Challenges in the extraction of TMDs: perturbative vs non-perturbative aspects

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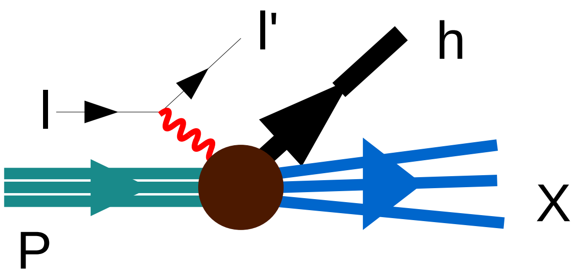
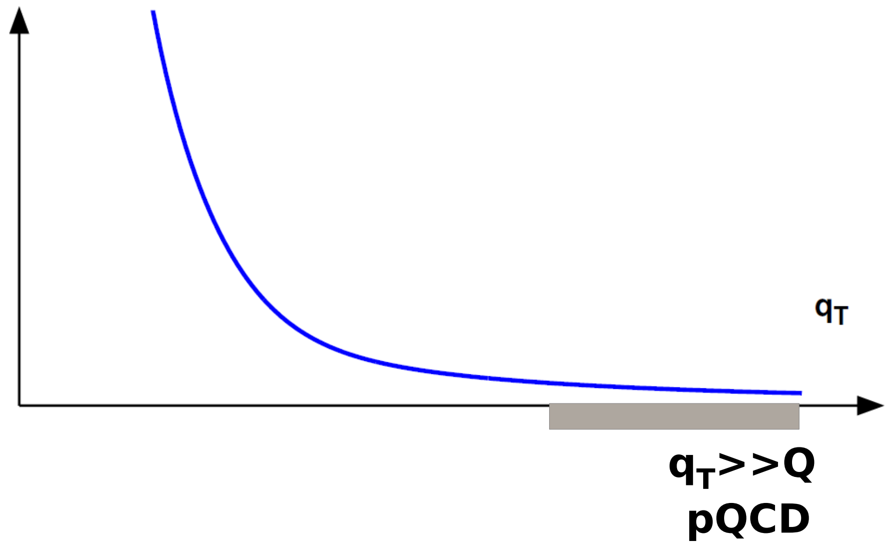
OUTLINE

- **SIDIS cross section and transverse momentum.**
- **Theory to be tested (Always try before you buy).**
- **Perturbative vs non-perturbative regime.**
- **The problem of matching.**
- **Final remarks.**

SIDIS cross section and transverse momentum



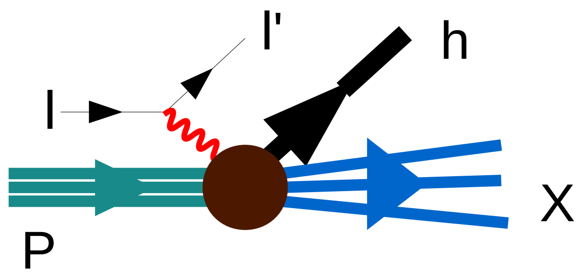


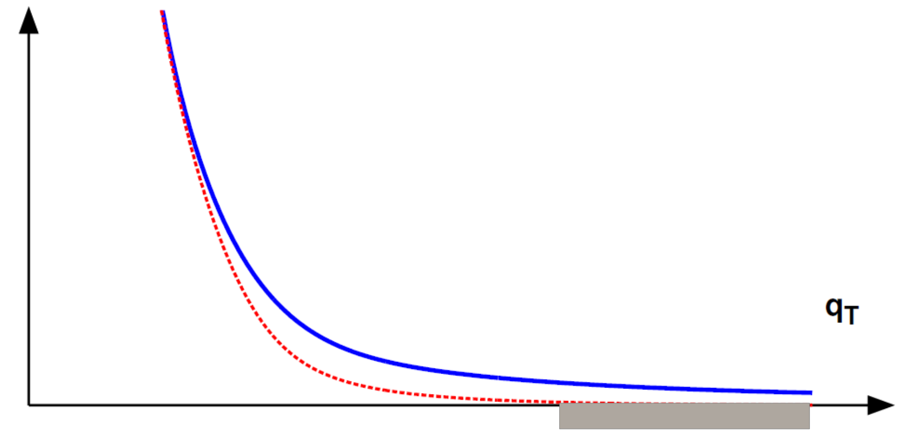
$$\frac{d^5\sigma}{dx_{bj}dQ^2dz_f dq_T^2 d\phi} = \frac{\alpha_{em}^2 \alpha_s}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \sum_k \mathcal{A}_k \int_{x_{min}}^1 \frac{dx}{x} \int_{z_{min}}^1 \frac{dz}{z} \left[f \circ D \circ \widehat{\sigma}_k \right]$$

$$\times \delta \left(\frac{q_T^2}{Q^2} - \left(\frac{1}{\widehat{x}} - 1 \right) \left(\frac{1}{\widehat{z}} - 1 \right) \right) ,$$

Koike, Nagashina, Vogelsang Nucl. Phys. **B 744** (2006)

SIDIS cross section and transverse momentum





$$\frac{d\sigma^{\mathrm{NLO}}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\mathrm{ASY}}}{dx\,dy\,dz\,dq_T^2} + Y$$

$$\sim \alpha_S^k \ln^{2k}\left(Q^2/q_T^2\right)/q_T^2$$

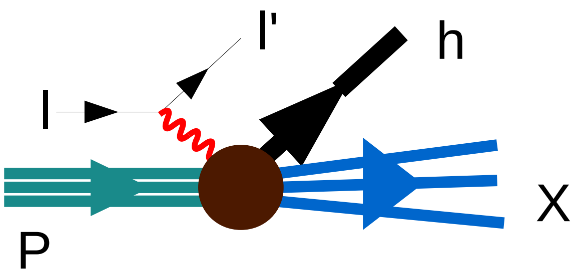
Resummation of divergent logarithms. CSS formalism.

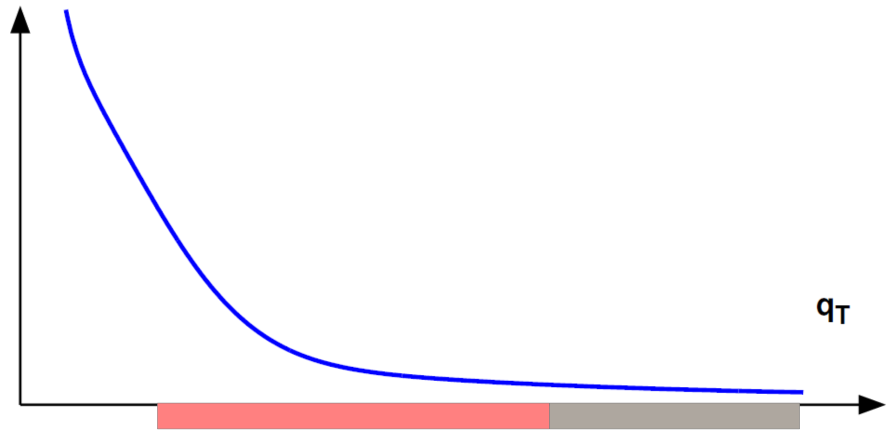
Collins, Soper, Sterman Nucl.. Phys. **B 250** (1985)

Nadolsky, Stump, Yuan Phys. Rev. **D 61** (2000)

Koike, Nagashina, Vogelsang Nucl. Phys. **B 744** (2006)

SIDIS cross section and transverse momentum



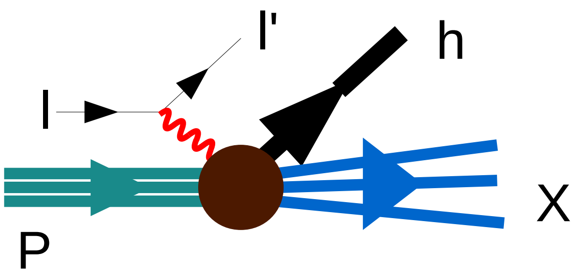


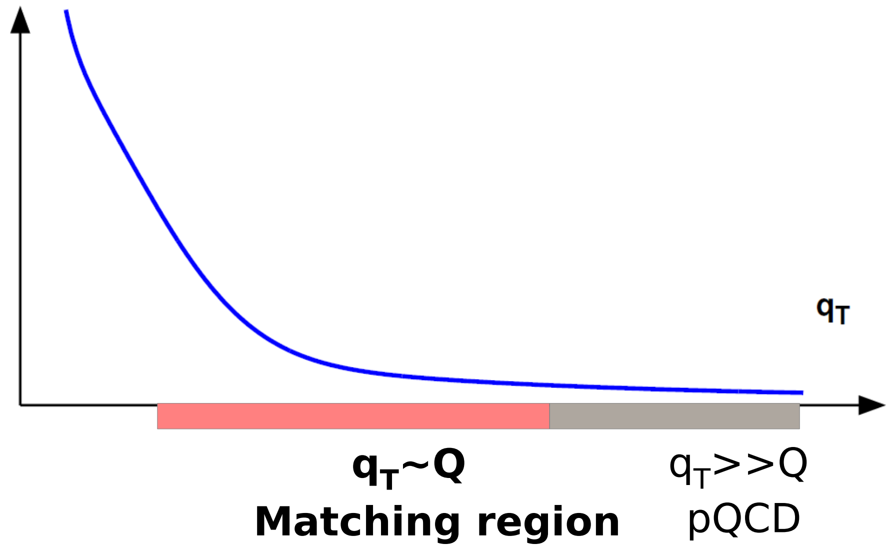
$$\frac{d\sigma^{\text{total}}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{\text{DIS}} \int \frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} W^{\text{SIDIS}}(x,z,b_T,Q) + Y^{\text{SIDIS}}(x,z,q_T,Q)$$

$$W^{\text{SIDIS}}(x,z,b_T,Q) = \exp\left[S_{\text{pert}}(b_T,Q)\right] \sum_j e_j^2 \sum_{i,k} C_{ji}^{\text{in}} \otimes f_i\left(x,\mu_b^2\right) C_{kj}^{\text{out}} \otimes D_k\left(z,\mu_b^2\right)$$

$$S_{\text{pert}}(b_T,Q) = -\int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[A(\alpha_s(\mu))\ln\left(\frac{Q^2}{\mu^2}\right) + B(\alpha_s(\mu))\right]$$

SIDIS cross section and transverse momentum





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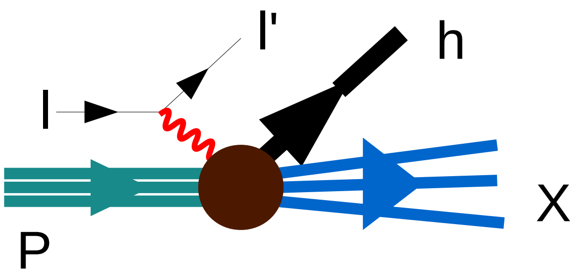
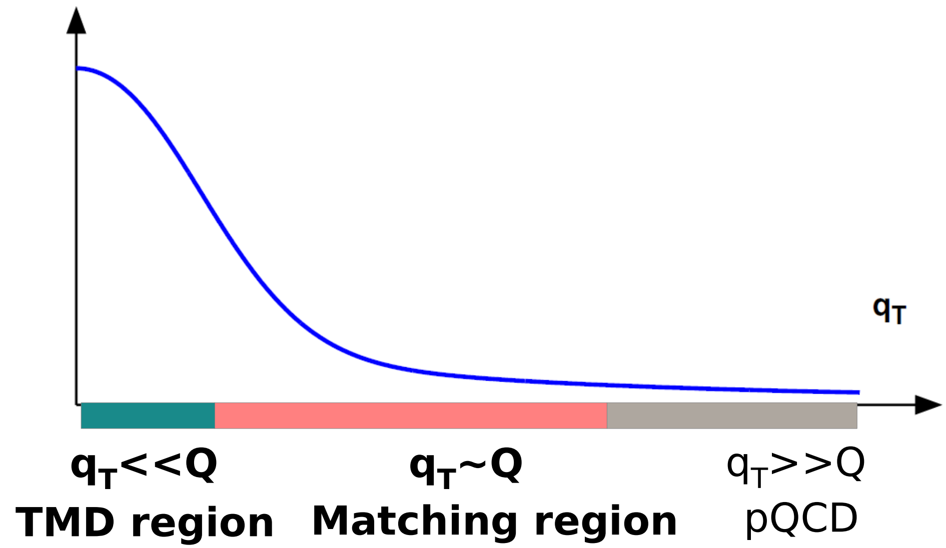
Fourier Transform spans all impact parameter space.

Must avoid Landau pole.

SIDIS cross section and transverse momentum



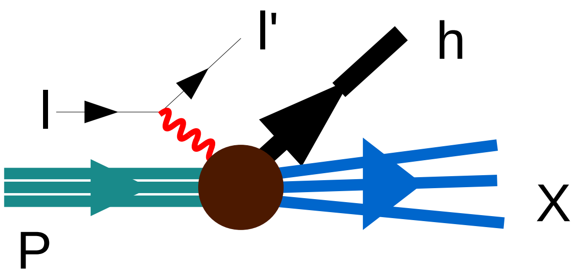


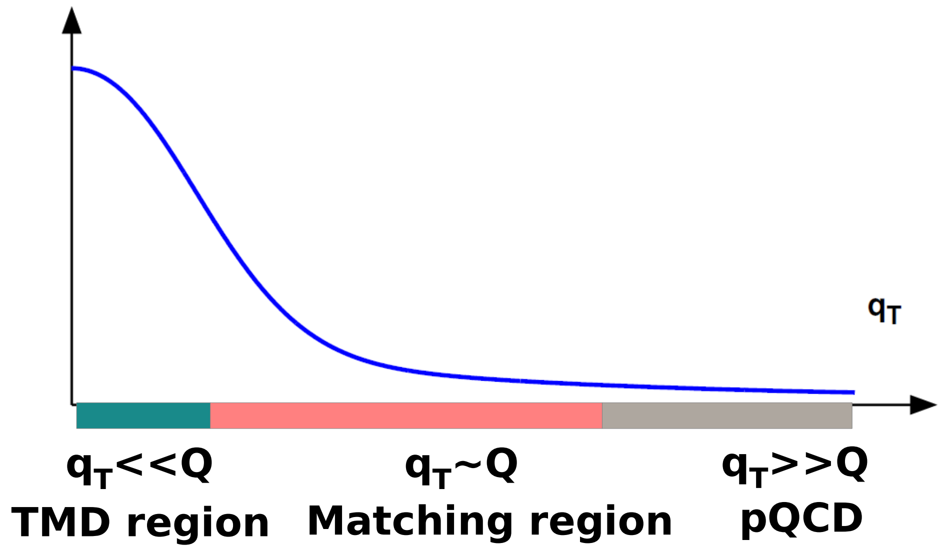
$$\frac{d\sigma^{\rm total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{\rm DIS}\int_0^\infty \frac{db_T b_T}{(2\pi)} J_0(q_T b_T) W^{\rm SIDIS}(x,z,b_*,Q)\exp\left[S_{\rm NP}(x,z,b_T,Q)\right]$$

$$+\,Y(x,z,q_T,Q)\,,$$

$$b_* = \frac{b_T}{\sqrt{1+b_T^2/b_{\rm max}^2}}$$

SIDIS cross section and transverse momentum



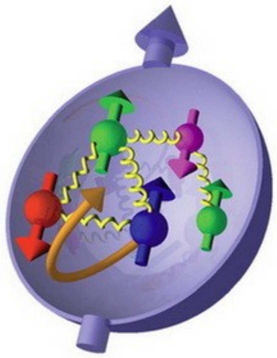


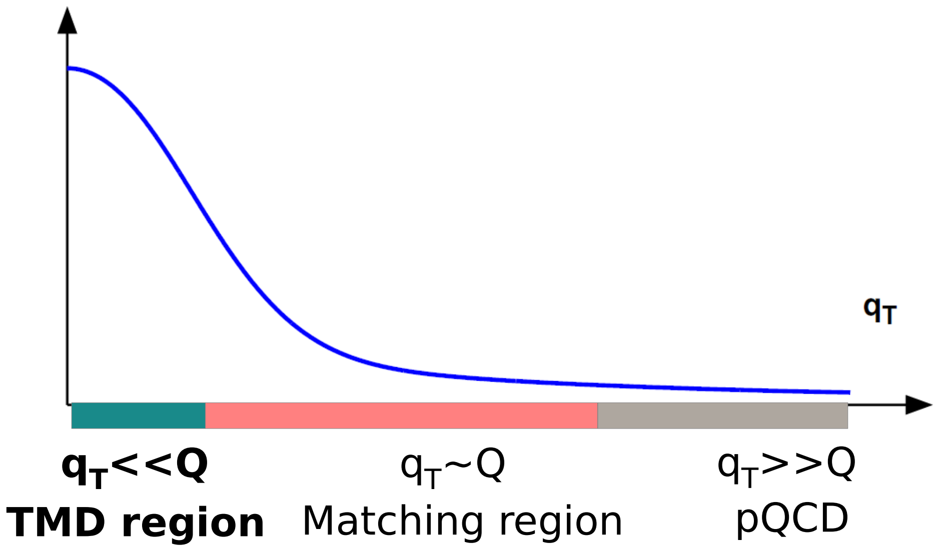
The goal is to describe data over all regions.

Theory to be tested
(Always try before you buy).

Non-perturbative

Hadron Structure



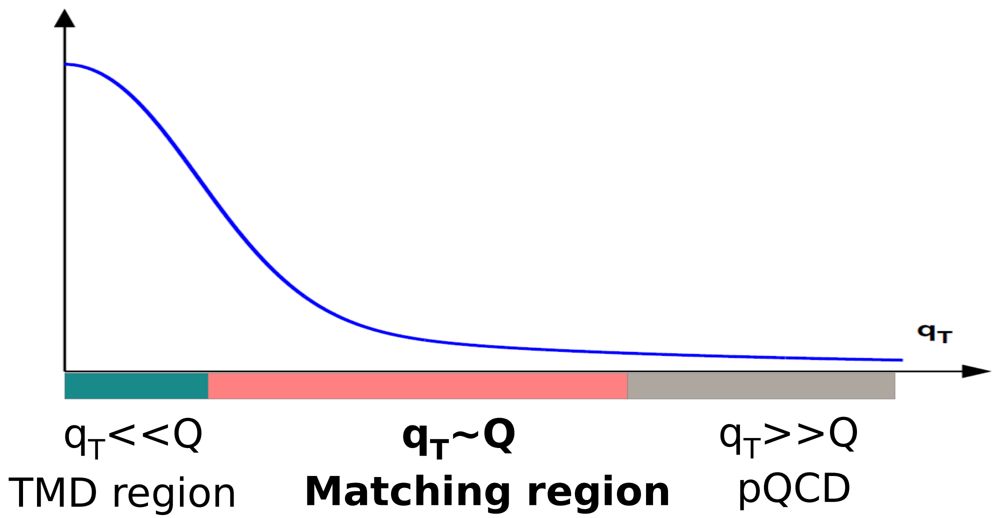


- The non-perturbative region contains information about the intrinsic structure of hadrons.
- Characterized by some Model / Prescription – dependence.
- Such dependence does not affect the pQCD region.

Y-term matching

Resummation of Large logs

$$\alpha_S^k \ln^{2k}\left(Q^2/q_T^2\right)/q_T^2$$



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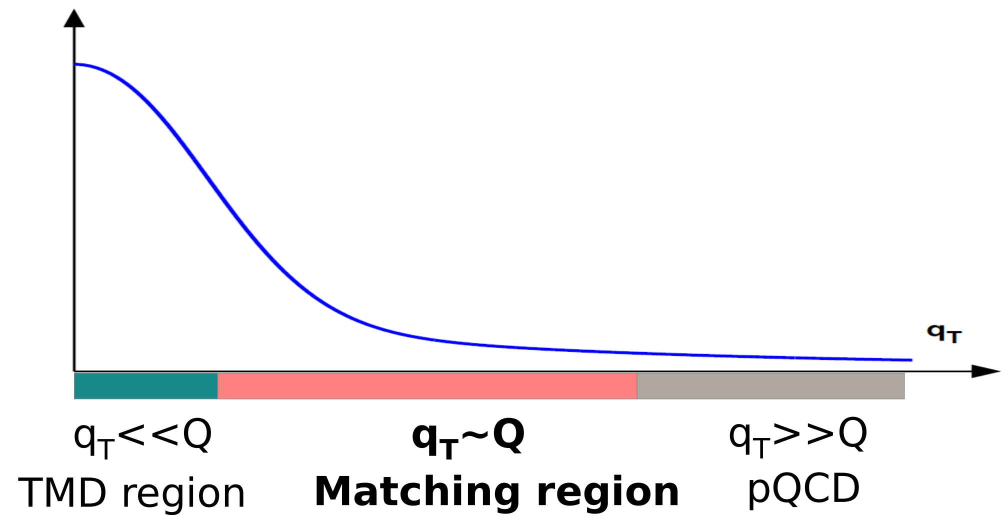


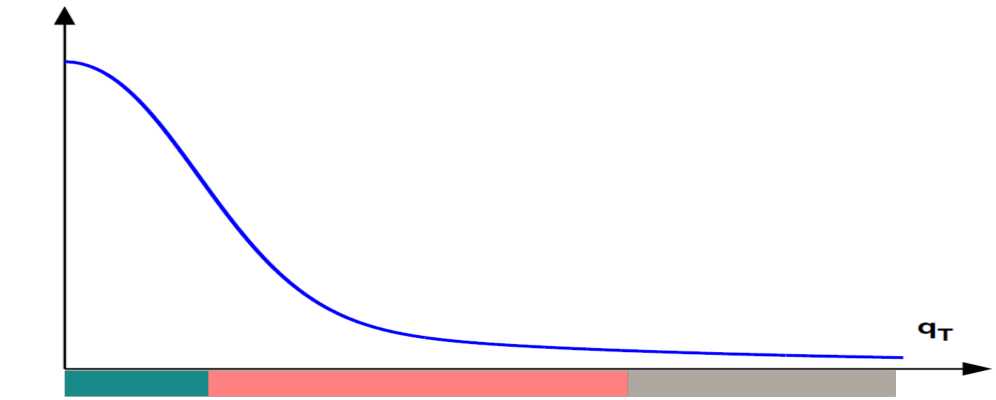
- Describes the region $\Lambda_{QCD} << qT < Q$.

- $q_T \sim Q$

$$\frac{d\sigma^{\rm NLO}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\rm ASY}}{dx\,dy\,dz\,dq_T^2} + Y \xrightarrow{\text{Resummation}} d\sigma^{\rm total} = W + Y$$

•Y-term matching

Matching to pQCD



- Describes the region $\Lambda_{QCD} << qT < Q$.

- **$q_T \sim Q \rightarrow \sigma^{res} \sim \sigma^{asy}$ (In principle, provides a matching scheme)**

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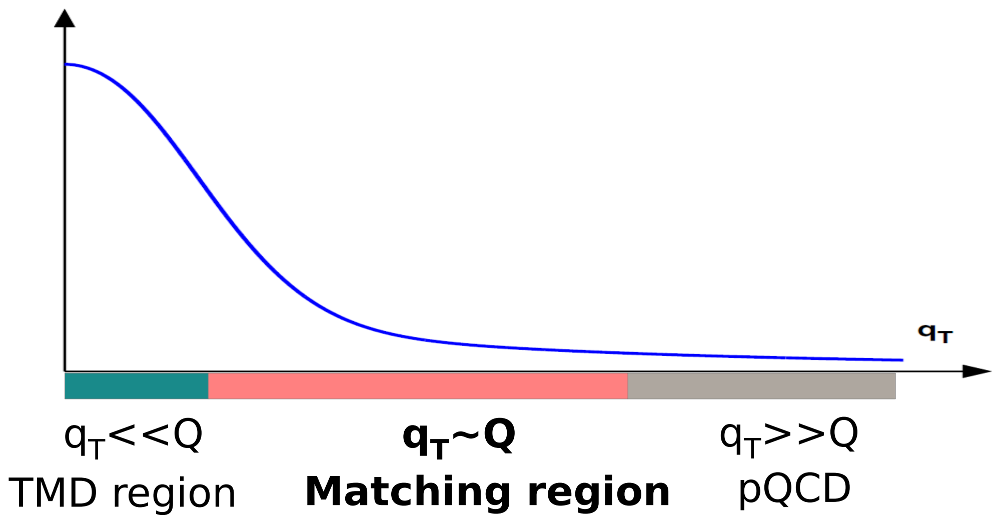

- Describes the region $\Lambda_{QCD} << qT < Q$.
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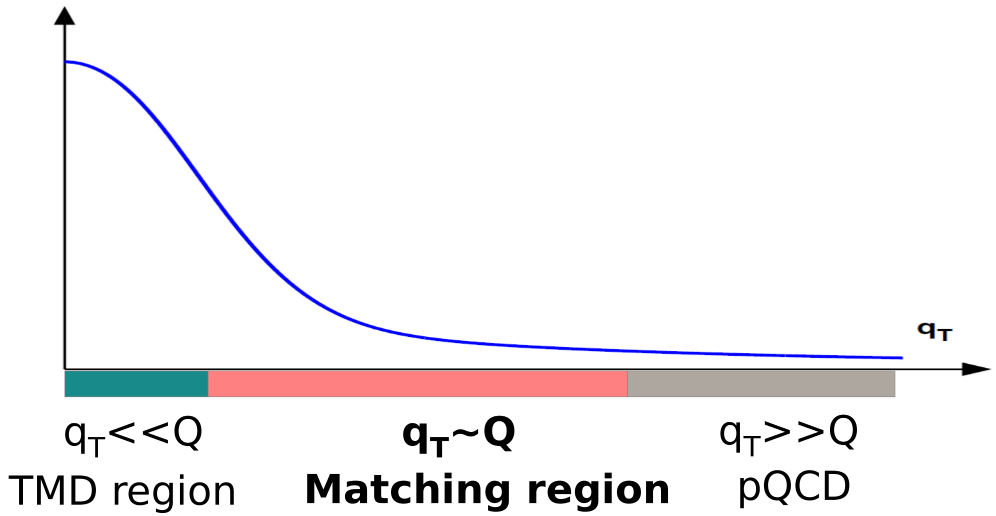
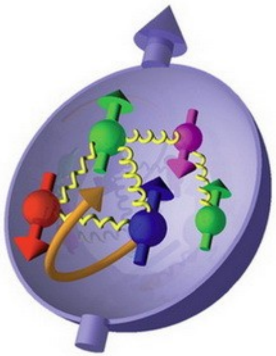
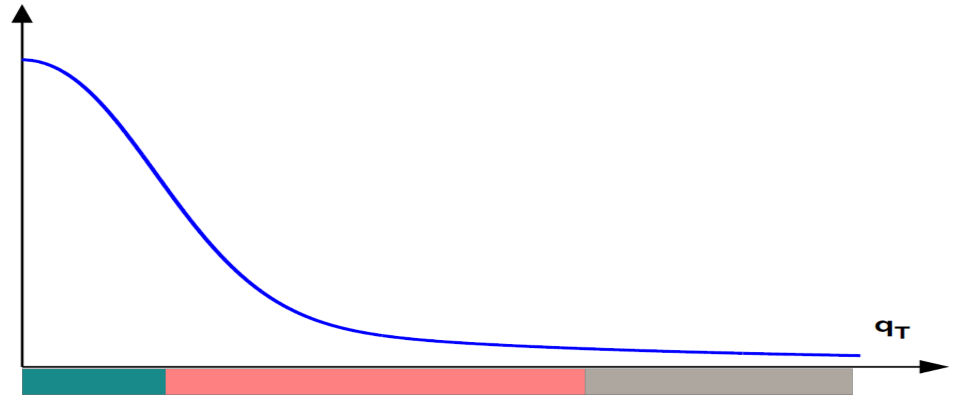


- Describes the region $\Lambda_{QCD} << qT < Q$.
- $\mathbf{q_T \sim Q \rightarrow \sigma^{res} \sim \sigma^{asy}}$ **(In principle, provides a matching scheme)**
- As $q_T \rightarrow 0$, the contribution from the Y-term is negligible.

Towards TMD extraction





- What is the role of the model-dependent part of the cross section?
- Is the b-prescription trivial to implement?
- Can the Y-term matching work?
- What data can we expect to be able to reproduce?

Perturbative vs Non-perturbative

- We studied some of these points in a recent publication

DOI:10.1007/JHEP02(2015)095

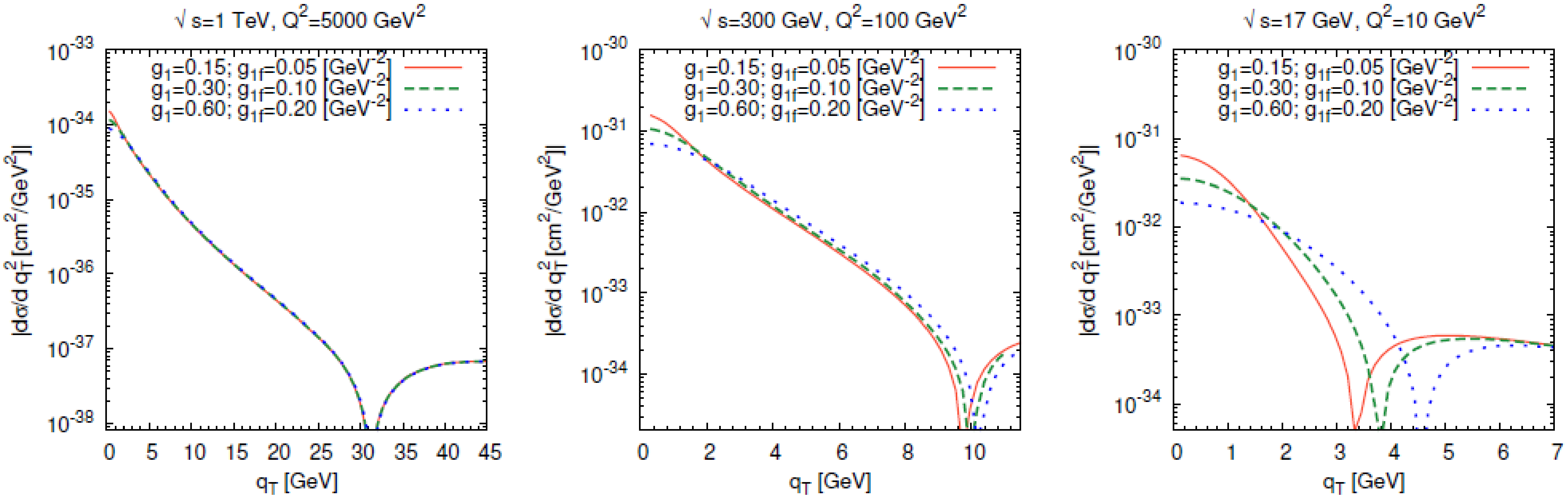
- Using the CSS formalism at next to leading log (NLL).
- The modern TMD formulation at leading order in the strong coupling, can be reduced to the NLL CSS scheme, by setting the rapidity divergence regulators $\zeta_F = \zeta_D \equiv Q^2$.
- We explored different kinematical regions: Extreme, HERA-like, COMPASS-like.

$$\frac{d\sigma^{\text{total}}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{\text{DIS}} \int_0^\infty \frac{db_T b_T}{(2\pi)} J_0(q_T b_T) W^{\text{SIDIS}}(x, z, b_*, Q) \exp\left[S_{\text{NP}}(x, z, b_T, Q)\right] + Y(x, z, q_T, Q)\,,$$

$$S_{\text{NP}} = \left(-\frac{g_1}{2} - \frac{g_{1f}}{2z^2} - g_2 \ln\left(\frac{Q}{Q_0}\right)\right) b_T^2$$

Model dependence

$$S_{\rm NP} = \left(-\frac{g_1}{2} - \frac{g_{1f}}{2z^2} - g_2 \ln\left(\frac{Q}{Q_0}\right)\right) b_T^2$$



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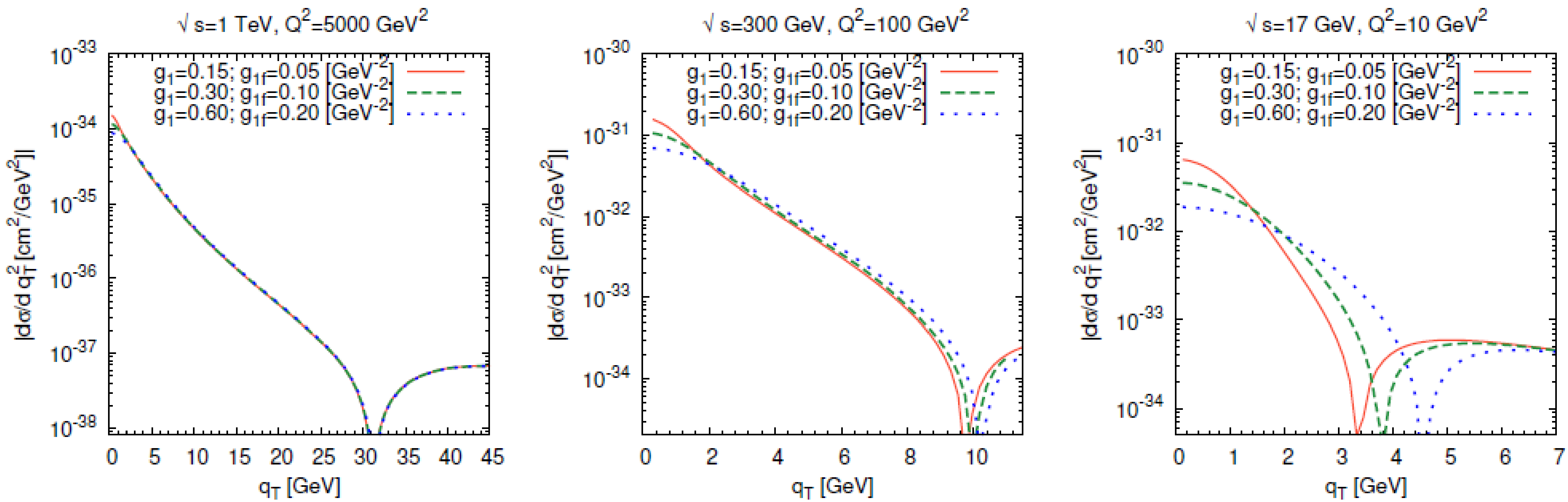
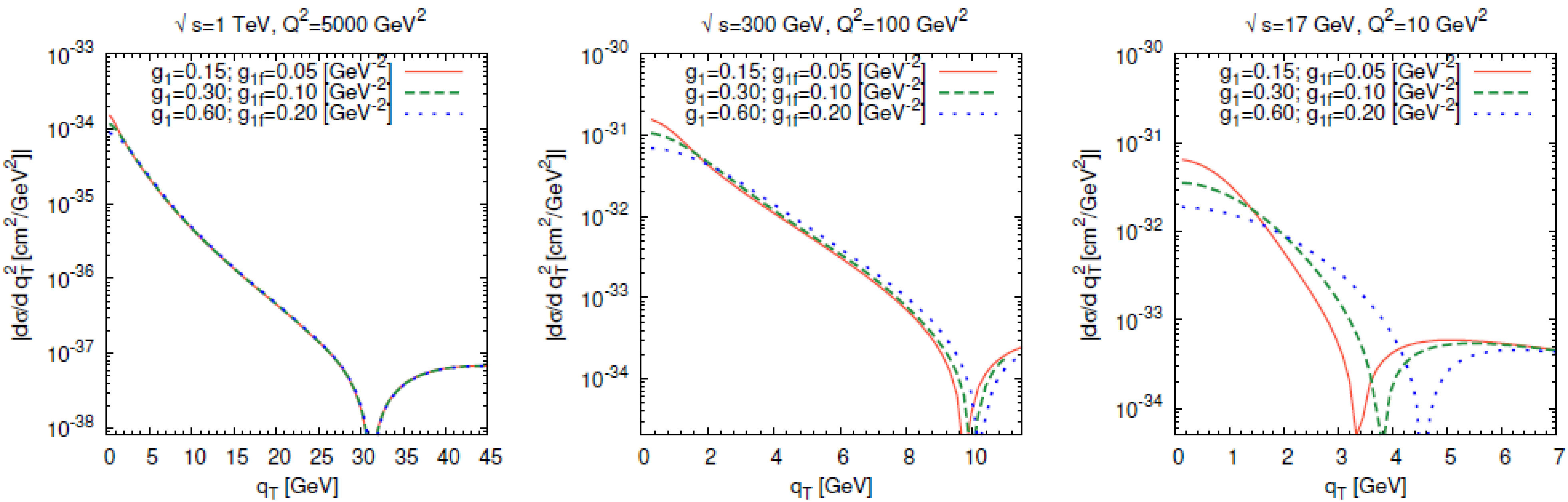


Extreme kinematics.
Non-perturbative part
has no effect at large qT

Model dependence

$$S_{\mathrm{NP}} = \left(-\frac{g_1}{2} - \frac{g_{1f}}{2z^2} - g_2 \ln\left(\frac{Q}{Q_0}\right)\right) b_T^2$$



Extreme kinematics.
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HERA-like kinematics.
Mild model dependence
From non-perturbative part

Model dependence

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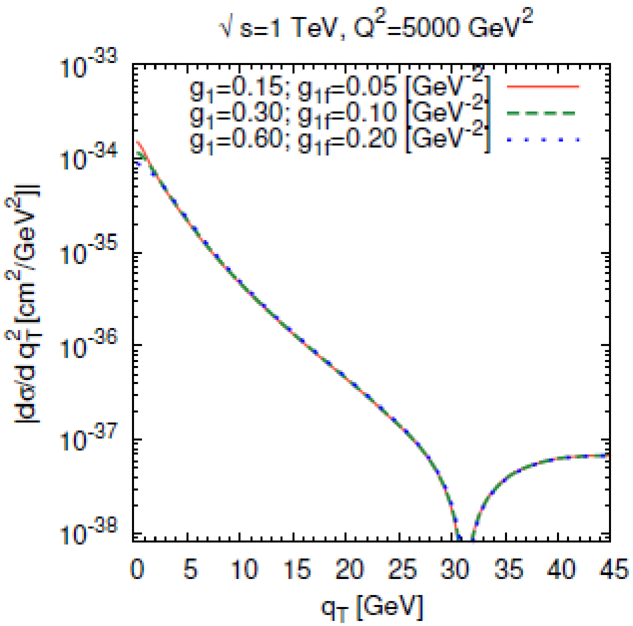
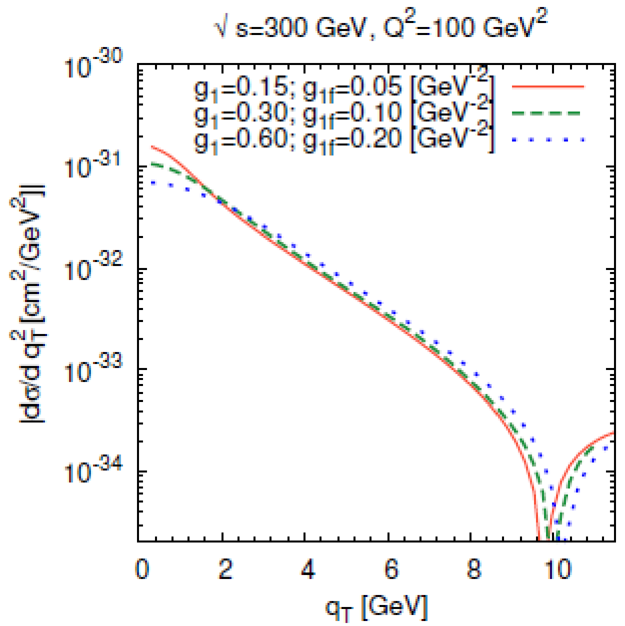
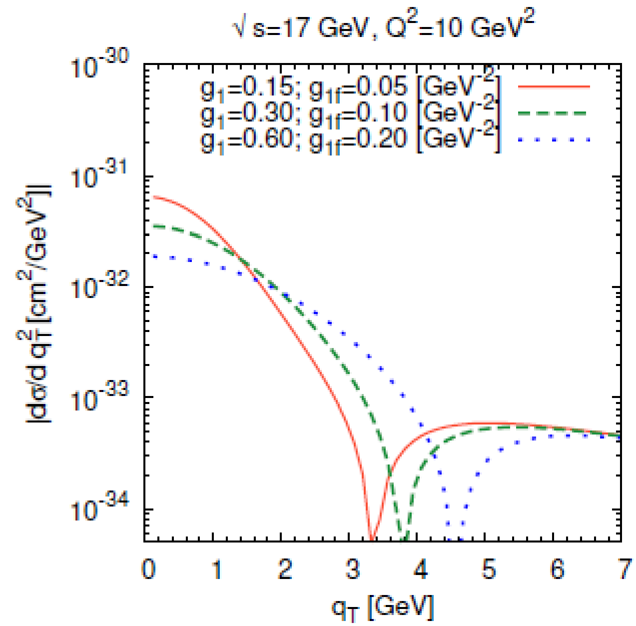






Extreme kinematics.
Non-perturbative part
has no effect at large qT

HERA-like kinematics.
Mild model dependence
From non-perturbative part

COMPASS-like kinematics.
Strong dependence on Model parameters!!!

b-prescription

Recall...

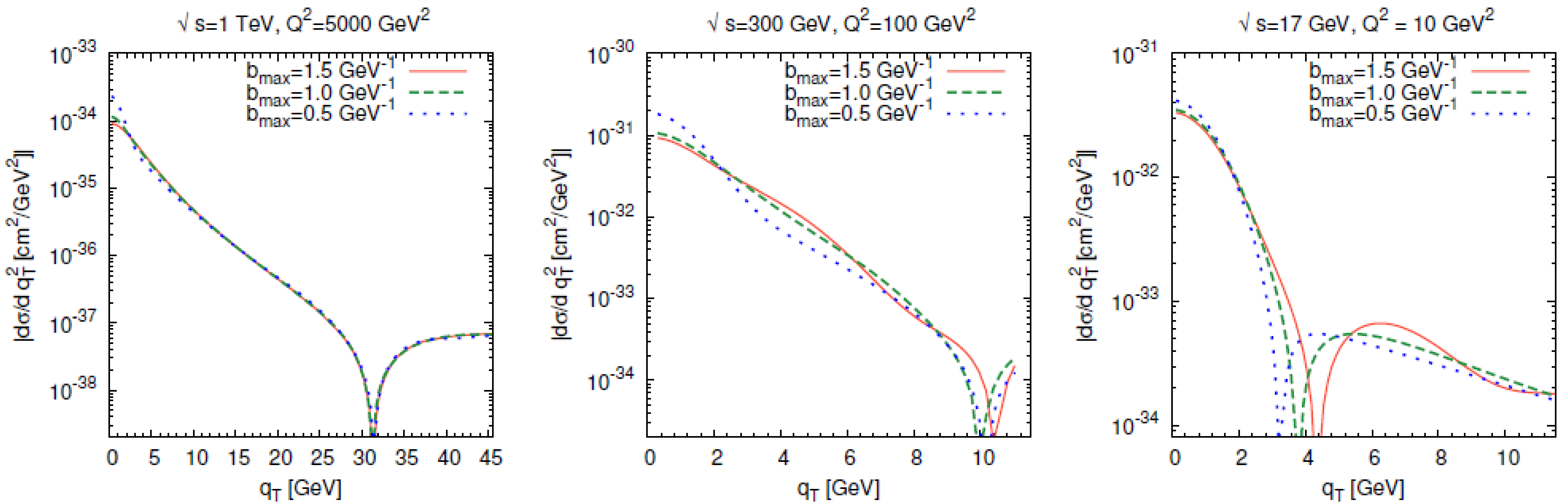
$$b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

$$\frac{d\sigma^{\rm total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{\rm DIS}\int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2}W^{\rm SIDIS}(x,z,b_T,Q) + Y^{\rm SIDIS}(x,z,q_T,Q)$$

How does the prescription to avoid the Landau pole affect calculations?

b-prescription

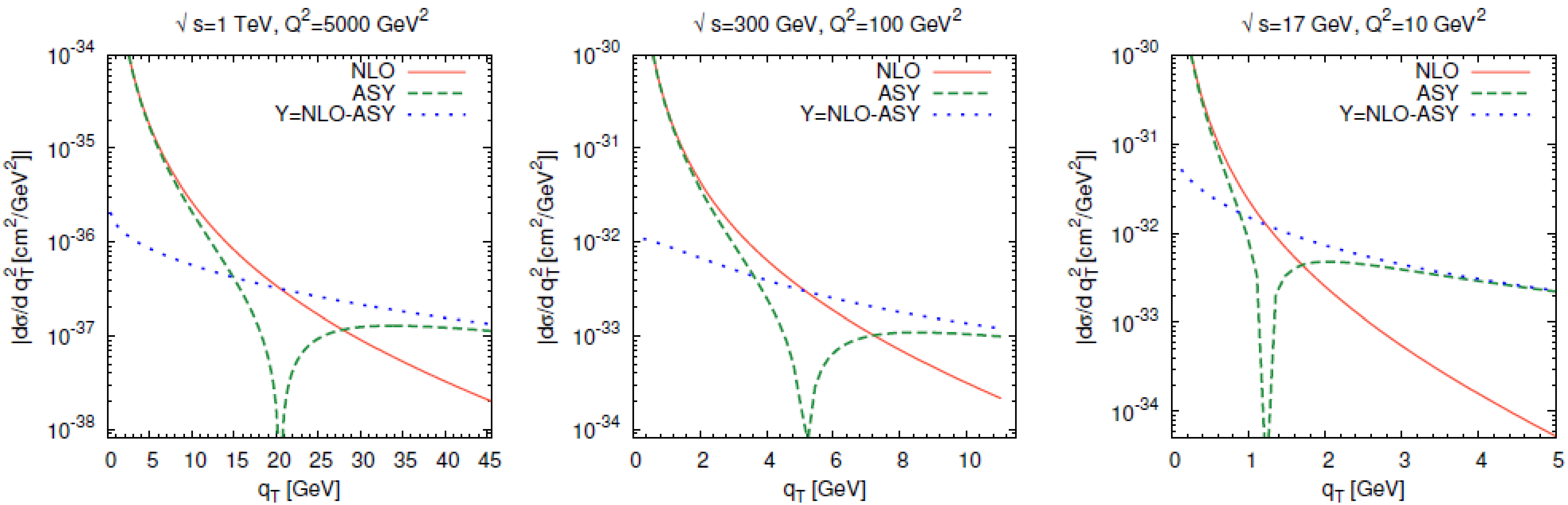
$$b_* = \frac{b_T}{\sqrt{1 + b_T^2/b_{\max}^2}}$$



How does the prescription to avoid the Landau pole affect calculations?

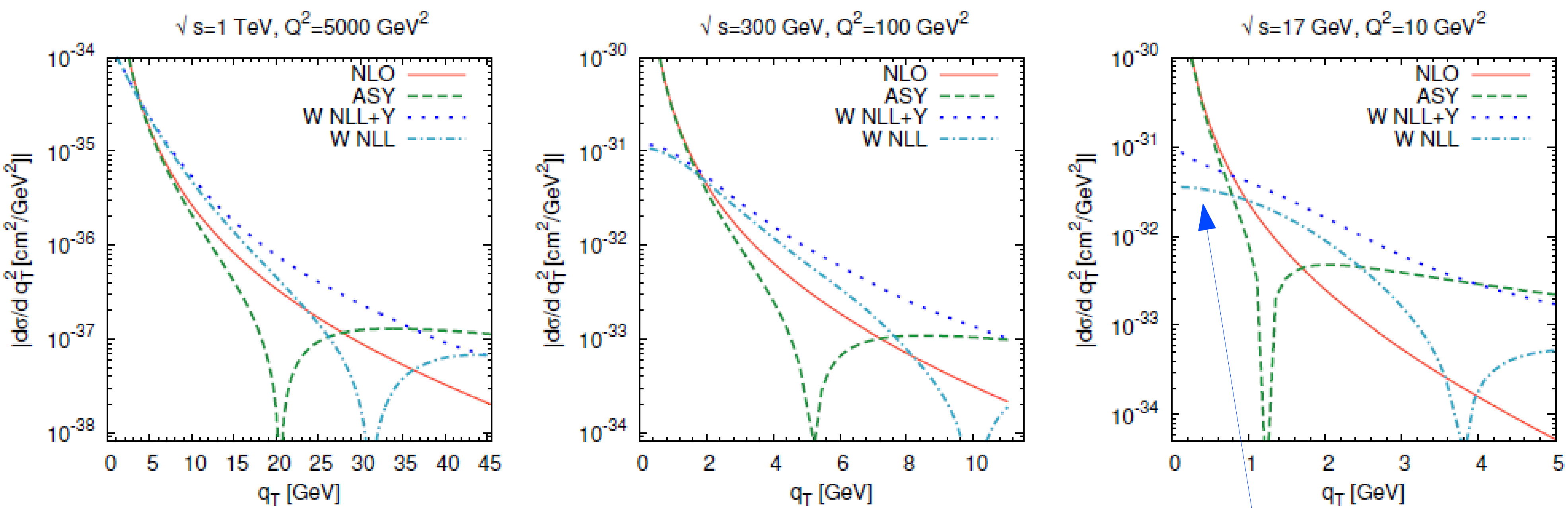
One has a similar effect as for the model parameters.

The Y-term



$$\frac{d\sigma^{\mathrm{NLO}}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\mathrm{ASY}}}{dx\,dy\,dz\,dq_T^2} + Y$$

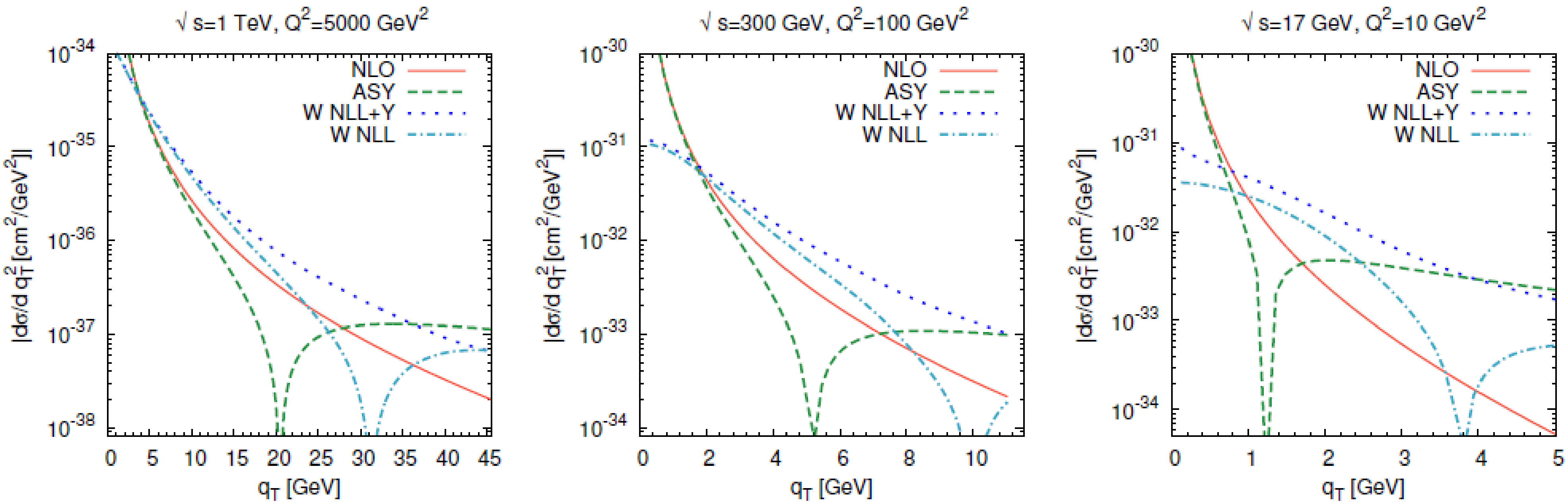
The Y-term



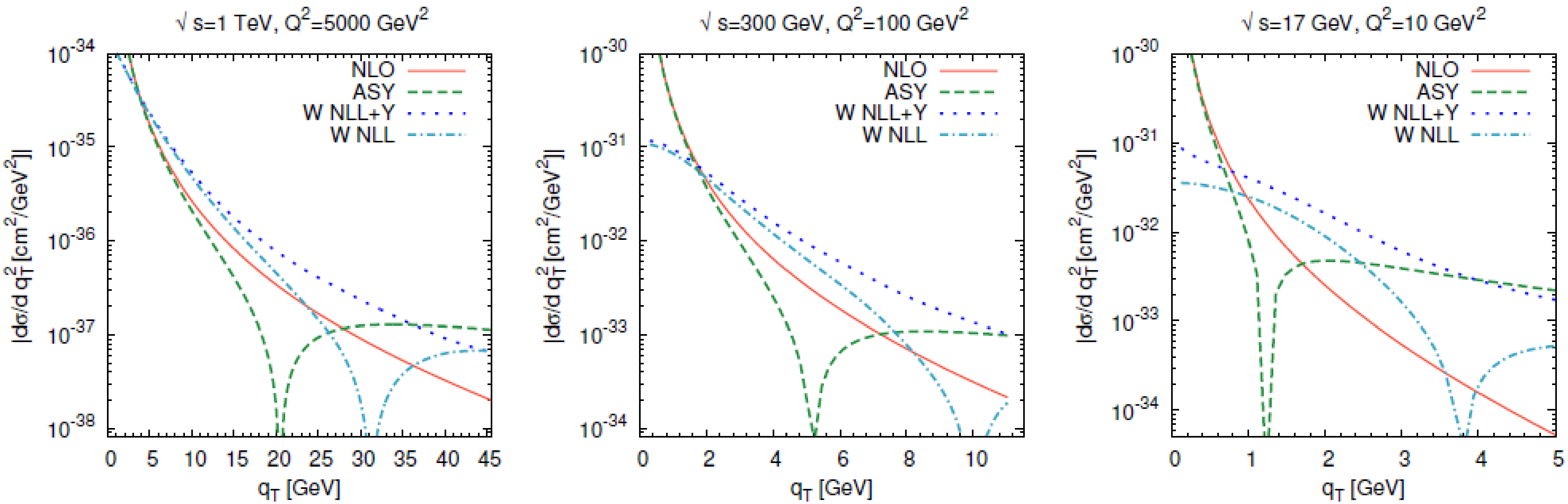
$$\frac{d\sigma^{\rm NLO}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\rm ASY}}{dx\,dy\,dz\,dq_T^2} + Y$$

sizable contribution from Y term

The problem of Matching



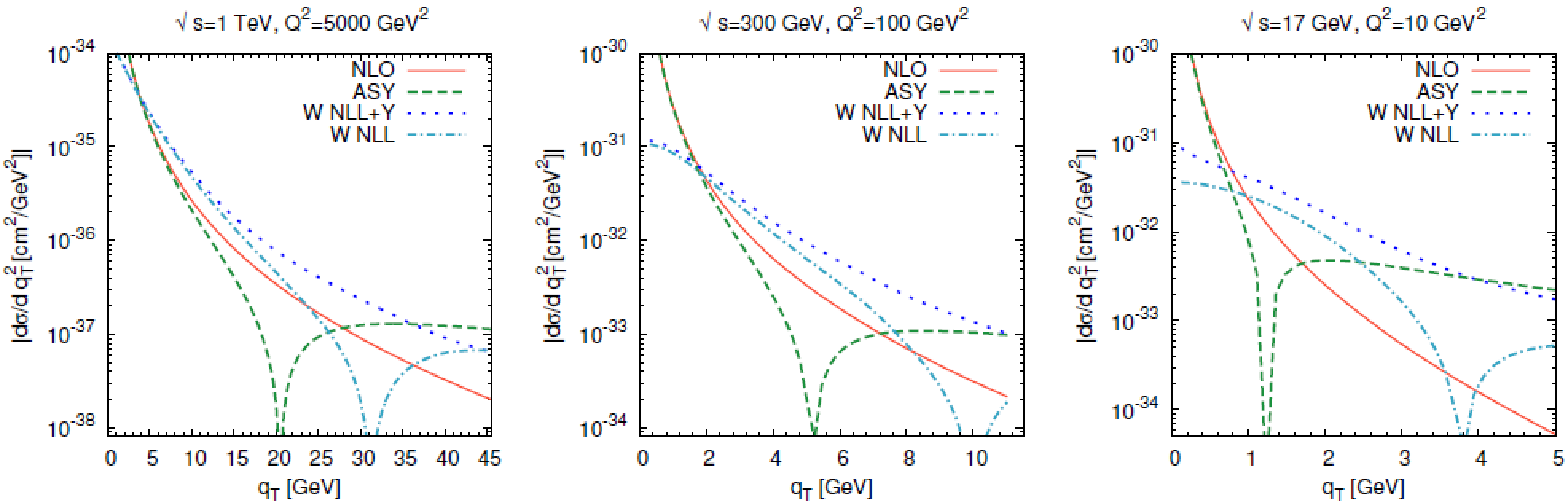
The problem of Matching



Assertion $q_T \sim Q \quad \rightarrow \quad \sigma^{res} \sim \sigma^{asy}$ fails.

No matching seem possible (NLL)

The problem of Matching

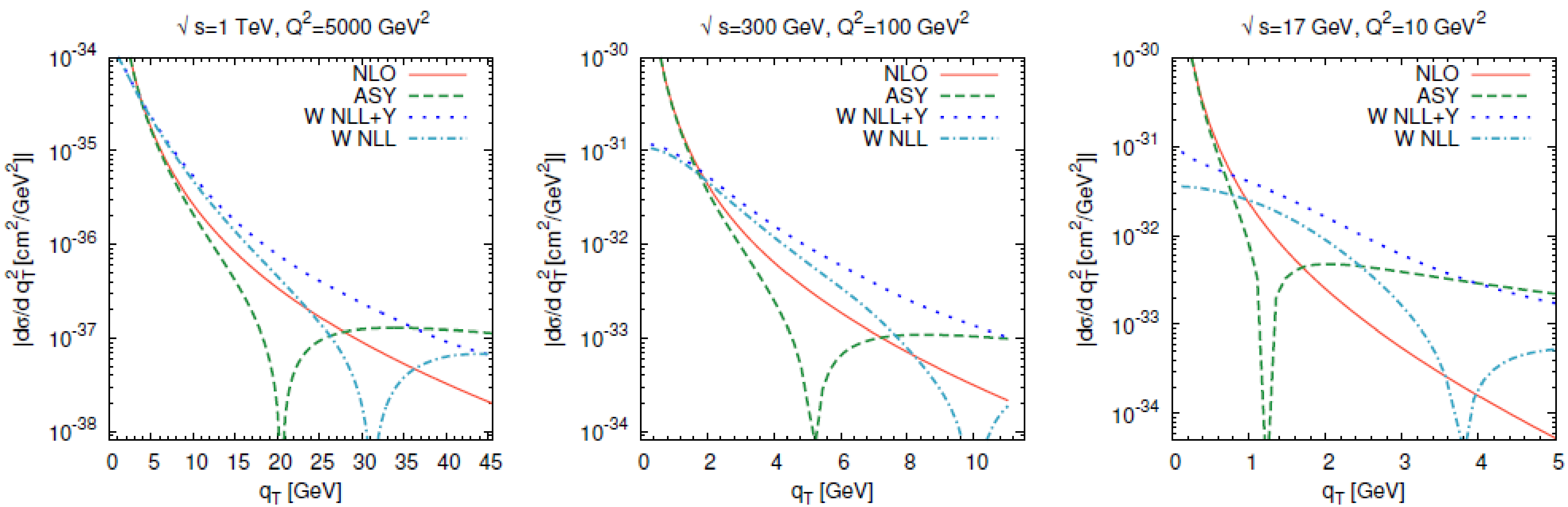


Several factors: - effect of non-perturbative function

- prescription to avoid Landau pole

- Behaviour of perturbative Sudakov

The problem of Matching



Several factors: - **effect of non-perturbative function**

- prescription to avoid Landau pole

- Behaviour of perturbative Sudakov

Lets try another way...

Consider the NLL resummed cross section and approximate at order α_s: W^{FXO}

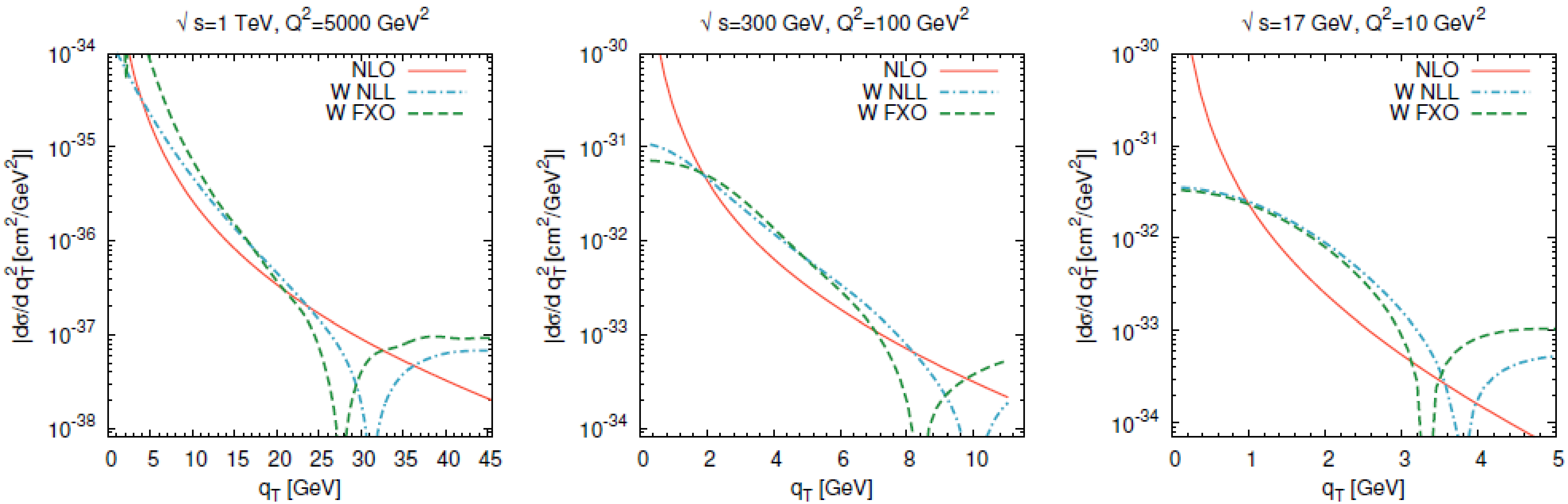
If a region exists where $W^{\mathrm{FXO}} \simeq W^{\mathrm{NLL}}$, then the quantity

$$d\sigma^{\mathrm{total}} = W^{\mathrm{NLL}} - W^{\mathrm{FXO}} + d\sigma^{\mathrm{NLO}}$$

May be used to match the perturbative calculation.

Here, W^{FXO} contains the same non-perturbative function as W^{NLL}

FXO matching



No matching seems possible, in general.

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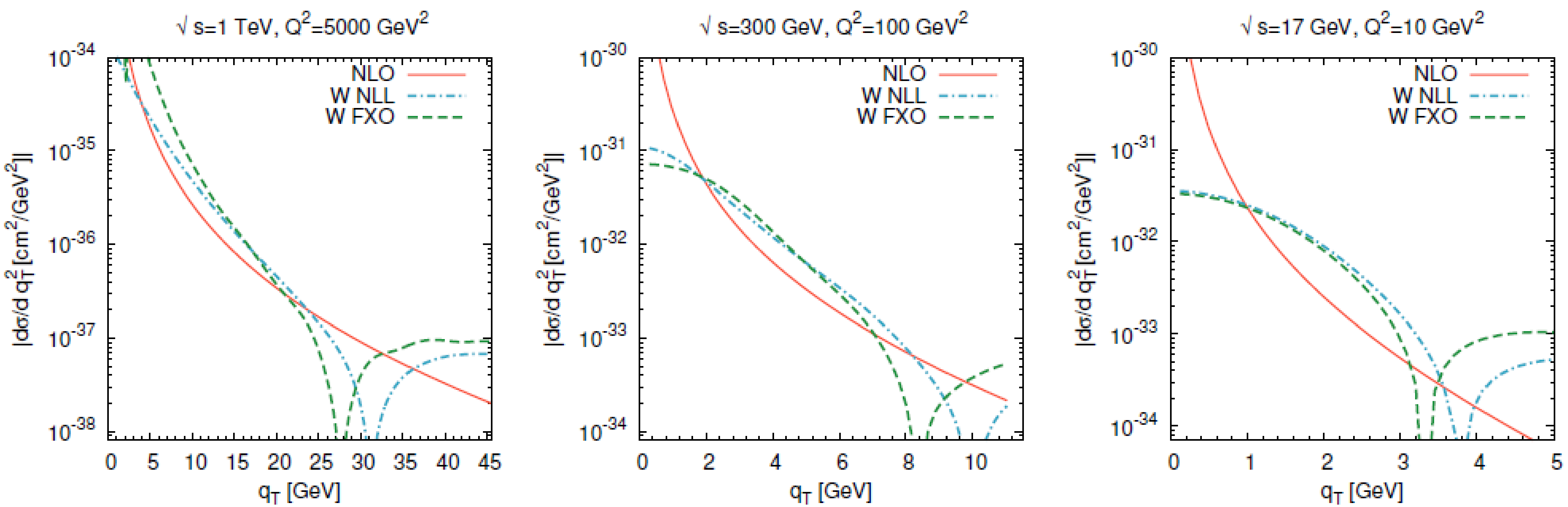
The problem of Matching



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Several factors: - effect of non-perturbative function

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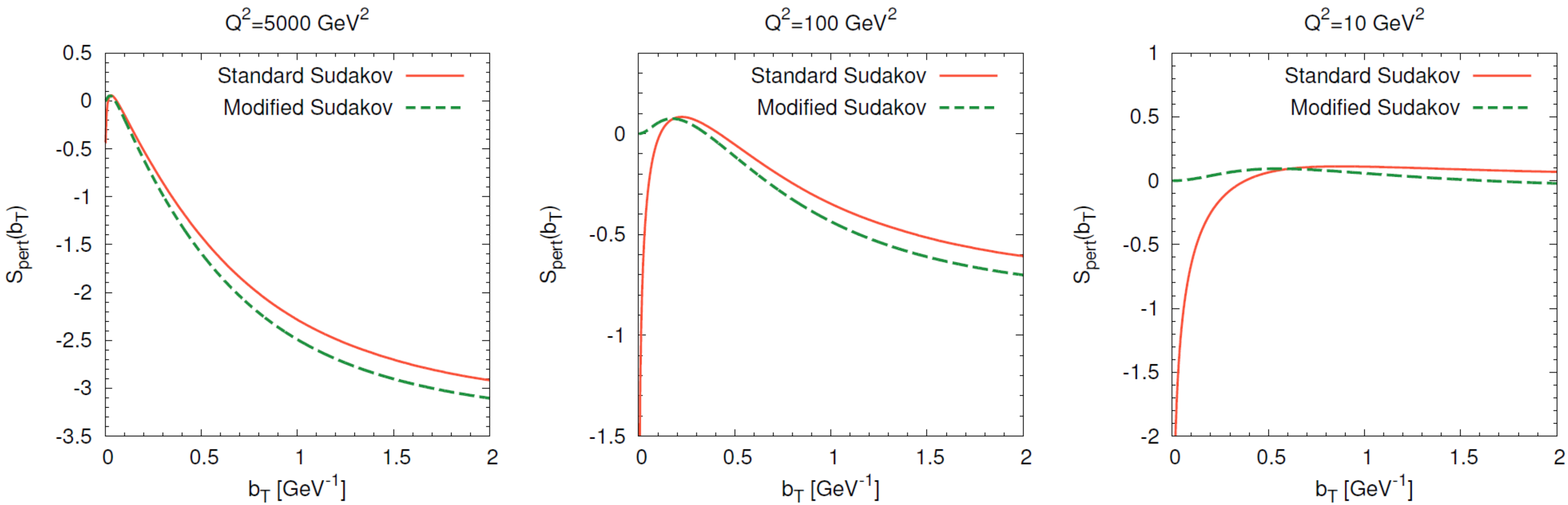
- **Behaviour of perturbative Sudakov**

The problem of Matching



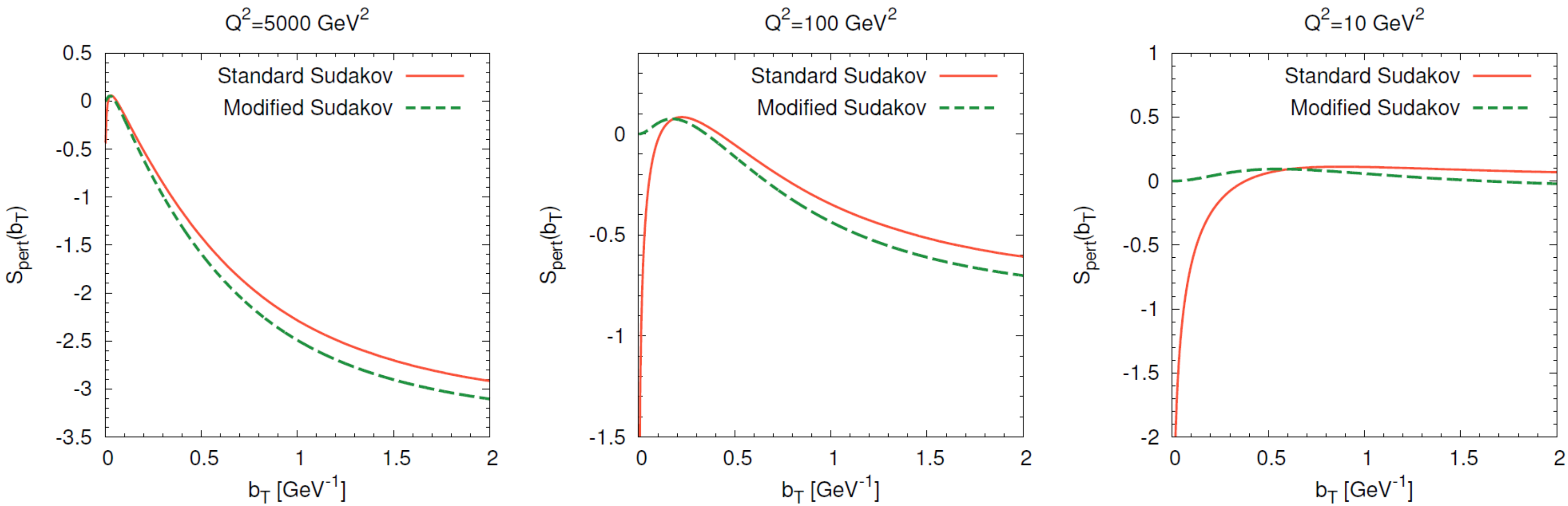
Sudakov factor becomes negative as $b_T \to 0$

Alternative prescription to ensure $S \to 0 \text{ at } b_T \to 0$

$$\log\left(Q^2/\mu_b^2\right) \to \log\left(1 + Q^2/\mu_b^2\right)$$

Koike, Nagashina, Vogelsang Nucl. Phys. **B 744** (2006)

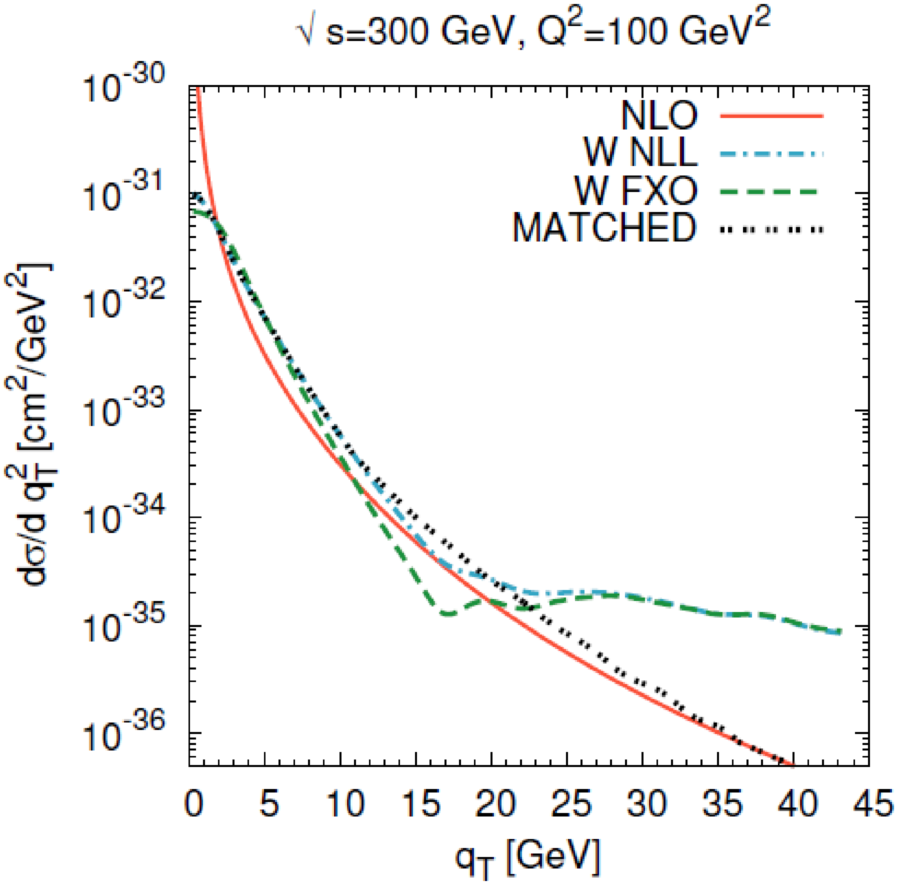
The problem of Matching



At both HERA and COMPASS-like kinematics Sudakov “recipees” render distinct results.

In the COMPASS case, the modified Sudakov is almost negligible.

A matching, rather by chance.
(a.k.a. Always say goodbye on a happy note)



$$d\sigma^{\text{total}} = W^{\text{NLL}} - W^{\text{FXO}} + d\sigma^{\text{NLO}}$$

Final Remarks

- The different regions of qT do not seem to be well defined in some kinematics.
- Model dependence effects are seen at larger values than qT than expected.
- b-prescriptions have also an important impact.
- A prescription for successful matching is still missing.
- Very important to think on a NNLL calculation.
- COMPASS-like kinematics are particularly sensitive to all the issues above.
- Are we ready to study TMD-evolution from SIDIS data at not so extreme energies?
- How to overcome the technical difficulties of the b-space formulation?
- How to estimate theoretical errors ?

Thanks.

Scale freezing...