On the resummation of non-global logarithms at finite N_c

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DIS 2015
Dallas, TX, 30th April 2015

Precision at the LHC

New physics @ LHC ⇒ Precision measurements. Require:

New tools to boost search for new physics:

e.g. jet substructure for boosted objects
aim: background/signal discrimination — e.g. Butterworth, et al '08

Analytic calculations

e.g. analytic understanding of jet substructure — Dasgupta et al '13

MC precision

- Use "reasonable" approximations
  - e.g. large-N_c approximation – how accurate?
- Tuned with data for NP parameters – mis-tuning?
- Compare with analytic results for systematic improvements

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QCD Resummation and its status

An important aspect is the resummation of large logs in QCD, typically present in exclusive observables:

$$\sigma(V) \propto \exp\left[L\, g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L)\right]$$

L: large log which depends on generic observable V.

- Why? Cuts $\Rightarrow$ miscancellation of real-virtual soft/collinear divergences
- Up to NNNLL (g_4) have been resummed (C parameter in e^+e^-, etc). Hoang et al '14
- Semi-analytical approaches available up to NLL and NNLL (CEASAR & ARES) Banfi et al '05, Banfi et al '14

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Non-global observables and non-global logarithms

Non-global observables

- **Non-global observables:** (typically) exclusive over angular phase space.
- They have non-global logs (NGLs). Dasgupta & Salam '01, '02

Many important observables are non-global:

- Some jet shapes (used in substructure techniques), jet mass, angularities...
- Gaps-between-jets observables (Interjet energy flow)
- Azimuthal decorrelations between jets (with certain jet definition)
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Resummation of NGLs

Non-trivial resummation:

- Gluon multiplicity (solution: large-N_c limit)
- Prohibitive integrations (non-iterative geometry) (solution: MC)

Analytic all-order treatment of NGLs @ large N_c is encoded in BMS equation — Banfi et al '02

- No proper analytic solution available (only MC)
- Until recently NGLs were only resummed numerically at large N_c

Why Precision is spoiled by these approximations?

- Large N_c: $\Rightarrow$ uncertainties $\mathcal{O}(1/N_c^2)$
  - solution: k_t clustering reduces NGLs — Appleby et al'03
  - however anti-k_t algorithm is preferred
  - Clustering logs introduced by k_t algorithm — Banfi et al'05, Delenda et al'06

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Recent progress in NGLs resummation

Numerical resummation @ finite N_c for gap energy Hatta & Ueda '13

- **Result:** finite N_c corrections negligible in e^+e^- collisions
- **Expect:** significant contributions from finite N_c correction in hadronic collision!

Analytic solution to the BMS equation at fixed order up to five loops for hemisphere mass distribution [defined later]. Schwartz & Zhu '14

$$\Sigma_{\rm SZ}^{\rm NG} = 1 - \frac{\pi^2}{24}\widehat{L}^2 + \frac{\zeta_3}{12}\widehat{L}^3 + \frac{\pi^4}{34\,560}\widehat{L}^4 + \left(-\frac{\pi^2\zeta_3}{360} + \frac{17}{480}\zeta_5\right)\widehat{L}^5 + \mathcal{O}(\alpha_s^6)\,,$$

with $\widehat{L} = N_c\alpha_s\ln(1/\rho)/\pi$

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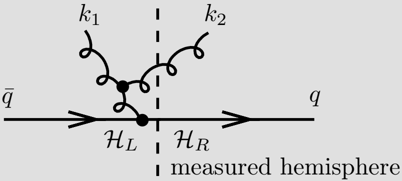
NGLs @ finite N_c

Here we are interested in the analytic resummaion of NGLs at finite N_c:

Do by c.f. hemisphere mass distribution in $e^+e^- \to$ di-jets



$$\rho = \left(p_q + \sum_{i\in\mathcal{H}_R} k_i\right)^2 /Q^2 \approx 2\sum_{i\in\mathcal{H}_R} k_i.p_q/Q^2 = \sum_i x_i\, e^{-\eta_i}\,,$$

Q: CoM energy, k_{ti} gluon transverse momenta, $x_i = k_{ti}/Q$, η_i: rapidity

Work with energy-ordering: $Q \gg k_{t1} \gg k_{t2} \gg \cdots \gg k_{tn}$.

Hemisphere mass distribution

Compute Born-normalised integrated hemisphere mass distribution:

$$\Sigma(\rho) = \int_0^{\rho} \frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\rho'}\,\mathrm{d}\rho' = 1 + \Sigma_1(\rho) + \Sigma_2(\rho) + \cdots\,,$$

$$\Sigma_m(\rho) = \sum_X \int \left(\frac{1}{m!}\prod_{i=1}^{m}\mathrm{d}\Phi_i\right)\hat{\mathcal{U}}_m\,\mathcal{W}^X_{12\cdots m}\,,$$

i.e. probability that measured hemisphere mass is less than ρ.

- $\mathcal{W}^X_{12\cdots m} = \mathcal{W}^X(k_1, k_2, \cdots, k_m)$: Born-normalised eikonal squared amplitudes for emission of m gluons of configuration X.
- X: real (R)/virtual (V) configurations of gluons.
- $\prod_{i=1}^{m}\mathrm{d}\Phi_i$: phase-space of emitted gluons
- $\hat{\mathcal{U}}_m$: measurement operator: cut events with hemisphere mass $> \rho$.

Brute force calculation method of NGLs at finite N_c

Issues to tackle:

- Evaluate eikonal squared amplitudes $\mathcal{W}^X_{12\cdots m}$:
  - Account for all gluon branchings
  - Calculate traces of color matrices for each diagram using `ColorMath` Sjödahl '12
  - Consider all real-virtual configurations and compute mis-match between them
  - See next talk for details
- Perform integrations (semi-)analytically [up to 7-D @ 4 loops]
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Results: factorisation of NGLs from global logs

Result: The hemisphere mass distrubution is:

$$\Sigma(\rho) = \Sigma^{\mathrm{P}}(\rho) \times \Sigma^{\mathrm{NG}}(\rho)\,,$$

Σ^{P}: Sudakov form factor (global logs):

$$\Sigma^{\mathrm{P}}(\rho) = \exp\left(\Sigma_1^{\mathrm{P}}\right) = \exp\left(-\mathrm{C_F}\bar{\alpha}_s L^2\right)\,,$$

$L = \ln(1/\rho)$, $\bar{\alpha}_s = \alpha_s/2\pi$.
Why double logs? $\Rightarrow$ soft and collinear logs.
The non-global factor:

$$\Sigma^{\mathrm{NG}}(\rho) = 1 + \Sigma_2^{\mathrm{NG}}(\rho) + \Sigma_3^{\mathrm{NG}}(\rho) + \cdots\,.$$

resums NGLs starting from two loops [only single logs].

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Results: NGLs series

Up to four loops (fully) and five loops (partially):

$$\begin{aligned}\Sigma^{\mathrm{NG}}(\rho) =& 1 - \frac{\bar{L}^2}{2!}\mathrm{C_F C_A}\zeta_2 + \frac{\bar{L}^3}{3!}\mathrm{C_F C_A^2}\zeta_3 + \\ & -\frac{\bar{L}^4}{4!}\left[\frac{25}{8}\mathrm{C_F C_A^3}\zeta_4 - \frac{13}{5}\mathrm{C_F^2 C_A^2}\zeta_2^2\right] + \\ & -\frac{\bar{L}^5}{2!3!}\mathrm{C_F^2 C_A^3}\zeta_2\zeta_3 + \frac{\bar{L}^5}{5!}\mathrm{C_F C_A^4}\zeta_5\left[\alpha + \beta\left(\frac{\mathrm{C_F}}{\mathrm{C_A}} - \frac{1}{2}\right)\right] + \\ & + \mathcal{O}(\alpha_s^6).\end{aligned}$$

$\bar{L} = \bar{\alpha}_s L$, α and β are undetermine fixed coefficients.

Observation 1: At four loop finite-N_c corrections are $\mathcal{O}(1.5\%)$ of large N_c result [agreement with findings of Hatta & Ueda '13]

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Observation 2: Our result at large N_c agrees with Schwartz and Zhu's result (obtained via BMS equation):

$$\Sigma^{\mathrm{NG}} = 1 - \frac{\pi^2}{24}(N_c\bar{L})^2 + \frac{\zeta_3}{12}(N_c\bar{L})^3 + \frac{\pi^4}{34\,560}(N_c\bar{L})^4 + \\ + \left(-\frac{\pi^2\zeta_3}{288} + \alpha\frac{\zeta_5}{240}\right)(N_c\bar{L})^5 + \mathcal{O}\left((N_c\bar{L})^6\right) ,$$

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deduce $\alpha = \frac{17}{2} + \frac{\zeta_2\zeta_3}{\zeta_5}$. Anstaz for $\beta = 2\frac{\zeta_2\zeta_3}{\zeta_5}$ based on observed pattern of zeta functions.

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Results: Exponentiation of NGLs

Observation 3: NGLs exhibit a pattern of exponentiation:

$$\Sigma^{\rm NG}(\rho) = \exp\left\{ -\frac{\bar{L}^2}{2!}\mathrm{C_F C_A}\zeta_2 + \frac{\bar{L}^3}{3!}\mathrm{C_F C_A^2}\zeta_3 - \frac{\bar{L}^4}{4!}\mathrm{C_F C_A^3}\,\zeta_4\left[\frac{29}{8} + \left(\frac{\mathrm{C_F}}{\mathrm{C_A}} - \frac{1}{2}\right)\right] + + \frac{\bar{L}^5}{5!}\mathrm{C_F C_A^4}\,\zeta_5\left[\alpha + \beta\left(\frac{\mathrm{C_F}}{\mathrm{C_A}} - \frac{1}{2}\right)\right] + \mathcal{O}(\alpha_s^6)\right\}.$$

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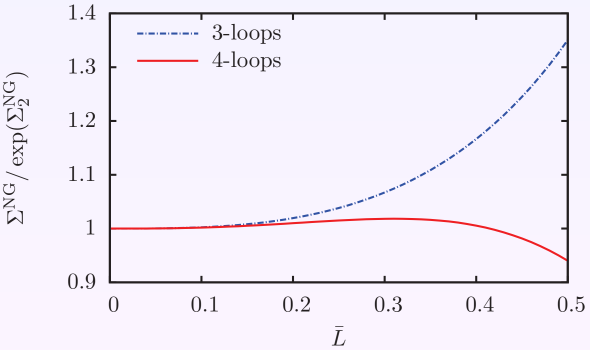
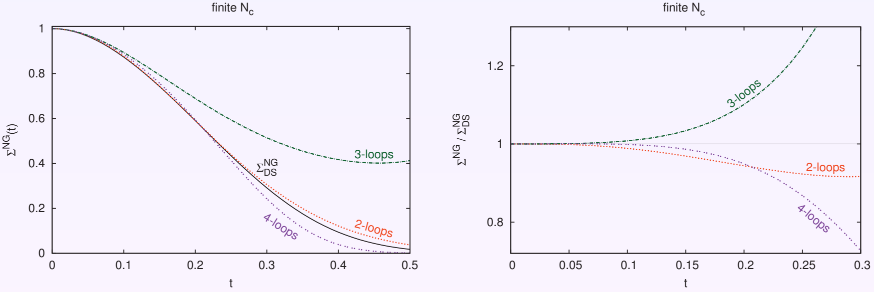
Convergence of the series



Up to 4-loops is insufficient for good convergence. A few more terms should suffice!

Comparison with Monte Carlo

At finite N_c



$$\Sigma_{\rm DS}^{\rm NG}(t) = \exp\left(-C_F C_A \frac{\pi^2}{3} \frac{1+(0.85 C_A t)^2}{1+(0.86 C_A t)^{1.33}}\, t^2\right).$$

$$t = \frac{1}{4\pi\beta_0} \ln \frac{1}{1-2\beta_0 \alpha_s L},$$

parametrisation of MC by Dasgupta & Salam '01

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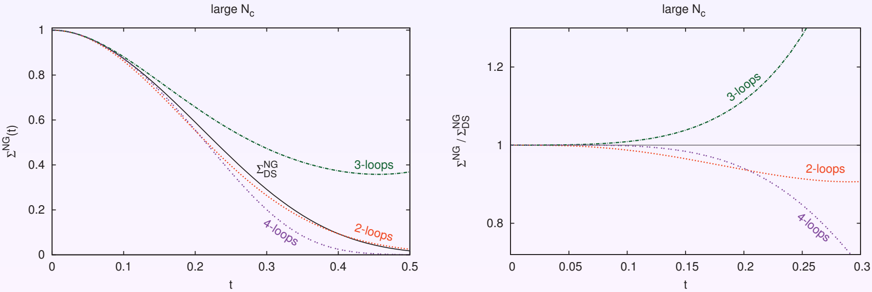

Conclusions and outlook

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More work is underway in this subject:

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