TMDs at small-x: What we learnt and What to expect

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Nice things about transverse momentum distributions (TMDs)

- Universality and a universal language
  - DIS/Drell-Yan
  - eA/pA/AA(?), small-x wave functions of nucleus
- QCD dynamics
  - TMD evolution
  - Small-x evolution

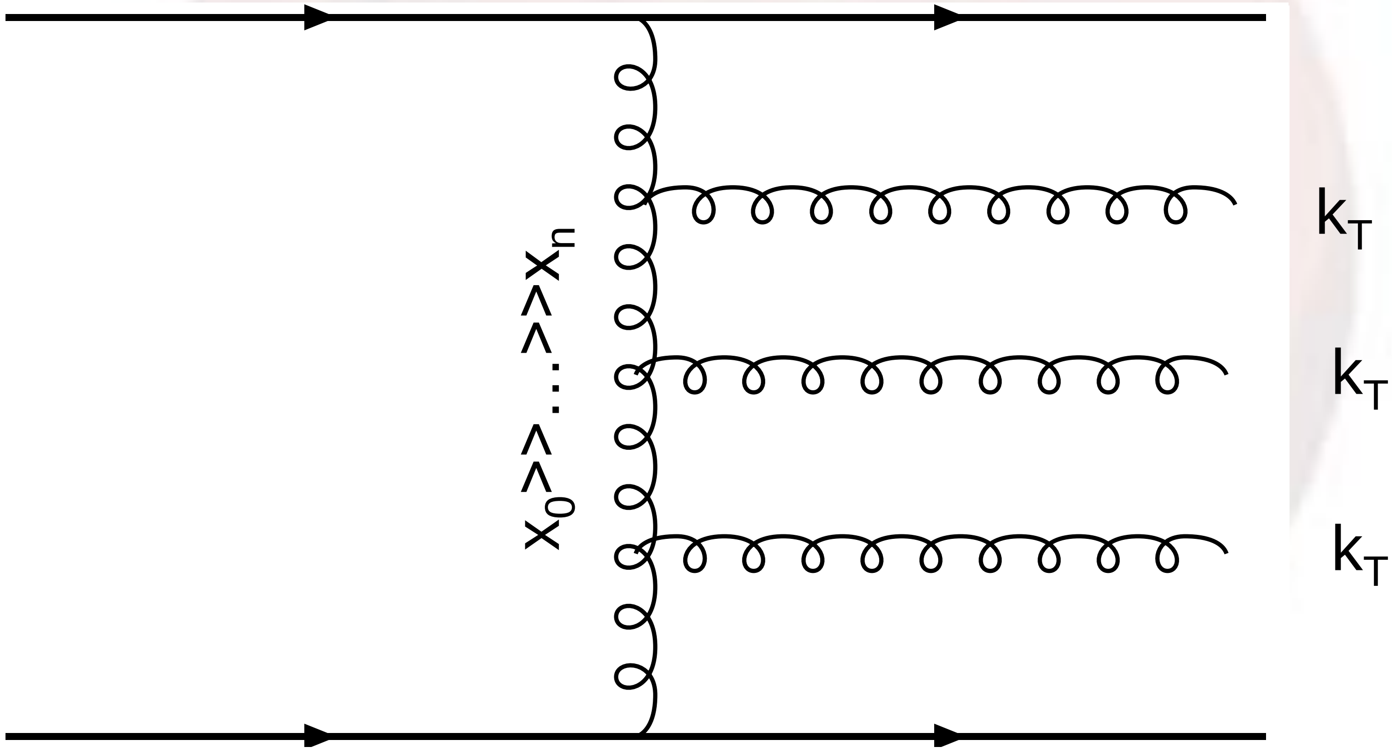
QCD evolution at high energy

- BFKL/BK-JIMWLK (small-x)
- Sudakov (TMD)

Mueller-Xiao-Yuan 2013
Balistky-Tarasov 2014

Tarasov’s talk on Wednesday

High energy scattering



BFKL: $\frac{\partial \mathcal{F}}{\partial \ln(1/x)} = \mathcal{K} \bigotimes \mathcal{F}$ Un-integrated gluon distribution

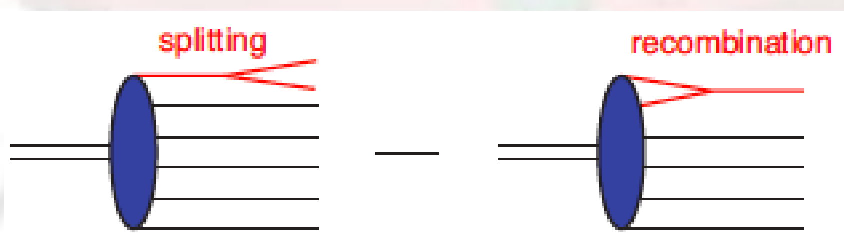
Non-linear term at high density

- Balitsky-Fadin-Lipatov-Kuraev, 1977-78

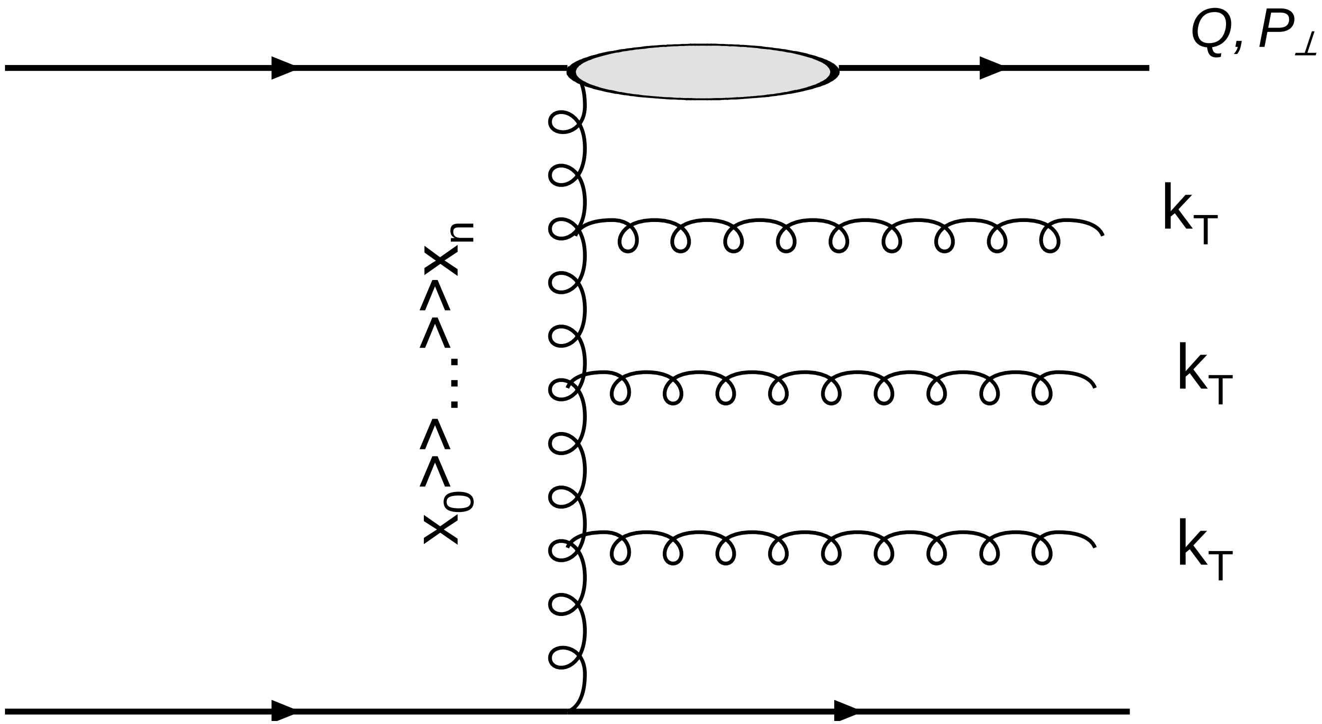
$$\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{\mathrm{BFKL}} \otimes N(x, r_T)$$

- Balitsky-Kovchegov: Non-linear term, 98

$$\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{\mathrm{BFKL}} \otimes N(x, r_T) - \alpha_s \left[N(x, r_T)\right]^2.$$

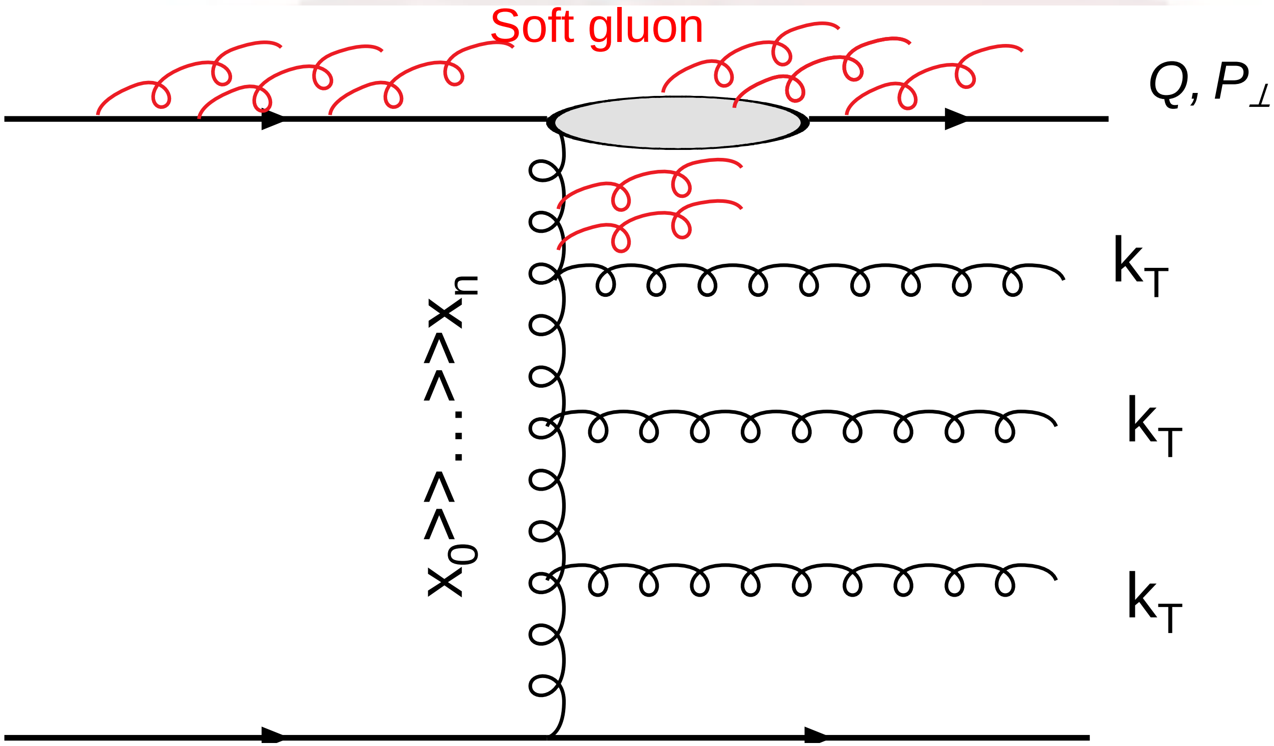


Hard processes at small-x



- Manifest dependence on un-integrated gluon distributions
  - Dominguiz-Marquet-Xiao-Yuan, 2010

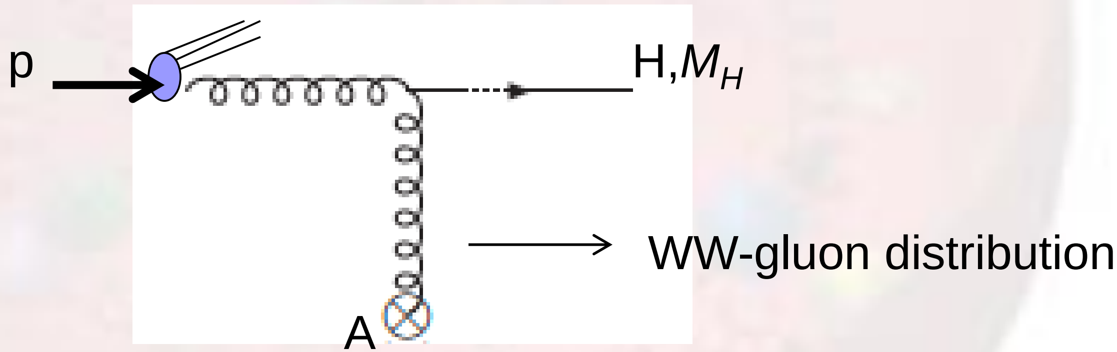
Additional dynamics comes in



- BFKL vs Sudakov resummations (LL)

Sudakov resummation at small-x

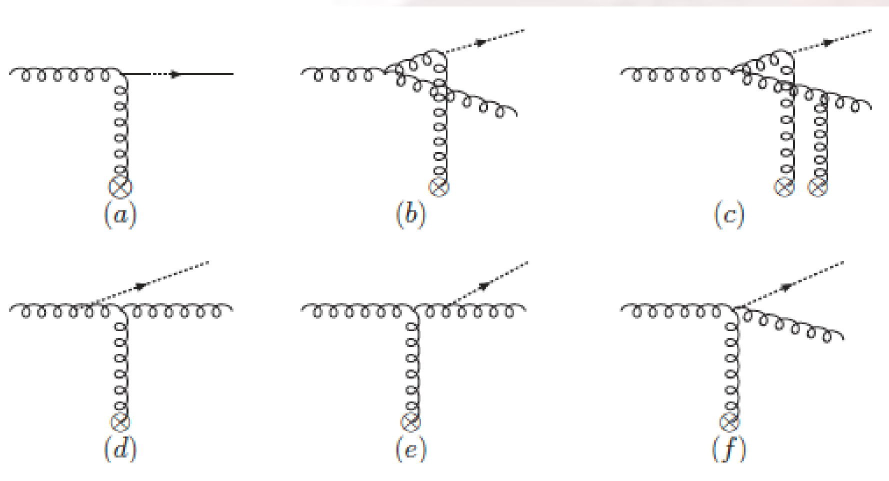
- Take massive scalar particle production p+A->H+X as an example to demonstrate the double logarithms, and resummation

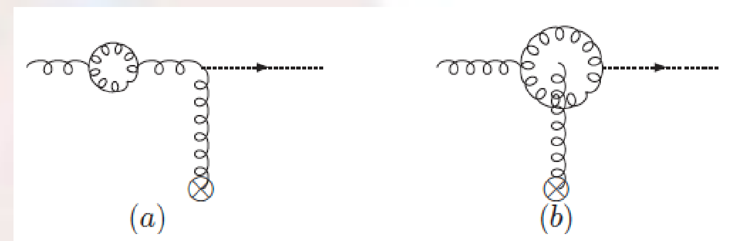


$$\frac{d\sigma^{(\mathrm{LO})}}{dyd^2k_\perp} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp\cdot r_\perp} x_0 g_p(x_0) S^{(WW)}(x_\perp, x'_\perp)$$

$$S_Y^{WW}(x_\perp, y_\perp) = -\left\langle \mathrm{Tr}\left[\partial_\perp^\beta U(x_\perp) U^\dagger(y_\perp) \partial_\perp^\beta U(y_\perp) U^\dagger(x_\perp)\right]\right\rangle_Y$$

Explicit one-loop calculations





$$x_0 g_p(x_0) \int \frac{d\xi}{\xi} \mathbf{K}_{DMMX} \otimes S^{WW}(x_\perp, y_\perp) + \left(-\frac{1}{\epsilon}\right) S^{WW}(x_\perp, y_\perp) \mathcal{P}_{g/g} \otimes x_0 g(x_0) \ ,$$

- Collinear divergence → DGLAP evolution
- Small-x divergence → BK-type evolution

Dominguiz-Mueller-Munier-Xiao, 2011

Soft vs Collinear gluons

- Radiated gluon momentum

$$k_g = \alpha_g p_1 + \beta_g p_2 + k_{g\perp} \ ,$$

- Soft gluon, $\alpha \sim \beta << 1$
- Collinear gluon, $\alpha \sim 1$, $\beta << 1$
- Small-x collinear gluon, $1-\beta << 1$, $\alpha \to 0$
  - Rapidity divergence

Final result

- Double logs at one-loop order

$$\frac{d\sigma^{(\mathrm{LO+NLO})}}{dyd^2k_\perp}|_{k_\perp\ll Q} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp\cdot r_\perp} S^{WW}_{Y=\ln 1/x_a}(x_\perp, x'_\perp) x g_p(x, \mu^2 = \frac{c_0^2}{r_\perp^2}) \left\{1 + \frac{\alpha_s}{\pi} C_A \left[\beta_0 \ln\frac{Q^2 r_\perp^2}{c_0^2} - \frac{1}{2}\left(\ln\frac{Q^2 r_\perp^2}{c_0^2}\right)^2 + \frac{\pi^2}{2}\right]\right\},$$

- Include both BFKL (BK) and Sudakov

$$\frac{d\sigma^{(\mathrm{resum})}}{dyd^2k_\perp}|_{k_\perp\ll Q} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp\cdot r_\perp} e^{-S_{sud}(Q^2, r_\perp^2)} S^{WW}_{Y=\ln 1/x_a}(x_\perp, x'_\perp) \times x g_p(x, \mu^2 = c_0^2/r_\perp^2)\left[1 + \frac{\alpha_s}{\pi}\frac{\pi^2}{2} N_c\right],$$

Mueller-Xiao-Yuan 2012

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Sudakov leading double logs+small-x logs in hard processes

- Each incoming parton contributes to a half of the associated color factor in Sudakov
  - Initial gluon radiation, aka, TMDs

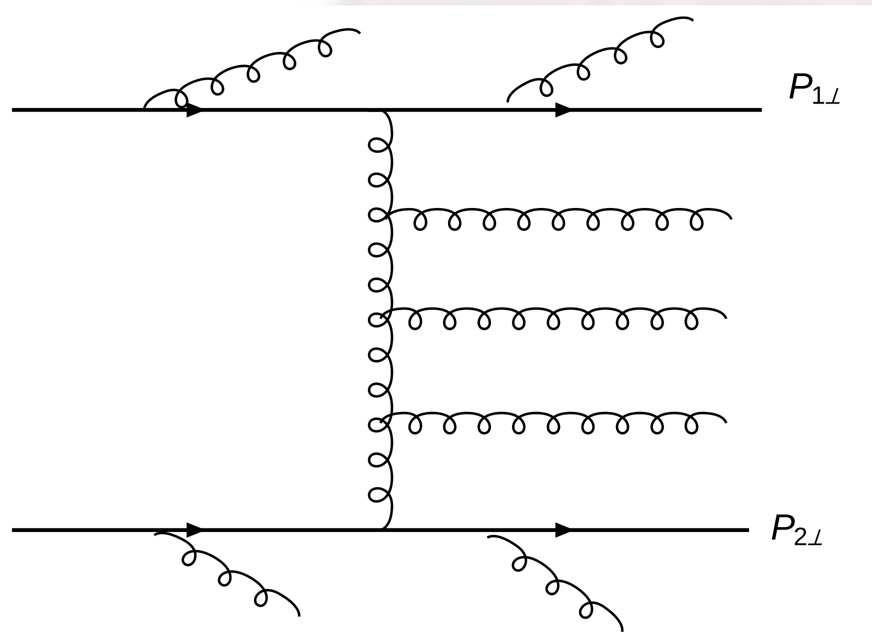
$$\frac{d\sigma}{dy_1 dy_2 dP_\perp^2 d^2k_\perp} \propto H(P_\perp^2) \int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot(x_\perp - y_\perp)} \widetilde{W}_{x_A}(x_\perp, y_\perp)$$

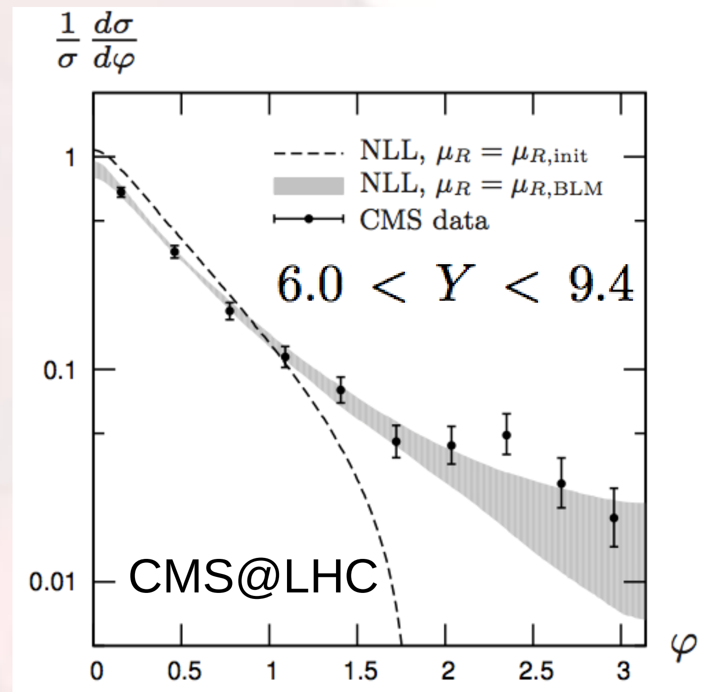
Sudakov →

$$H(P_\perp^2) \int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot R_\perp} e^{-S_{sud}(P_\perp, R_\perp)} \widetilde{W}_{x_A}(x_\perp, y_\perp)$$

Mueller-Xiao-Yuan 2013

Dijet with large rapidity gap





Sudakov resummation will dominate
Small angle distribution (Mueller-Xiao-Yuan)

Ducloue,Szymanowski,Wallon
1309.3229, only take into account
BFKL

Work in progress,

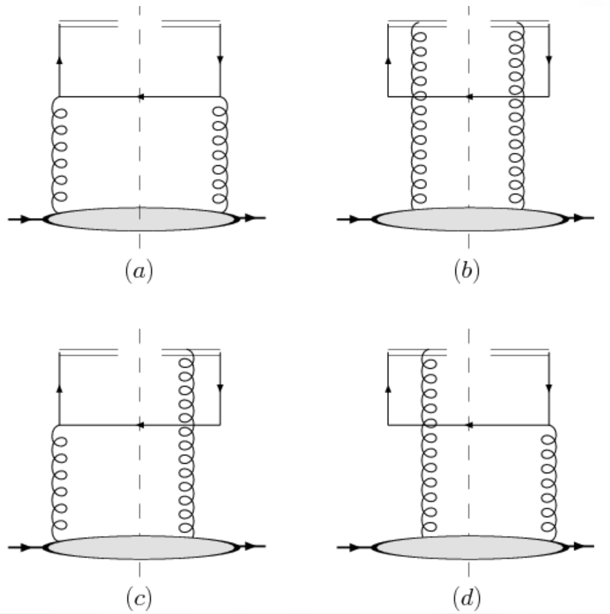
TMDs at small-x: what we do we know?

- We hope to extract these TMDs from experiments by applying the resummation formula

$$\frac{d\sigma}{dy_1 dy_2 dP_\perp^2 d^2k_\perp} \propto H(P_\perp^2) \int d^2x_\perp d^2y_\perp e^{ik_\perp \cdot R_\perp} e^{-S_{sud}(P_\perp, R_\perp)} \widetilde{W}_{x_A}(x_\perp, y_\perp)$$

- Small-x TMDs can be calculated from CGC, instead of parameterization
  - Prediction power

TMD quark at small-x



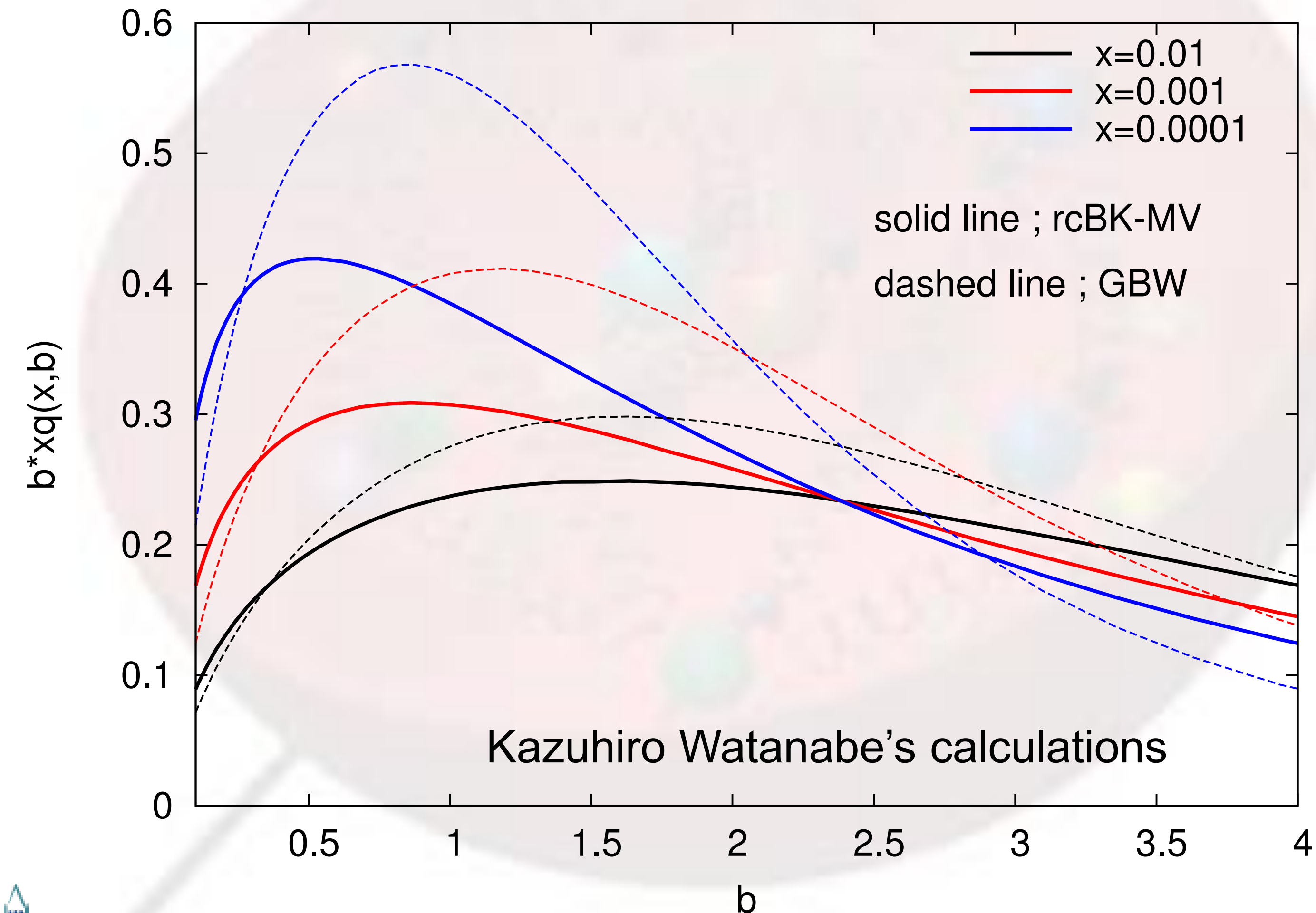
McLerran-Venugopalan 98

$$q(x,k_\perp)=\frac{N_c}{8\pi^4}\int\frac{dx'}{x'^2}\int d^2bd^2q_\perp F(q_\perp,x')A(q_\perp,k_\perp)$$

- Can be calculated from the dipole amplitude, and can be applied to DIS and Drell-Yan processes
  - Universality

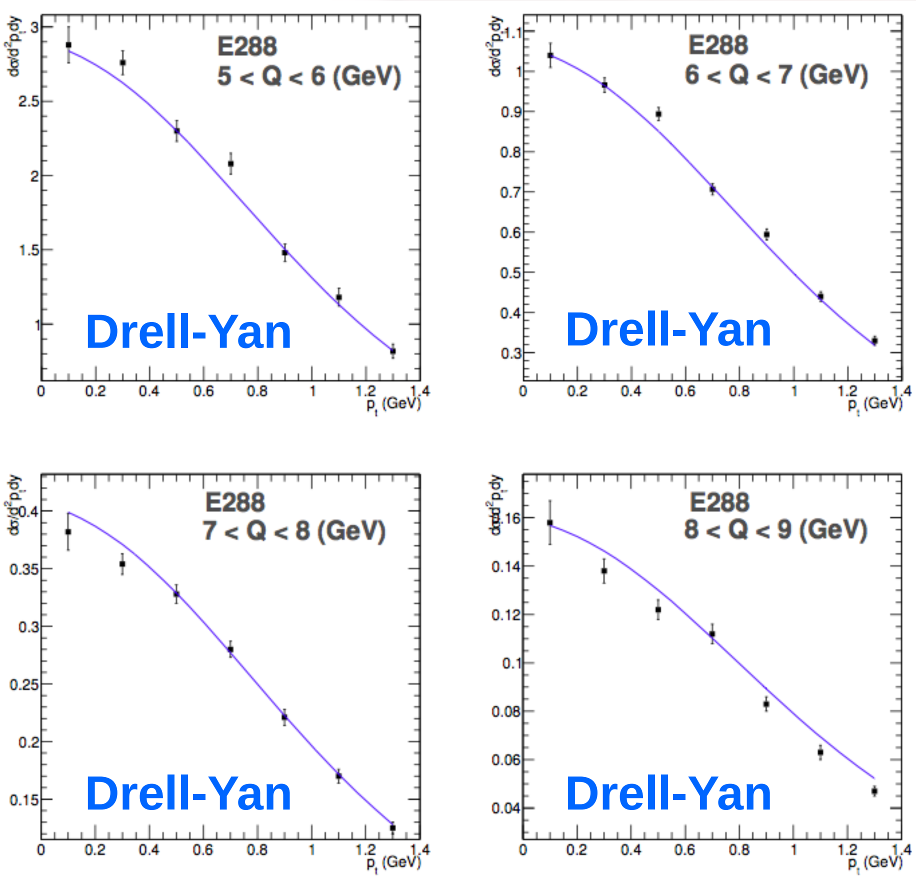
Marquet-Xiao-Yuan 2009

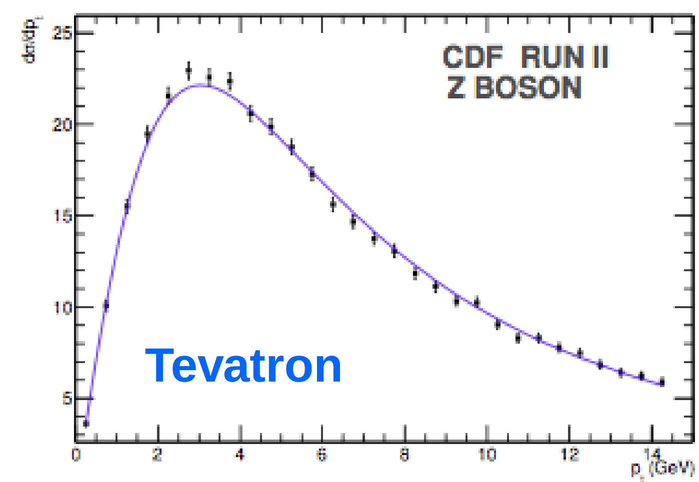
TMD quark at small-x

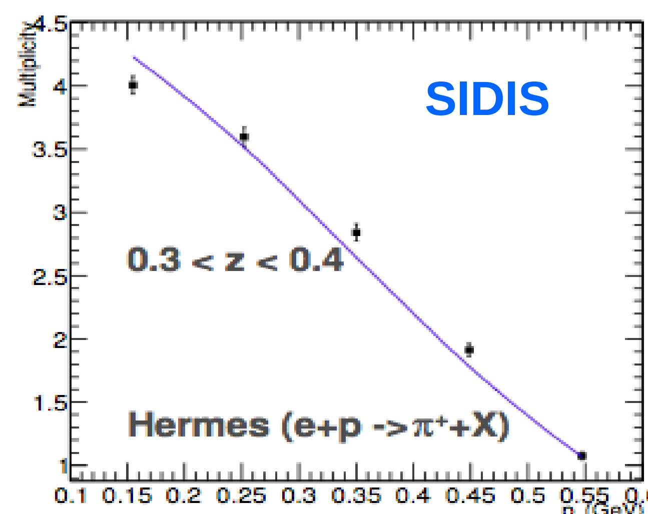


What we know the TMD quarks (not small-x)

Sun-Issacson-Yuan-Yuan, 2014

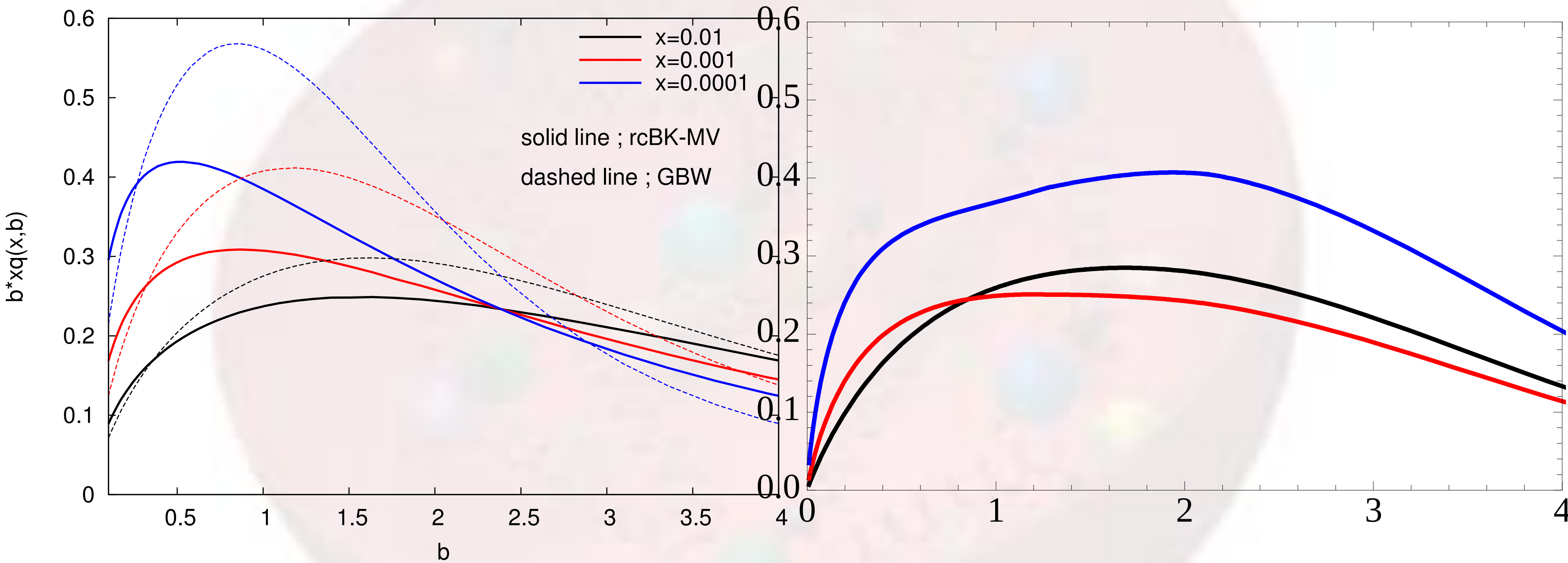






See also, BLNY 2002

TMD quark at small-x: CGC vs Collinear



- Realistic comparison will shed light on the TMD quarks at small-x (work in progress)

- Where can we get gluon TMDs?
  - Dijet production
  - Heavy quark pair/heavy quarkonium production
  - …

TMD gluon from quarkonium productions

Sun,C.-P.Yuan,F.Yuan, PRD 2013
earlier work: Bergers-Qiu-Wang, 2004

- Low pt distribution

$$\frac{d\sigma}{d^2P_\perp dy}|_{P_\perp \ll M} = \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{P}_\perp\cdot\vec{b}}W(b,M,x_1,x_2)$$

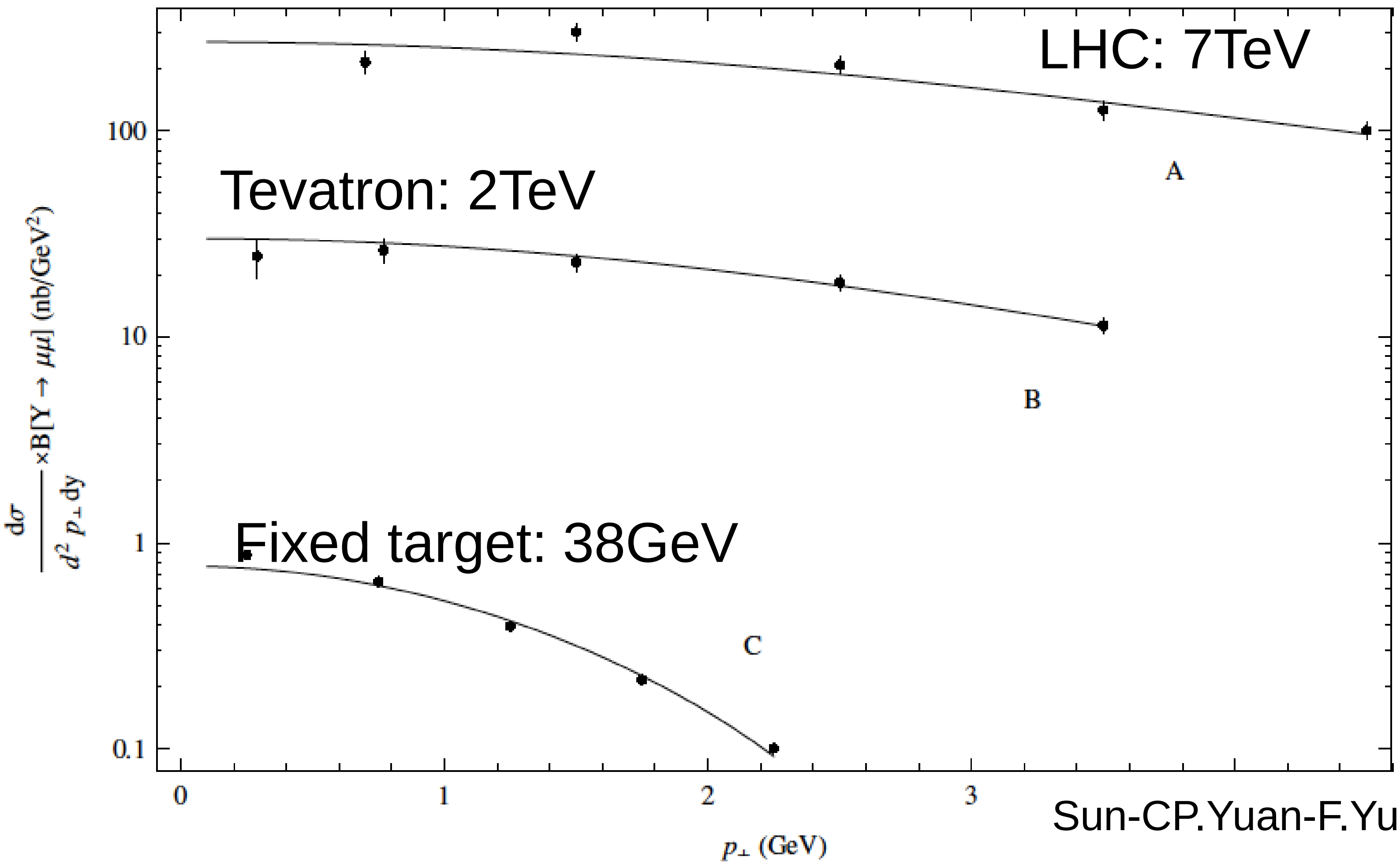
$$W(b,M^2) = e^{-S_{Sud}(M^2,b,C_1,C_2)}W(b,C_1,C_2)$$

- Non-perturbative input

$$W(b) = W(b_*)W^{NP}(b)$$

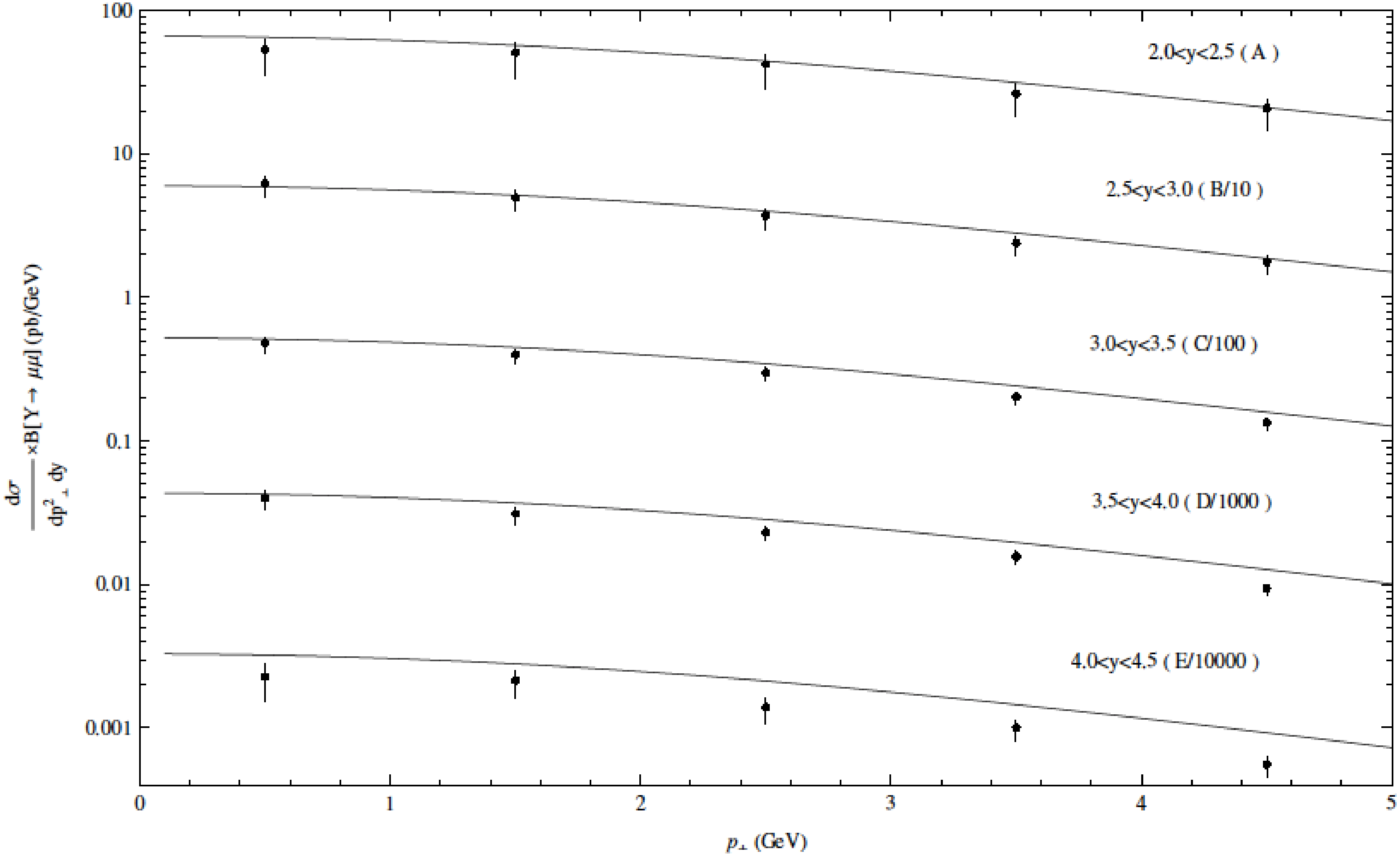
$$W^{NP}(b) = \exp\left[-g_1 - g_2\ln\left(\frac{Q}{2Q_0}\right) - g_1g_3\ln(100x_1x_2)\right]b^2$$

Quarkonium production as a probe to the gluon TMDs (Υ)



Sun-CP.Yuan-F.Yuan, 2013

More data from LHC



Sun-CP.Yuan-F.Yuan, 2013

Gluon TMD: CGC vs Collinear

Model-dependent way

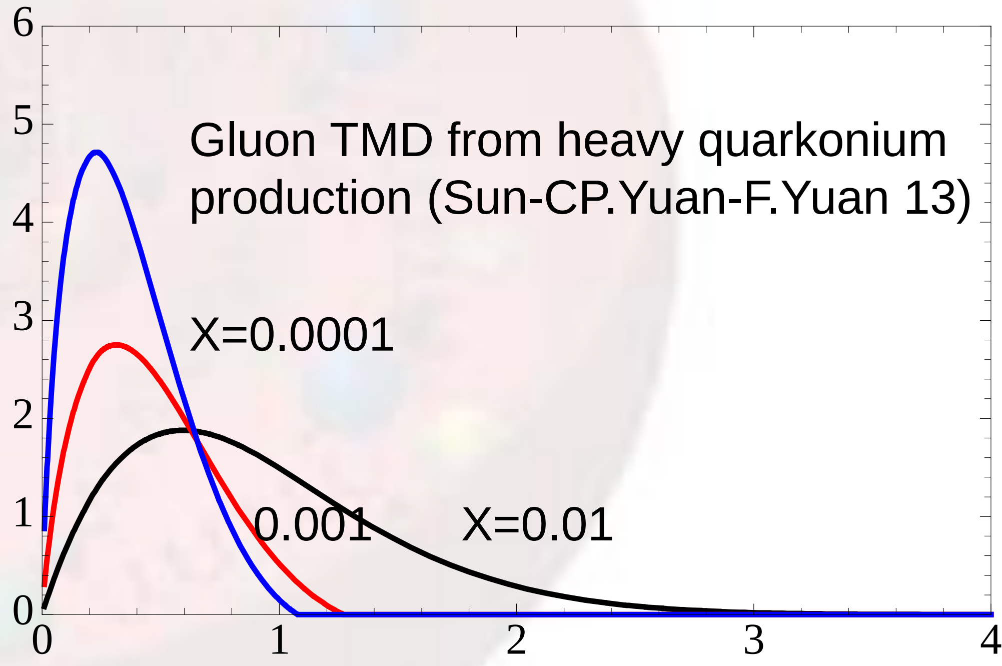
??

Qiu-Sun-Xiao-Yuan 14

Ma-Venugopalan 14/15

Ducloué-Lappi-Mäntysaari 15

(work in progress)



Conclusion

- We start to have a quantitative picture on TMDs at small-x
  - Which connect different physics
- More theoretical works are needed
- Experiments are crucial to answer some of the important questions
  - LHC, and EIC