Generalised Parton Distributions: A Dyson-Schwinger approach for the pion

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In collaboration with:
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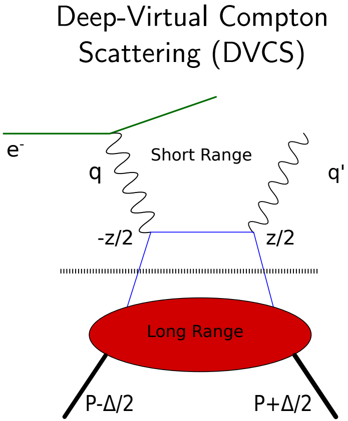
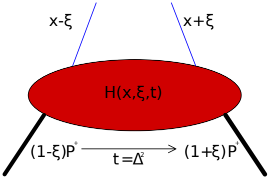
The Golden Channel: Deep-Virtual Compton Scattering



- Short range → perturbation theory.
- Long range → non pertubative objects: GPDs.
- Encode the hadrons 3D partonic structure and the spin structure.
- *Universality*

Kinematic Variables



- H stands for GPD,
- depending on 3 variables: x, ξ, t.

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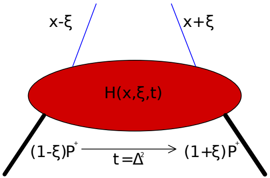

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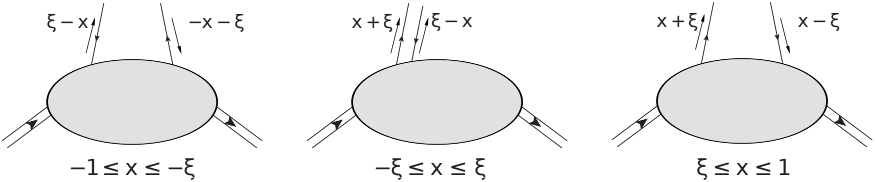

Formal Definition

- Proton case:

$$
\begin{aligned}
&\frac{1}{2}\int \frac{e^{ixP^+z^-}}{2\pi}\langle P+\frac{\Delta}{2}|\bar{q}(-\frac{z}{2})\gamma^+ q(\frac{z}{2})|P-\frac{\Delta}{2}\rangle \mathrm{d}z^-|_{z^+=0,\mathbf{z}=0} \\
= \quad & \frac{1}{2P^+}[H^q(x,\xi,t)\bar{u}(P+\frac{\Delta}{2})\gamma^+ u(P-\frac{\Delta}{2}) \\
& +E^q(x,\xi,t)\bar{u}(P+\frac{\Delta}{2})\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}u(P-\frac{\Delta}{2})].
\end{aligned}
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X. Ji,1997
D. Müller et al.,1994
A. Radyushkin,1997

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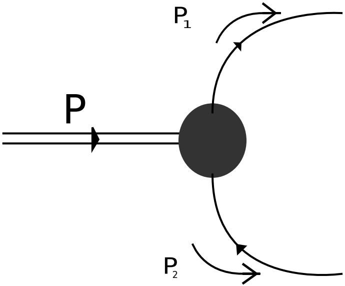
- Pion case:

$$H(x,\xi,t)=\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\langle P+\frac{\Delta}{2}|\bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)|P-\frac{\Delta}{2}\rangle_{z^+=0,z_\perp=0}$$

The pion? But why?



- Advantages :
  - Two-body system.
  - Pseudo-scalar meson.
  - Valence quarks u and d.
  - Isospin symmetry.
- Drawbacks
  - Very few experimental data available.
  - No data at $\xi \neq 0$
    $\rightarrow$ The model can be compared only at $\xi = 0$, *i.e.* to the Parton Distribution Function (PDF) and to the form factor.
  - *Amrath et al., Eur. Phys. J. C58 179*

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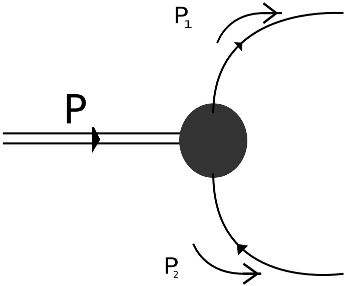

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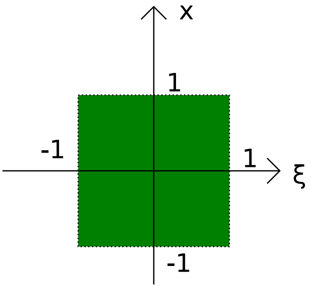
Good starting point before dealing with more complicated objects.

GPD properties



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  $|x| \leq 1$ and $|\xi| \leq 1$
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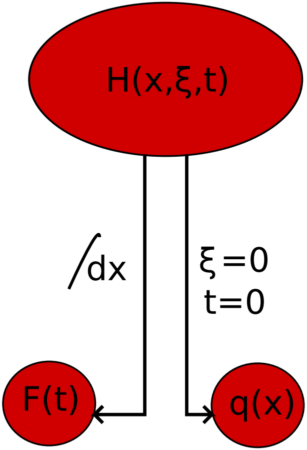

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  - if $t = 0$ and $\xi = 0 \rightarrow$ PDF,
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GPD properties

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$$\mathcal{M}_m(\xi,t) = \sum_{i=0}^{\frac{m}{2}} c_{2i}(t)\xi^{2i} + mod(m,2)c_{m+1}\xi^{m+1}$$

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Those properties make GPD modeling a challenge.

Models of GPDs

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- Quark-diquark models:
  - G. Goldstein, J. Hernandez, S. Liuti (2010)

Alternative ideas

- Lattice QCD:
  - Computations of Mellin Moments.
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Dyson-Schwinger equations seem to be a very promising approach to model GPDs!

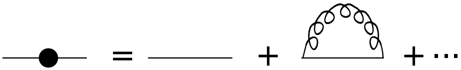
Dyson-Schwinger Equations

- Equations between non pertubative Green functions.
- Infinite number of coupled equations $\rightarrow$ no one has solved it until now!
- This requires approximations. In QCD, there are mainly two:
  - Rainbow Ladder (RL), resumming over a certain class of diagrams,
  - Dynamical Chiral Symmetry Breaking (DCSB).

  See for instance *L.Chang et al.*,PRC87,2013 for details about truncation schemes.

Example: the quark propagator

Pertubative case:



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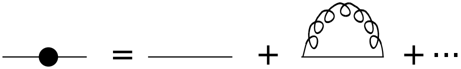

Dyson-Schwinger case:

Solutions of the BSE-DSE

DSE-BSE equations have been solved numerically and solutions have been fitted on specific parametrisations (*L. Chang et al.*, 2013).

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- Propagator → linear combination of free propagators using complex conjugate poles:

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- Pion Bethe-Salpeter amplitude → use the Nakanishi representation:

$$\Gamma_\pi(k,P) = c_j \int_{-1}^{1} \mathrm{d}z \frac{\rho_\nu(z)\Lambda_j^{2\nu}}{\left[\left(k - \frac{1-z}{2}P\right)^2 + \Lambda_j^2\right]^\nu} + \ldots$$

Algebraic model for pion GPD

Propagator:

$$S(p^2) = \frac{-ip\cdot\gamma + M}{p^2 + M^2}$$

- p is the quark momentum,
- M is the effective mass of the constituent quark.

Vertex:

$$\Gamma_\pi \propto i\gamma_5 \int \frac{\mathrm{d}z\ M^2\rho_\nu(z)}{(q(k,\Delta,P)^2 + M^2)^\nu}$$

- $\rho_\nu(z) \propto (1-z^2)^\nu$ is the z distribution.
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Those functions are building blocks of the realistic Bethe-Salpeter computations.

Pion GPD model

$$\mathcal{M}_m(\xi, t) = \int_{-1}^{1} \mathrm{d}x \; x^m \; H(x, \xi, t)$$
$$= \frac{1}{2(P \cdot n)^{m+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{\psi}(0) \gamma \cdot n (i \overleftrightarrow{D} \cdot n)^m \psi(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle .$$

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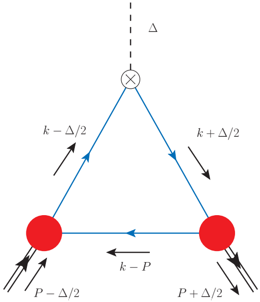

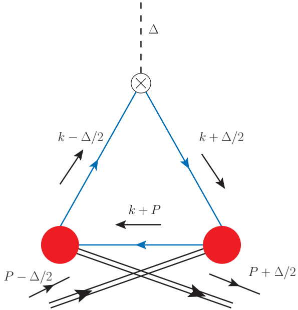

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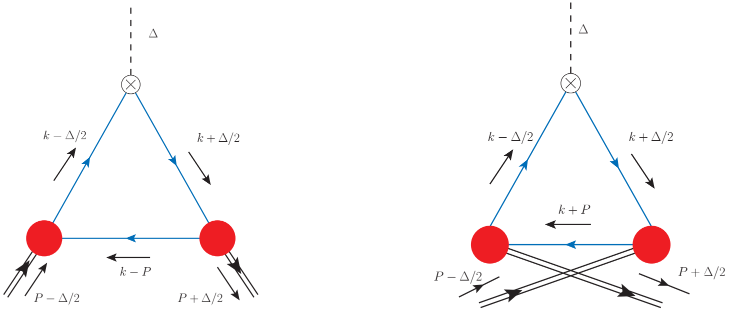

$$2(P\cdot n)^{m+1}\mathcal{M}_m(\xi,t) = \mathrm{tr}_{CFD}\int \frac{\mathrm{d}^4k}{(2\pi)^4}(k\cdot n)^m \; i\Gamma_\pi(k-\frac{\Delta}{2},P-\frac{\Delta}{2})\; S(k-\frac{\Delta}{2})$$
$$i\gamma\cdot n\; S(k+\frac{\Delta}{2})\; i\bar{\Gamma}_\pi(k+\frac{\Delta}{2},P+\frac{\Delta}{2})\; S(k-P)$$

Form factor

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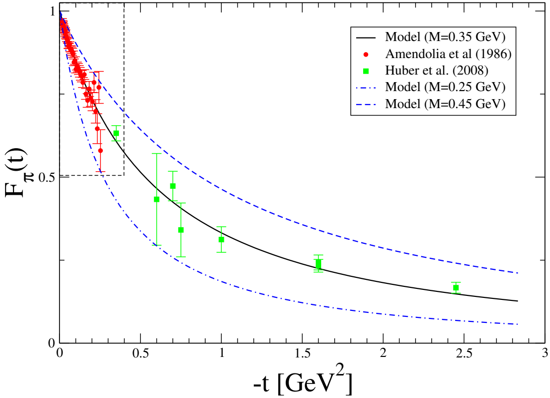

C. Mezrag et al., arXiv 1406.7425

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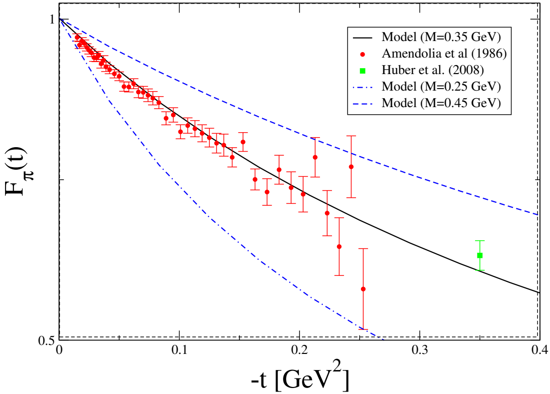

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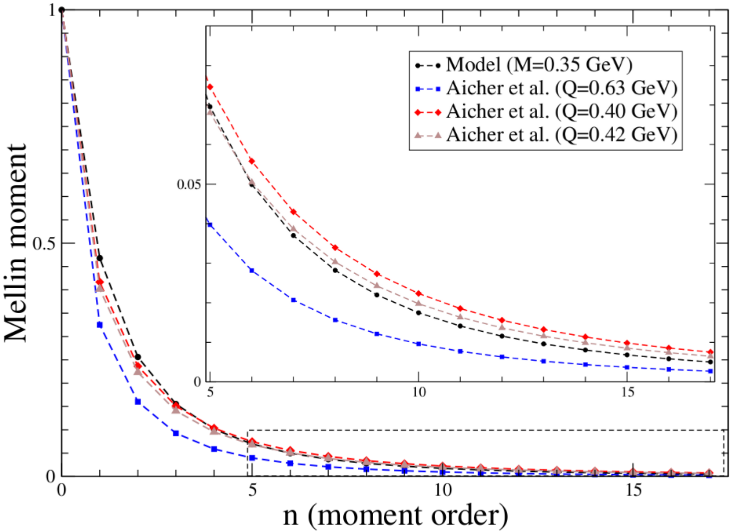
PDF's Mellin moments



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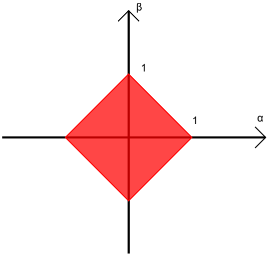
Double Distributions

Müller et al. (1994), Radyushkin (1996), Teryaev (2001)

Double Distributions are formally the Radon transform of the GPDs.

$$H(x,\xi) = \int_\Omega \mathrm{d}\alpha \mathrm{d}\beta (F(\beta,\alpha) + \xi G(\beta,\alpha))\delta(x - \beta - \xi\alpha)$$



$$\Omega = \{(\alpha,\beta) || \alpha| + |\beta| \leq 1\}$$

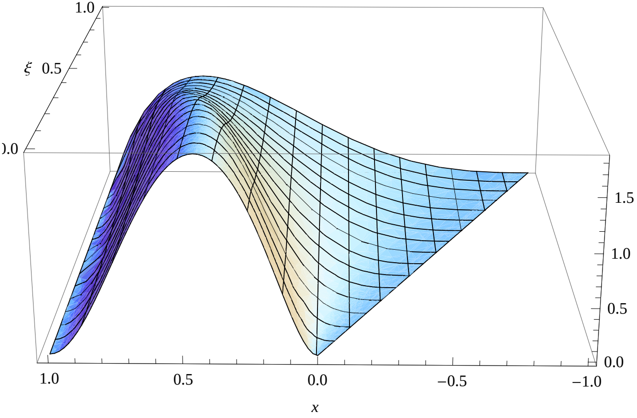
Advantage:

Easy way to respect the polynomiality in ξ

$$\int_{-1}^{1} x^n H(x,\xi)\mathrm{d}x$$

$$= \int_\Omega (\beta + \xi\alpha)^n (F(\beta,\alpha) + \xi G(\beta,\alpha))\mathrm{d}\Omega$$

Reconstruction ($t = 0$)



We get back the support properties!

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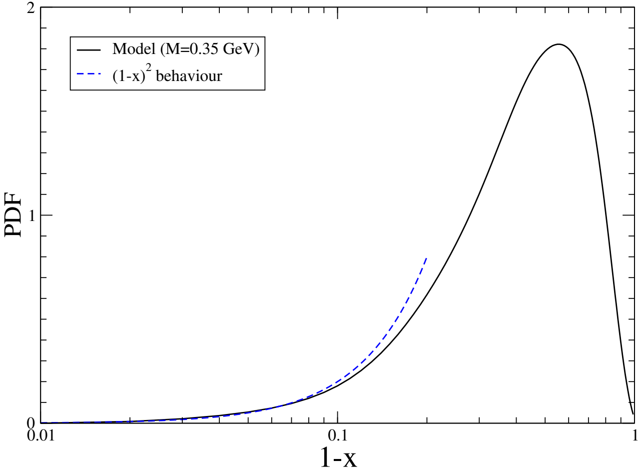
- Double Distributions ensure polynomiality and parity in ξ
- We can get analytic expressions. For the PDF($\nu = 1$):

$$
\begin{aligned}
q(x) \quad = \quad & \frac{72}{25}\left(x^3(x(-2(x-4)x-15)+30)\log(x)\right. \\
& + \left(2x^2+3\right)(x-1)^4\log(1-x) \\
& \left. +x(x(x(2x-5)-15)-3)(x-1)\right)
\end{aligned}
$$

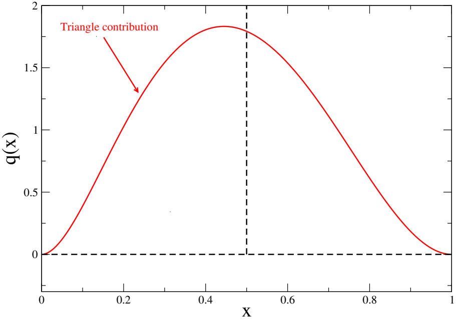
Large x behavior



At large x:

$$q(x) \approx (1-x)^2$$

Limits of triangle diagrams



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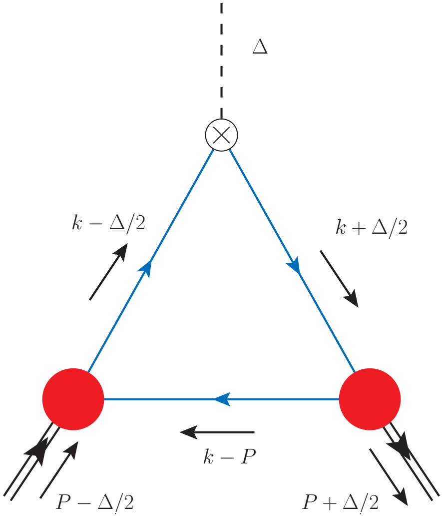

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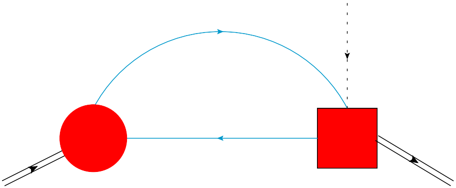

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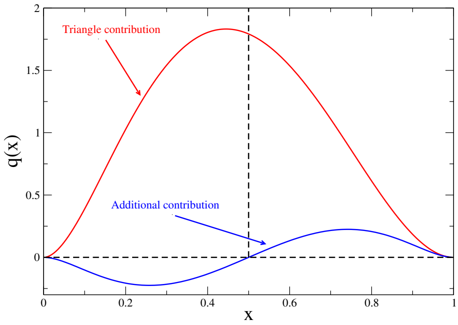

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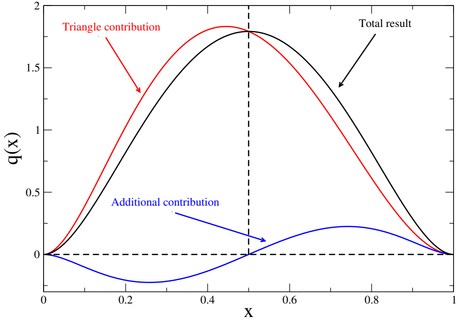

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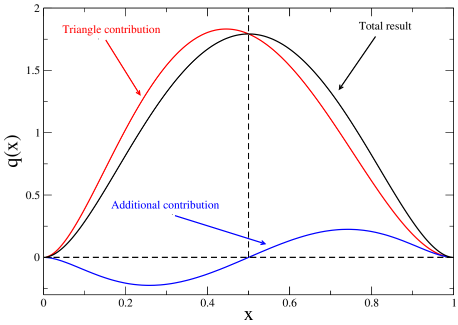

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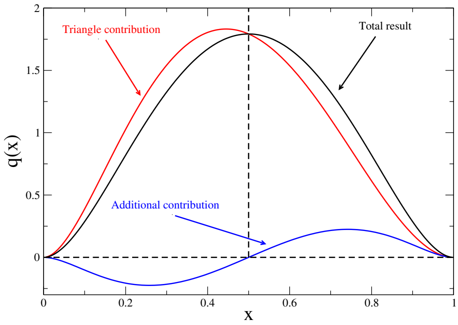

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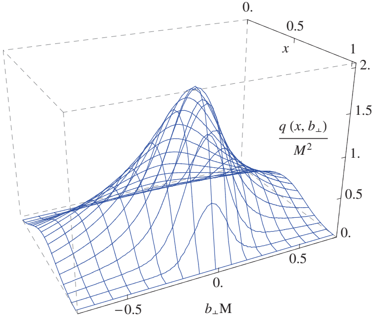
It gives us some insight to go to non zero t.

Sketching the pion 3D structure

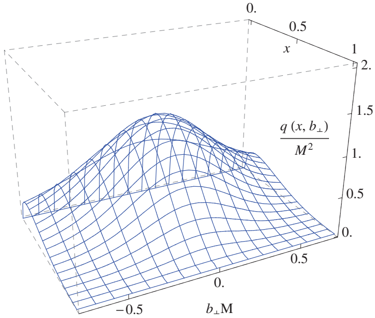
$$\rho^q(x, b_\perp) = \int_0^\infty \frac{\mathrm{d}|\Delta_\perp|}{2\pi} |\Delta_\perp| J_0\big(|b_\perp| \cdot |\Delta_\perp|\big) H^q(x, 0, -\Delta_\perp^2),$$

- $b_\perp$ is the Fourier conjugate of $\Delta_\perp$.
- $b_\perp$ is the position in the plane transverse to the hadron direction.
- J_0 is the first kind Bessel function.
- $\rho^q(x, b_\perp)$ is the probability density to find a quark q at a given position $b_\perp$ in the transverse plane and with a given longitudinal momentum fraction x.

Sketching the pion 3D structure



$\zeta = 0.5 GeV$



$\zeta = 2 GeV$

Plots from *C. Mezrag et. al.*, PLB 741

Soft Pion Theorem

Polyakov soft pion theorem: if $\xi = 1$ and $t = 0$ then $H \propto$ Pion DA.

(*Polyakov*,1999)

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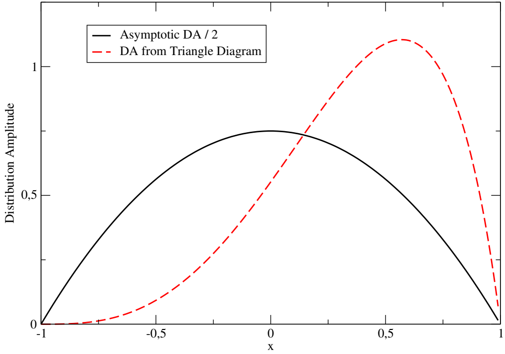

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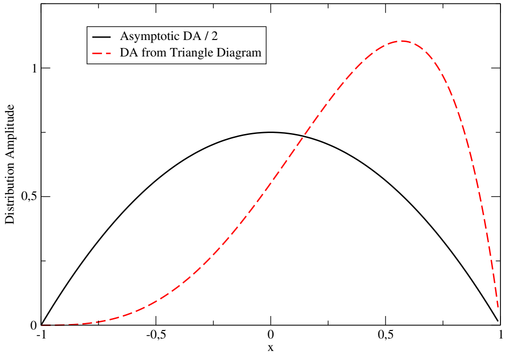

Here the soft pion theorem is violated. **Why?**

Relation between Pion and quark propagator

- Propagator:

$$S^{-1}(k) = -i\gamma \cdot k \; A(k^2) + B(k^2)$$

- Vertex:

$$\Gamma_\pi(k,P) = \gamma_5 \left(iE_\pi(k,P) + \gamma \cdot P \; F_\pi(k,P) + \gamma \cdot k \; P \cdot k \; G_\pi(k,P) + \ldots \right)$$

- AVWTI leads to the relation:

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Freezing the mass leads to a violation of AVWTI!

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Is it sufficient to respect the AVWTI to get back the soft pion theorem?

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Answer

Yes, providing that the truncation scheme is consistent enough.

Soft Pion Theorem: the solution

- Generaly speaking:

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The soft pion theorem will be automatically implemented when modeling the pion GPD from the full solutions of the BSE-DSE.

(*C. Mezrag et al.*, PLB 741).

Summary and conclusions

- We presented a new model for pion GPD which fulfills most of the required symmetry properties.
- Double Distributions make the full problem analytic.
- Our comparisons with available experimental data are very encouraging.
- Limitations highlight physics key points.

Summary and conclusions

- We presented a new model for pion GPD which fulfills most of the required symmetry properties.
- Double Distributions make the full problem analytic.
- Our comparisons with available experimental data are very encouraging.
- Limitations highlight physics key points.

If the GPDs remain the good objects to understand the physics, DDs are the good objects to deal with support properties and full reconstruction.

Outlooks

- We want to reconstruct the GPD thanks to DD in the realistic case, *i.e.* with vertices and propagators coming from numerical solutions of the Dyson-Schwinger equations.
- Compare our model with the existing phenomenological DD models, *i.e.* Radyushkin Ansatz.
- The proton case remains the Holy Grail...

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- Compare our model with the existing phenomenological DD models, *i.e.* Radyushkin Ansatz.
- The proton case remains the Holy Grail... which may be reached in the valence region using a quark-diquark model.

Thank You!

Back up

Kroll - Goloskokov model.

- **Factorised Ansatz**. For $i = g$, sea or val :

$$H_i(x,\xi,t) = \int_{|\alpha|+|\beta|\leq 1} d\beta d\alpha\, \delta(\beta+\xi\alpha - x) f_i(\beta,\alpha,t)$$

$$f_i(\beta,\alpha,t) = e^{b_i t}\frac{1}{|\beta|^{\alpha' t}} h_i(\beta)\pi_{n_i}(\beta,\alpha)$$

$$\pi_{n_i}(\beta,\alpha) = \frac{\Gamma(2n_i+2)}{2^{2n_i+1}\Gamma^2(n_i+1)}\frac{(1-|\beta|)^2-\alpha^2]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

- Expressions for h_i and n_i :

$$\begin{aligned} h_g(\beta) &= |\beta| g(|\beta|) & n_g &= 2 \\ h^q_{\mathrm{sea}}(\beta) &= q_{\mathrm{sea}}(|\beta|)\mathrm{sign}(\beta) & n_{\mathrm{sea}} &= 2 \\ h^q_{\mathrm{val}}(\beta) &= q_{\mathrm{val}}(\beta)\Theta(\beta) & n_{\mathrm{val}} &= 1 \end{aligned}$$

Goloskokov and Kroll, Eur. Phys. J. **C42**, 281 (2005)

- Comparison to **existing DVCS measurements** at LO.

Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Double Distribution Ambiguity

Teryaev Phys. Lett. B **510** (2001) 125
Tiburzi Phys. Rev. D **70** (2004) 057504

Rewrite the non forward matrix element in terms of DD :

$$\begin{aligned} &\langle P - \frac{r}{2} | \bar{\psi}(-\frac{z}{2}) \not{z} \psi(\frac{z}{2}) | P + \frac{r}{2} \rangle \\ = \quad & \int_\Omega e^{-i\beta(Pz) - i\alpha\frac{(rz)}{2}} (2(Pz)F(\beta,\alpha) + (rz)G(\beta,\alpha)) \mathrm{d}\alpha \mathrm{d}\beta \end{aligned}$$

Matrix element **invariant** under the following transformation :

$$\begin{aligned} F(\beta,\alpha) &\rightarrow F(\beta,\alpha) + \frac{\partial\sigma}{\partial\alpha} \\ G(\beta,\alpha) &\rightarrow G(\beta,\alpha) - \frac{\partial\sigma}{\partial\beta} \\ \sigma(\beta,\alpha) &= -\sigma(\beta,-\alpha) \end{aligned}$$

This invariance allows for **different** methods to parametrise GPDs.

Positivity

- Positivity condition in the DGLAP region:

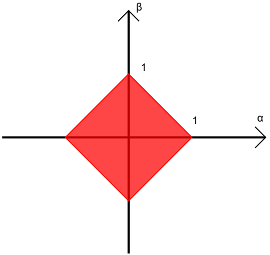
$$|H(x,\xi,t)| \leq \sqrt{q(\frac{x-\xi}{1-\xi})q(\frac{x+\xi}{1+\xi})}$$

Pire, Soffer, Teryaev, 1999

- In our two-body problem, $q(x) \propto x^2$ at small x.
- Consequently $H(x,\xi,t)$ should vanish on the line $x = \xi$.
- We'll see how the more realistic model behaves.

Double Distributions are formally the Radon transform of the GPDs.

$$H(x,\xi) = \int_\Omega \mathrm{d}\alpha \mathrm{d}\beta (F(\beta,\alpha) + \xi G(\beta,\alpha))\delta(x - \beta - \xi\alpha)$$



$$\Omega = \{(\alpha,\beta) | |\alpha| + |\beta| \leq 1\}$$

Advantage:
Easy way to respect the polynomiality in ξ

$$\int_{-1}^{1} x^n H(x,\xi)\mathrm{d}x$$
$$= \int_\Omega (\beta + \xi\alpha)^n (F(\beta,\alpha) + \xi G(\beta,\alpha))\mathrm{d}\Omega$$

From Mellin moments to Double Distributions (DD)

$$H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \left(F(\beta,\alpha,t) + \xi G(\beta,\alpha,t)\right) \delta(x-\beta-\alpha\xi)$$

- Time reversal invariance is encoded in the parity in α:
  - $F(\beta,\alpha)$ must be even in α
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- Form Factor case:

$$\mathcal{F}(t) = \int_{-1}^{1} \mathrm{d}x \ H(x,\xi,t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \ F(\beta,\alpha,t)$$

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$G(\beta,\alpha)$ **does not** play any role in those cases.

Properties of Mellin moments

Polynomiality:

$$
\begin{aligned}
&\mathcal{M}_m(\xi, t) \\
= \;& \frac{1}{2(P\cdot n)^{m+1}} \left\langle \pi, P+\frac{\Delta}{2} \middle| \bar{\psi}(0)\gamma\cdot n(i\overleftrightarrow{D}\cdot n)^m\psi(0) \middle| \pi, P-\frac{\Delta}{2} \right\rangle \\
= \;& \frac{n_\mu n_{\mu_1}\ldots n_{\mu_m}}{(P\cdot n)^{m+1}} P^{\{\mu} \sum_{j=0}^{m} \binom{m}{j} F_{m,j}(t) P^{\mu_1}\ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}} \\
& - n_\mu n_{\mu_1}\ldots n_{\mu_m} \frac{\Delta}{2}^{\{\mu} \sum_{j=0}^{m} \binom{m}{j} G_{m,j}(t) P^{\mu_1}\ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}}
\end{aligned}
$$

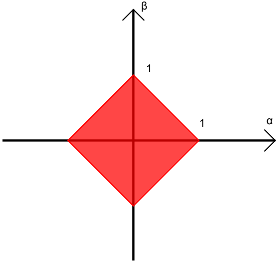
$\xi = -\frac{\Delta\cdot n}{2P\cdot n} \Rightarrow \mathcal{M}_m(\xi, t)$ is a polynomial in ξ of order $m+1$.

Properties of Mellin moments

Double distributions:

$$F_{m,j}(t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha\ \beta^{m-j}\alpha^{j}\ F(\beta, \alpha, t)$$

$$G_{m,j}(t) = \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha\ \beta^{m-j}\alpha^{j}\ G(\beta, \alpha, t)$$



Properties of Mellin moments

$$\mathcal{M}_m(\xi, t) = n_\mu n_{\mu_1} \ldots n_{\mu_m} \sum_{j=0}^{m} \binom{m}{j} \int_{-1}^{1} \mathrm{d}\beta \int_{-1+|\beta|}^{1-|\beta|} \mathrm{d}\alpha \; \beta^{m-j} \alpha^j$$
$$F(\beta, \alpha, t) P^{\{\mu} P^{\mu_1} \ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$
$$-G(\beta, \alpha, t) \frac{\Delta}{2}^{\{\mu} P^{\mu_1} \ldots P^{\mu_j} \left(-\frac{\Delta}{2}\right)^{\mu_{j+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$

Analytic Results

$$
\begin{aligned}
F^u(\beta,\alpha,t) \quad = \quad & \frac{48}{5}\Bigg\{-\frac{18M^4 t(\beta-1)(\alpha-\beta+1)(\alpha+\beta-1)\left(\left(\alpha^2-(\beta-1)^2\right)\tanh^{-1}\left(\frac{2\beta}{-\alpha^2+\beta^2+1}\right)+2\beta\right)}{\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^3} \\
& +\frac{9M^4(\alpha-\beta+1)\left(-4\beta\left(-\alpha^2+\beta^2+1\right)+2\tanh^{-1}\left(\frac{2\beta}{-\alpha^2+\beta^2+1}\right)\right)}{4(\alpha-\beta-1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& +\frac{9M^4(\alpha-\beta+1)\left(\left(\alpha^4-2\alpha^2\left(\beta^2+1\right)+\beta^2\left(\beta^2-2\right)\right)\log\left(\frac{(\alpha-\beta-1)(\alpha+\beta+1)}{\alpha^2-(\beta-1)^2}\right)\right)}{4(\alpha-\beta-1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& +\frac{9M^4(\alpha+\beta-1)\left(-4\beta\left(-\alpha^2+\beta^2+1\right)+2\tanh^{-1}\left(\frac{2\beta}{-\alpha^2+\beta^2+1}\right)\right)}{4(\alpha+\beta+1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& +\frac{9M^4(\alpha+\beta-1)\left(\left(\alpha^4-2\alpha^2\left(\beta^2+1\right)+\beta^4-2\beta^2\right)\log\left(\frac{(\alpha-\beta-1)(\alpha+\beta+1)}{\alpha^2-(\beta-1)^2}\right)\right)}{4(\alpha+\beta+1)\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& +\frac{9M^4\beta(\alpha-\beta+1)^2(\alpha+\beta-1)^2\left(\frac{2\left(\alpha^2\beta-\beta^3+\beta\right)}{\alpha^4-2\alpha^2(\beta^2+1)+(\beta^2-1)^2}\right)}{\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2} \\
& +\frac{9M^4\beta(\alpha-\beta+1)^2(\alpha+\beta-1)^2\left(-\tanh^{-1}(\alpha-\beta)+\tanh^{-1}(\alpha+\beta)\right)}{\left(4M^2+t\left((\beta-1)^2-\alpha^2\right)\right)^2}\Bigg\},
\end{aligned}
$$

Analytic Results

$$
\begin{aligned}
H^{u}_{x\geq\xi}(x,\xi,0) \quad = \quad & \frac{48}{5}\left\{ \frac{3\left(-2(x-1)^4\left(2x^2-5\xi^2+3\right)\log(1-x)\right)}{20\left(\xi^2-1\right)^3} \right. \\
& \frac{3\left(+4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\tanh^{-1}\left(\frac{(x-1)\xi}{x-\xi^2}\right)\right)}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(x^3(x(2(x-4)x+15)-30)-15(2x(x+5)+5)\xi^4\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(-5x(x(x(x+2)+36)+18)\xi^2-15\xi^6\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(2(x-1)\left((23x+58)\xi^4+(x(x(x+67)+112)+6)\xi^2+x(x((5-2x)x+15)+3)\right)\right)}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(\left(15(2x(x+5)+5)\xi^4+10x(3x(x+5)+11)\xi^2\right)\log\left(1-\xi^2\right)\right)}{20\left(\xi^2-1\right)^3} \\
& \left. +\frac{3\left(2x(5x(x+2)-6)+15\xi^6-5\xi^2+3\right)\log\left(1-\xi^2\right)}{20\left(\xi^2-1\right)^3} \right\},
\end{aligned}
$$

Analytic Results

$$
\begin{aligned}
H^{u}_{|x|\leq\xi}(x,\xi,0) = \frac{48}{5}\Bigg\{ & \frac{6\xi(x-1)^4\left(-\left(2x^2-5\xi^2+3\right)\right)\log(1-x)}{40\xi\left(\xi^2-1\right)^3} \\
& + \frac{6\xi\left(-4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\log(2\xi)\right)}{40\xi\left(\xi^2-1\right)^3} \\
& + \frac{6\xi(\xi+1)^3\left((38x+13)\xi^2+6x(5x+6)\xi+2x(5x(x+2)-6)+15\xi^3-9\xi+3\right)\log(\xi+1)}{40\xi\left(\xi^2-1\right)^3} \\
& + \frac{6\xi(x-\xi)^3\left((7x-58)\xi^2+6(x-4)x\xi+x(2(x-4)x+15)+15\xi^3+75\xi-30\right)\log(\xi-x)}{40\xi\left(\xi^2-1\right)^3} \\
& + \frac{3(\xi-1)(x+\xi)\left(4x^4\xi-2x^3\xi(\xi+7)+x^2(\xi((119-25\xi)\xi-5)+15)\right)}{40\xi\left(\xi^2-1\right)^3} \\
& + \frac{3(\xi-1)(x+\xi)\left(x\xi(\xi(\xi(71\xi+5)+219)+9)+2\xi(\xi(2\xi(34\xi+5)+9)+3)\right)}{40\xi\left(\xi^2-1\right)^3}\Bigg\}.
\end{aligned}
$$