Challenges in the extraction of TMDs: perturbative vs non-perturbative aspects

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OUTLINE

- SIDIS cross section and transverse momentum.
- Theory to be tested (Always try before you buy).
- Perturbative vs non-perturbative regime.
- The problem of matching.
- Final remarks.
SIDIS cross section and transverse momentum

\[ \frac{d^5\sigma}{dx_bj dQ^2 dz_f dq_T^2 d\phi} = \frac{\alpha_{em}^2 \alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \sum_k A_k \int_{x_{min}}^1 \frac{dx}{x} \int_{z_{min}}^1 \frac{dz}{z} \left[ f \circ D \circ \hat{\sigma}_k \right] \]

\[ \times \delta \left( \frac{q_T^2}{Q^2} - \left( \frac{1}{\bar{x}} - 1 \right) \left( \frac{1}{\bar{z}} - 1 \right) \right), \]

Resummation of divergent logarithms. CSS formalism.

Collins, Soper, Sterman

Nadolsky, Stump, Yuan

Koike, Nagashina, Vogelsang
SIDIS cross section and transverse momentum

\[
\frac{d\sigma^{\text{total}}}{dx \, dy \, dz \, dq_T^2} = \pi \sigma_0^{\text{DIS}} \int \frac{d^2 b_T e^{i q_T \cdot b_T}}{(2\pi)^2} W^{\text{SIDIS}}(x, z, b_T, Q) + Y^{\text{SIDIS}}(x, z, q_T, Q)
\]

\[
W^{\text{SIDIS}}(x, z, b_T, Q) = \exp \left[ S_{\text{pert}}(b_T, Q) \right] \sum_j e_j^2 \sum_{i,k} C_{j,i}^{\text{in}} \otimes f_i(x, \mu_b^2) \ C_{k,j}^{\text{out}} \otimes D_k(z, \mu_b^2)
\]

\[
S_{\text{pert}}(b_T, Q) = - \int \frac{d\mu^2}{\mu^2} \left[ A(\alpha_s(\mu)) \ln \left( \frac{Q^2}{\mu^2} \right) + B(\alpha_s(\mu)) \right]
\]
Fourier Transform spans all impact parameter space.

Must avoid Landau pole.
SIDIS cross section and transverse momentum

\[
\frac{d\sigma^{\text{total}}}{dx \, dy \, dz \, dq_T^2} = \pi \sigma_0^{\text{DIS}} \int_0^\infty \frac{db_T b_T}{(2\pi)} J_0(q_T b_T) W^{\text{SIDIS}}(x, z, b_*, Q) \exp[S_{NP}(x, z, b_T, Q)] + Y(x, z, q_T, Q),
\]

\[b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}}\]
The goal is to describe data over all regions.
Theory to be tested
(Always try before you buy).
The non-perturbative region contains information about the intrinsic structure of hadrons.

Characterized by some Model / Prescription – dependence.

Such dependence does not affect the pQCD region.
Resummation of Large logs

$$\alpha_s^k \ln^{2k} \left( \frac{Q^2}{q_T^2} \right) / q_T^2$$
~ Y-term matching ~

Resummation of Large logs

\[ \alpha_s^k \ln^{2k} \left( \frac{Q^2}{q_T^2} \right) / q_T^2 \]

- Describes the region \( \Lambda_{QCD} << q_T < Q \).

- \( q_T \sim Q \)

\[
\frac{d\sigma^{NLO}}{dx \ dy \ dz \ dq_T^2} = \frac{d\sigma^{ASY}}{dx \ dy \ dz \ dq_T^2} + Y \quad \xrightarrow{\text{Resummation}} \quad d\sigma^{total} = W + Y
\]
Y-term matching

Matching to pQCD

- Describes the region $\Lambda_{QCD} \ll q_T < Q$.

- $q_T \sim Q \rightarrow \sigma^{\text{res}} \sim \sigma^{\text{asy}}$ (In principle, provides a matching scheme)

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\frac{d\sigma^{\text{NLO}}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\text{ASY}}}{dx\,dy\,dz\,dq_T^2} + Y
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Resummation

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Resummation
- Describes the region \( \Lambda_{QCD} << q_T < Q \).

- \( q_T \sim Q \rightarrow \sigma^{\text{res}} \sim \sigma^{\text{asy}} \) (In principle, provides a matching scheme)

- As \( q_T \rightarrow 0 \), the contribution from the Y-term is negligible.
Towards TMD extraction

- What is the role of the model-dependent part of the cross section?
- Is the b-prescription trivial to implement?
- Can the Y-term matching work?
- What data can we expect to be able to reproduce?
We studied some of these points in a recent publication

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Using the CSS formalism at next to leading log (NLL).

The modern TMD formulation at leading order in the strong coupling, can be reduced to the NLL CSS scheme, by setting the rapidity divergence regulators $\zeta_F = \zeta_D = Q^2$.

We explored different kinematical regions: Extreme, HERA-like, COMPASS-like.

\[
\frac{d\sigma_{\text{total}}}{dx \, dy \, dz \, dq_T^2} = \pi \sigma_0^{\text{DIS}} \int_0^\infty \frac{db_T b_T}{(2\pi)} J_0(q_T b_T) W^{\text{SIDIS}}(x, z, b_*, Q) \exp[S_{\text{NP}}(x, z, b_T, Q)]
\]

\[
+ Y(x, z, q_T, Q),
\]

\[
S_{\text{NP}} = \left(-\frac{g_1}{2} - \frac{g_1 f}{2z^2} - g_2 \ln \left(\frac{Q}{Q_0}\right)\right) b_T^2.
\]
$S_{NP} = \left( -\frac{g_1}{2} - \frac{g_1 f}{2z^2} - g_2 \ln \left( \frac{Q}{Q_0} \right) \right) \hat{b}_T^2$
Model dependence

\[ S_{NP} = \left( -\frac{g_1}{2} - \frac{g_1 f}{2 z^2} - g_2 \ln \left( \frac{Q}{Q_0} \right) \right) b_T^2 \]

Extreme kinematics. Non-perturbative part has no effect at large qT
Model dependence

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HERA-like kinematics. Mild model dependence From non-perturbative part
Model dependence

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Extreme kinematics. Non-perturbative part has no effect at large qT

HERA-like kinematics. Mild model dependence From non-perturbative part

COMPASS-like kinematics. Strong dependence on Model parameters!!!
Recall...

\[ b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}} \]

\[
\frac{d\sigma^{\text{total}}}{dx \, dy \, dz \, dq_T^2} = \pi \sigma_0^{\text{DIS}} \int \frac{d^2 b_T e^{i q_T \cdot b_T}}{(2\pi)^2} W^{\text{SIDIS}}(x, z, b_T, Q) + Y^{\text{SIDIS}}(x, z, q_T, Q)
\]

How does the prescription to avoid the Landau pole affect calculations?
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One has a similar effect as for the model parameters.
The Y-term

\[
\frac{d\sigma^{\text{NLO}}}{dx \, dy \, dz \, dq_T^2} = \frac{d\sigma^{\text{ASY}}}{dx \, dy \, dz \, dq_T^2} + Y
\]
The Y-term

\[
\frac{d\sigma^{\text{NLO}}}{dx \, dy \, dz \, dq_T^2} = \frac{d\sigma^{\text{ASY}}}{dx \, dy \, dz \, dq_T^2} + Y
\]

sizable contribution from Y term
The problem of Matching

\[ \sqrt{s} = 1 \text{ TeV}, \quad Q^2 = 5000 \text{ GeV}^2 \]

\[ \sqrt{s} = 300 \text{ GeV}, \quad Q^2 = 100 \text{ GeV}^2 \]

\[ \sqrt{s} = 17 \text{ GeV}, \quad Q^2 = 10 \text{ GeV}^2 \]
The problem of Matching

Assertion $q_T \sim Q \rightarrow \sigma^{\text{res}} \sim \sigma^{\text{asy}}$ fails.

No matching seem possible (NLL)
The problem of Matching

Several factors:
- effect of non-perturbative function
- prescription to avoid Landau pole
- Behaviour of perturbative Sudakov
Several factors: - **effect of non-perturbative function**

- prescription to avoid Landau pole

- Behaviour of perturbative Sudakov
Consider the NLL resummed cross section and approximate at order $\alpha_s$: $W^{FXO}$.

If a region exists where $W^{FXO} \approx W^{NLL}$, then the quantity

$$d\sigma^{\text{total}} = W^{NLL} - W^{FXO} + d\sigma^{\text{NLO}}$$

May be used to match the perturbative calculation.

Here, $W^{FXO}$ contains the same non-perturbative function as $W^{NLL}$. 
No matching seems possible, in general.

Several factors:
- effect of non-perturbative function
- prescription to avoid Landau pole
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The problem of Matching

No matching seems possible, in general.

Several factors: - effect of non-perturbative function
- prescription to avoid Landau pole
- Behaviour of perturbative Sudakov
Sudakov factor becomes negative as $b_T \to 0$

Alternative prescription to ensure $S \to 0$ at $b_T \to 0$

$$\log \left( \frac{Q^2/\mu_b^2}{1} \right) \to \log \left( 1 + \frac{Q^2/\mu_b^2}{Q^2/\mu_b^2} \right)$$

At both HERA and COMPASS-like kinematics Sudakov “recipes” render distinct results.

In the COMPASS case, the modified Sudakov is almost negligible.
A matching, rather by chance.
(a.k.a. Always say goodbye on a happy note)

\[ d\sigma^{\text{total}} = W^{\text{NLL}} - W^{\text{FXO}} + d\sigma^{\text{NLO}} \]
Final Remarks

- The different regions of qT do not seem to be well defined in some kinematics.
- Model dependence effects are seen at larger values than qT than expected.
- b-prescriptions have also an important impact.
- A prescription for successful matching is still missing.
- Very important to think on a NNLL calculation.
- COMPASS-like kinematics are particularly sensitive to all the issues above.
- Are we ready to study TMD-evolution from SIDIS data at not so extreme energies?
- How to overcome the technical difficulties of the b-space formulation?
- How to estimate theoretical errors?
Thanks.
Scale freezing...