Higgs Production at N3LO
from SM to $\mathcal{N}=4$ SYM

Ye Li
in collaboration with
A. von Manteuffel, R. Schabinger, H.X. Zhu

SLAC National Accelerator Lab

DIS2015, Dallas, TX
Higgs Discovered!

- LHC just resumed running this year

\[ m_H = 125.09 \pm 0.24 \text{ (GeV)} \]
Why care about N3LO?

- Current experimental precision is around 20%

arXiv:1412.8662
Why care about N3LO?

• Theoretically

  • New physics can hide behind Higgs production in many common BSM scenarios

  ggH coupling is sensitive to new particles running in the loop, for example, a top partner
Why care about N3LO?

- Theoretically
  - NNLO uncertainty on Higgs production in gluon fusion channel is larger than future experimental uncertainty

**Table: Higgs Production Uncertainty**

<table>
<thead>
<tr>
<th>L (fb⁻¹)</th>
<th>γγ</th>
<th>WW</th>
<th>ZZ</th>
<th>bb</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>[6%, 12%]</td>
<td>[6%, 11%]</td>
<td>[7%, 11%]</td>
<td>[11%, 14%]</td>
</tr>
<tr>
<td>3000</td>
<td>[4%, 8%]</td>
<td>[4%, 7%]</td>
<td>[4%, 7%]</td>
<td>[5%, 7%]</td>
</tr>
</tbody>
</table>

**Graph:**

- Higgs working group report
  - a. NNLO scale
  - b. NLO EW
  - c. Large m_t approximation
  - d. Quark mass input
  - e. PDF

**Notes:**

- CMS snowmass workgroup report
  - NNLO scale uncertainty: MUST BE REDUCED!
Why care about N3LO?

• Theoretically
  • Will perturbative series converge for gluon fusion?
    • $\alpha_s/4/\pi \sim 0.01$ but N(N)LO/(N)LO $\gg 0.1$
  • Does the true result lie within NNLO uncertainty band?

\[ \sqrt{s} = 8 \text{ TeV LHC} \]

\[ \delta \text{NNLO} = 2.9 \text{ pb} \]

\[ \delta \text{NLO} = 7.1 \text{ pb} \]

\[ \text{LO} = 9.6 \text{ pb} \]
### Current Status

- Higgs effective theory works well to 1% precision
  - $\lambda_t$ known to five loops (2005)
  - $L_{eff} = -\frac{1}{4}\lambda_t H G_{\mu\nu}^{\alpha} G_{\mu\nu}^\alpha$

### Timeline

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<tbody>
<tr>
<td>HEFT</td>
<td>NLO</td>
<td>NNLO</td>
<td>NNNLO</td>
</tr>
</tbody>
</table>

- Full top mass effect known up to NLO, and NNLO top mass dependence well estimated (2009)

**Contributors**

- Shifman, Vainshtein, Voloshin, Zakharov
- Dawson, Djouadi, Spira, Zerwas
- Harlander, Kilgore, Anastasiou, Melnikov, Ravindran, Smith, van Neerven
- Anastasiou, Duhr, Dulat, Herzog, Mistlberger
- Schroder, Steinhauser, Chetyrkin, Kuhn, Sturm
- Spira, Anastasiou, Bucherer, Kunszt
NNNLO: how it is done?

\[ \sigma = \int dz \, dx_1 \, dx_2 \, f_i(x_1) f_j(x_2) \, \hat{\sigma}_{ij}^{H} \left( z \right) \delta \left( z - \frac{M_H^2}{x_1 x_2 S_{CM}} \right) \]

- Higgs Inclusive cross section

- *phase space* integration is “simplified” to *loop* integration by reverse unitarity

\[ \delta(p^2 - m^2) \to \frac{i}{p^2 - m^2 + i0} - c.c. \]

Apply standard loop techniques:
1. integration-by-part (IBP) to reduce integrals;
2. differential equations (DEs) to solve master integrals (MIs)
Despite all the “fancy” techniques, still a daunting task

- $O(10^4)$ diagrams, $O(10^2)$ real/virtual integrals

Need approximation as the first step!
Soft-Virtual Approximation

- N(N)(N)LO is in theory dominated by terms more singular in the expansion around $z=1$

- soft-virtual approximation: only keep delta function and plus distributions of $1-z$

\[
\hat{\sigma}(gg \to HX) = \frac{\alpha_s^2(\mu)}{576\pi v^2} \left\{ \delta(1-z) + \frac{\alpha_s(\mu)}{\pi} \left[ h(z) + \bar{h}(z)\log\left(\frac{M_H^2}{\mu^2} \right) \right] \right\}. \tag{3.33}
\]

We have then:

\[
\begin{align*}
\hat{h}(z) &= \delta(1-z)\left[ \pi^2 + \frac{11}{2} \right] - \frac{11}{2} (1-z)^3 \\
&\quad + 6(1+z^4 + (1-z)^4)\left( \log(1-z) \right)_{+} \\
&\quad - 6\left[ \frac{z^2}{(1-z)_+} + (1-z) + z^2(1-z) \right] \log(z), \\
\bar{h}(z) &= 6\left[ \frac{z^2}{(1-z)_+} + (1-z) + z^2(1-z) \right]. \tag{3.34}
\end{align*}
\]

LO corresponds to $z=1$

Dawson, 91
Soft-Virtual Approximation

- Soft-virtual cross section can be factorized

\[ \sigma_{s.v.}(z) = H(Q) \ S(z) \]

- Hard function is essentially the form factor of ggH effective vertex

- In soft function, interaction between soft gluons and on-shell hard parton factorize into Wilson line

\[ S(z): \text{interactions of the cloud of soft gluons} \]

\[ H(Q): \text{short-distance physics} \]
Hard Function

- Includes all virtual/multi-loop correction to the LO process

- Extracted from three-loop gluon form factor (2009)

References:

Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus

Diagram of master integrals:

- $B_{4,1}$
- $B_{5,1}$
- $B_{5,2}$
- $B_{6,1}$
- $B_{6,2}$
- $B_{6,3}$
- $C_{6,1}$
- $C_{6,2}$
- $C_{6,3}$
- $A_{5,1}$
- $A_{5,2}$
- $A_{6,1}$
- $A_{6,2}$
- $A_{6,3}$
- $A_{7,1}$
- $A_{7,2}$
- $A_{7,3}$
- $A_{7,4}$
- $A_{7,5}$
- $A_{8,1}$
- $A_{8,2}$
- $A_{9,1}$
- $A_{9,2}$
- $A_{9,4}$
Soft Function

• Only cares about color structure of LO process

• Defined as vacuum expectation value of two intersecting semi-infinite light-like Wilson lines

\[
S(1 - z) \equiv \sum_{X_s} \langle 0|T\{Y_n Y_n^\dagger\}|X_s\rangle \delta(2E_{X_s} - (1 - z)M_H)\langle 0|T\{Y_n Y_n^\dagger\}|0\rangle
\]

\[
Y_n(x) = P \exp \left[ i g_s \int_{-\infty}^{0} ds \, n \cdot A(x + sn) \right]
\]

Wilson lines used here follow the directions of two incoming partons
Soft Function

- Three parts of the calculation at N3LO

Two-loop one emission (2013)
YL, Zhu; Duhr, Gehrmann

One-loop two emission (2014)
Anastasiou, Durh, Dulat, Furlan, Gehrmann, Herzog, Mistlberger; YL, von Manteuffel, Schabinger, Zhu

Tree-level three emission (2013)
Anastasiou, Duhr, Dulat, Mistlberger; YL, von Manteuffel, Schabinger, Zhu
Soft Virtual Results

- Obtain Higgs at N3LO in SVA by putting together hard and soft

- Same soft function can be Casimir-scaled to get DY at N3LO in SVA

\[
\ln(S_{\text{DY}}) = \frac{C_F}{C_A} \ln(S_{\text{Higgs}})
\]

\[
\begin{pmatrix}
3
\end{pmatrix}
\begin{pmatrix}
8
\end{pmatrix}
= \left(\begin{array}{c}
\frac{C_F}{C_A}
\end{array}\right) \ln \left(\begin{array}{c}
\frac{C_F}{C_A}
\end{array}\right)
\]
Beyond Standard Model

- Straightforward to extend to $\mathcal{N}=4$ SYM
  - Hard function known to three loops (2011)
  - Add scalar contribution and Yukawa interaction between fermion and scalars for soft function
    - 1 vector, 4 fermion, 3 scalars and 3 pseudo-scalars in adjoint representation

$$G_{\mathcal{N}=4}^{(3)}(z, 0) = a_s^3 \left\{ N_c^3 \left[ 8 \mathcal{D}_6 - 56 \zeta_2 \mathcal{D}_4 + 181 \zeta_3 \mathcal{D}_3 - 77 \zeta_4 \mathcal{D}_2 - \left( \frac{725}{6} \zeta_2 \zeta_3 - 186 \zeta_5 \right) \mathcal{D}_1 
+ \left( \frac{413}{6} \zeta_3^2 - \frac{2003}{48} \zeta_6 \right) \mathcal{D}_0 \right] \right\}$$

1. uniform transcendentality displayed
2. match maximum transcendental part of the SM result
From SVA to Full N3LO

• The SVA is far from enough to provide reliable N3LO

\[ \hat{\sigma}(\bar{z})|_{\bar{z}=1-z} = \sum_n \left[ \ln^n \frac{n(\bar{z})}{\bar{z}} \right] + \delta(\bar{z}) + \hat{\sigma}^{(0)}(\bar{z}) + \bar{z}\hat{\sigma}^{(1)}(\bar{z}) + \ldots \]

\( \text{SVA} \quad \text{Finite at } z=1 \)

• Within SVA, less singular terms produce similar contribution compared to more singular terms

• Still useful as

\( \text{Essential input for soft resummation at N3LL (2014)} \)

\( \text{Integrals calculated can serve as boundary conditions for full integral} \)

\( \text{First several terms in soft virtual expansion to achieve full result (MIs can be recycled for higher order terms)} \)
N3LO Higgs

- Recently additional 38 finite terms have been calculated

2014.11.

2015.03.

2014.03.

SVA

LHC @ 13TeV
gg→h+X subchannel
MSTW08 68c
μ=μR=μF=\text{m}_h

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger

Moriond 2015 by Mistlberger
N3LO Higgs

• NNNLO is within NNLO uncertainty band!

$\sqrt{s} = 13 TeV$

$\mu = M_H/2$ seems to be the ideal choice of scale

Moriond 2015 by Mistlberger
Conclusion

• Dawn of N3LO precision era

• Soft-virtual corrections known from two independent calculation

• Full result is well approached by soft expansion

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger; YL, von Manteuffel, Schabinger, Zhu