Higgs Production at N3LO

from SM to $\mathcal{N}$=4 SYM

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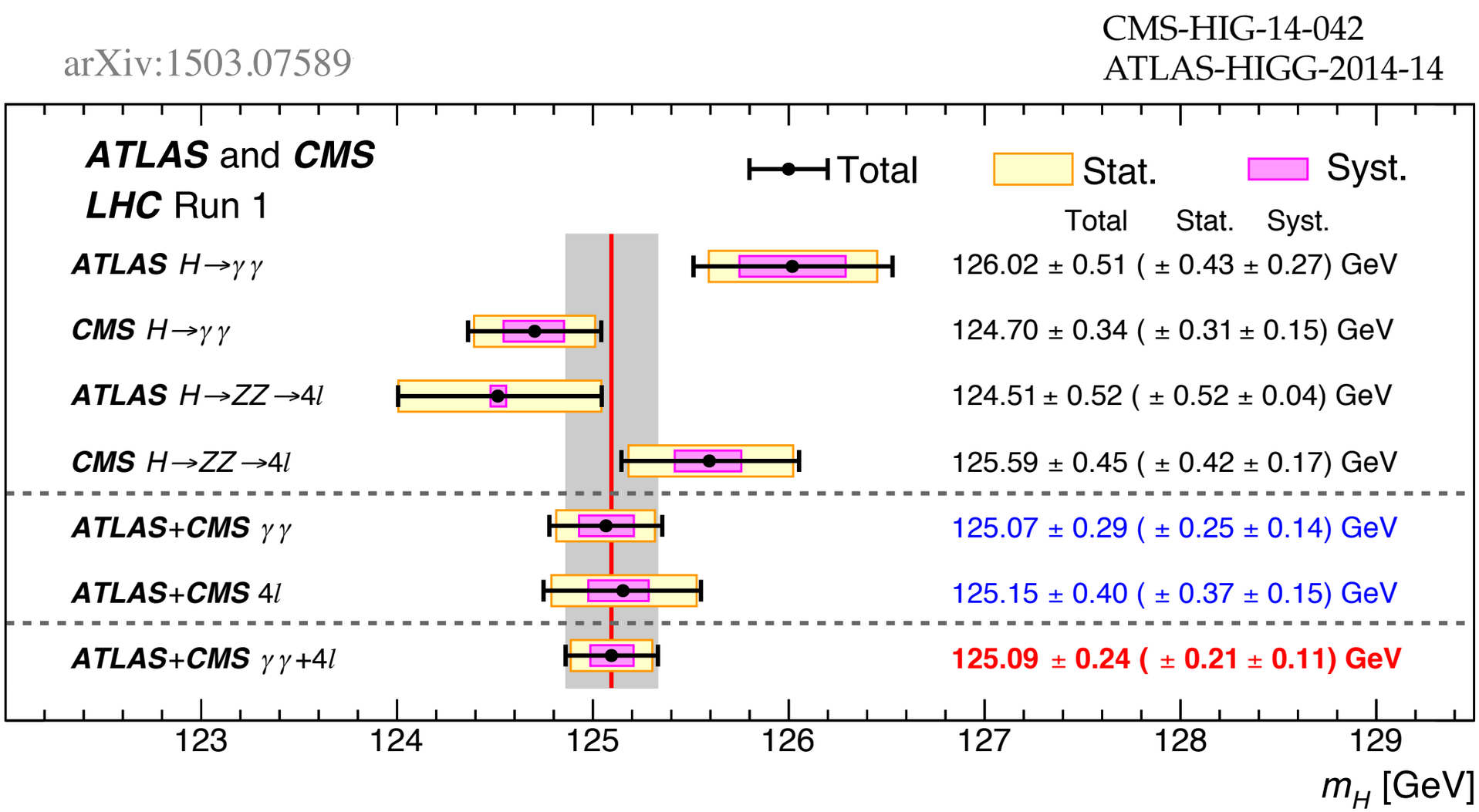
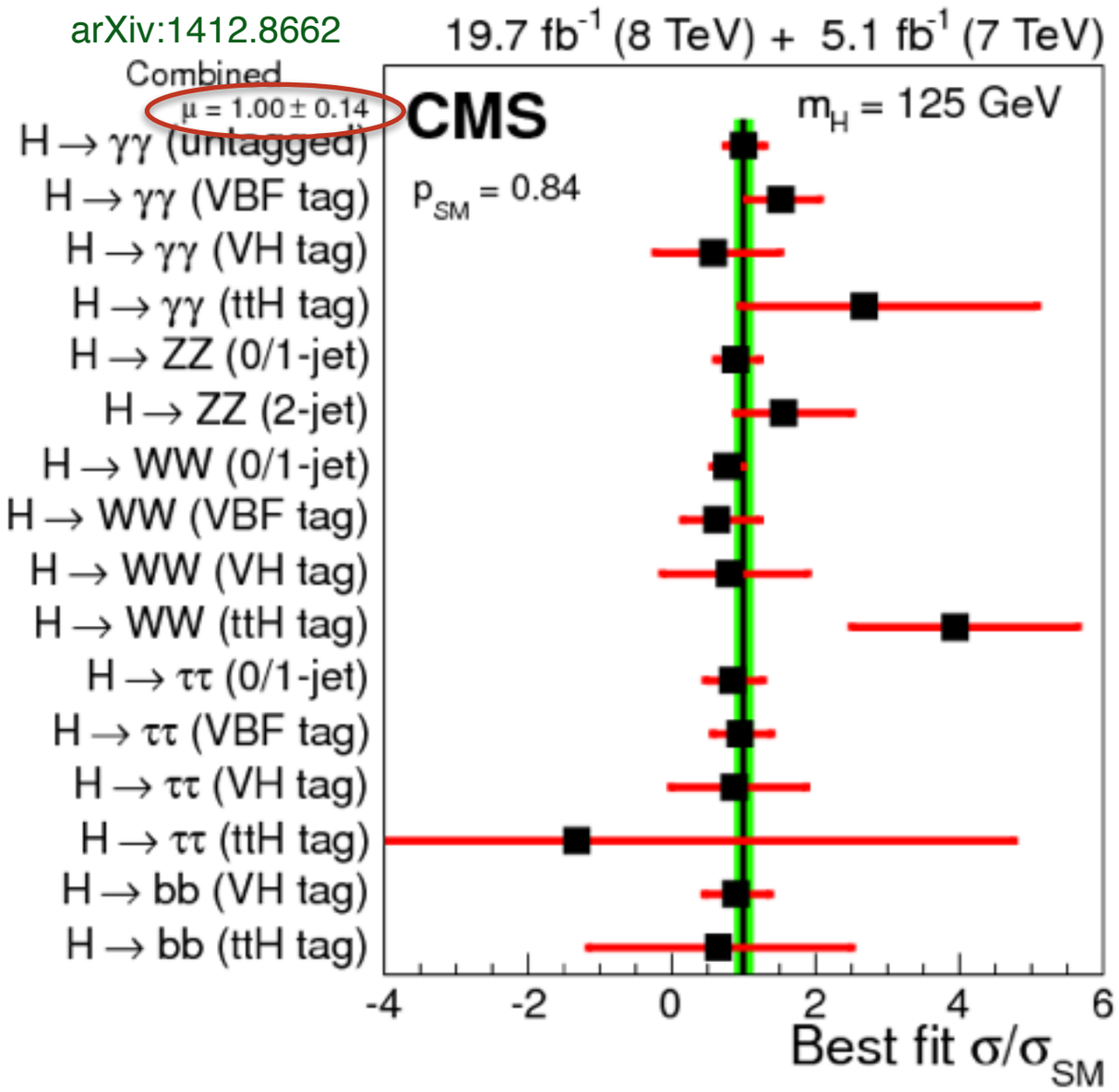
Higgs Discovered !

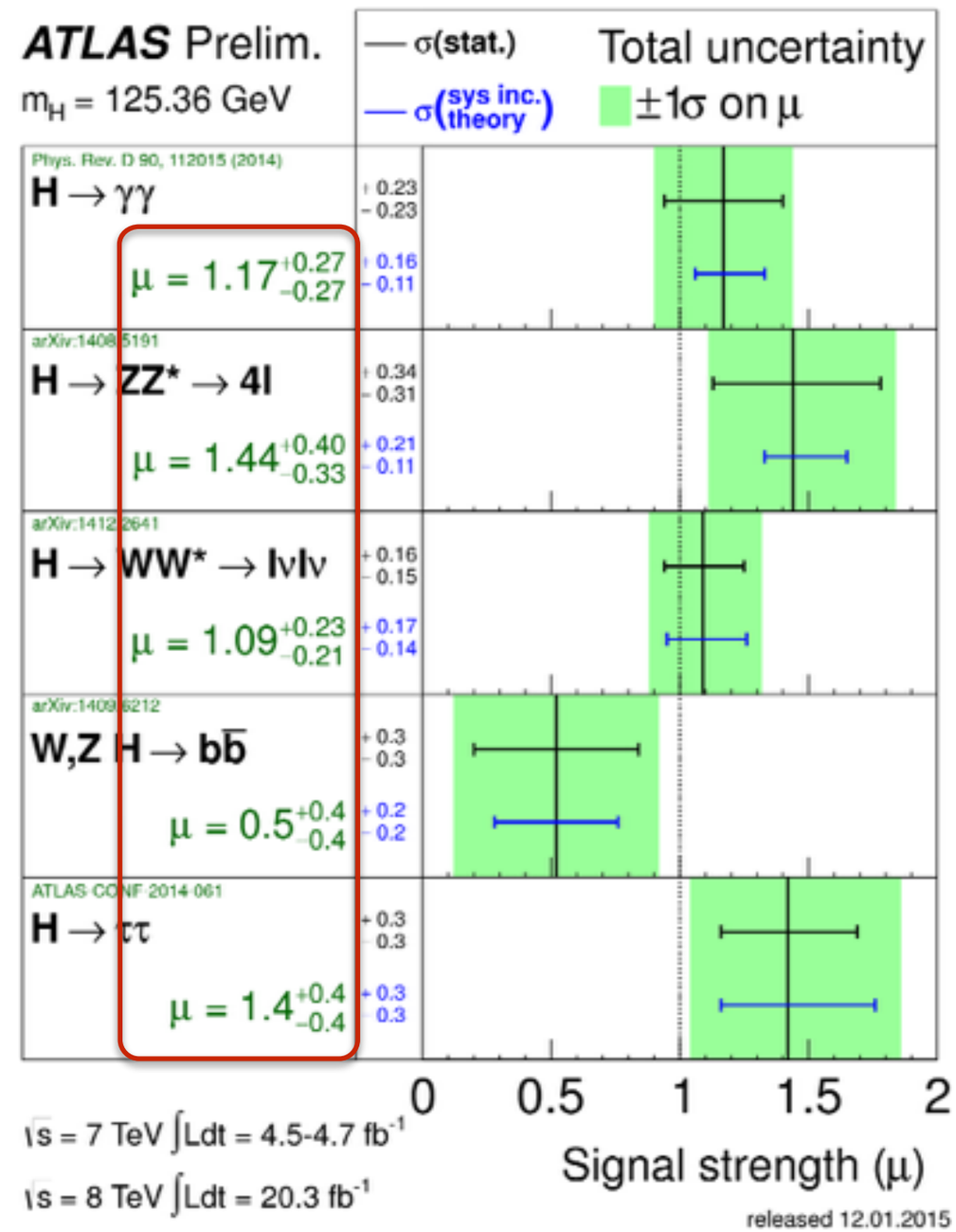
- LHC just resumed running this year



Why care about N3LO?

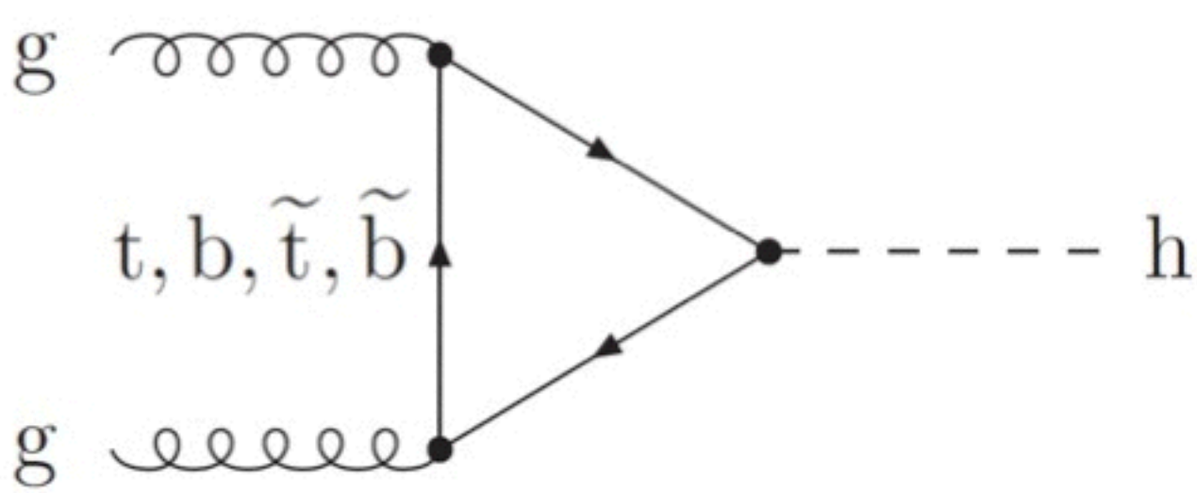
- Current experimental precision is around 20%





Why care about N3LO?

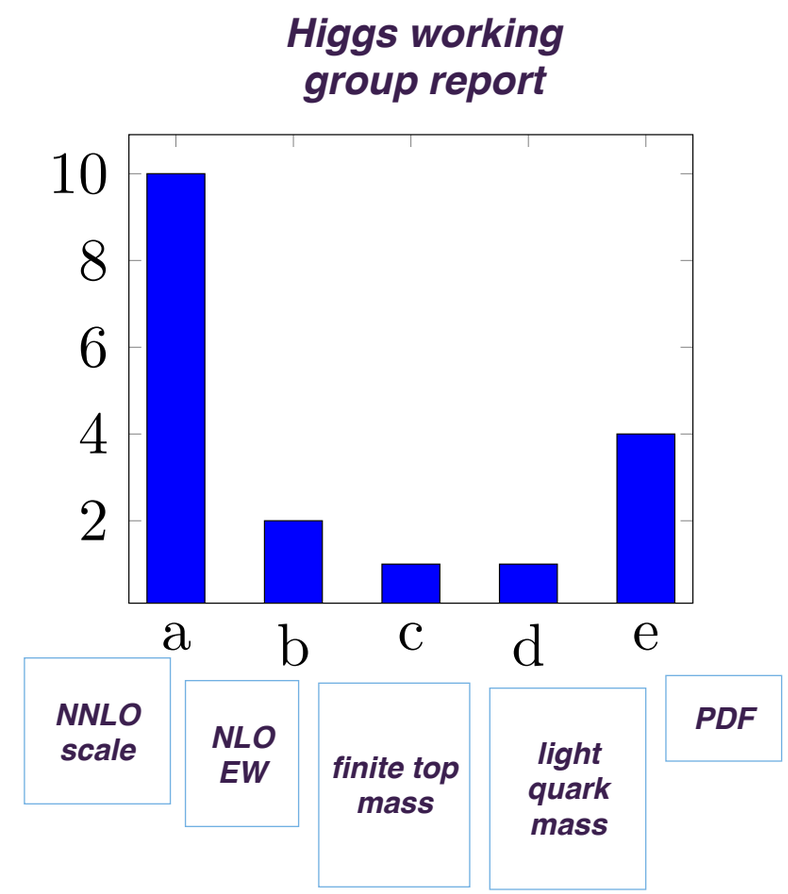
- Theoretically
  - New physics can hide behind Higgs production in many common BSM scenarios



ggH coupling is sensitive to new particles running in the loop, for example, a top partner

Why care about N3LO?

- Theoretically
  - NNLO uncertainty on Higgs production in gluon fusion channel is larger than future experimental uncertainty

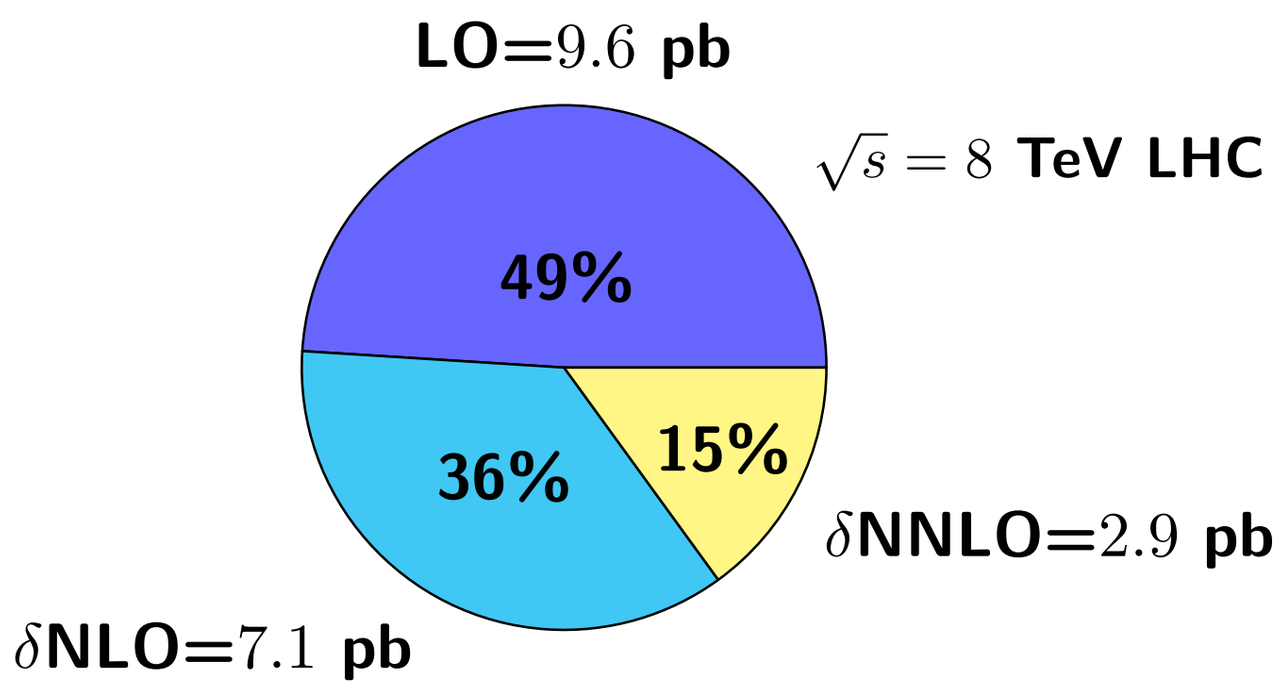


CMS snowmass workgroup report

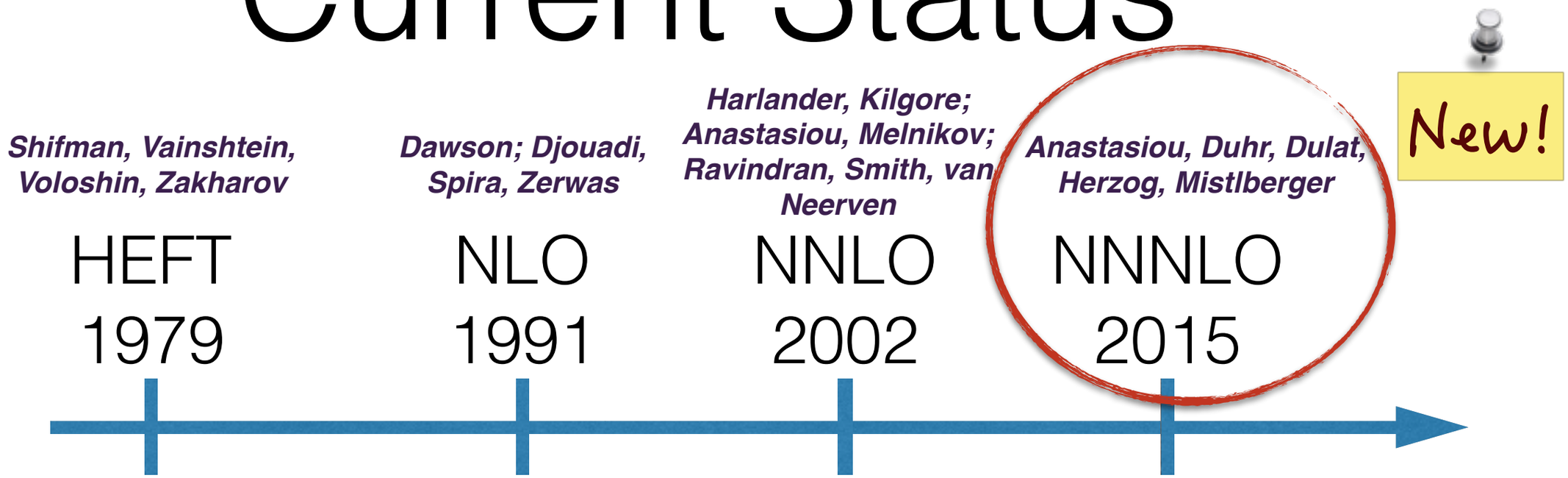
L (fb^{-}1)	$\gamma\gamma$	WW	ZZ	$b\bar{b}$
300	[6%, 12%]	[6%, 11%]	[7%, 11%]	[11%, 14%]
3000	[4%, 8%]	[4%, 7%]	[4%, 7%]	[5%, 7%]

Why care about N3LO?

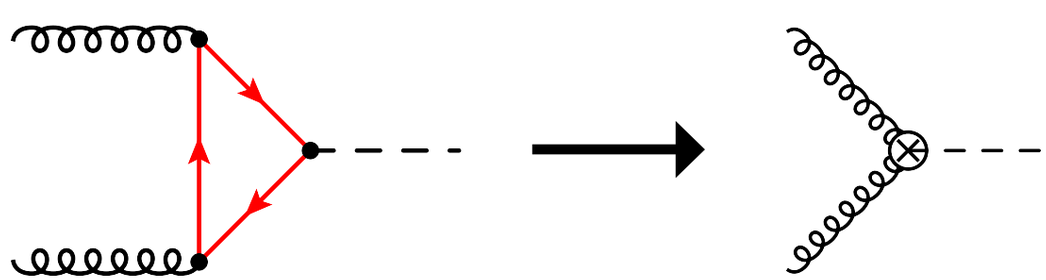
- Theoretically
  - Will perturbative series converge for gluon fusion?
    - $\alpha_S/4/\pi \sim 0.01$ but N(N)LO/(N)LO $\gg 0.1$
  - Does the true result lie within NNLO uncertainty band?



Current Status



- Higgs effective theory works well to 1% precision
  - λ_t known to five loops (2005) *Schroder, Steinhauser; Chetyrkin, Kuhn, Sturm*



$$\mathcal{L}_{eff} = -\frac{1}{4}\lambda_t H G^{\mu\nu,a} G^a_{\mu\nu}$$

- Full top mass effect known up to NLO, and NNLO top mass dependence well estimated (2009) *Spira; Anastasiou, Bucherer, Kunszt*

NNNLO: how it is done?

$$\sigma = \int dz\, dx_1\, dx_2\; f_i(x_1) f_j(x_2)\; \hat{\sigma}_{ij\to HX}(z)\; \delta(z - \frac{M_H^2}{x_1 x_2 S_{CM}})$$

single parameter dependence

- Higgs Inclusive cross section

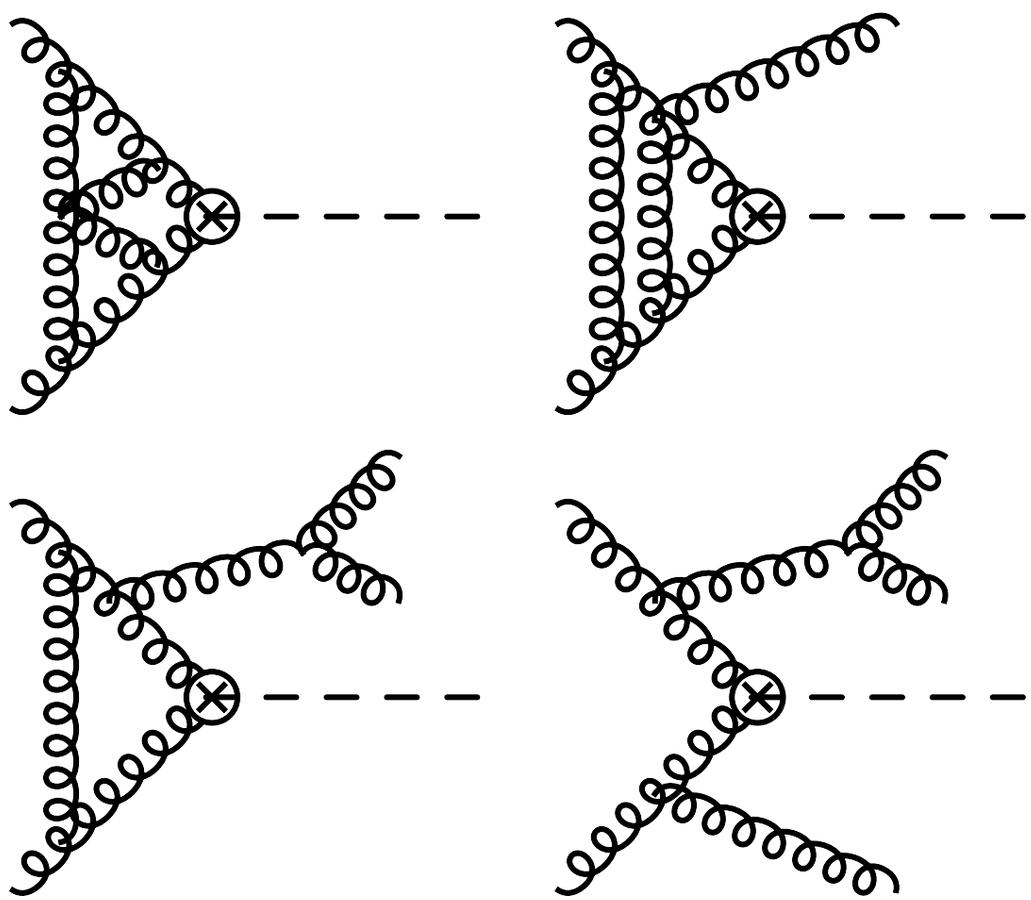
- *phase space* integration is "simplified" to *loop* integration by reverse unitarity

$$\delta(p^2 - m^2) \to \frac{i}{p^2 - m^2 + i0} - c.c.$$

Apply standard loop techniques:
1. integration-by-part (IBP) to reduce integrals;
2. differential equations (DEs) to solve master integrals (MIs)

NNNLO

- Despite all the “fancy” techniques, still a daunting task
  - $O(10^4)$ diagrams, $O(10^2)$ real/virtual integrals



Need approximation as the first step !

Soft-Virtual Approximation

- N(N)(N)LO is in theory dominated by terms more singular in the expansion around *z=1*
  - soft-virtual approximation: only keep delta function and plus distributions of *1-z*

$$\hat{\sigma}(\mathrm{gg} \to \mathrm{HX}) = \frac{\alpha_s^2(\mu)}{576\pi v^2}\left\{\delta(1-z) + \frac{\alpha_s(\mu)}{\pi}\left[h(z) + \bar{h}(z)\log\left(\frac{M_H^2}{\mu^2}\right)\right]\right\}. \quad (3.33)$$

We have then*

$$h(z) = \delta(1-z)\left[\pi^2 + \tfrac{11}{2}\right] - \tfrac{11}{2}(1-z)^3$$
$$+ 6\left(1 + z^4 + (1-z)^4\right)\left(\frac{\log(1-z)}{1-z}\right)_+$$
$$- 6\left[\frac{z^2}{(1-z)_+} + (1-z) + z^2(1-z)\right]\log(z),$$

$$\bar{h}(z) = 6\left[\frac{z^2}{(1-z)_+} + (1-z) + z^2(1-z)\right]. \quad (3.34)$$

LO corresponds to *z=1*

Dawson, 91

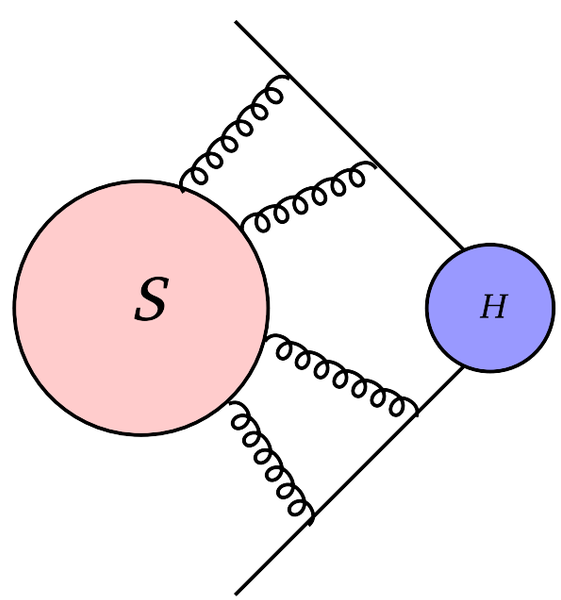
Soft-Virtual Approximation

- Soft-virtual cross section can be factorized

$$\sigma_{s.v.}(z) = H(Q)\, S(z)$$

  - Hard function is essentially the form factor of ggH effective vertex
  - In soft function, interaction between soft gluons and on-shell hard parton factorize into Wilson line

S(z): interactions of the cloud of soft gluons



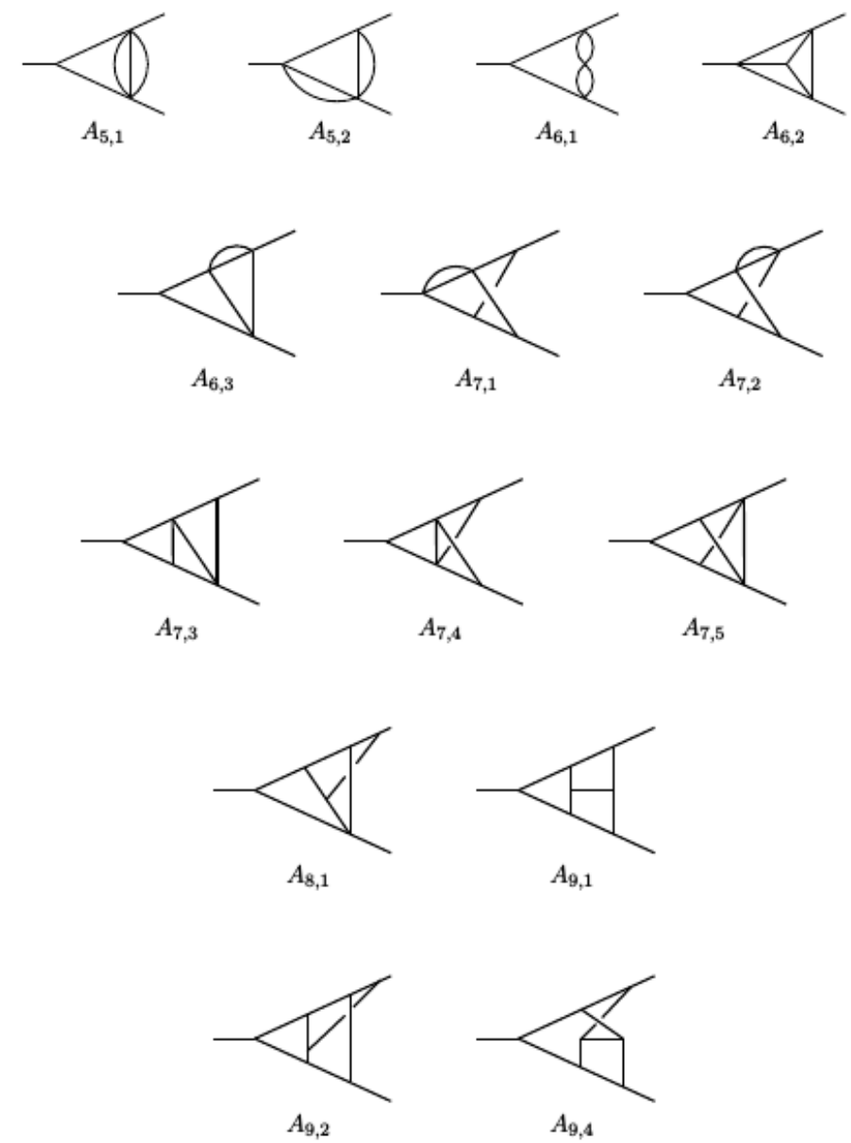
H(Q): short-distance physics

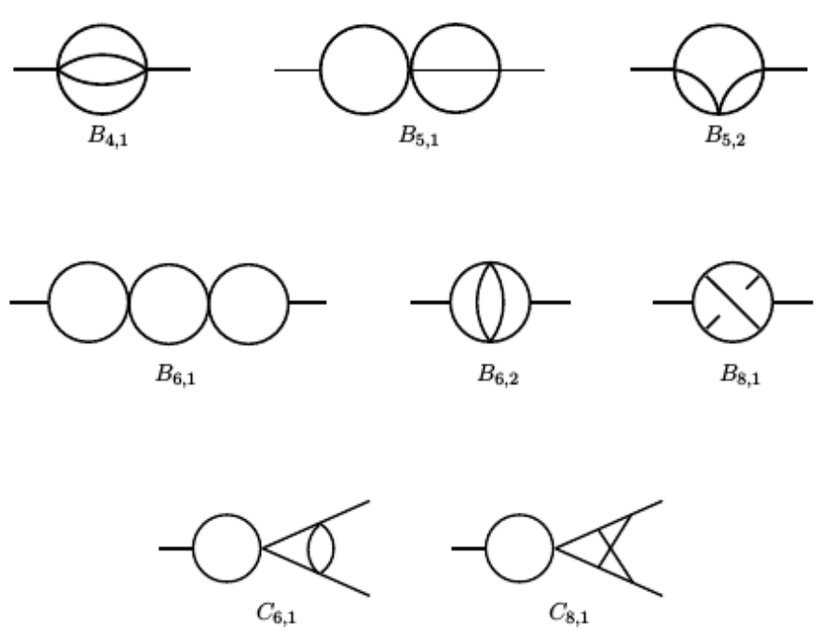
Hard Function

- Includes all virtual/multi-loop correction to the LO process

Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus

- Extracted from three-loop gluon form factor (2009)





Soft Function

- Only cares about color structure of LO process
- Defined as vacuum expectation value of two intersecting semi-infinite light-like Wilson lines

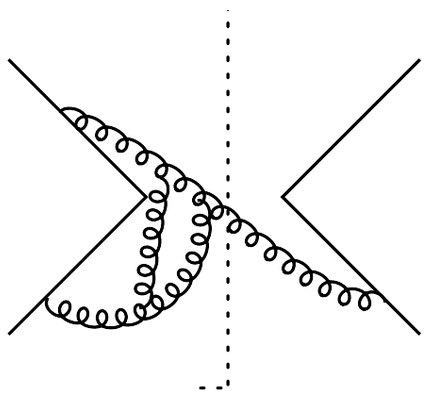
$$\mathcal{S}(1-z) \equiv \sum_{X_s} \langle 0|T\{Y_n Y_{\bar{n}}^\dagger\}|X_s\rangle \delta(2E_{X_s} - (1-z)M_H) \langle 0|T\{Y_{\bar{n}} Y_n^\dagger\}|0\rangle$$

$$Y_n(x) = \mathrm{P}\exp\left[ig_s \int_{-\infty}^{0} \mathrm{d}s\, n\cdot \mathbf{A}(x+sn)\right]$$

Wilson lines used here follow the directions of two incoming partons

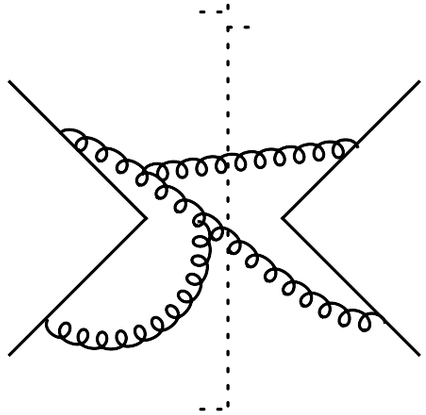
Soft Function

- Three parts of the calculation at N3LO



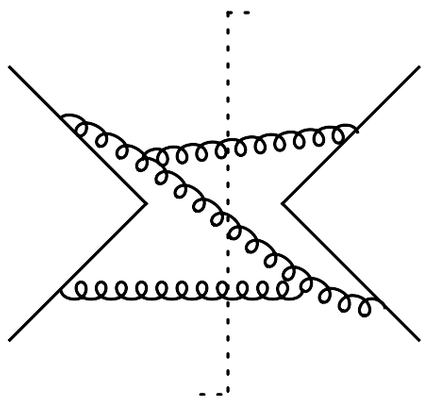
Two-loop one emission (2013)

YL, Zhu; Duhr, Gehrmann



One-loop two emission (2014)

Anastasiou, Durh, Dulat, Furlan, Gehrmann, Herzog, Mistlberger; YL, von Manteuffel, Schabinger, Zhu



Tree-level three emission (2013)

Anastasiou, Duhr, Dulat, Mistlberger; YL, von Manteuffel, Schabinger, Zhu

Soft Virtual Results

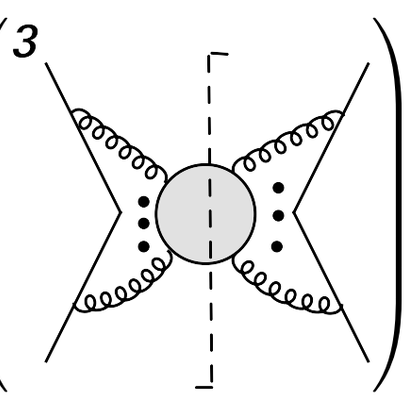
Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger; YL, von Manteuffel, Schabinger, Zhu

- Obtain Higgs at N3LO in SVA by putting together hard and soft

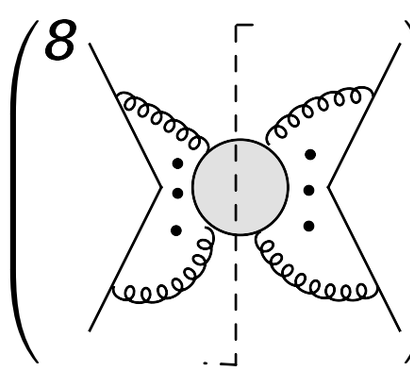
- Same soft function can be Casimir-scaled to get DY at N3LO in SVA

$$\ln(\mathcal{S}_{\rm DY}) \quad = \quad \frac{C_F}{C_A}\ln(\mathcal{S}_{\rm Higgs})$$

$$\ln\Bigg($$



$$\Bigg) = \frac{C_F}{C_A}\ln\Bigg($$



$$\Bigg)$$

$$\begin{aligned}
\mathcal{G}_{\rm s.v.}(z,0) =& \mathcal{D}_0 + a_s\Big\{C_A\Big[\frac{1}{4}\mathcal{D}_2 + \frac{1}{8}\zeta_2\mathcal{D}_0\Big]\Big\} + a_s^2\Big\{C_A^2\Big[2\mathcal{D}_4 - \frac{11}{12}\mathcal{D}_3 + \frac{1}{36}\Big(67 - 90\zeta_2\Big)\mathcal{D}_2 \\
&+ \frac{1}{216}\Big(-202 + 198\zeta_2 + 1053\zeta_3\Big)\mathcal{D}_1 + \frac{1}{576}\Big(837 + 536\zeta_2 - 660\zeta_3 - 18\zeta_4\Big)\mathcal{D}_0\Big] \\
&+ C_A N_f\Big[\frac{1}{6}\mathcal{D}_3 - \frac{5}{18}\mathcal{D}_2 + \frac{1}{54}\Big(7 - 9\zeta_2\Big)\mathcal{D}_1 + \frac{1}{72}\Big(-30 - 10\zeta_2 - 3\zeta_3\Big)\mathcal{D}_0\Big] \\
&+ C_F N_f\Big[\frac{1}{1728}\Big(-603 + 432\zeta_3\Big)\mathcal{D}_0\Big]\Big\} + a_s^3\Big\{C_A^3\Big[8\mathcal{D}_6 - \frac{110}{9}\mathcal{D}_5 \\
&+ \frac{1}{27}\Big(925 - 1512\zeta_2\Big)\mathcal{D}_4 + \frac{1}{27}\Big(-1051 + 1683\zeta_2 + 4887\zeta_3\Big)\mathcal{D}_3 \\
&+ \frac{1}{648}\Big(30569 - 32832\zeta_2 - 76032\zeta_3 - 49896\zeta_4\Big)\mathcal{D}_2 + \frac{1}{23328}\Big(-297029 \\
&+ 616536\zeta_2 + 1931256\zeta_3 + 245916\zeta_4 - 2818800\zeta_2\zeta_3 + 4339008\zeta_5\Big)\mathcal{D}_1 \\
&+ \frac{1}{5184}\Big(215131 + 64604\zeta_2 - 265356\zeta_3 - 91542\zeta_4 + 211464\zeta_2\zeta_3 - 272844\zeta_5 \\
&+ 356832\zeta_3^2 - 216324\zeta_6\Big)\mathcal{D}_0\Big] + C_A^2 N_f\Big[\frac{20}{9}\mathcal{D}_5 - \frac{164}{27}\mathcal{D}_4 + \frac{1}{54}\Big(457 - 612\zeta_2\Big)\mathcal{D}_3 \\
&+ \frac{1}{324}\Big(-4211 + 3384\zeta_2 + 4968\zeta_3\Big)\mathcal{D}_2 + \frac{1}{11664}\Big(31313 - 78228\zeta_2 - 153900\zeta_3 \\
&- 16524\zeta_4\Big)\mathcal{D}_1 + \frac{1}{5184}\Big(-98059 - 4480\zeta_2 + 29544\zeta_3 + 31548\zeta_4 - 54000\zeta_2\zeta_3 \\
&+ 62568\zeta_5\Big)\mathcal{D}_0\Big] + C_A N_f^2\Big[\frac{4}{27}\mathcal{D}_4 - \frac{10}{27}\mathcal{D}_3 + \frac{1}{81}\Big(25 - 36\zeta_2\Big)\mathcal{D}_2 \\
&+ \frac{1}{729}\Big(-58 + 270\zeta_2 + 135\zeta_3\Big)\mathcal{D}_1 + \frac{1}{5184}\Big(7545 - 2128\zeta_2 + 2064\zeta_3 \\
&- 2736\zeta_4\Big)\mathcal{D}_0\Big] + C_A C_F N_f\Big[\frac{1}{2}\mathcal{D}_3 + \frac{3}{8}\Big(-21 + 16\zeta_3\Big)\mathcal{D}_2 + \frac{1}{864}\Big(1711 - 432\zeta_2 \\
&- 912\zeta_3 - 432\zeta_4\Big)\mathcal{D}_1 + \frac{1}{5184}\Big(-63991 - 10224\zeta_2 + 33696\zeta_3 + 792\zeta_4 + 15552\zeta_2\zeta_3 \\
&+ 12960\zeta_5\Big)\mathcal{D}_0\Big] + \frac{1}{36}C_F^2 N_f\Big(38 + 111\zeta_3 - 180\zeta_5\Big)\mathcal{D}_0 \\
&+ \frac{1}{2592}C_F N_f^2\Big(4481 - 828\zeta_2 - 3024\zeta_3 - 72\zeta_4\Big)\mathcal{D}_0\Big\}\,. \qquad (4.3)
\end{aligned}$$

Beyond Standard Model

- Straightforward to extend to $\mathcal{N}$=4 SYM

  Gehrmann, Henn, Huber

  - Hard function known to three loops (2011)
  - Add scalar contribution and Yukawa interaction between fermion and scalars for soft function
    - 1 vector, 4 fermion, 3 scalars and 3 pseudo-scalars in adjoint representation

$$G^{(3)}_{\mathcal{N}=4}(z,0) = a_s^3 \Big\{ N_c^3 \Big[8\mathcal{D}_6 - 56\zeta_2\mathcal{D}_4 + 181\zeta_3\mathcal{D}_3 - 77\zeta_4\mathcal{D}_2 - \Big(\frac{725}{6}\zeta_2\zeta_3 - 186\zeta_5\Big)\mathcal{D}_1 + \Big(\frac{413}{6}\zeta_3^2 - \frac{2003}{48}\zeta_6\Big)\mathcal{D}_0 \Big] \Big\}$$

1. *uniform transcendentality* displayed
2. match *maximum transcendental* part of the SM result

From SVA to Full N3LO

- The SVA is far from enough to provide reliable N3LO

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

$$\hat{\sigma}(\bar{z})|_{\bar{z}=1-z} = \underbrace{\sum_n \left[\frac{\ln^n(\bar{z})}{\bar{z}}\right]_+ + \delta(\bar{z})}_{\text{SVA}} + \underbrace{\hat{\sigma}^{(0)}(\bar{z}) + \bar{z}\hat{\sigma}^{(1)}(\bar{z}) + \ldots}_{\text{Finite at z=1}}$$

- Within SVA, less singular terms produce similar contribution compared to more singular terms

- Still useful as

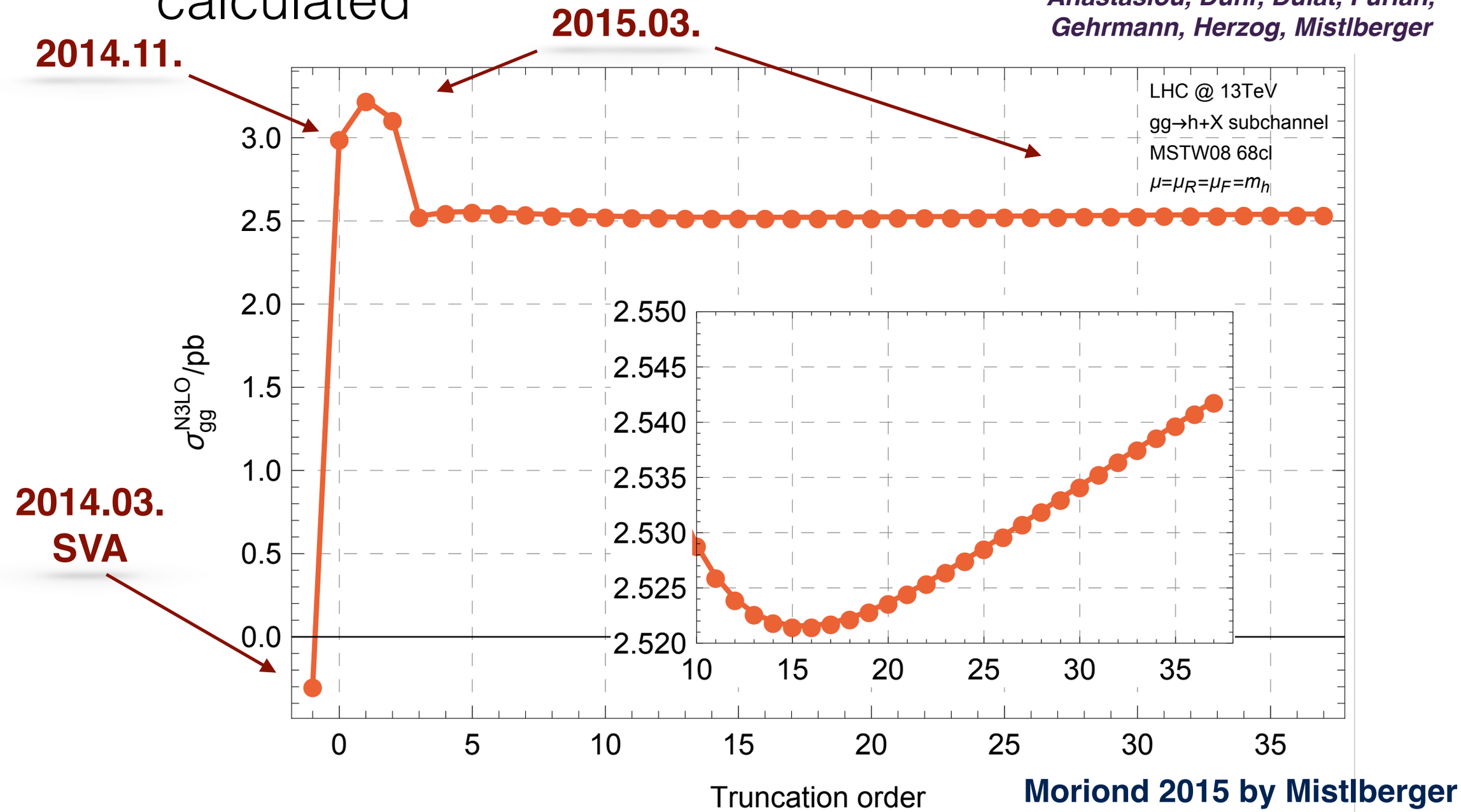
  Bonvini, Marzani; Catani, Cieri, de Florian, Gerrera, Grazzini

  - Essential input for soft resummation at N3LL (2014)
  - Integrals calculated can serve as boundary conditions for full integral
  - First several terms in soft virtual expansion to achieve full result (MIs can be recycled for higher order terms)

N3LO Higgs

- Recently additional 38 finite terms have been calculated

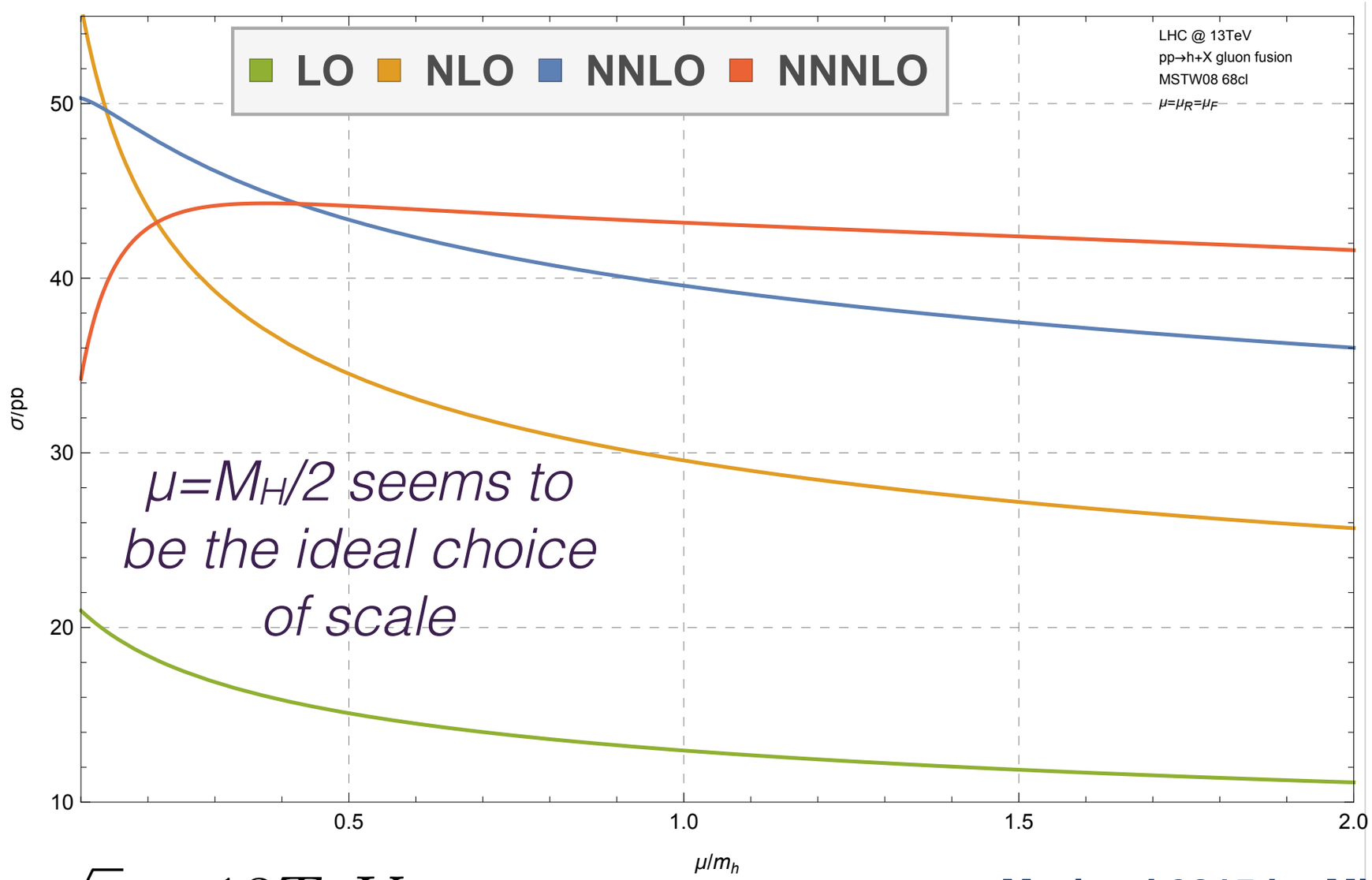
Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger



Moriond 2015 by Mistlberger

N3LO Higgs

- NNNLO is within NNLO uncertainty band !



$\sqrt{s} = 13TeV$

Moriond 2015 by Mistlberger

Conclusion

- Dawn of N3LO precision era
- Soft-virtual corrections known from two independent calculation

  Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger; YL, von Manteuffel, Schabinger, Zhu

- Full result is well approached by soft expansion