TMDs at small-x: What we learnt and What to expect

Feng Yuan
Lawrence Berkeley National Laboratory
Nice things about transverse momentum distributions (TMDs)

- Universality and a universal language
  - DIS/Drell-Yan
  - eA/pA/AA(?), small-x wave functions of nucleus
- QCD dynamics
  - TMD evolution
  - Small-x evolution
QCD evolution at high energy

- BFKL/BK-JIMWLK (small-x)
- Sudakov (TMD)

Mueller-Xiao-Yuan 2013
Balistky-Tarasov 2014

Tarasov’s talk on Wednesday
High energy scattering

\[ \frac{\partial F}{\partial \ln(1/x)} = \kappa \otimes F \]  
Un-integrated gluon distribution

BFKL:
Non-linear term at high density

- Balitsky-Fadin-Lipatov-Kuraev, 1977-78

\[
\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x, r_T)
\]

- Balitsky-Kovchegov: Non-linear term, 98

\[
\frac{\partial N(x, r_T)}{\partial \ln(1/x)} = \alpha_s K_{BFKL} \otimes N(x, r_T) - \alpha_s [N(x, r_T)]^2.
\]
Hard processes at small-\(x\)

- Manifest dependence on un-integrated gluon distributions

- Dominguez-Marquet-Xiao-Yuan, 2010
Additional dynamics comes in

- BFKL vs Sudakov resummations (LL)
Sudakov resummation at small-\(x\)

- Take massive scalar particle production \(p+A \rightarrow H+X\) as an example to demonstrate the double logarithms, and resummation

\[
\frac{d\sigma^{(LO)}}{dyd^2k_{\perp}} = \sigma_0 \int \frac{d^2x_{\perp}d^2x'_{\perp}}{(2\pi)^2} e^{ik_{\perp} \cdot r_{\perp}} x_0 g_p(x_0) S^{(WW)}(x_{\perp}, x'_{\perp})
\]

\[
S_Y^{WW}(x_{\perp}, y_{\perp}) = - \left\langle \text{Tr} \left[ \partial_{\perp}^\beta U(x_{\perp}) U^\dagger(y_{\perp}) \partial_{\perp}^\beta U(y_{\perp}) U^\dagger(x_{\perp}) \right] \right\rangle_Y
\]
Explicit one-loop calculations

\[ x_0 g_p(x_0) \int \frac{d\xi}{\xi} K_{DMX} \otimes S^{WW}(x_\perp, y_\perp) + \left( -\frac{1}{\epsilon} \right) S^{WW}(x_\perp, y_\perp) P_{g/g} \otimes x_0 g(x_0) , \]

- Collinear divergence \( \Rightarrow \) DGLAP evolution
- Small-x divergence \( \Rightarrow \) BK-type evolution

Soft vs Collinear gluons

- Radiated gluon momentum
  \[ k_g = \alpha_g p_1 + \beta_g p_2 + k_g \perp , \]
- Soft gluon, \( \alpha \sim \beta \ll 1 \)
- Collinear gluon, \( \alpha \sim 1, \beta \ll 1 \)
- Small-\( x \) collinear gluon, \( 1 - \beta \ll 1, \alpha \to 0 \)
  - Rapidity divergence
Final result

- Double logs at one-loop order

\[ \frac{d\sigma^{(\text{LO+NLO})}}{dyd^2k_{\perp}} \bigg|_{k_{\perp} \ll Q} = \sigma_0 \int \frac{d^2x_{\perp}d^2x'_{\perp}}{(2\pi)^2} e^{i k_{\perp} \cdot r_{\perp}} S_{Y=\ln 1/x_a}(x_{\perp}, x'_{\perp}) x g_p(x, \mu^2 = c_0^2/r_{\perp}^2) \]

\[ \left\{ 1 + \frac{\alpha_s}{\pi} C_A \left[ \beta_0 \ln \frac{Q^2 r_{\perp}^2}{c_0^2} - \frac{1}{2} \left( \ln \frac{Q^2 r_{\perp}^2}{c_0^2} \right)^2 + \frac{\pi^2}{2} \right] \right\} , \]

- Include both BFKL (BK) and Sudakov

\[ \frac{d\sigma^{(\text{resum})}}{dyd^2k_{\perp}} \bigg|_{k_{\perp} \ll Q} = \sigma_0 \int \frac{d^2x_{\perp}d^2x'_{\perp}}{(2\pi)^2} e^{i k_{\perp} \cdot r_{\perp}} e^{-S_{\text{sud}}(Q^2,r_{\perp}^2)} S_{Y=\ln 1/x_a}(x_{\perp}, x'_{\perp}) \]

\[ \times x g_p(x, \mu^2 = c_0^2/r_{\perp}^2) \left[ 1 + \frac{\alpha_s}{\pi} \frac{\pi^2}{2} N_c \right] , \]

Mueller-Xiao-Yuan 2012
Sudakov leading double logs+small-x logs in hard processes

- Each incoming parton contributes to a half of the associated color factor in Sudakov

$\frac{d\sigma}{dy_1 dy_2 dP^2 \perp d^2 k_\perp} \propto H(P^2_\perp) \int d^2 x_\perp d^2 y_\perp e^{i k_\perp \cdot (x_\perp - y_\perp)} \tilde{W}_{x_A}(x_\perp, y_\perp)$

Sudakov

$H(P^2_\perp) \int d^2 x_\perp d^2 y_\perp e^{i k_\perp \cdot R} e^{-S_{sud}(P_\perp, R_\perp)} \tilde{W}_{x_A}(x_\perp, y_\perp)$

Mueller-Xiao-Yuan 2013
Dijet with large rapidity gap

Sudakov resummation will dominate
Small angle distribution (Mueller-Xiao-Yuan)

Ducloue, Szymanowski, Wallon
1309.3229, only take into account BFKL

Work in progress,
TMDs at small-x: what we do we know?

- We hope to extract these TMDs from experiments by applying the resummation formula

\[
\frac{d\sigma}{dy_1 dy_2 dP^2 d^2 k} \propto H(P^2) \int d^2x d^2y e^{ik \cdot R} e^{-S_{sud}(P, R)} \widetilde{W}_{x_A}(x, y)
\]

- Small-x TMDs can be calculated from CGC, instead of parameterization
  - Prediction power
TMD quark at small-$x$

Can be calculated from the dipole amplitude, and can be applied to DIS and Drell-Yan processes.

Universality
TMD quark at small-x

Kazuhiro Watanabe’s calculations

solid line; rcBK-MV

dashed line; GBW
What we know the TMD quarks (not small-x)

Sun-Issacson-Yuan-Yuan, 2014

See also, BLNY 2002
Realistic comparison will shed light on the TMD quarks at small-$x$ (work in progress)
Where can we get gluon TMDs?

- Dijet production
- Heavy quark pair/heavy quarkonium production
- ...
TMD gluon from quarkonium productions

- Low pt distribution

\[ \frac{d\sigma}{d^2P_\perp dy} \bigg|_{P_\perp \ll M} = \frac{1}{(2\pi)^2} \int d^2b \ e^{i\vec{P}_\perp \cdot \vec{b}} W(b, M, x_1, x_2) \]

\[ W(b, M^2) = e^{-S_{Sud}(M^2, b, C_1, C_2)} W(b, C_1, C_2) \]

- Non-perturbative input

\[ W(b) = W(b_*) W^{NP}(b) \]

\[ W^{NP}(b) = \exp \left[ -g_1 - g_2 \ln \left( \frac{Q}{2Q_0} \right) - g_1 g_3 \ln (100x_1 x_2) \right] b^2 \]
Quarkonium production as a probe to the gluon TMDs ($\Upsilon$)

- Fixed target: 38GeV
- Tevatron: 2TeV
- LHC: 7TeV

Sun-CP.Yuan-F.Yuan, 2013
More data from LHC
Gluon TMD: CGC vs Collinear

Model-dependent way ??
Qiu-Sun-Xiao-Yuan 14
Ma-Venugopalan 14/15
Ducloué-Lappi-Mäntysaari 15

(work in progress)
Conclusion

- We start to have a quantitative picture on TMDs at small-$x$
  - Which connect different physics
- More theoretical works are needed
- Experiments are crucial to answer some of the important questions
  - LHC, and EIC