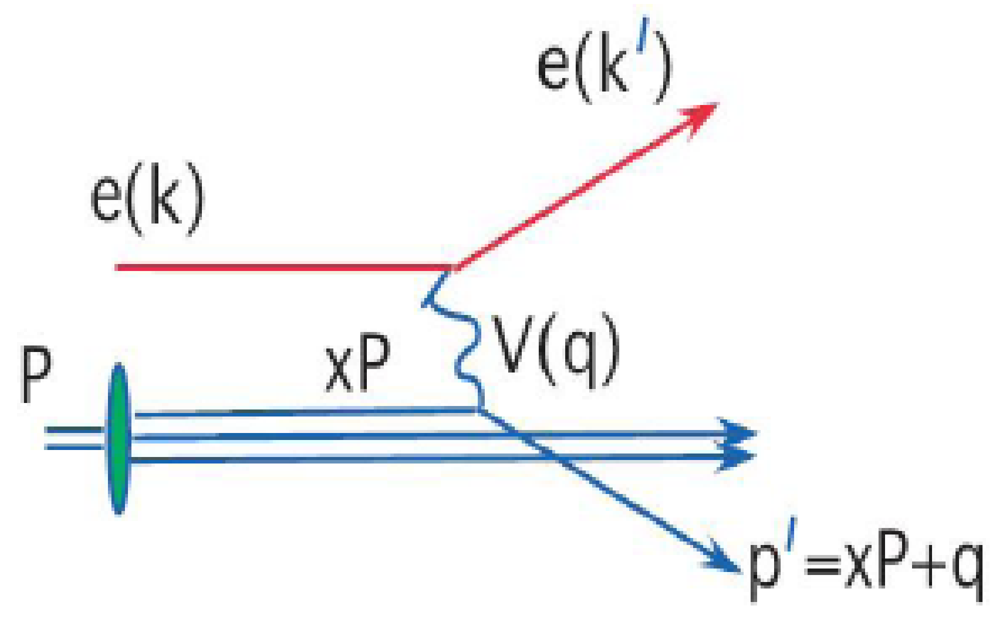
III. Deep Inelastic Scattering (DIS) at small-x:

Conformal Invariance?

Confinement ?

Satuation ?

$$Deep\ Inelastic\ Scattering\ (DIS)$$



$$F_2(x, Q2) = \frac{Q^2}{4\pi^2\alpha_{em}}\left[\sigma_T(\gamma^* p) +_L (\gamma^* p)\right]$$

$$x \equiv \frac{Q^2}{s}$$

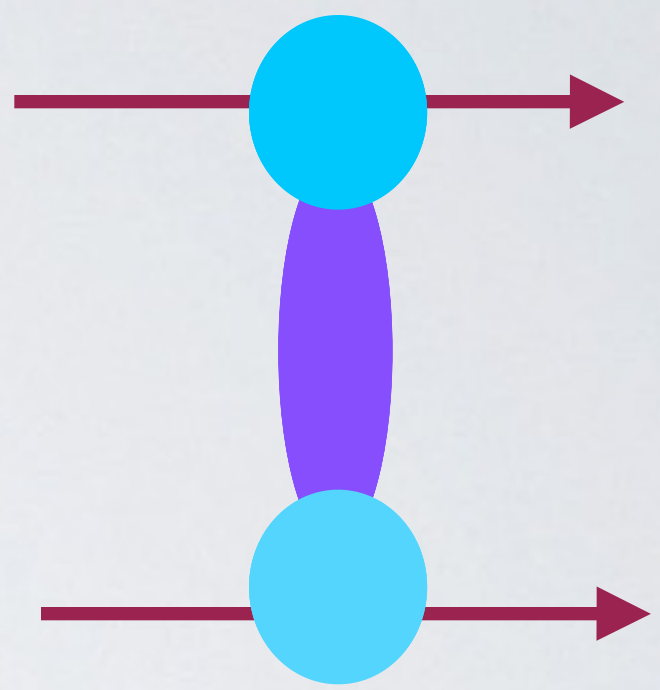
$$Small\ x: \ \frac{Q^2}{s} \to 0$$

$$Optical\ Theorem$$

$$\sigma_{total}(s, Q^2) = (1/s)\mathrm{Im}\ A(s, t = 0; Q^2)$$

ELASTIC VS DIS ADS BUILDING BLOCKS

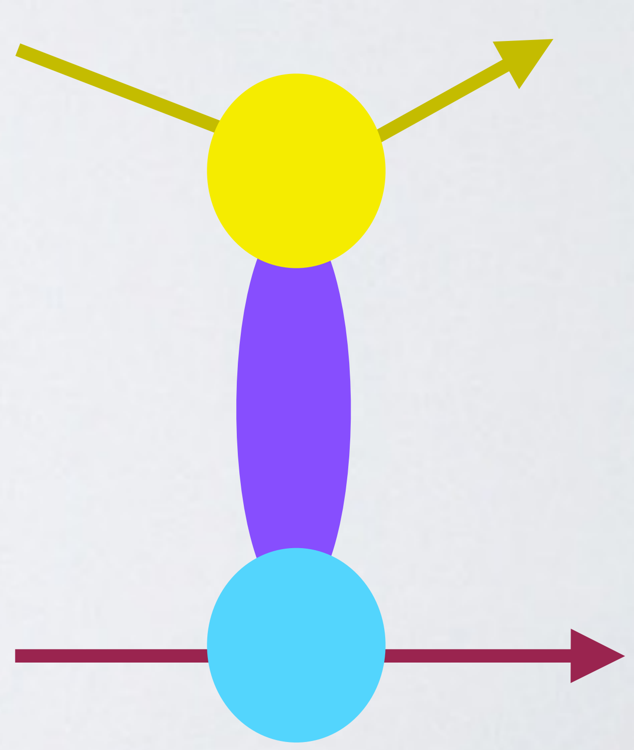
$$A(s, x_\perp - x'_\perp) = g_0^2 \int d^3\mathbf{b} d^3\mathbf{b}' \Phi_{12}(z) G(s, x_\perp - x'_\perp, z, z') \Phi_{34}(z')$$



$$\sigma_T(s) = \frac{1}{s} Im A(s, 0)$$

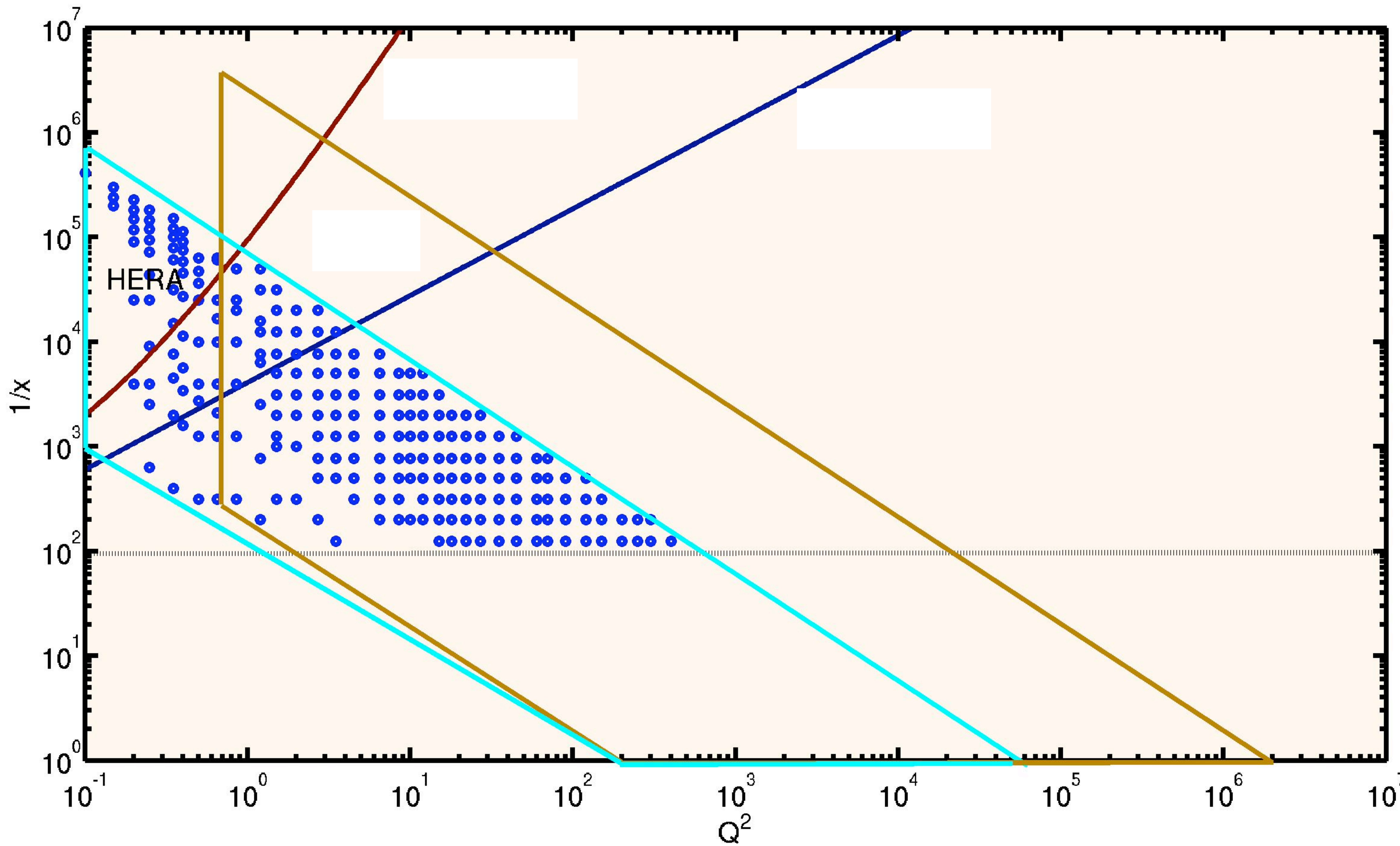
for $\quad F_2(x, Q)$

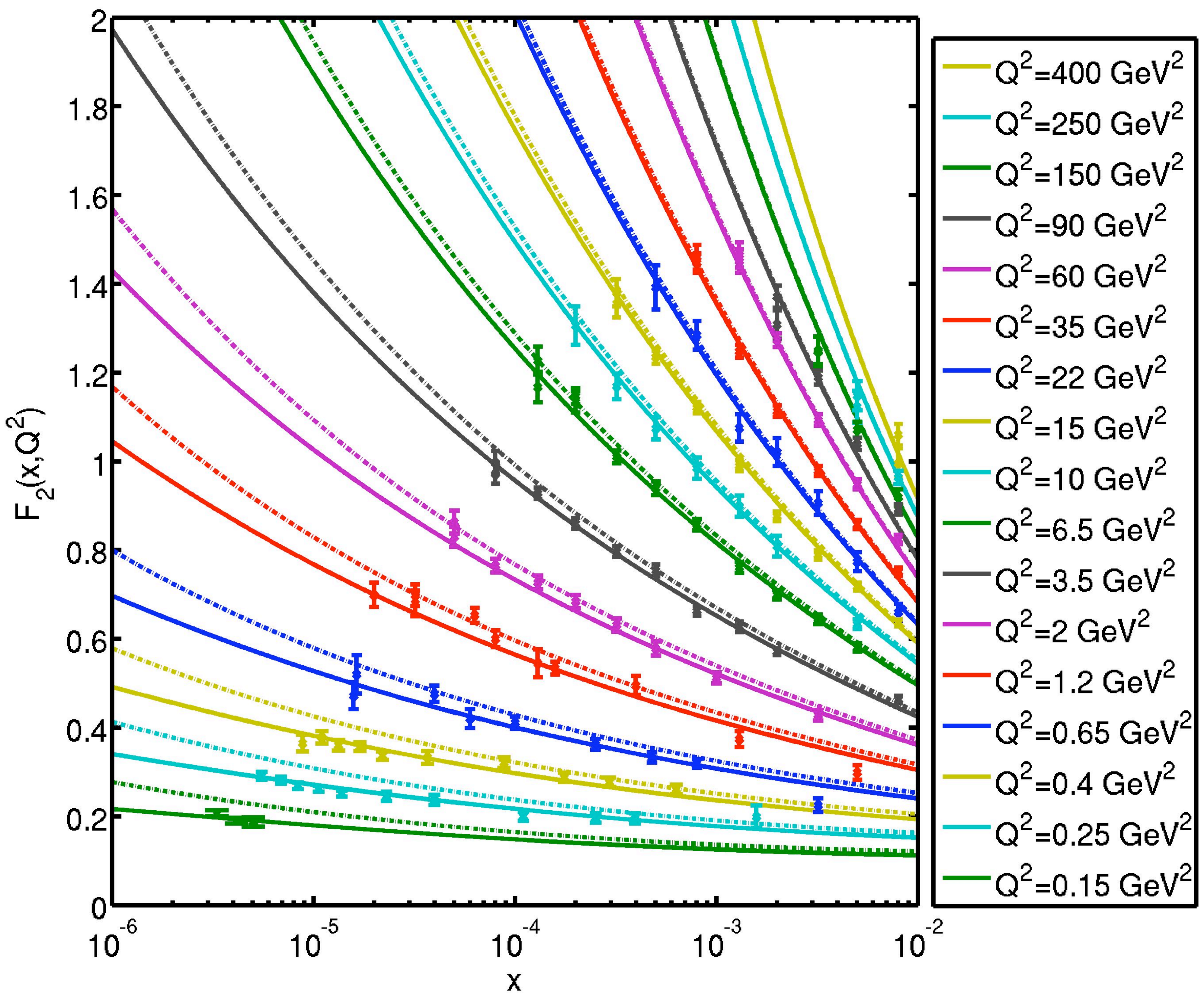
$$\Phi_{13}(z) \to \Phi_{\gamma^*\gamma^*}(z, Q) = \frac{1}{z}[Qz)^4 (K_0^2(Qz) + K_1^2(Qz)]$$



$$d^3\mathbf{b} \equiv dz d^2 x_\perp \sqrt{-g(z)} \quad \text{where} \quad g(z) = \det[g_{nm}] = -e^{5A(z)}$$

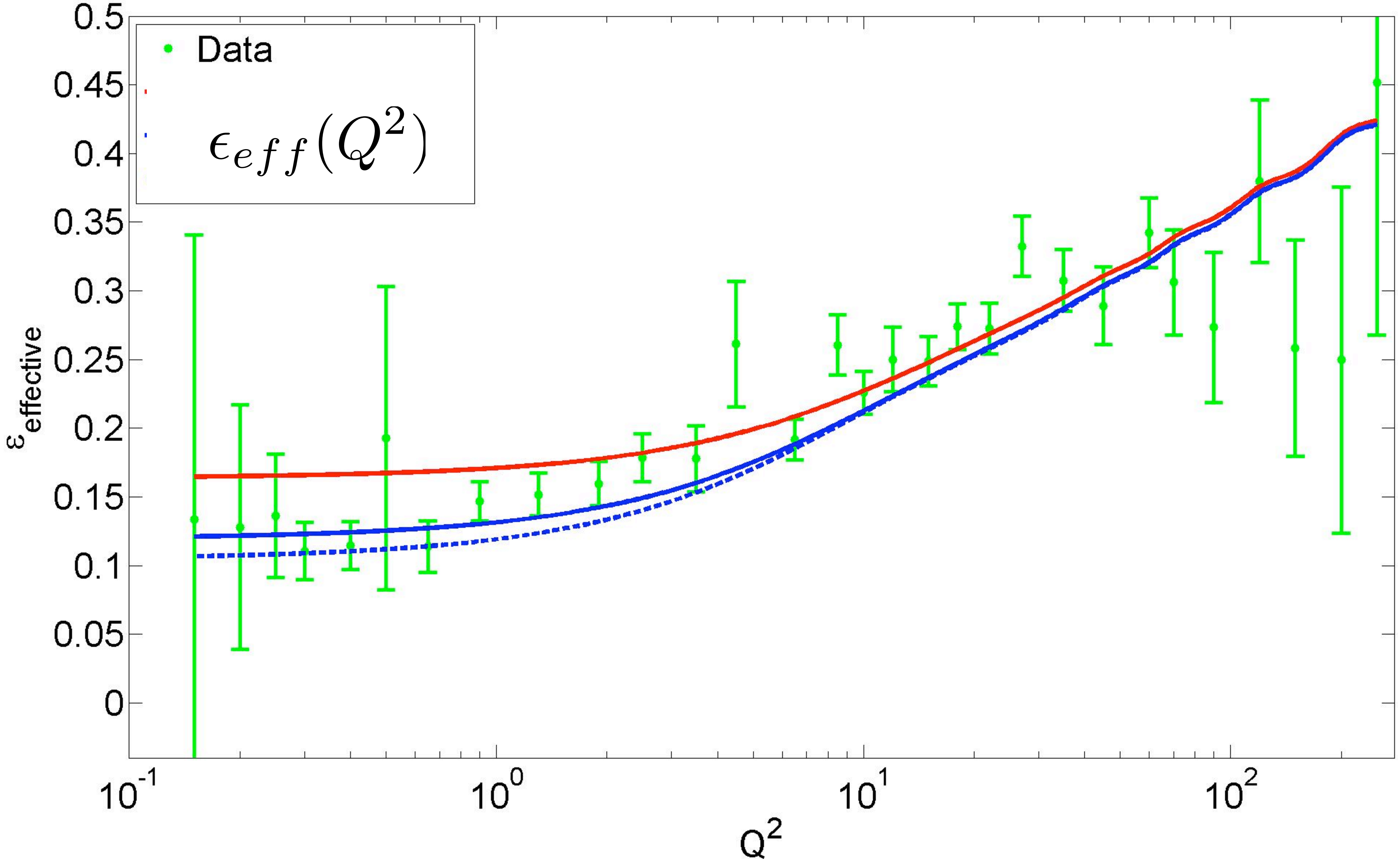
dots are H1-ZEUS small-x data points





$F_2(x,Q^2)$
x
Q^2=400 GeV^2
Q^2=250 GeV^2
Q^2=150 GeV^2
Q^2=90 GeV^2
Q^2=60 GeV^2
Q^2=35 GeV^2
Q^2=22 GeV^2
Q^2=15 GeV^2
Q^2=10 GeV^2
Q^2=6.5 GeV^2
Q^2=3.5 GeV^2
Q^2=2 GeV^2
Q^2=1.2 GeV^2
Q^2=0.65 GeV^2
Q^2=0.4 GeV^2
Q^2=0.25 GeV^2
Q^2=0.15 GeV^2

$$F_2(x,Q^2) \sim (1/x)^{\epsilon_{effective}}$$



Why are various fits generally so good?

Questions on HERA DIS small-x data:

- Why $\alpha_{eff} = 1 + \epsilon_{eff}(Q^2)$?
- Confinement? (Perturbative vs. Non-perturbative?)
- Saturation? (evolution vs. non-linear evolution?)

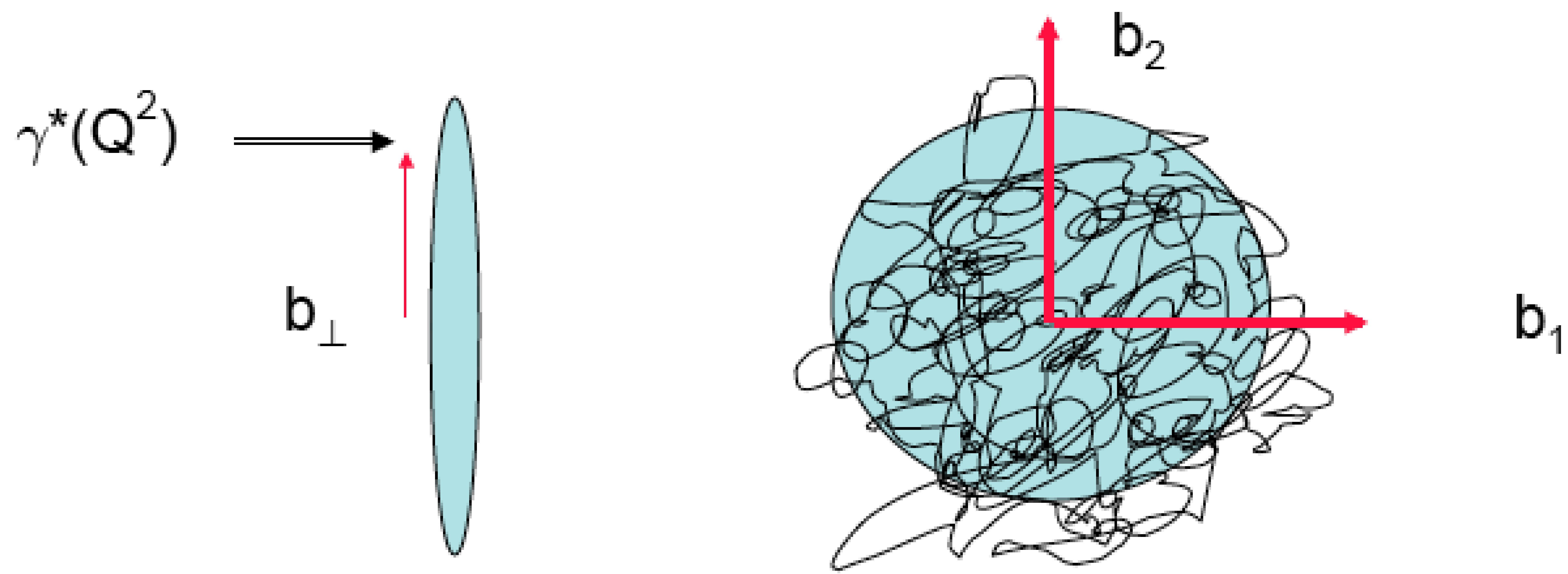
Issues: AdS/CFT for QCD

- Is strong coupling appropriate?
    - In many regimes, DIS can be treated perturbatively, but at small enough x, (for fixed Q^2), the physics is inclusive and becomes generically non-perturbative.
- Is confinement important?
    - Even for single Pomeron exchange, we will see confinement playing a role in determining the onset of saturation.
- Conformal Pomeron and OPE: Pomeron Spectral Curve and Graviton
- Conformal Pomeron/Odderon Intercepts in strong coupling:

IIIa. Conformal Pomeron, DIS at small-x, Anomalous Dimensions

QCD EMERGENCE OF 5-DIM ADS

"Fifth" co-ordinate is size z / z' of proj/target



5 kinematical Parameters:

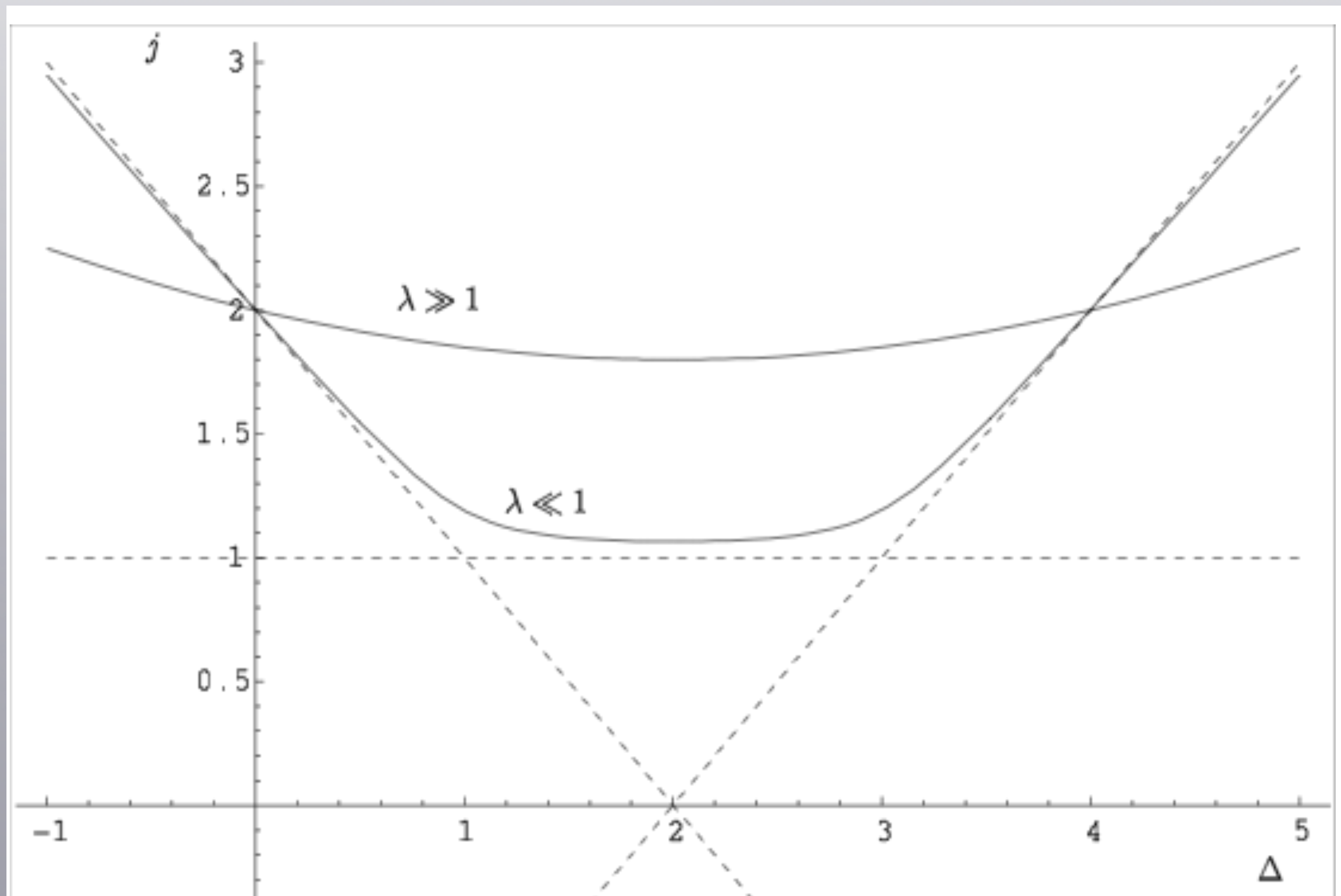
2-d Longitudinal $p^\pm = p^0 \pm p^3 \simeq \exp[\pm \log(s/\Lambda_{qcd})]$

2-d Transverse space: $x'_\perp - x_\perp = b_\perp$

1-d Resolution: $z = 1/Q$ (or $z' = 1/Q'$)

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F_2(x, Q^2) \rightarrow Q^{-\gamma_n}$$



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Simultaneous compatible large Q^2 and small x evolutions!

Energy-Momentum Conservation built-in automatically.

Graviton/Pomeron Regge trajectory [Brower, Polchinski, Strassler, Tan 06]

- Operators that contribute are the twist 2 operators

$$\mathcal{O}_J \sim F_{\alpha[\beta_1} D_{\beta_2} \dots D_{\beta_{J-1}} F_{\beta_J]}{}^{\alpha}$$

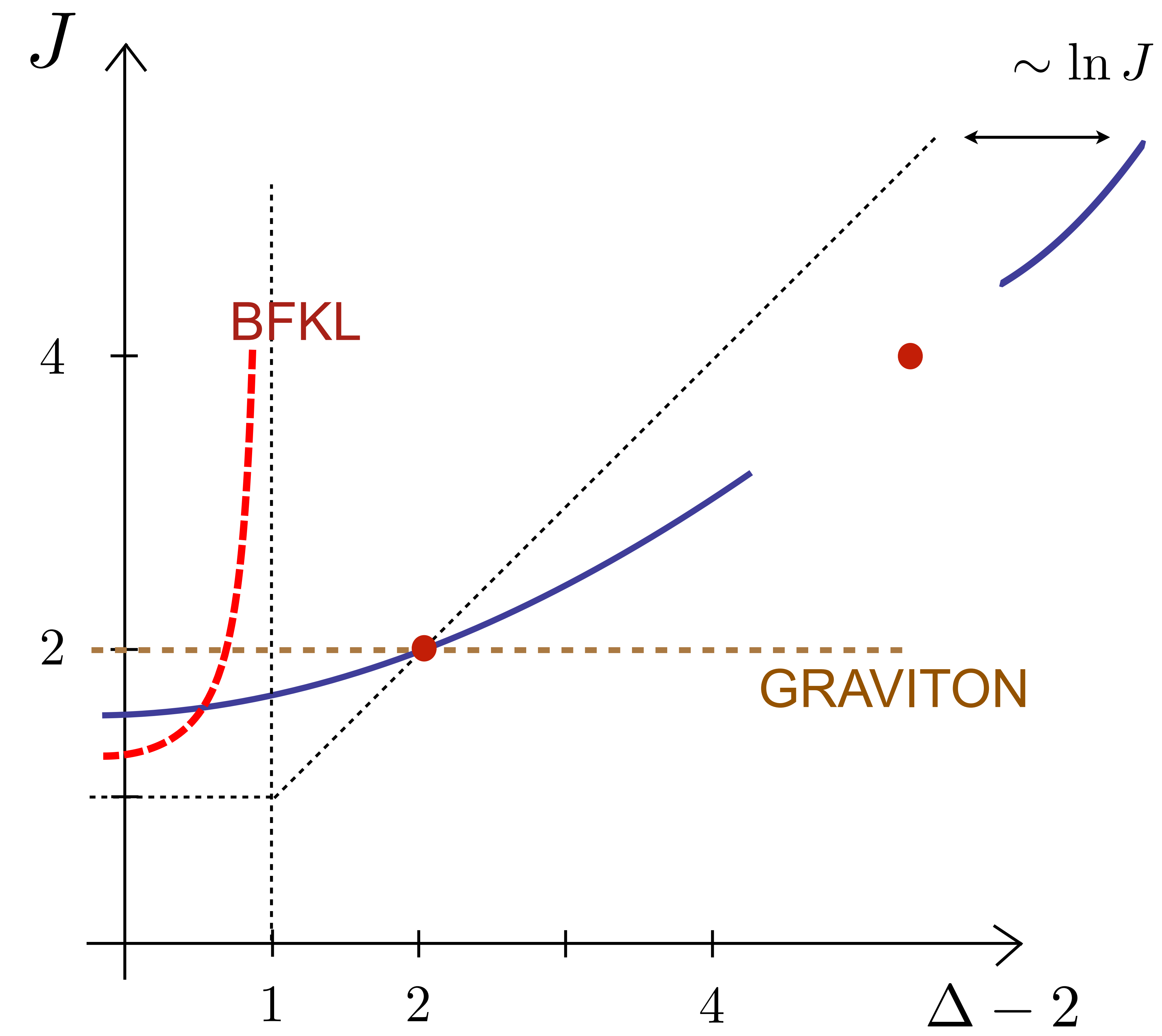
- Dual to string theory spin J field in leading Regge trajectory

$$\boxed{\left(D^2 - m^2\right) h_{a_1 \dots a_J} = 0}$$

$$m^2 = \Delta(\Delta - 4) - J\,, \qquad \Delta = \Delta(J)$$

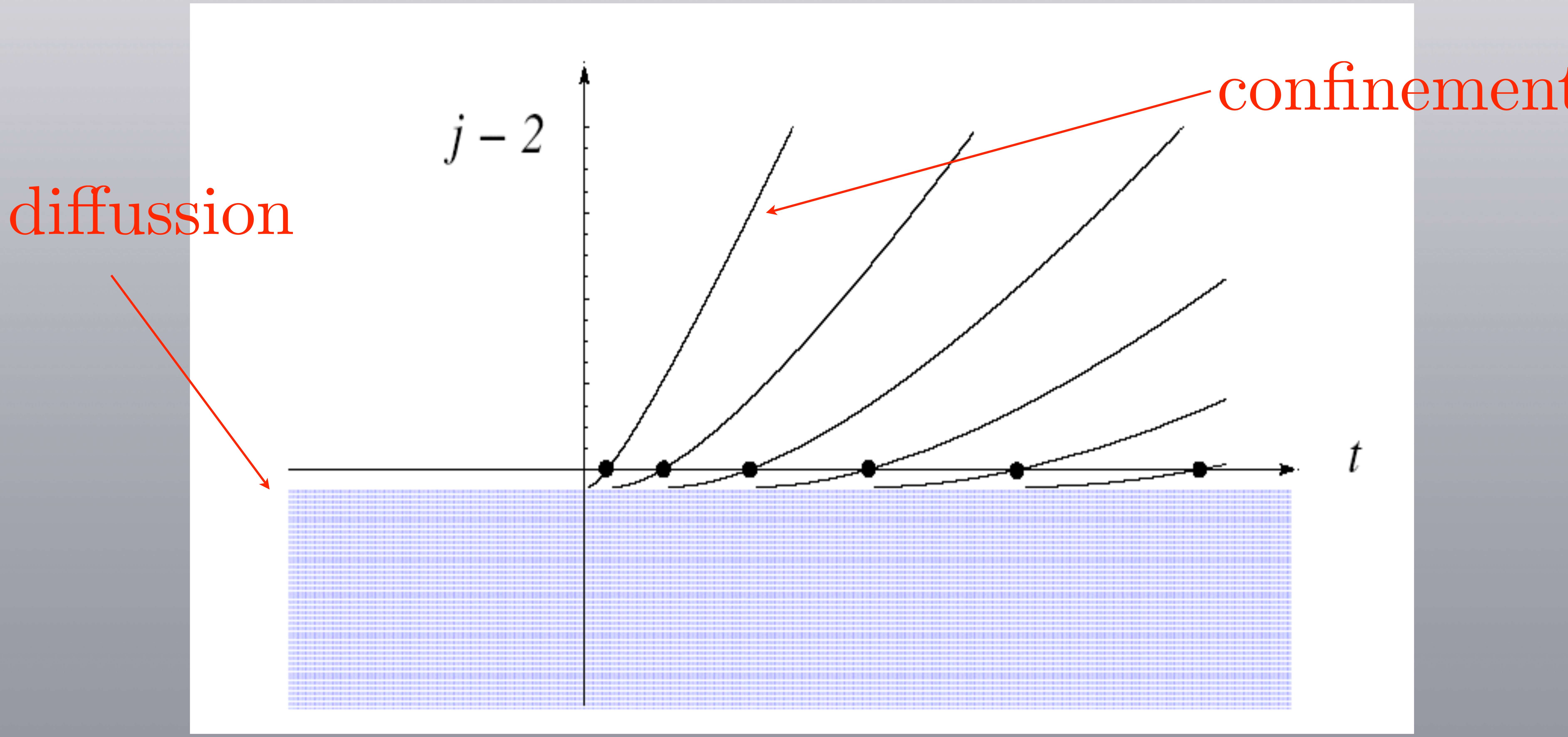
- Diffusion limit

$$J(\Delta) = J_0 + \mathcal{D}\,(\Delta - 2)^2 \implies m^2 = \frac{2}{\alpha'}(J-2) - \frac{J}{L^2}$$



IIIa. Unified Hard (conformal) and Soft (confining) Pomeron

At finite λ, due to Confinement in AdS, *at* $t > 0$ aymptotical linear Regge trajectories



Soft Wall Basics

In order to confine the theory one must effectively deform the AdS geometry. This can be done via:

- Sharp cutoff – $z = z_0 \approx 1/\Lambda_{QCD}$ (Hard Wall Model) [Polchinski, Strassler],[Brower, Djuric, Sarcevic, Tan]
- Gradual increase in length scales / large effective potential boundary for large z leads to possible bound states: confinement

For our geometric softwall, the deformation function becomes A(z)$\rightarrow \Lambda^2 z^2 -$Log(z/R). This leads to a metric

$$ds^2 \rightarrow \frac{e^{2\Lambda^2 z^2} R^2}{z^2} \left[dz^2 + dx \cdot dx \right]$$

We wish to use this soft wall model to describe deep inelastic scattering at leading order in the regge-limit. The object of interest is the AdS-pomeron, which was identified to be the Regge trajectory of the graviton [Brower, Polchinski, Strassler, Tan]. For us, it is sufficient to consider a purely geometric confinement deformation. However, to describe mesons it will be required to consider other dynamical fields in the bulk. [Karch, Katz, Son, Stephanov], [de Teramond, Brodsky], [Batell, Gherghetta]

IIIb. Saturation — Eikonal Sum

- Sum over all Pomeron graph (string perturbative, $1/N^2$)
- Eikonal summation in AdS_3
- Constraints from Conformal Invariance, Unitarity, Analyticity, Confinement, Universality, etc.
- Froissart Bound?
- “non-perturbative” (e.g., blackhole production)

Scattering in Conformal Limit:

Use the condition: $\chi(s, x^\perp - x'^\perp, z, z') = O(1)$

Elastic Ring: $b_{\rm diff} \sim \sqrt{zz'}\,(zz's/N^2)^{1/6}$

No Froissart $\boxed{\sigma_{total} \sim s^{1/3}}$

Inner Absorptive Disc:

$b_{\rm black} \sim \sqrt{zz'}\;\dfrac{(zz's)^{(j_0-1)/2}}{\lambda^{1/4}N}$ $\qquad$ $b_{\rm black} \sim \sqrt{zz'}\;\left(\dfrac{(zz's)^{j_0-1}}{\lambda^{1/4}N}\right)^{1/\sqrt{2\sqrt{\lambda}}(j_0-1)}$

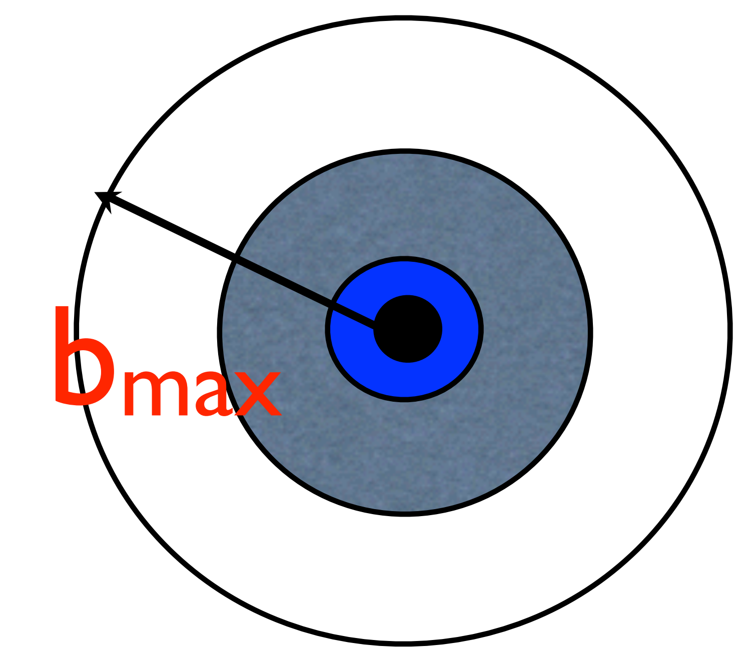
Inner Core: "black hole" production ?

Saturation of Froissart Bound

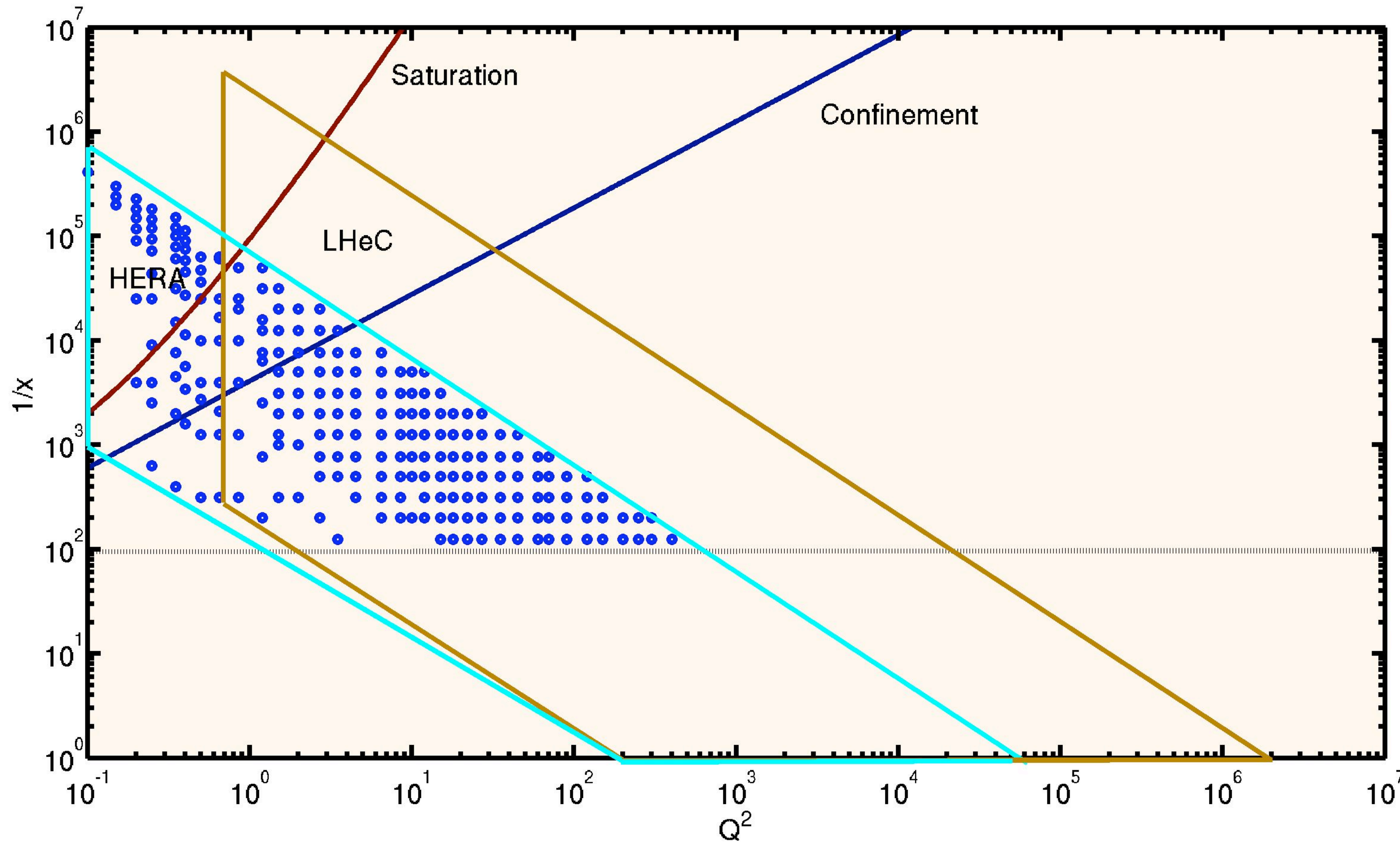
- The Confinement deformation gives an exponential cutoff for $b > b_{max} \sim c \log (s/s_0)$,
- Coefficient $c \sim 1/m_0$, m_0 being the mass of lightest tensor glueball.
- Froissart is respected and saturated.

$$\Delta b \sim \log(s/s_0)$$

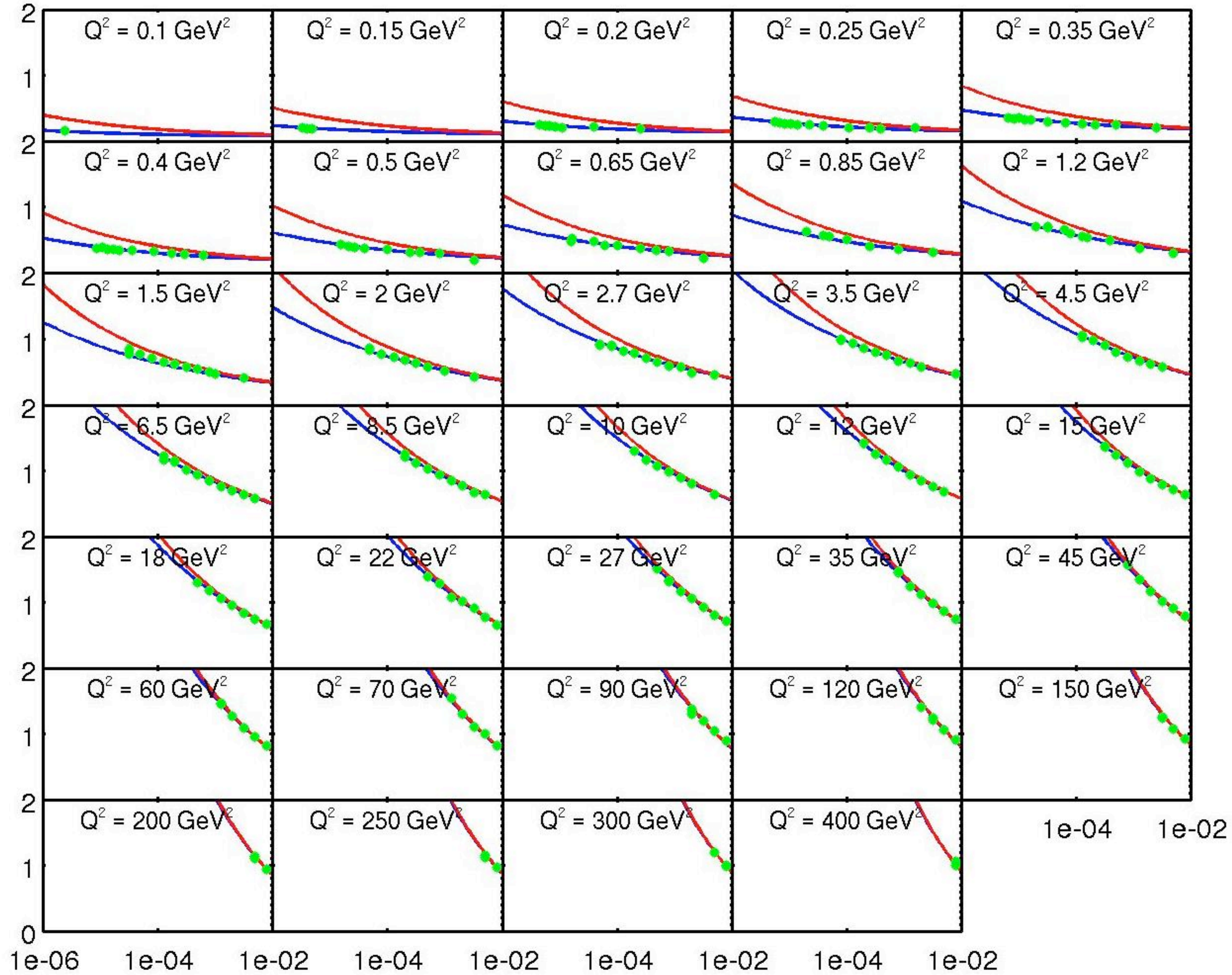
Disk picture



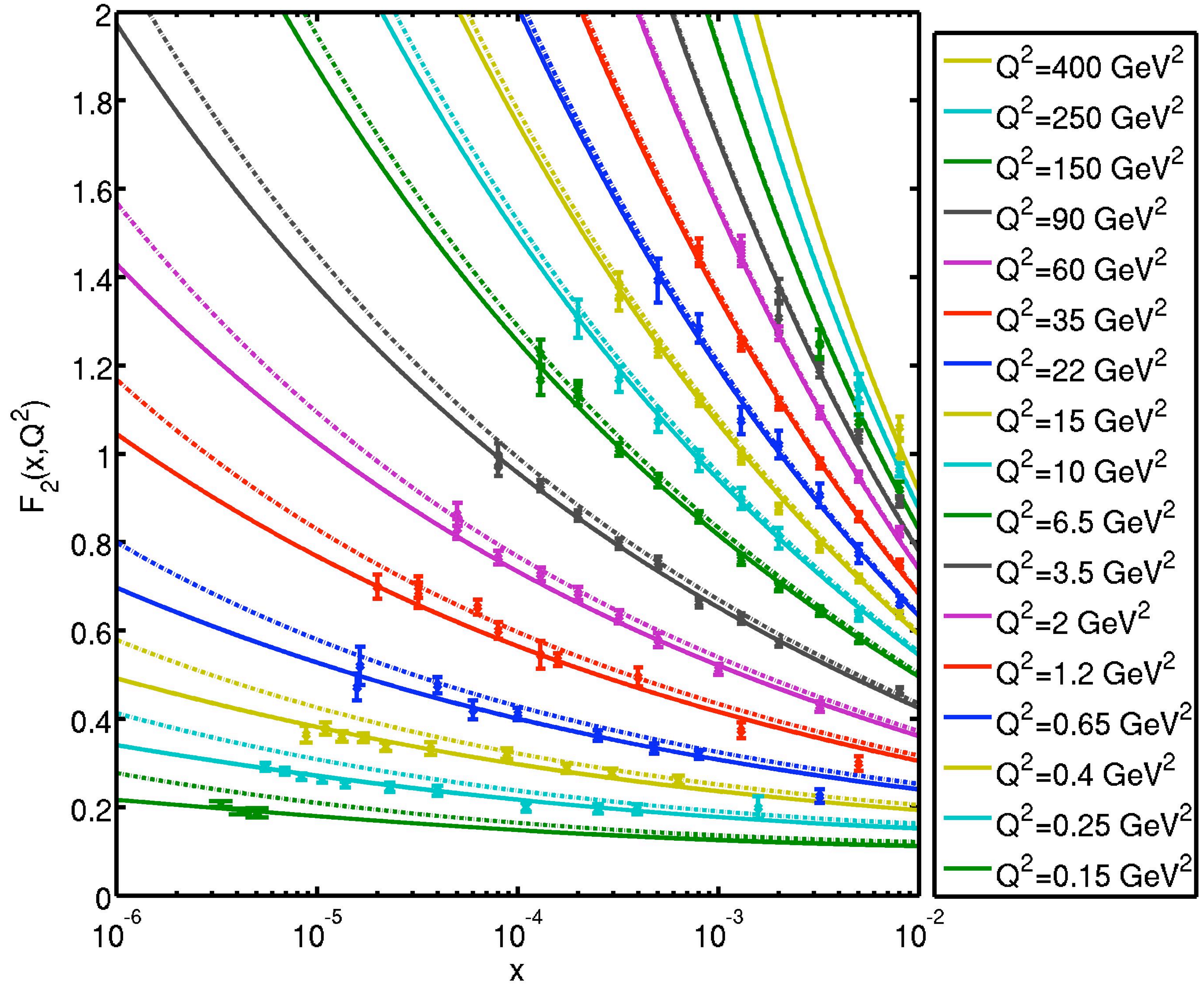
b_{max} determined by confinement.



Saturation
Confinement
LHeC
HERA
1/x
Q^2



Q² = 0.1 GeV²
Q² = 0.15 GeV²
Q² = 0.2 GeV²
Q² = 0.25 GeV²
Q² = 0.35 GeV²
Q² = 0.4 GeV²
Q² = 0.5 GeV²
Q² = 0.65 GeV²
Q² = 0.85 GeV²
Q² = 1.2 GeV²
Q² = 1.5 GeV²
Q² = 2 GeV²
Q² = 2.7 GeV²
Q² = 3.5 GeV²
Q² = 4.5 GeV²
Q² = 6.5 GeV²
Q² = 8.5 GeV²
Q² = 10 GeV²
Q² = 12 GeV²
Q² = 15 GeV²
Q² = 18 GeV²
Q² = 22 GeV²
Q² = 27 GeV²
Q² = 35 GeV²
Q² = 45 GeV²
Q² = 60 GeV²
Q² = 70 GeV²
Q² = 90 GeV²
Q² = 120 GeV²
Q² = 150 GeV²
Q² = 200 GeV²
Q² = 250 GeV²
Q² = 300 GeV²
Q² = 400 GeV²
1e-06
1e-04
1e-02
0
1
2



$F_2(x,Q^2)$
x
Q^2=400 GeV^2
Q^2=250 GeV^2
Q^2=150 GeV^2
Q^2=90 GeV^2
Q^2=60 GeV^2
Q^2=35 GeV^2
Q^2=22 GeV^2
Q^2=15 GeV^2
Q^2=10 GeV^2
Q^2=6.5 GeV^2
Q^2=3.5 GeV^2
Q^2=2 GeV^2
Q^2=1.2 GeV^2
Q^2=0.65 GeV^2
Q^2=0.4 GeV^2
Q^2=0.25 GeV^2
Q^2=0.15 GeV^2

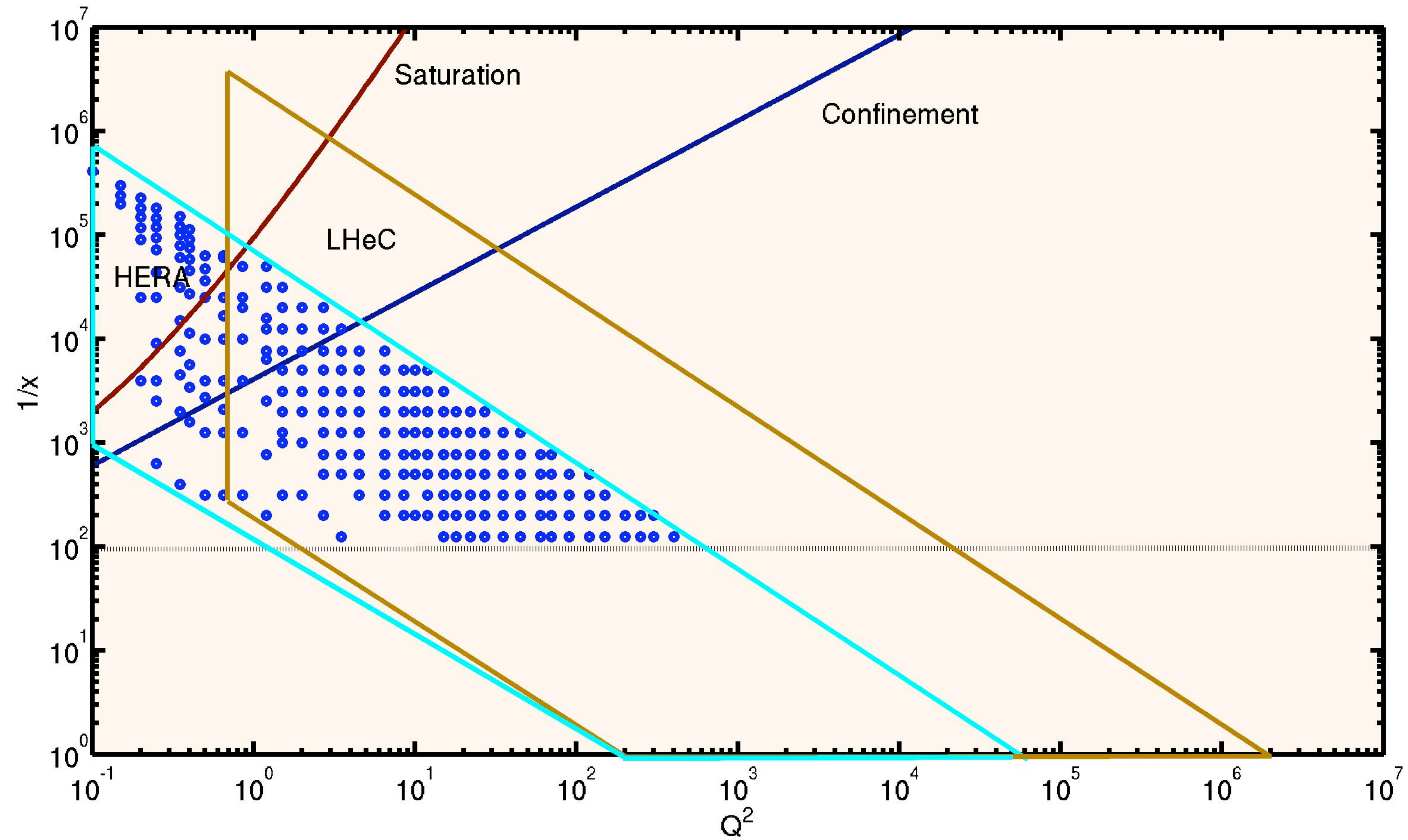
Comparison With Previous Work

Model	ρ	g_0^2	z_0	Q'	χ^2_{dof}
conformal	0.774*	110.13*	–	0.5575* GeV	11.7 (0.75*)
hard wall	0.7792	103.14	4.96 GeV^{-1}	0.4333 GeV	1.07 (0.69*)
softwall	0.7774	108.3616	8.1798 GeV^{-1}	0.4014 GeV	1.1035
softwall*	0.6741	154.6671	8.3271 GeV^{-1}	0.4467 GeV	1.1245

Comparison of the best fit (including a χ sieve) values for the conformal, hard wall, and soft wall AdS models. The final row includes the soft wall with improved intercept. [Costa, Goncalves, Penedones][Gromov, Levkovich-Maslyuk, Sizov, Valatka]The statistical errors (omitted) are all $\sim 1\%$ of fit parameters.

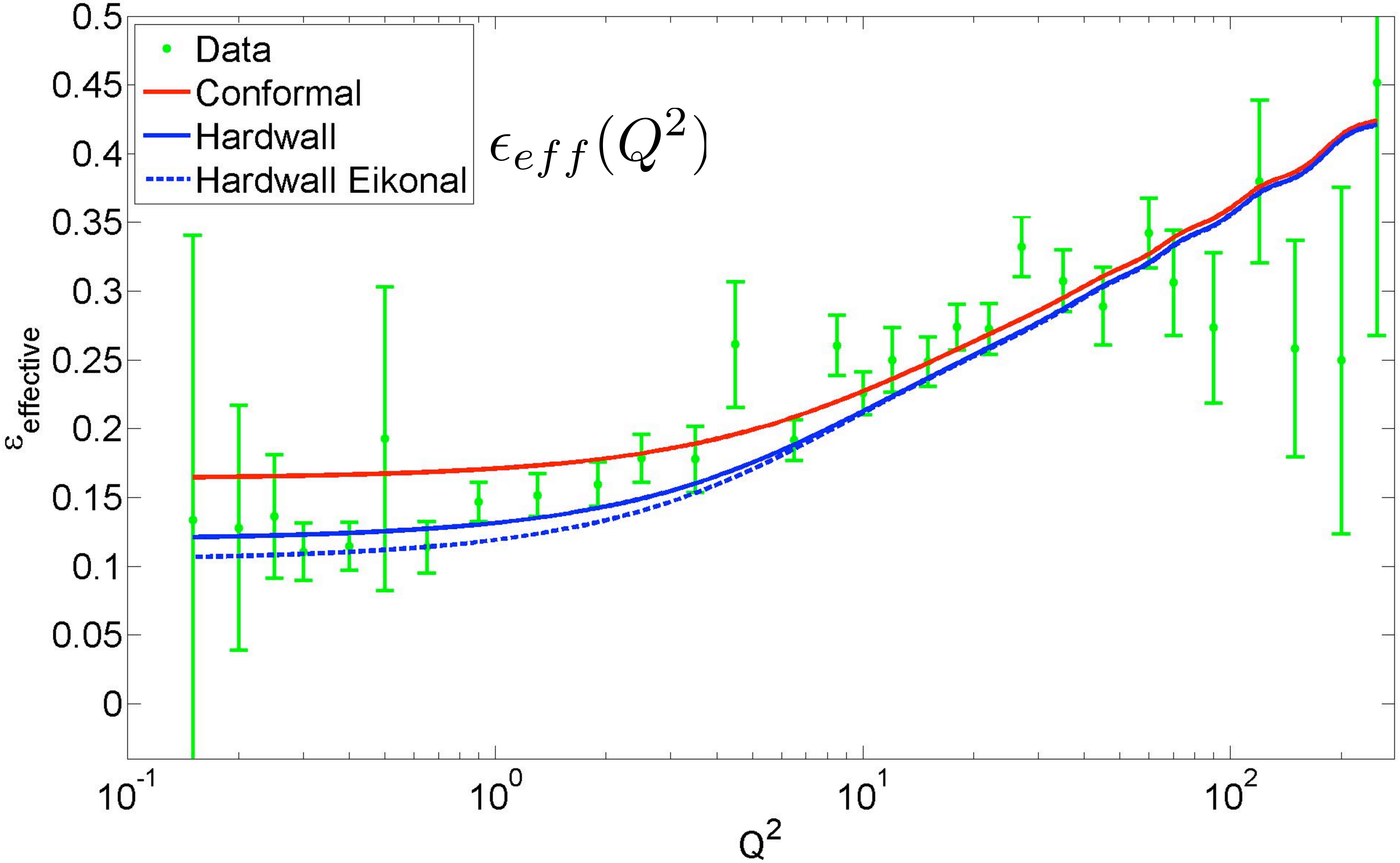
As expected, best fit values imply

$$\rho \to \lambda > 1 \qquad 1/z_0 \sim \Lambda_{QCD} \quad \text{and} \quad Q' \sim m_{proton}$$



Saturation
Confinement
LHeC
HERA
1/x
Q^2
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
10^{-1} 10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7

$$F_2(x, Q^2) \sim (1/x)^{\epsilon_{effecti}}$$



IV: Pomeron in the conformal Limit, OPE, and Anomalous Dimensions

$$G_{mn} = g^0_{mn} + h_{mn}$$

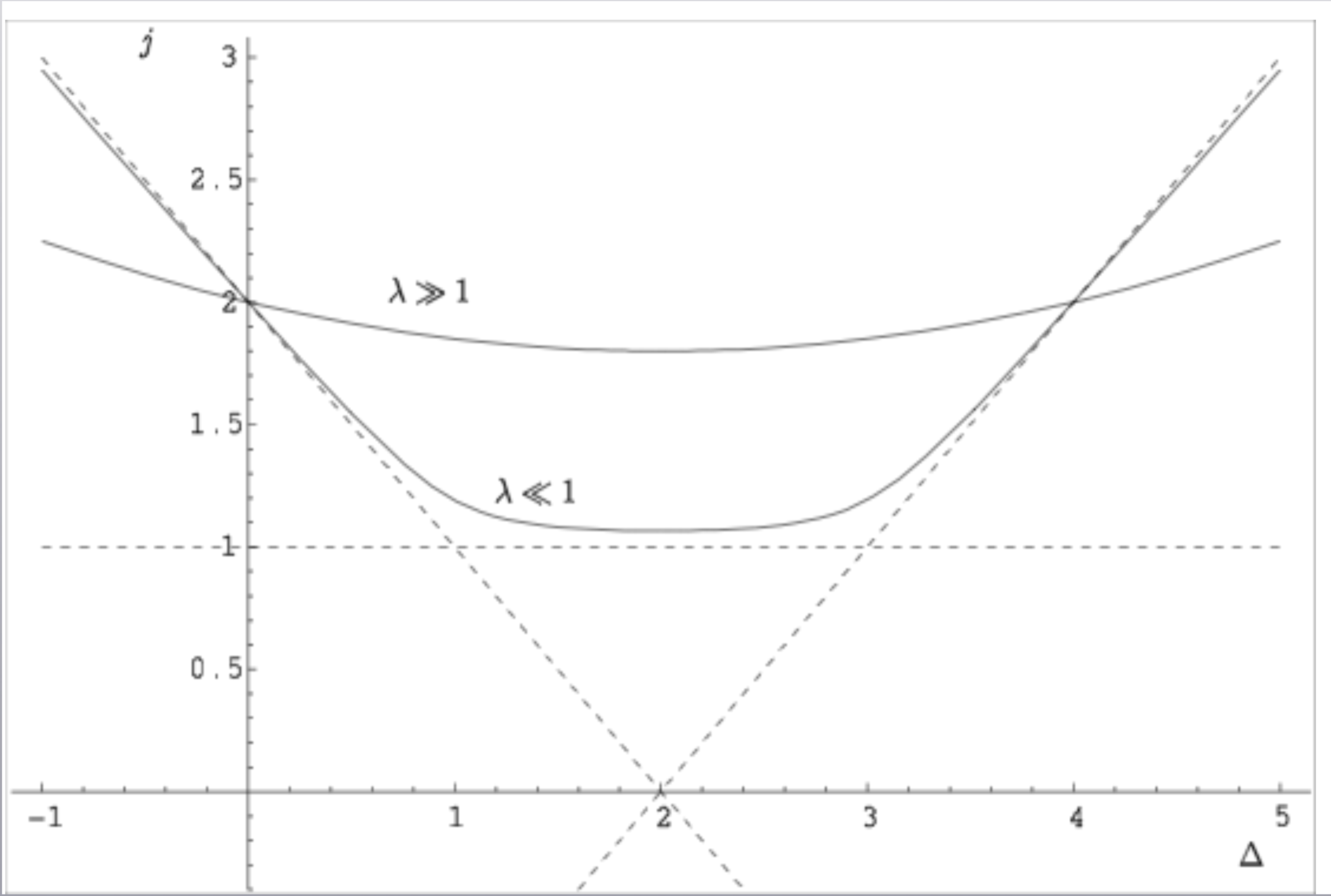
Massless modes of a closed string theory:

Need to keep higher string modes

As CFT, equivalence to OPE in strong coupling: using AdS

MOMENTS AND ANOMALOUS DIMENSION

$$M_n(Q^2) = \int_0^1 dx\, x^{n-2} F_2(x, Q^2) \to Q^{-\gamma_n}$$



$$\gamma_2 = 0$$

$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Simultaneous compatible large Q^2 and small x evolutions!

Energy-Momentum Conservation built-in automatically.

Graviton/Pomeron Regge trajectory [Brower, Polchinski, Strassler, Tan 06]

- Operators that contribute are the twist 2 operators

$$\mathcal{O}_J \sim F_{\alpha[\beta_1} D_{\beta_2} \ldots D_{\beta_{J-1}} F_{\beta_J]}{}^{\alpha}$$

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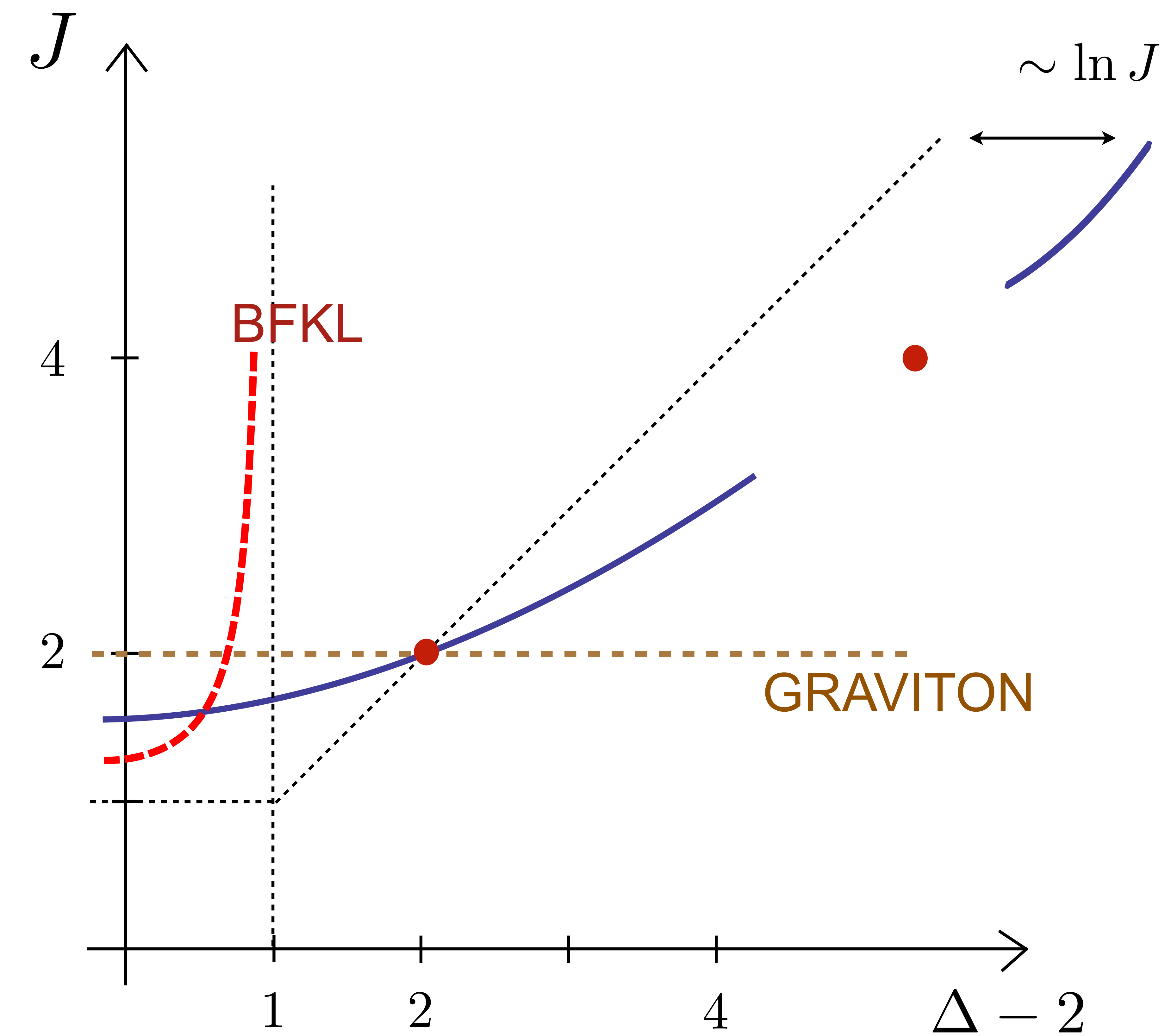
$$m^2 = \Delta(\Delta-4) - J\,, \qquad \Delta = \Delta(J)$$

- Diffusion limit

$$J(\Delta) = J_0 + \mathcal{D}\,(\Delta-2)^2 \;\Longrightarrow\; m^2 = \frac{2}{\alpha'}(J-2) - \frac{J}{L^2}$$



V: More on Pomeron and Odderon in the conformal Limit

Massless modes of a closed string theory:

metric tensor, $G_{mn} = g^0_{mn} + h_{mn}$

Kolb-Ramond anti-sym. tensor, $b_{mn} = -b_{nm}$

dilaton, etc. $\phi, \chi, \cdots$

POMERON AND ODDERON IN STRONG COUPLING:

$$\widetilde{\Delta}(S)^2 = \tau^2 + a_1(\tau,\lambda)S + a_2(\tau,\lambda)S^2 + \cdots$$

B.Basso, 1109.3154v2

POMERON

$$\alpha_p = 2 - \frac{2}{\lambda^{1/2}}$$

Brower, Polchinski, Strassler, Tan

Kotikov, Lipatov, et al.

ODDERON

Solution-a: $\alpha_O = 1 - \frac{8}{\lambda^{1/2}} -$

Solution-b: $\alpha_O = 1 - \frac{0}{\lambda^{1/2}} -$

Brower, Djuric, Tan

Avsar, Hatta, Matsuo

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POMERON

$$\alpha_p = 2 - \frac{2}{\lambda^{1/2}} - \frac{1}{\lambda} + \frac{1}{4\lambda^{3/2}} + \frac{6\zeta(3)+2}{\lambda^2} + \frac{18\zeta(3)+\frac{361}{64}}{\lambda^{5/2}} + \frac{39\zeta(3)+\frac{447}{32}}{\lambda^3} + \cdots$$

Brower, Polchinski, Strassler, Tan

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Costa, Goncalves, Penedones (1209.4355)

Kotikov, Lipatov (1301.0882)

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Solution-a:

$$\alpha_O = 1 - \frac{8}{\lambda^{1/2}} - \frac{4}{\lambda} + \frac{13}{\lambda^{3/2}} + \frac{96\zeta(3)+41}{\lambda^2} + \frac{288\zeta(3)+\frac{1823}{16}}{\lambda^{5/2}} + \frac{720\zeta(5)+1344\zeta(3)-\frac{3585}{4}}{\lambda^3}$$

Solution-b:

$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} -$$

Brower, Djuric, Tan

Avsar, Hatta, Matsuo

Brower, Costa, Djuric, Raben, Tan

POMERON AND ODDERON IN STRONG COUPLING:

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Solution-b:

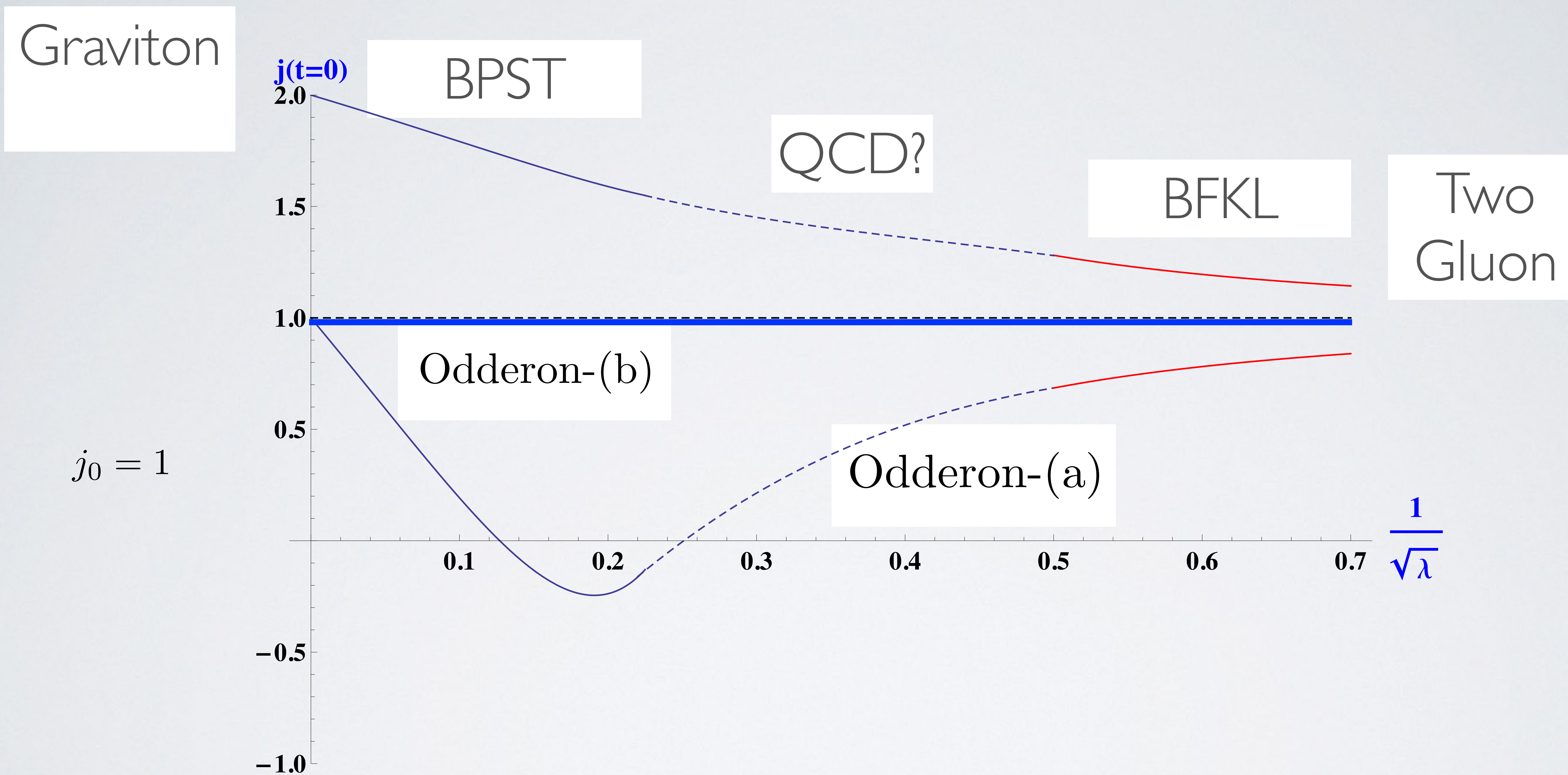
$$\alpha_O = 1 - \frac{0}{\lambda^{1/2}} - \frac{0}{\lambda} + \frac{0}{\lambda^{3/2}} + \frac{0}{\lambda^2} + \frac{0}{\lambda^{5/2}} + \frac{0}{\lambda^3} + \cdots$$

Brower, Djuric, Tan

Avsar, Hatta, Matsuo

Brower, Costa, Djuric, Raben, Tan

$\mathcal{N} = 4$ Strong vs Weak $g^2 N_c$



$j_0 = 1$

VII. Summary and Outlook

- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- First principle description of elastic/total cross sections, DIS at small-x, Central Diffractive Glueball production at LHC, etc.

Backup Slides

Conformal Invariance as Isometry of AdS

Longitudinal Boost: $\tau = \log(\rho z z' s/2)$ $\qquad \mathcal{K}(s,\vec{b},z,z') = \int \frac{dj}{2\pi i}\Big(\frac{e^{-i\pi j}+1}{\sin \pi j}\Big) e^{j\tau}\mathcal{K}(j,\vec{b},z,z')$

Conformal Invariance in Transverse AdS_3: $\xi = \sinh^{-1}\Big(\frac{b^2+(z-z')^2}{2zz'}\Big)$

$$\mathcal{K}(j,\vec{b},z,z') = \int \frac{d\nu}{2\pi}\Big(\frac{e^{i\nu\xi}}{\sinh\xi}\Big)G(j,\nu)$$

Pomeron as a pole in AdS: $\qquad G(j,\nu) = \frac{1}{j - j_0 + \nu^2/2\sqrt{\lambda}}$

Full Conformal Invariance:

$$\mathrm{Im}\,\mathcal{K}(s,\vec{b},z,z') = \int \frac{dj}{2\pi i}\int \frac{d\nu}{2\pi}\Big(\frac{e^{j\tau}e^{i\nu\xi}}{\sinh\xi}\Big)G(j,\nu)$$

$$\boxed{\Delta(j) = 2 + 2\sqrt{(j-j_0)/\rho}}$$

$$\mathcal{K}(j,\vec{b},z,z') \sim \frac{e^{(2-\Delta(j))\xi}}{\sinh\xi} \qquad\qquad \mathcal{K}(s,b,z,z') \sim e^{j_0}\Big(\frac{\xi}{\sinh\xi}\,\frac{\exp(-\frac{\xi^2}{\rho\tau})}{\tau^{3/2}}\Big)$$

Propagators and Wave functions

In this framework the pomeron propagator obeys:

$$\Big[-\partial_z^2 + \Lambda^4 z^2 + (2\Lambda^2 - t) + \frac{\alpha^2(j) - 1/4}{z^2} \Big] \chi_P(j, z, z', t) = \delta(z - z')$$

$$\alpha(j) = \Delta(j) - 2$$

Where as for a continuous t spectrum the solution becomes a combination of Whittaker's functions (generalized hyper geometric functions)

$$\chi_P \sim ...M_{\kappa,\mu}(z_<)W_{\kappa,\mu}(z_>) \tag{2}$$

for $\kappa = \kappa(t)$ and $\mu = \mu(j)$

$$\kappa(t) = t/4\Lambda^2 - 1/2 \qquad \mu(j) = \alpha(j)/2$$

Special Limits, Behavior, and Symmetry

- Λ controls the strength of the soft wall and in the limit $\Lambda \to 0$ one recovers the conformal solution

$$\mathrm{Im}\chi_P^{conformal}(t=0) = \frac{g_0^2}{16}\sqrt{\frac{\rho^3}{\pi}}(zz')\frac{e^{(1-\rho)\tau}}{\tau^{1/2}}exp\left(\frac{-(\mathrm{Log}z - \mathrm{Log}z')^2}{\rho\tau}\right)$$

  where $\tau = \mathrm{Log}(\rho zz' s/2)$ and $\rho = 2 - j_0$. Note: this has a similar behavior to the weak coupling BFKL solution where

$$\mathrm{Im}\chi(p_\perp, p'_\perp, s) \sim \frac{s^{j_0-1}}{\sqrt{\pi\mathcal{D}\mathrm{Log}s}}\exp(-(\mathrm{Log}p'_\perp - \mathrm{Log}p_\perp)^2/\mathcal{D}\mathrm{Log}s)$$

- If we look at the energy dependence of the pomeron propagator, we can see a softened behavior in the forward regge limit.

$$\chi_{conformal} \sim \frac{s^{j_0-1}}{\sqrt{\log s}} \to \chi_{HW} \sim \frac{s^{j_0-1}}{(\log s)^{3/2}}$$

  Analytically, this corresponded to the softening of a j-plane singularity from $1/\sqrt{j-j_0} \to \sqrt{j-j_0}$. Again, we see this same softened behavior in the soft wall model.

For two-to-two scattering involving on-shell hadrons, it is convenient to express the amplitude as

$$A_4(s,t) \simeq 2s \int d^2b e^{-i\mathbf{b}\mathbf{q}_\perp} \int dz dz' P_{13}(z) P_{24}(z')\, \chi(s,b,z,z'),$$

where, for scalar glueball states,

$$P_{ij}(z) = \sqrt{-g(z)}(z/R)^2 \phi_i(z)\phi_j(z)$$

involves a product of two external normalizable wave functions. We have introduced function $\chi(s,b,z,z')$, the "eikonal", where

$$\chi(s,b,z,z') = \frac{g_0^2 R^4}{2(zz')^2 s} \mathcal{K}(s,b,z,z')$$

and $\mathcal{K}(s,b,z,z')$ is the BPST Pomeron kernel.

High Energy Scattering and DIS in String Theory

AdS space continued

- We are interested in calculating the structure function $F_2(x, Q^2)$, which is simply the cross section for an off-shell photon. Using the optical theorem we obtain

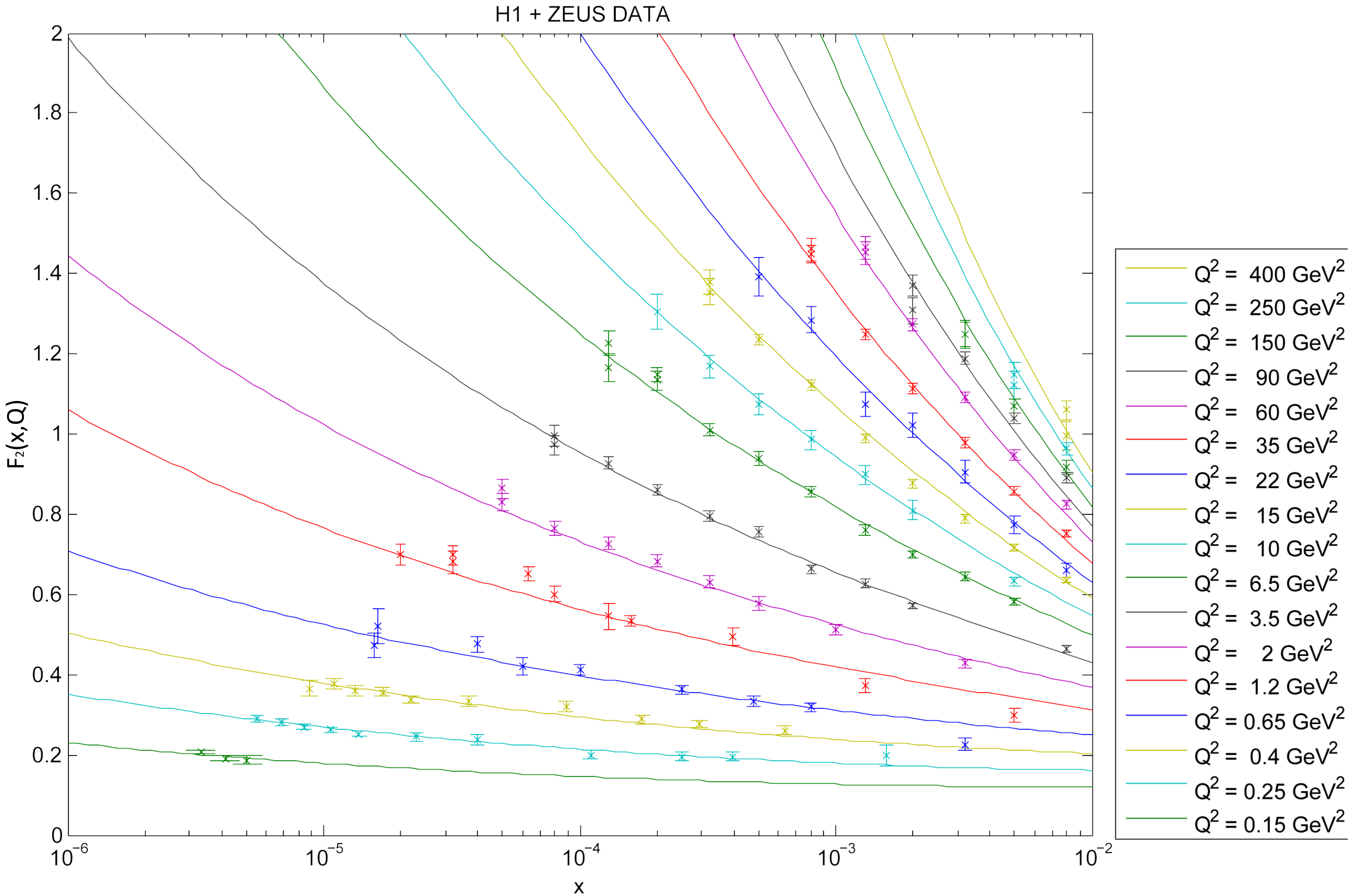
$$\sigma_{tot} \simeq 2 \int d^2b \int dz dz' P_{13}(z) P_{24}(z')\ Im\ \chi(s, b, z, z')$$

- For DIS, P_{13} should present a photon on the boundary that couples to a spin 1 current in the bulk. This current then propagates through the bulk, and scatters off the target.
- The wave function, in the conformal limit, is

$$P_{13}(z) \to P_{13}(z, Q) = \frac{1}{z}(Qz)^4 (K_0^2(Qz) + K_1^2(Qz))$$

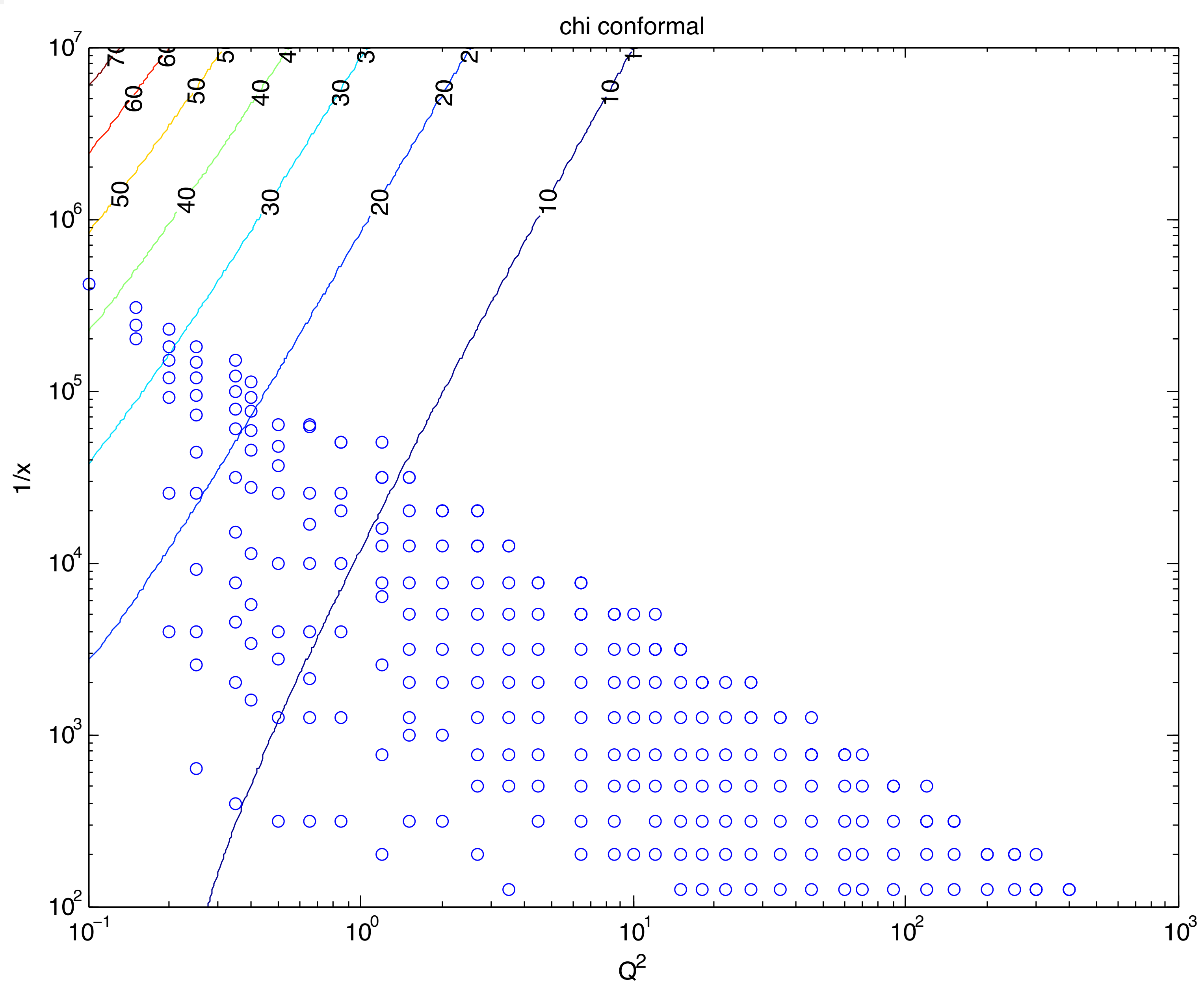
- For the proton, one for now treats it as a glueball of mass $\sim \Lambda = 1/Q'$.

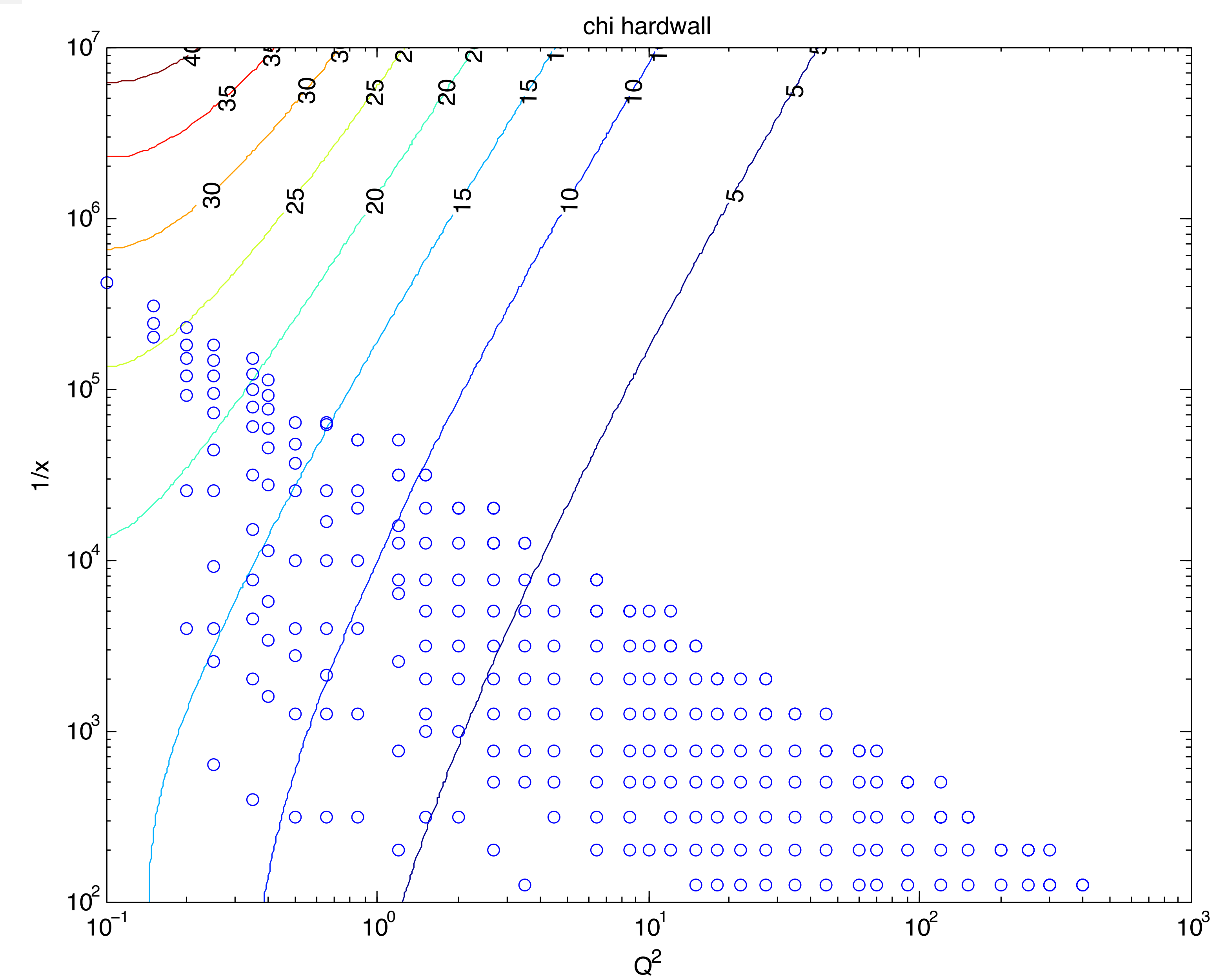
Plots

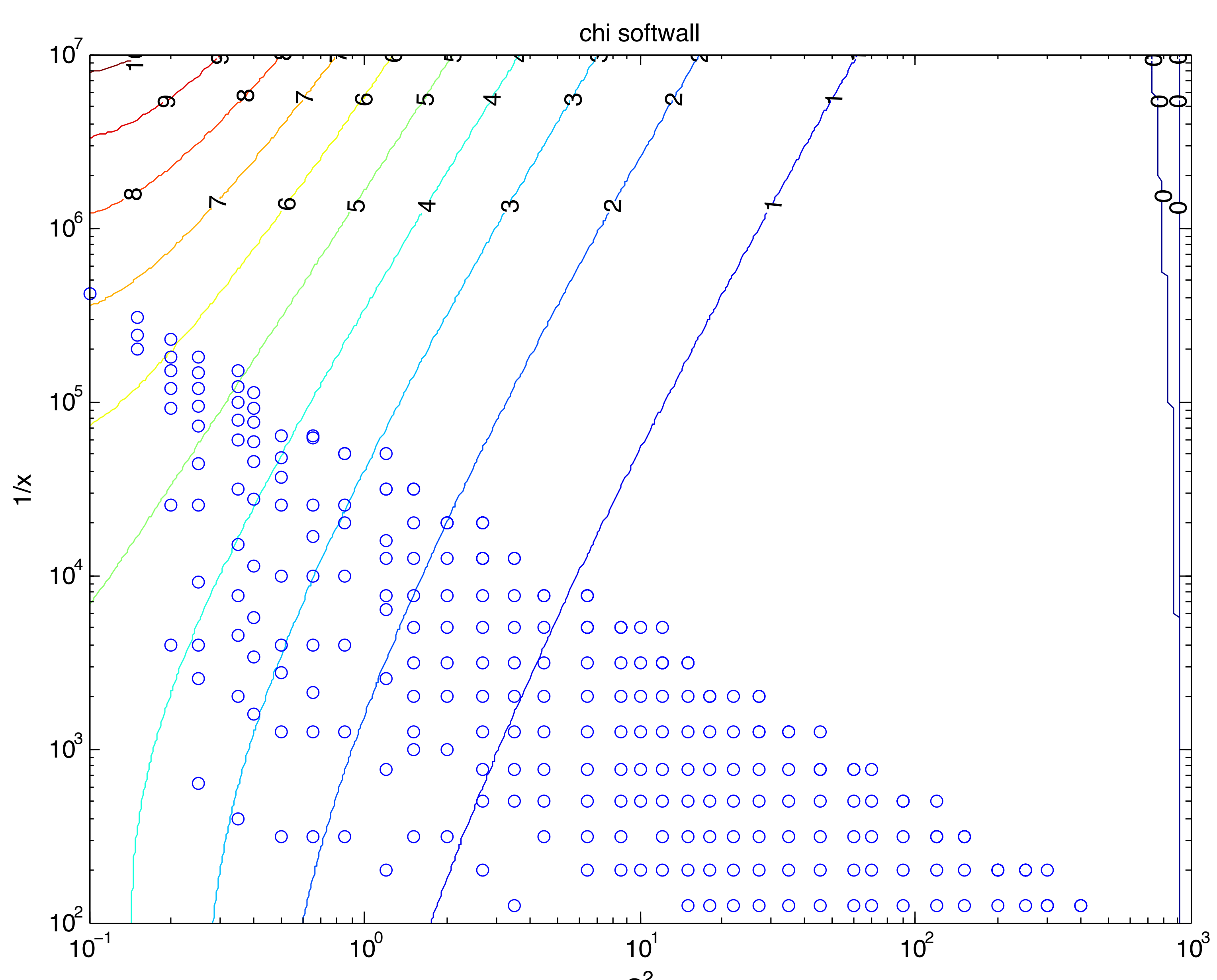


The structure function $F_2(x, Q^2)$ plotted for farious values of Q^2. The data points are from the H1-Zeus collaboration and the solid lines are the soft wall fit values.

Plots Cont.



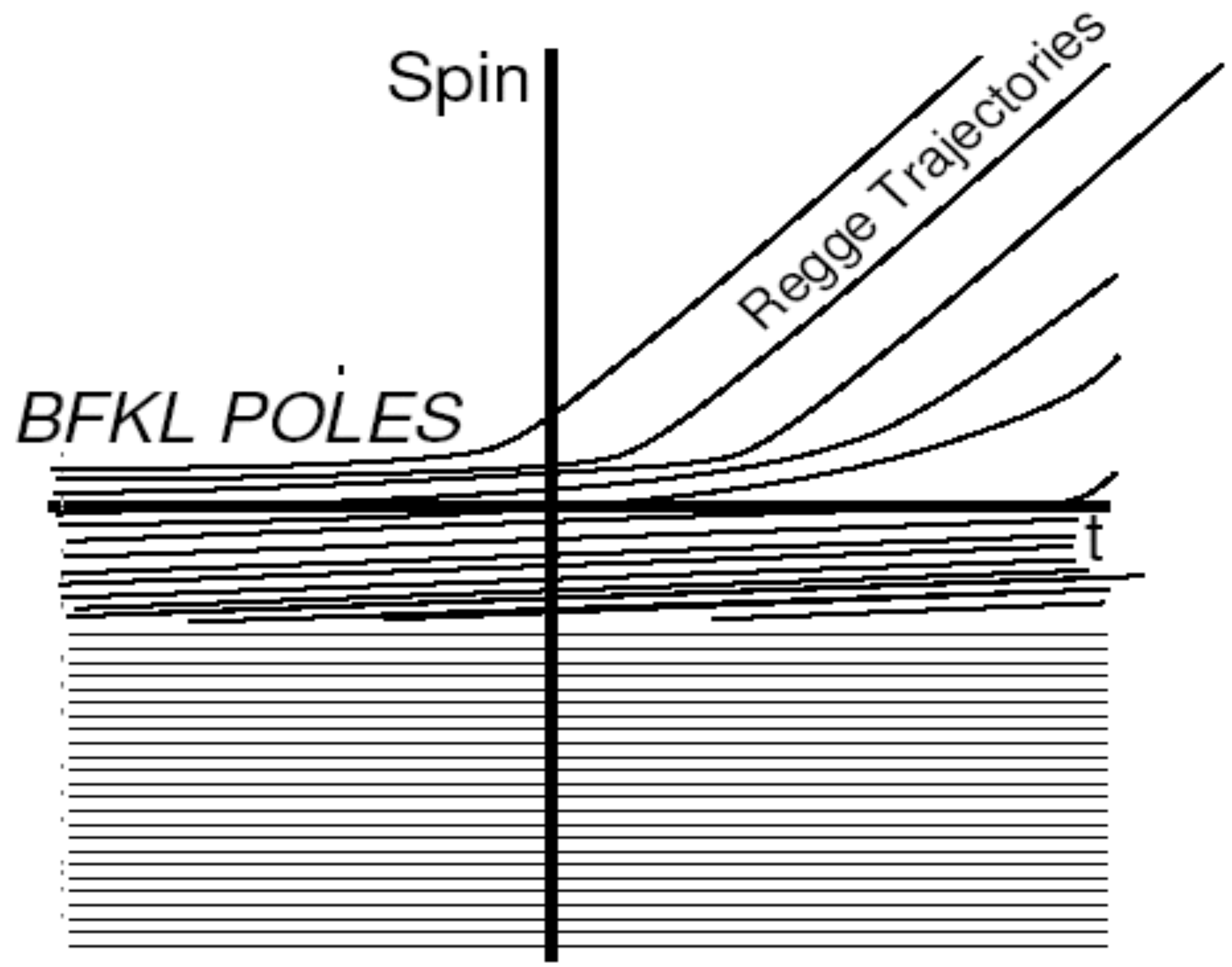




Contour plots of Im$[\chi]$ as a function of $1/x$ vs Q^2 (Gev) for conformal, hard-wall, and softwall models. These plots are all in the forward limit, but the impact parameter representation can tell us about the onset of non-linear eikonal effects. The similar behavior for the softwall implies a similar conclusion about confinement vs saturation.

Pomeron in QCD

Running UV, Confining IR (large N)



The hadronic spectrum is little changed, as expected.

The BFKL cut turns into a set of poles, as expected.

CFT correlate function – coordinate representation

$$\langle \phi_1(x_1)\phi_2(x_2)\phi_3(x_3)\phi_4(x_4)\rangle$$

OPE: $$\phi(x_1)\phi_2(x_2) \simeq \sum_k C_{1,2;k}(x_{12},\partial_1)\mathcal{O}_k(x_1)$$

Bootstrap: s-channel OPE = t-channel OPE

unitarity, positivity, locality, analyticity, etc.

Dynamics: $\boxed{\mathcal{O}_{(\Delta,j)_k}(x)}$ Conformal Dimension, Spin

Dynamics

$$\boxed{a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \to \quad \frac{1}{\Delta - \Delta(j)}}$$

Single Trace Gauge Invariant Operators of $\mathcal{N} = 4$ SYM,

$$\boxed{Tr[F^2], \quad Tr[F_{\mu\rho}F_{\rho\nu}], \quad Tr[F_{\mu\rho}D_{\pm}^{S}F_{\rho\nu}], \quad Tr[Z^{\tau}], \quad Tr[D_{\pm}^{S}Z^{\tau}], \cdots}$$

Super-gravity in the $\lambda \to \infty$:

$$\boxed{Tr[F^2] \leftrightarrow \phi, \quad Tr[F_{\mu\rho}F_{\rho\nu}] \leftrightarrow G_{\mu\nu}, \quad \cdots}$$

Symmetry of Spectral Curve:

$$\boxed{\Delta(j) \leftrightarrow 4 - \Delta(j)}$$

Graviton Spectral Curve:

$$\boxed{a_j(\Delta) \sim \frac{1}{\Delta - \Delta_j} \quad \to \quad \frac{1}{\Delta - \Delta(j)}}$$

Single Trace Gauge Invariant Operators of $\mathcal{N} = 4$ SYM,

$$\boxed{Tr[F_{\pm\perp} D_{\pm}^{j-2} F_{\perp\pm}], \quad j = 2, 4, \cdots}$$

Super-gravity in the $\lambda \to \infty$:

$$\boxed{\Delta(2) = 4; \quad \Delta(j) = O(\lambda^{1/4}) \to \infty, \quad j > 2}$$

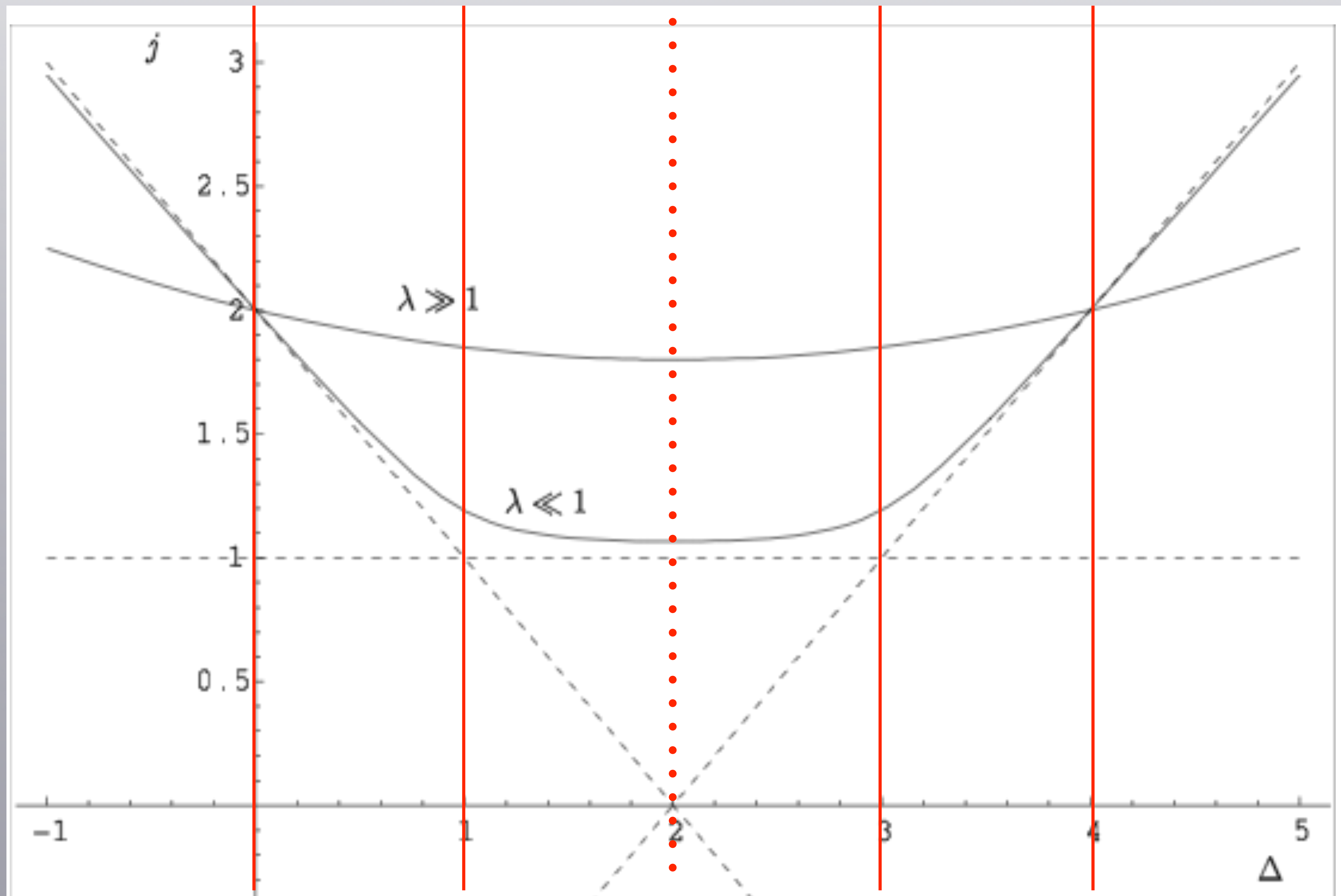
Symmetry of Spectral Curve:

$$\boxed{\Delta(j) \leftrightarrow 4 - \Delta(j)}$$

ANOMALOUS DIMENSIONS:

$$\gamma(j,\lambda) = \Delta(j,\lambda) - j - 2$$

$$\gamma_2 = 0$$



$$\Delta(j) = 2 + \sqrt{2}\sqrt{\sqrt{g^2 N_c}(j - j_0)}$$

$$\gamma_n = 2\sqrt{1 + \sqrt{g^2 N}(n-2)/2} - n$$

Energy-Momentum Conservation built-in automatically.