JIMWLK evolution at NLO

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arXiv:1310.0378 (PRD), arXiv:1401.0374 (JHEP), arXiv:1405.0418 (JHEP)

Inspired by Ian Balitsky

High Energy Scattering

Target (ρ^t) **Projectile (ρ^p)**

$$\langle \mathbf{T}| \quad \rightarrow \qquad\qquad \leftarrow \quad |\mathbf{P}\rangle$$

S-matrix:

$$\mathbf{S}(\mathbf{Y}) \;=\; \langle \mathbf{T}\,\langle\,\mathbf{P}|\;\; \hat{\mathbf{S}}(\rho^{\mathrm{t}},\,\rho^{\mathrm{p}})\;\; |\mathbf{P}\rangle\,\mathbf{T}\rangle$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathrm{t}},\,\rho^{\mathrm{p}})$

$$\langle\hat{\mathcal{O}}\rangle_{\mathbf{Y}} \;=\; \langle \mathbf{T}\,\langle\,\mathbf{P}|\;\; \hat{\mathcal{O}}(\rho^{\mathrm{t}},\,\rho^{\mathrm{p}})\;\; |\mathbf{P}\rangle\,\mathbf{T}\rangle$$

The question we pose is how these averages change with increase in energy of the process

Projectile averaged operators:

$$\langle\, \mathbf{P} |\ \hat{\mathcal{O}}(\rho^{\mathrm{t}},\ \rho^{\mathrm{p}})\ |\mathbf{P}\rangle \ = \ \int \mathbf{D}\rho^{\mathrm{p}}\ \ \hat{\mathcal{O}}(\rho^{\mathrm{t}},\ \rho^{\mathrm{p}})\ \ \mathbf{W}^{\mathbf{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

evolve with rapidity as $\mathbf{H}\ \rightarrow$ the HE effective Hamiltonian

$$\frac{\mathbf{d}\langle \mathbf{P}|\hat{\mathcal{O}}|\mathbf{P}\rangle}{\mathbf{dY}} \ = \ -\int \mathbf{D}\rho^{\mathrm{p}}\ \ \ \hat{\mathcal{O}}(\rho^{\mathrm{t}},\ \rho^{\mathrm{p}})\ \ \mathbf{H}[\rho^{\mathrm{p}},\ \delta/\delta\rho^{\mathrm{p}}]\ \ \ \mathbf{W}^{\mathbf{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

or in other words

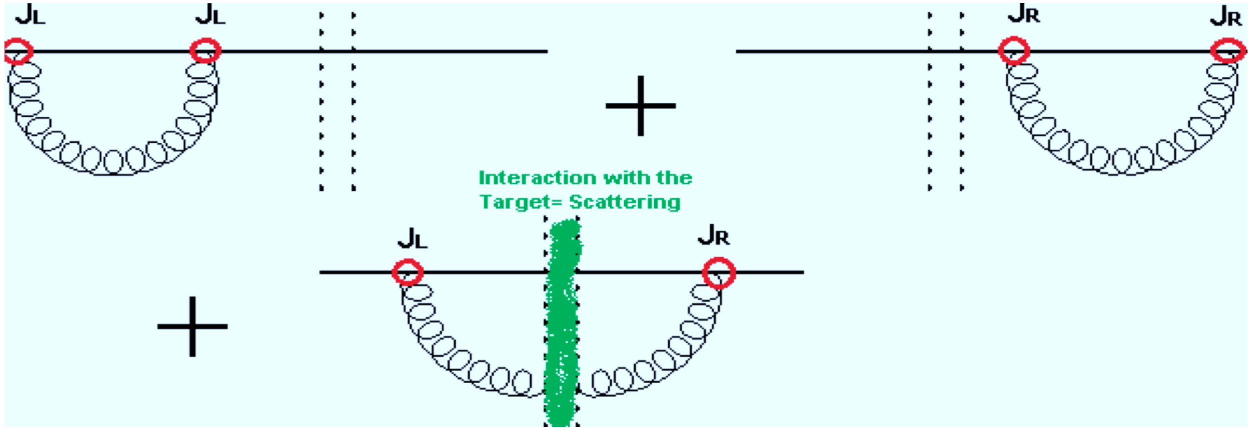
$$\frac{\mathbf{dW}^{\mathbf{p}}}{\mathbf{dY}} \ = \ -\,\mathbf{H}\ \ \mathbf{W}^{\mathbf{p}}$$

Spectrum of $\mathbf{H}$ defines the energy dependence of the average.

JIMWLK Hamiltonian

The JIMWLK Hamiltonian is a limit of $\mathbf{H}$ for dilute partonic system ($\rho_p \to 0$) which scatters on a dense target. It accounts for linear gluon emission + multiple rescatterings.

$$H^{JIMWLK} = \frac{\alpha_s}{2\pi^2} \int_{x,y,z} \frac{(x-z)_i(y-z)_i}{(x-z)^2(y-z)^2} \left\{ J_L^a(x) J_L^a(y) + J_R^a(x) J_R^a(y) - 2 J_L^a(x) S_A^{ab}(z) J_R^b(y) \right\}$$



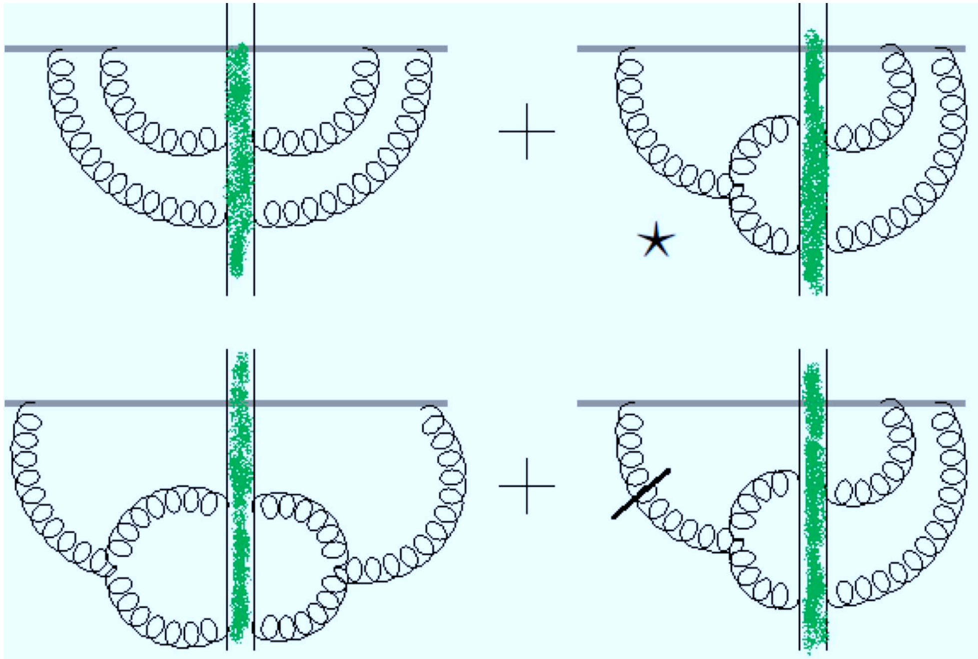
$$\mathbf{S_A^{cd}(z) \;=\; \mathcal{P}\,\exp\left\{ i \int dx^+\, T^a\, \alpha_t^a(z, x^+) \right\}^{cd}} . \qquad "\boldsymbol{\Delta}"\,\alpha_t \;=\; \rho_t \qquad \mathbf{(YM)}$$

The left and right SU(N) generators:

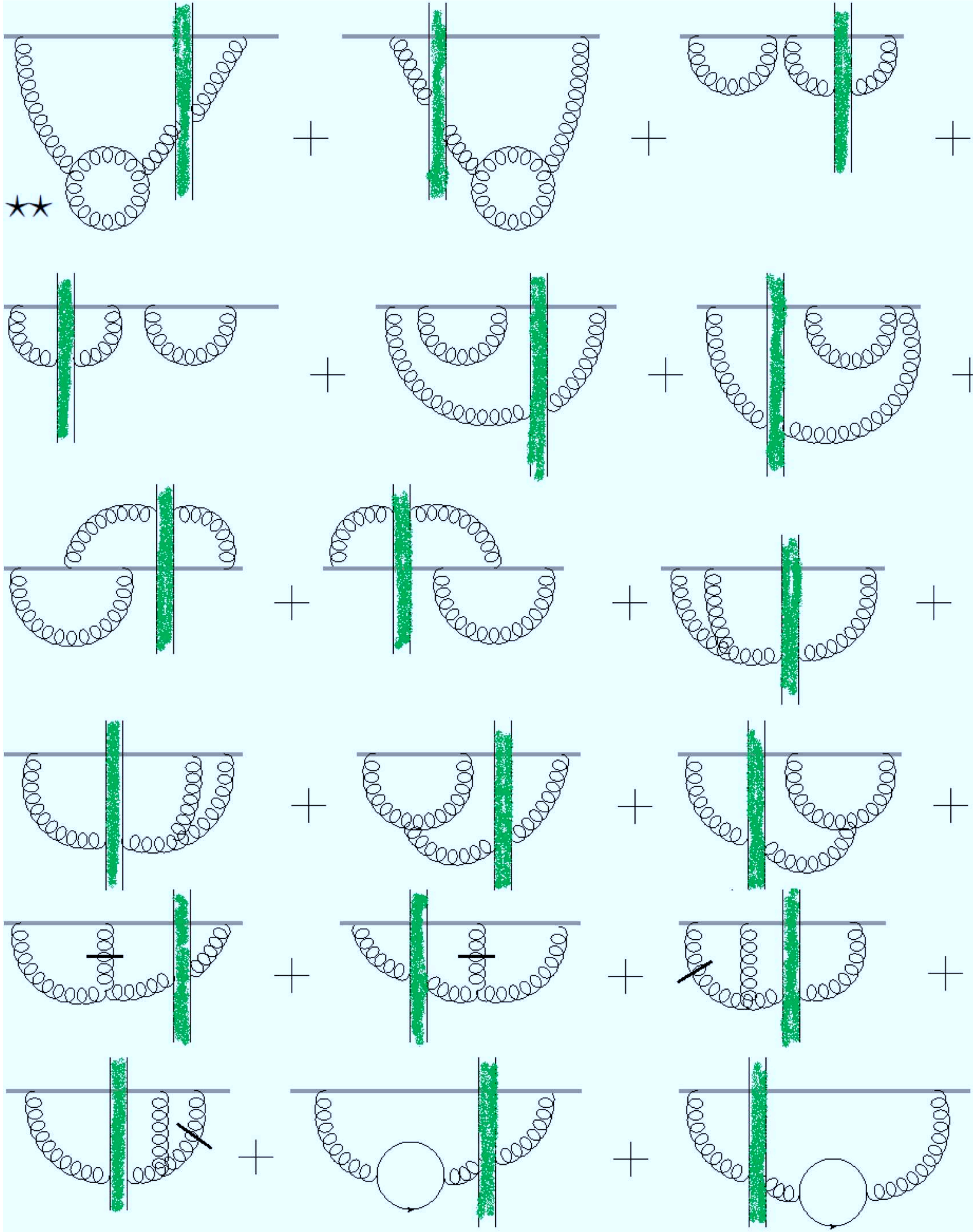
$$\mathbf{J_L^a(x) S_A^{ij}(z) = (T^a S_A(z))^{ij}\, \delta^2(x-z)} \qquad \mathbf{J_R^a(x) S_A^{ij}(z) = (S_A(z) T^a)^{ij}\, \delta^2(x-z)}$$

Towards JIMWLK Hamiltonian @ NLO

Some 30 diagrams of the kind:



Symmetries: $\mathbf{SU_L(N) \times SU_R(N)}$ **CPT,** **Unitarity**



JIMWLK Hamiltonian @ NLO

$$H^{NLO\ JIMWLK} = \int_{x,y,z} K_{JSJ}(x,y;z)\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z')\left[f^{abc}f^{def}J_L^a(x)S_A^{be}(z)S_A^{cf}(z')J_R^d(y) - N_c J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$
$$+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\left[2\,J_L^a(x)\,tr[S^\dagger(z)\,t^a\,S(z')t^b]\,J_R^b(y) \;-\; J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right]$$
$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\Big[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w)$$
$$- J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y)\Big]$$
$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\left[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w) - J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y)\right]$$
$$+ \int_{w,x,y} K_{JJJ}(w;x,y)f^{deb}\left[J_L^d(x)\,J_L^e(y)\,J_L^b(w) \;-\; J_R^d(x)\,J_R^e(y)\,J_R^b(w)\right].$$

Shortcuts to the Kernels

Step 1: Compute evolution of 3-quark Wilson loop operator in SU(3) (baryon)

$$\mathbf{B(u,v,w) = \epsilon^{ijk}\epsilon^{lmn} S_F^{il}(u) S_F^{jm}(v) S_F^{kn}(w)}$$

$$\partial_{\mathbf{Y}} \mathbf{B(u,v,w) = -H^{NLO\,JIMWLK}\, B(u,v,w)}$$

and compare with Grabovsky (hep-ph/1307.5414) $\rightarrow \mathbf{K_{JJSSJ},\ K_{JJSJ}}$

Step 2: Compute evolution of quark dipole operator

$$\mathbf{s(u,v) = tr[S_F(u) S_F^{\dagger}(v)]/N_c}$$

$$\partial_{\mathbf{Y}} \mathbf{s(u,v) = -H^{NLO\,JIMWLK}\, s(u,v)}$$

and compare with Balitsky and Chirilli (hep-ph/0710.4330) $\rightarrow \mathbf{K_{JSSJ},\ K_{JSJ},\ K_{qq}}$

NLO Kernels (for gauge invariant operators)

$$K_{JJSSJ}(w;x,y;z,z') = -i\frac{\alpha_s^2}{2\,\pi^4}\left(\frac{X_i Y'_j}{X^2 Y'^2}\right)$$
$$\times\Big(\frac{\delta_{ij}}{2(z-z')^2} + \frac{(z'-z)_i W'_j}{(z'-z)^2 W'^2} + \frac{(z-z')_j W_i}{(z-z')^2 W^2} - \frac{W_i W'_j}{W^2 W'^2}\Big)\ln\frac{W^2}{W'^2}$$

$$K_{JJSJ}(w;x,y;z) \;=\; -\,i\,\frac{\alpha_s^2}{4\,\pi^3}\left[\frac{X\cdot W}{X^2\,W^2} - \frac{Y\cdot W}{Y^2\,W^2}\right]\ln\frac{Y^2}{(x-y)^2}\;\ln\frac{X^2}{(x-y)^2},$$

$$K_{q\bar{q}}(x,y;z,z') = -\frac{\alpha_s^2\,n_f}{8\,\pi^4}\Big\{\frac{X'^2Y^2 + Y'^2X^2 - (x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2 - X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2} - \frac{2}{(z-z')^4}\Big\}$$

$$X = x - z,\quad X' = x - z',\quad Y = y - z,\quad Y' = y - z',\quad W = w - z$$

$$K_{JSSJ}(x,y;z,z') = \frac{\alpha_s^2}{16\,\pi^4}\Big[-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2+X'^2Y^2-4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2-X'^2Y^2]}$$

$$+\frac{(x-y)^4}{X^2Y'^2-X'^2Y^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big]+\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Big]+\tilde{K}(x,y,z,z').$$

$$K_{JSJ}(x,y;z) = -\frac{\alpha_s^2}{16\pi^3}\frac{(x-y)^2}{X^2Y^2}\Big[b\ln(x-y)^2\mu^2-b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2}+(\frac{67}{9}-\frac{\pi^2}{3})N_c-\frac{10}{9}n_f\Big]$$

$$-\frac{N_c}{2}\int_{z'}\tilde{K}(x,y,z,z').$$

Here μ is the normalization point, $b=\frac{11}{3}N_c-\frac{2}{3}n_f$

$$\tilde{K}(x,y,z,z') = \frac{i}{2}\Big[K_{JJSSJ}(x;x,y;z,z')-K_{JJSSJ}(y;x,y;z,z')-K_{JJSSJ}(x;y,x;z,z')$$

$$+K_{JJSSJ}(y;y,x;z,z')\Big]$$

The kernels are not unique though...

NLO Kernels for color non-singlets

"By inspection" of Balitsky and Chirilli (arXiv:1309.7644 [hep-ph])

$$K_{JSJ}(x,y,z) \to \bar{K}_{JSJ}(x,y,z) \equiv K_{JSJ}(x,y,z)+$$
$$+\frac{\alpha_s^2}{16\pi^3}\left\{\left[\frac{1}{X^2}+\frac{1}{Y^2}\right]\left[\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c-\frac{10}{9}n_f\right]+\frac{b}{X^2}\ln X^2\mu^2+\frac{b}{Y^2}\ln Y^2\mu^2\right\};$$

$$K_{JSSJ}(x,y;z,z') \to \bar{K}_{JSSJ}(x,y;z,z')+$$
$$\equiv K_{JSSJ}(x,y;z,z')+\frac{\alpha_s^2}{8\pi^4}\left[\frac{4}{(z-z')^4}-\frac{I(x,z,z')}{(z-z')^2}-\frac{I(y,z,z')}{(z-z')^2}\right];$$

$$K_{q\bar{q}}(x,y;z,z') \to \bar{K}_{q\bar{q}}(x,y;z,z') \equiv K_{q\bar{q}}(x,y;z,z')-\frac{\alpha_s^2 n_f}{8\pi^4}\left[\frac{I_f(x,z,z')}{(z-z')^2}+\frac{I_f(y,z,z')}{(z-z')^2}\right],$$

$$I(x,z,z')=\frac{1}{X^2-(X')^2}\ln\frac{X^2}{(X')^2}\left[\frac{X^2+(X')^2}{(z-z')^2}-\frac{X\cdot X'}{X^2}-\frac{X\cdot X'}{(X')^2}-2\right];$$
$$I_f(x,z,z')=\frac{2}{(z-z')^2}-\frac{2X\cdot X'}{(z-z')^2(X^2-(X')^2)}\ln\frac{X^2}{(X')^2}.$$

Comparing with Balitsky and Chirilli (arXiv:1309.7644 [hep-ph])

Compute evolution of Wilson lines with open color indices:

$$\partial_{\mathbf{Y}} \left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\right] = -\mathbf{H}^{\mathbf{NLO\,JIMWLK}} \left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\right]$$

$$\partial_{\mathbf{Y}} \left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\mathbf{S}^{\mathbf{cd}}(\mathbf{y})\right] = -\mathbf{H}^{\mathbf{NLO\,JIMWLK}} \left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\mathbf{S}^{\mathbf{cd}}(\mathbf{y})\right]$$

$$\partial_{\mathbf{Y}} \left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\mathbf{S}^{\mathbf{cd}}(\mathbf{y})\mathbf{S}^{\mathbf{ef}}(\mathbf{z})\right] = -\mathbf{H}^{\mathbf{NLO\,JIMWLK}} \left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\mathbf{S}^{\mathbf{cd}}(\mathbf{y})\mathbf{S}^{\mathbf{ef}}(\mathbf{z})\right]$$

100% agreement!

Is the JIMWLK Hamiltonian Conformally invariant?

Scale invariance is trivial. Lets focus on inversion. Introduce $\mathbf{x_\pm = x_1 \pm i\, x_2}$

$$\text{Inversion transformation}: \quad x_+ \to 1/x_- \, ; \qquad\qquad x_- \to 1/x_+$$

A “naive” representation $\mathcal{I}_0$ of the inversion transformation is

$$\mathcal{I}_0 : \quad \mathbf{S(x_+, x_-) \to S(1/x_-, 1/x_+)} \ , \qquad \mathbf{J_{L,R}(x_+, x_-) \to \frac{1}{x_+ x_-} J_{L,R}(1/x_-, 1/x_+)} \, .$$

Conformal invariance (in the gauge invariant sector) @LO:

$$\mathcal{I}_0 \ \mathbf{H}^{\mathbf{LO\,JIMWLK}} \ \mathcal{I}_0 \ = \ \mathbf{H}^{\mathbf{LO\,JIMWLK}}$$

No (naive) Conformal invariance @NLO:

$$\mathcal{I}_0 \ \mathbf{H}^{\mathbf{NLO\,JIMWLK}} \ \mathcal{I}_0 \ = \ \mathbf{H}^{\mathbf{NLO\,JIMWLK}} \ + \ \mathcal{A}$$

QCD is not conformally invariant beyond tree level, but $\mathcal{N} = 4$ SUSY is.

JIMWLK Hamiltonian IS conformally invariant! (in $\mathcal{N} = 4$)

S **forms a non-trivial representation of the conformal group:**

$$\mathcal{I} : S(x) \to S(1/x) + \delta S(x)\,, \qquad \mathcal{I} : H^{LO} \to H^{LO} - \mathcal{A}$$

Here δS is of order α_s. The condition is that the net anomaly cancels:

$$\mathcal{I}\,(\mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}})\,\mathcal{I} \;=\; \mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}}$$

We have constructed $\mathcal{I}$ perturbatively: $\qquad \mathcal{I} \;=\; (\mathbf{1} + \mathcal{C})\,\mathcal{I}_{\mathbf{0}}\,.$

$$\mathcal{C} \;=\; -\,\frac{\mathbf{1}}{\mathbf{2}}\,\frac{\alpha_{\mathrm{s}}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathbf{x,y,z}} \ln\left[\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}\mathbf{a}^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\right]\times$$

$$\times\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\left\{\mathbf{J}^{\mathbf{a}}_{\mathbf{L}}(\mathbf{x})\mathbf{J}^{\mathbf{a}}_{\mathbf{L}}(\mathbf{y}) + \mathbf{J}^{\mathbf{a}}_{\mathbf{R}}(\mathbf{x})\mathbf{J}^{\mathbf{a}}_{\mathbf{R}}(\mathbf{y}) - \mathbf{2}\mathbf{J}^{\mathbf{a}}_{\mathbf{L}}(\mathbf{x})\mathbf{S}^{\mathbf{ab}}_{\mathbf{A}}(\mathbf{z})\mathbf{J}^{\mathbf{b}}_{\mathbf{R}}(\mathbf{y})\right\}$$

For an arbitrary operator $\mathcal{O}$ (s, B, H^{JIMWLK},...) we define its conformal extension:

$$\mathcal{O}^{conf} \;=\; \mathcal{O} + \frac{1}{2}\,[\mathcal{C},\,\mathcal{O}]\,; \qquad [s^{conf}\text{ by Balitsky and Chirilli (arXiv : 0903.5326)}]$$

CONCLUSIONS

- We have constructed the JIMWLK Hamiltonian at NLO. It fully reproduces and generalizes (all?) previously known low x evolution equations at NLO, including Balitsky's hierarchy at NLO

- We have proven the conformal invariance of the NLO JIMWLK Hamiltonian (in $\mathcal{N} = 4$). For any operator, we can construct its perturbative extension, such that the resulting operator evolves with conformal kernels.

- Once expanded in the dilute limit, the NLO JIMWLK makes it possible to study evolution of any multi-gluon BKP state and transition vertices at NLO.