

On the use of physical DGLAP evolution of structure functions and its use for detecting saturation effects

Martin Hentschinski

Instituto Ciencias Nucleares
Universidad Nacional Autónoma de México
México, D.F. 04510, MX



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based on common work with Marco Stratmann

[arXiv:1311.2825]

Collinear factorization: factorization for $Q^2 \rightarrow \infty$

- ✓ very precise theoretical formulation
factorization into (perturbative) coefficients & (non-perturbative) pdfs
- ✓ perturbative corrections known up to NNLO

Ambiguities remain:

► pdfs a theory definition:

factorization into bare (= divergent)
coefficients & pdfs

$$F_2(x, Q^2) = \sum_{k,q,g} \hat{C}_{2,k} \otimes \hat{f}_k$$



→ cancelation introduces
(factorization) scheme & scale
dependence

$$F_2(x, Q^2) = \sum_{k,q,g} C_{2,k} \otimes f_k$$

physical evolution equations

idea: don't care about pdfs



evolve observable itself

[Furmanski, Petronzio, ZP C 11, 293(1982)],

[Catani, ZP C 75, 665 (1997)],

[Blümlein, Ravindran, van Neerven, NPB 586,

349 (2000)]

$$Q^2 \frac{d}{dQ^2} F(x, Q^2) = K \otimes F(x, Q^2)$$



observable itself!

evolution kernels K

- ▶ physical
- ▶ no factorization scheme ambiguity;
only renormalization scale



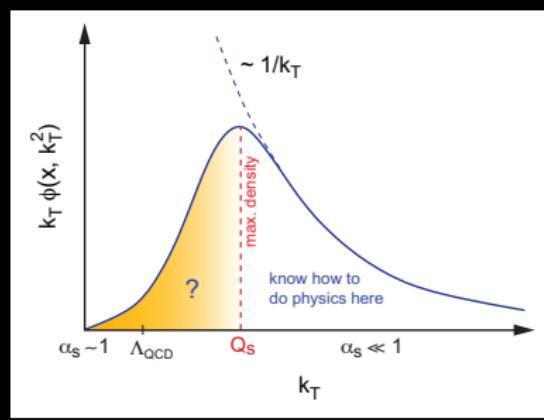
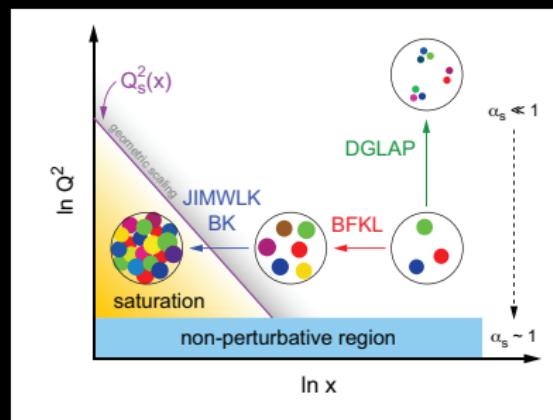
equivalent to [Catani, ZP C 75, 665
(1997)]

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

lose universality of pdfs, but gain precision

Possible applications

- Determination of α_s from DIS data (unlike pdf determination no factorization scale ambiguities)
- Search for breakdown of linear DGLAP evolution due to higher twist effects → signal for saturation (relevant for searches at eRHIC, MEIC, LHeC, ...)



realization: from pdfs to physical evolution

Physical evolution: write DGLAP equations not in terms of pdfs, but observables e.g. structure functions

- ▶ Need for every independent active pdf an observable

- ▶ inclusive neutral current DIS:

$$\text{gluon} + n_f(q + \bar{q})$$

make use of well known flavor decomposition:

flavor singlet & non-singlets

$$\Sigma = \sum_k^{n_f} (q_k + \bar{q}_k)$$

$$q_3 = u + \bar{u} - d - \bar{d}$$

$$q_8 = u + \bar{u} + d + \bar{d} - 2(s + \bar{s})$$

only flavor singlet Σ mixes under DGLAP evolution with the gluon

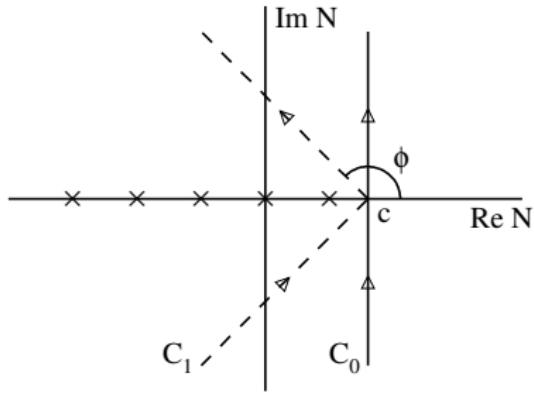
$$d_{\ln \mu^2} \begin{pmatrix} \Sigma \\ g \end{pmatrix} = \begin{pmatrix} P_{qq} & P_{\Sigma g} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$

matrix valued DGLAP
evolution decouples: evolution
for non-singlets & 2-dim
evolution for singlet

Work in conjugate Mellin space

In moment space, $a(N) = \int_0^1 dx x^{N-1} a(x)$, convolutions turn into products

$$F(x) = \iint_0^1 dz_1 dz_2 C(z_1) f(z_2) \delta(z_1 z_2 - x) \Leftrightarrow F(N) = C(N) \cdot f(N)$$



turns analysis into linear algebra
inverse Mellin transform:
numerically

$$a(x) = \int_{c-i\infty}^{c+i\infty} \frac{dN}{2\pi i} x^{-N} a(N)$$

This study

- ▶ Concentrate on color singlet sector → most relevant for small x analysis
- ▶ some aspects of non-singlet sectors have been studied previously

[van Neerven, Vogt, NPB 568:263 (2000)]

- ▶ suitable doublet of observables:
$$\begin{pmatrix} F_2^{(S)} \\ F_L^{(S)} \end{pmatrix} = \underbrace{\begin{pmatrix} C_{2q} & C_{2g} \\ C_{Lq} & C_{Lg} \end{pmatrix}}_{\text{coeff. matrix } C} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}$$
- ▶ alternative
$$\left(F_2^{(S)}, F_S^{(S)} = \frac{dF_2^{(S)}}{d \ln Q^2} \right);$$

[sometimes also use
$$\left(F_2^{(S)}, F_D^{(S)} = -\frac{\beta_0}{2\beta(\alpha_s(Q^2))} F_S^{(S)} \right)$$
]

Physical evolution kernels – master formula

For a suitable doublet of observables determine (with $a_s = \frac{\alpha_s}{4\pi}$):

$$d_{\ln Q^2} \begin{pmatrix} F_A \\ F_B \end{pmatrix} = d_{\ln Q^2} \left[C \cdot \begin{pmatrix} \Sigma \\ g \end{pmatrix} \right]$$

$$\begin{aligned} &= \left[\beta \frac{dC}{da_s} + C \cdot P \right] \cdot \begin{pmatrix} \Sigma \\ g \end{pmatrix} \\ &= \left[\beta \frac{dC}{da_s} + C \cdot P \right] C^{-1} \begin{pmatrix} F_A \\ F_B \end{pmatrix} \equiv K \cdot \begin{pmatrix} F_A \\ F_B \end{pmatrix} \end{aligned}$$

master formula

$$K = \left[\beta \frac{dC}{da_s} + C \cdot P \right] C^{-1} = a_s K^{(0)} + a_s^2 K^{(1)} + a_s^3 K^{(2)} + \dots$$

- kernel K independent of factorization scheme & scale order by order in perturbation theory
[Blümlein, Ravindran, van Neerven, NPB 586, 349 (2000)]
- finite order: dependence on renormalization scale & scheme remains

Numerical implementation

- initial condition: to test framework

→ toy input at $Q_0^2 = 2\text{GeV}^2$,

(PEGASUS [A. Vogt, CPC 170, 65 (2005)] default intial parton distributions)

fix $n_f = 3$ and $\alpha_s(Q_0^2) = 0.35$

$$xu_v(x, Q_0^2) = 5.10722x^{0.8}(1-x)^3$$

$$xd_v(x, Q_0^2) = 3.064320x^{0.8}(1-x)^4$$

$$xg(x, Q_0^2) = 1.70000x^{-0.1}(1-x)^5$$

$$x\bar{d}(x, Q_0^2) = 0.1939875x^{-0.1}(1-x)^6$$

$$x\bar{u}(x, Q_0^2) = (1-x)x\bar{d}(x, Q_0^2)$$

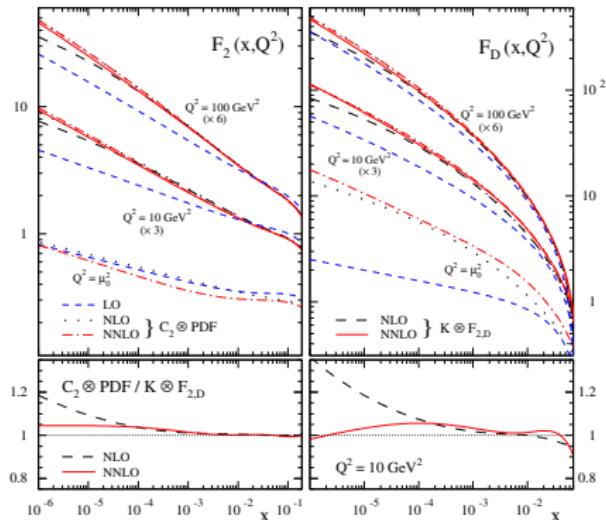
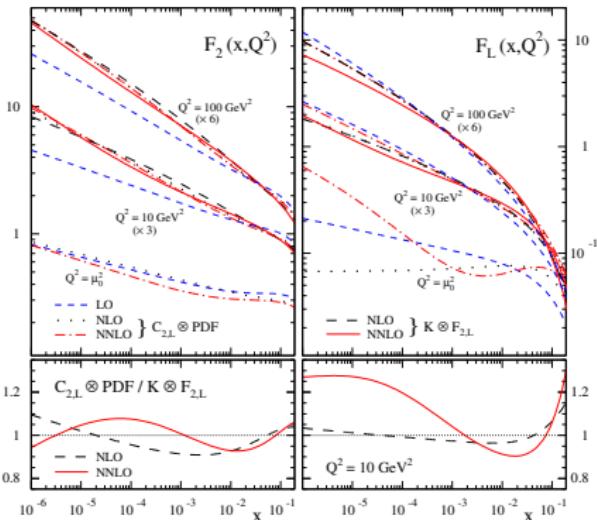
$$xs(x, Q_0^2) = x\bar{s}(x, Q_0^2) = 0.2(\bar{u} + \bar{d})(x, Q_0^2)$$

- structure functions at input scale from pdfs

$$F_I(x, Q_0^2) = \sum_k C_{I,k}(Q_0^2) \cdot f_k(x, Q_0^2)$$

at LO pdf and physical evolution identical

→ at small/large x : pdf & physical evolution differ at NLO, NNLO



Difference pdf/physical anomalous dimensions due to spurious higher order terms

Resolving differences (here up to NLO, also for NNLO)

pdf implementation (schematic)

$$F(Q^2) = (1 + a_s C^{(1)}) \left(\frac{a_s}{a_0} \right)^{R^{(0)}} \left["1" + (a_0 - a_s) R_{\text{P}}^{(1)} \right] f(Q_0^2)$$

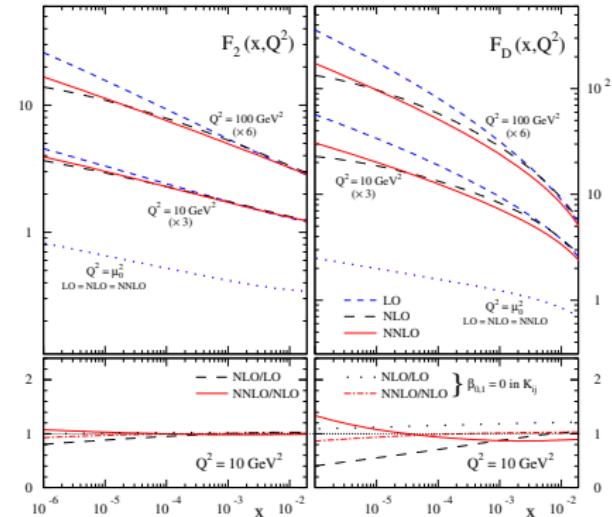
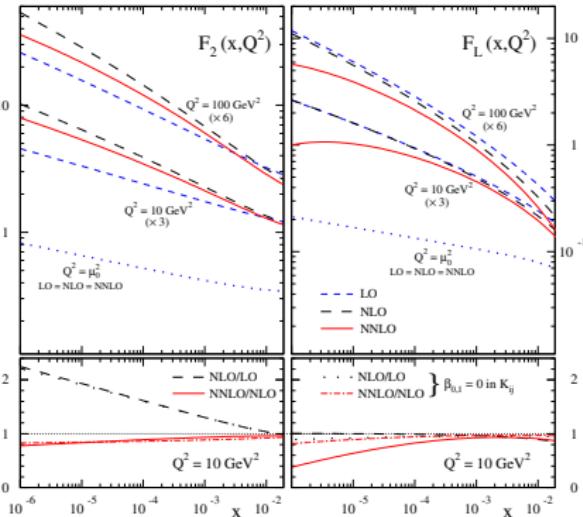
implementation of physical evolution (schematic)

$$F(Q^2) = \left(\frac{a_s}{a_0} \right)^{R^{(0)}} \left["1" + (a_0 - a_s) R_{\text{K}}^{(1)} \right] (1 + a_0 C^{(1)}) f(Q_0^2)$$

contains spurious NNLO terms

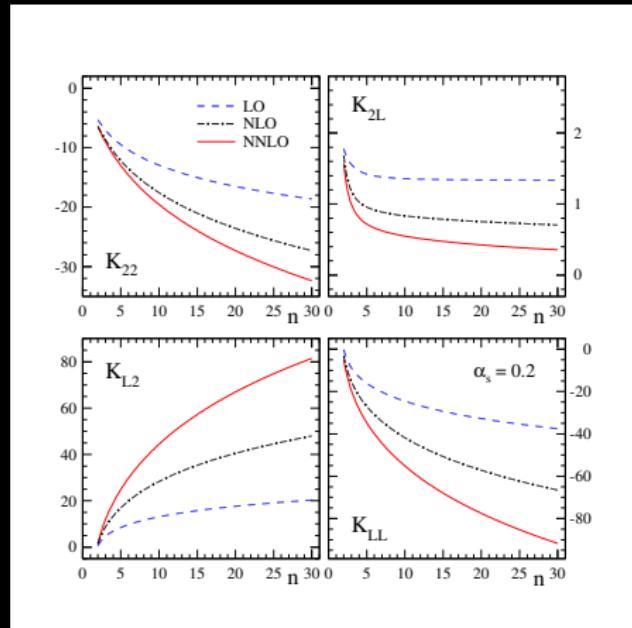
- ▶ convent. DGLAP: $\mathcal{O}(a_s a_0)$, $\mathcal{O}(a_s^2)$ – final scale Q^2
- ▶ phys. evolution: $\mathcal{O}(a_s a_0)$, $\mathcal{O}(a_0^2)$ – intital scale Q_0^2
- ▶ can exclude them in toy input → precise agreement
(higher order zero solution [Glück, Grassie, Reya; PRD 30, 1447 (1984)],
[Glück, Reya, Vogt; PRD 46 (1992)])

K-factors: evolve LO input with LO, NLO, NNLO \simeq real experimental data



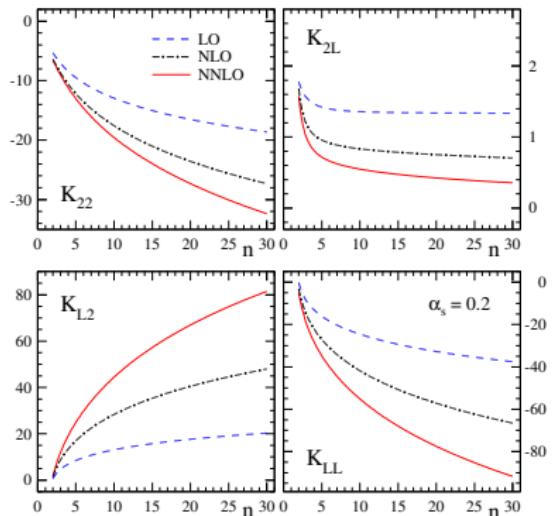
3-loop correction to F_L coefficient very large (esp. at small x) → related instability also reported in pure pdf studies e.g. [Thorne; arXiv:0808.1845]

NLO & NNLO phys. anom. dimensions are large



significantly larger than e.g.
splitting functions

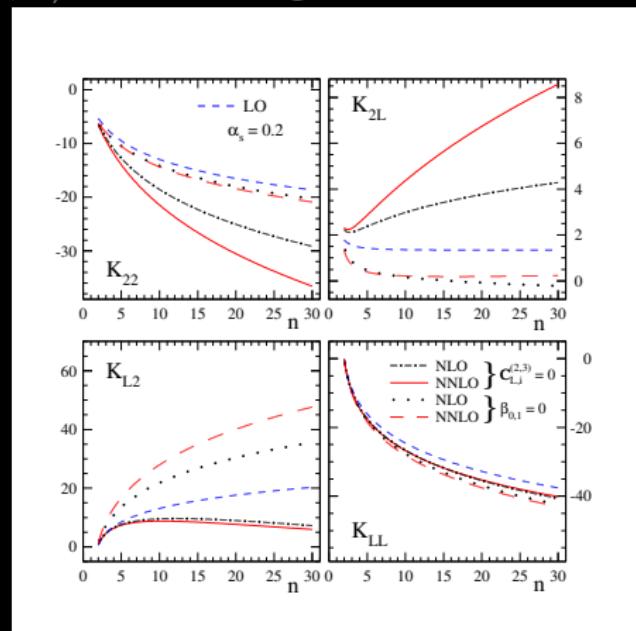
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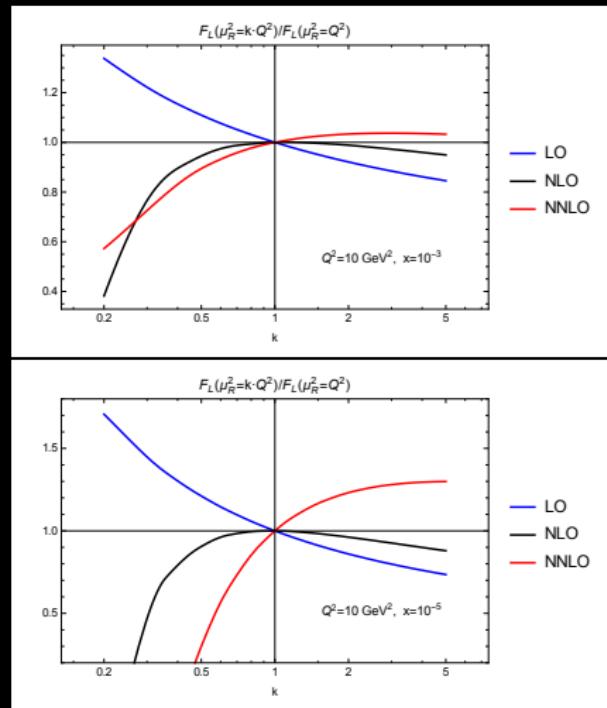
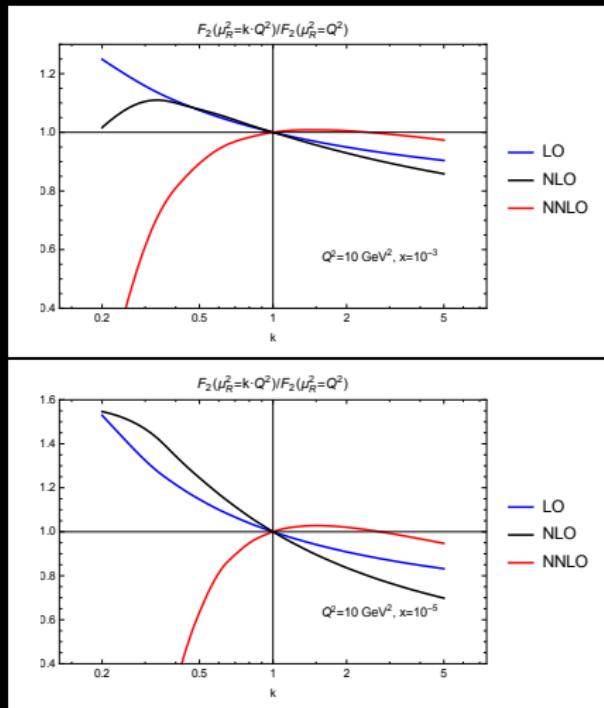
significantly larger than e.g.
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source:

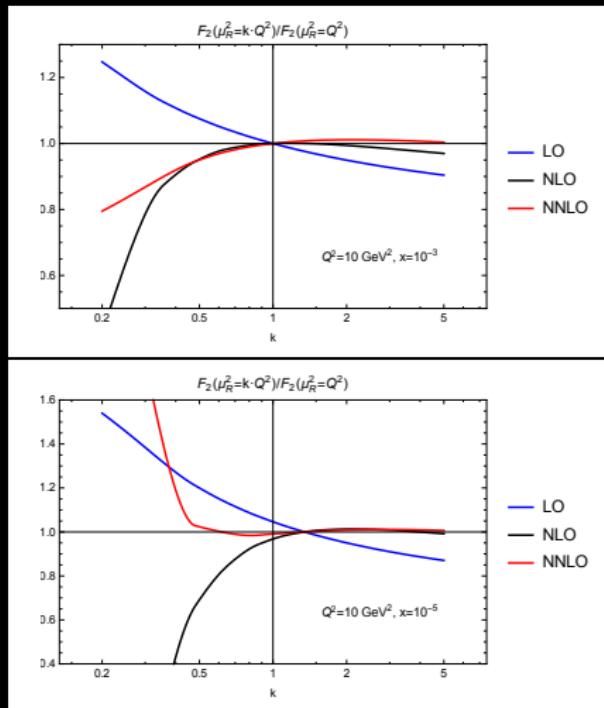
$\beta_{0,1}$ terms & large F_L coefficients



Renormalization scale dependence – (F_2, F_L)



Renormalization scale dependence – (F_2, F_S)



What's going on?

Reason for large scale uncertainty

$$F(Q^2) = \left(\frac{a_s}{a_0}\right)^{R^{(0)}} \left["1" + (a_0 - a_s) R_{\text{K}}^{(1)} + \dots \right] F(Q_0^2)$$

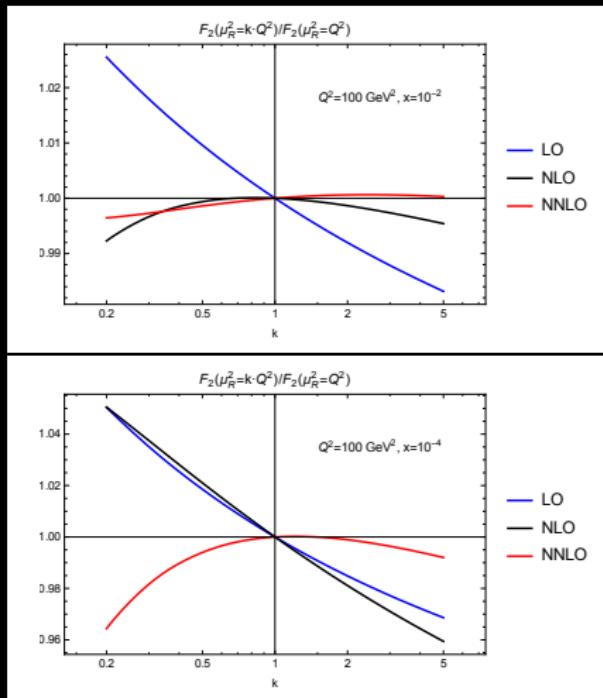
- ▶ large kernels combined at initial scale with $\alpha_s(k \cdot Q_0^2) \in [0.24, 0.81]$
→ problems mainly due to large coupling at initial scale
- ▶ verified this using toy input at $Q_0^2 = 30 \text{GeV}^2$ with $\alpha_s(30 \text{GeV}^2) = 0.2$
(taken from [Moch, Vermaseren, Vogt, PLB 606 (2005) 123])

$$x\Sigma(x) = .6x^{-0.3}(1-x)^{3.5}(1+5x^{0.8})$$

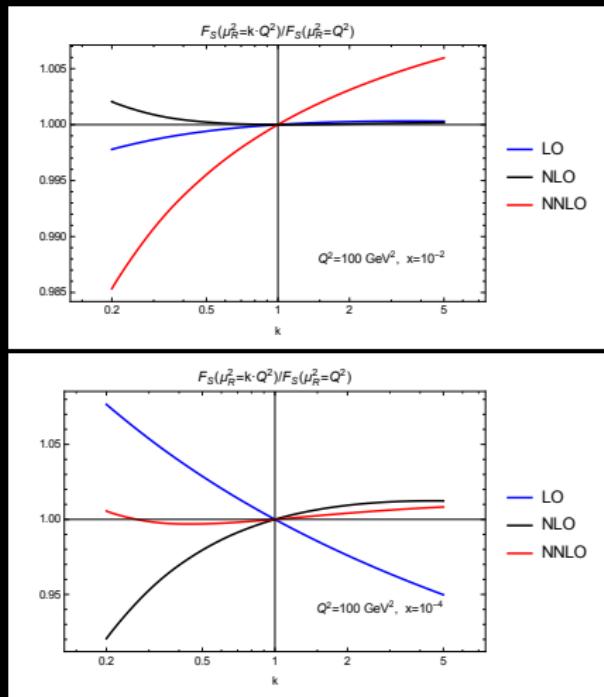
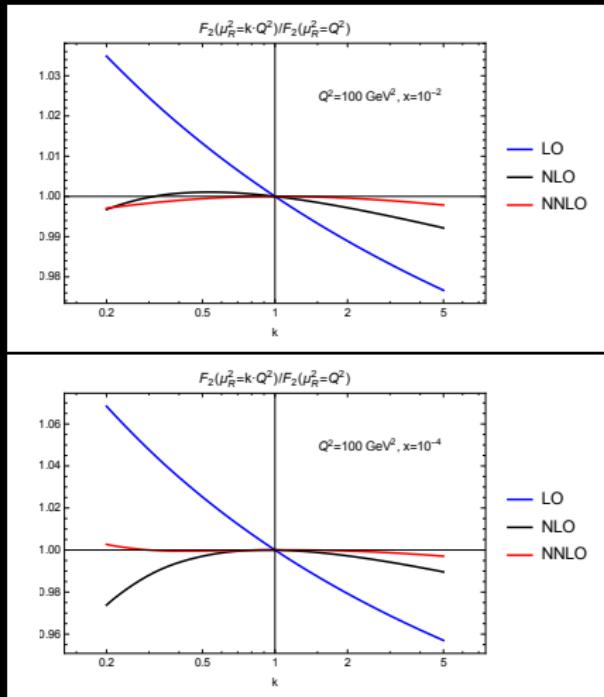
$$xg(x) = 1.6x^{-0.3}(1-x)^{4.5}(1-.6x^{0.3})$$

- ▶ find generally reasonably small renormalization scale dependence
→ not a failure etc. of physical anomalous dimensions
- ▶ but problems at small initial scale $Q_0^2 \simeq 1 - 2 \text{GeV}^2$ remain

Renormalization scale dependence with $Q^2 = 30 \text{ GeV}^2$ input – (F_2, F_L)

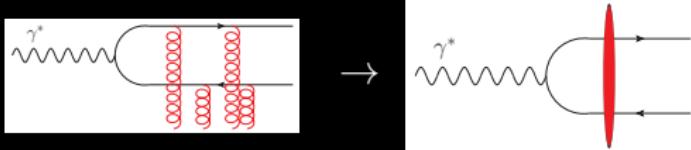


Renormalization scale dependence with $Q^2 = 30 \text{ GeV}^2$ input – (F_2, F_S)



1st application: evolution of saturation model input

Saturation physics \equiv high gluon densities \rightarrow multiple scatterings



$x \rightarrow 0$: a single interaction with strong & Lorentz contracted gluon field

$$\sigma_{L,T}^{\gamma^* A}(x, Q^2) = 2 \sum_f \int d^2 b d^2 r \int_0^1 dz \left| \psi_{L,T}^{(f)}(r, z; Q^2) \right|^2 \mathcal{N}(x, r, b)$$

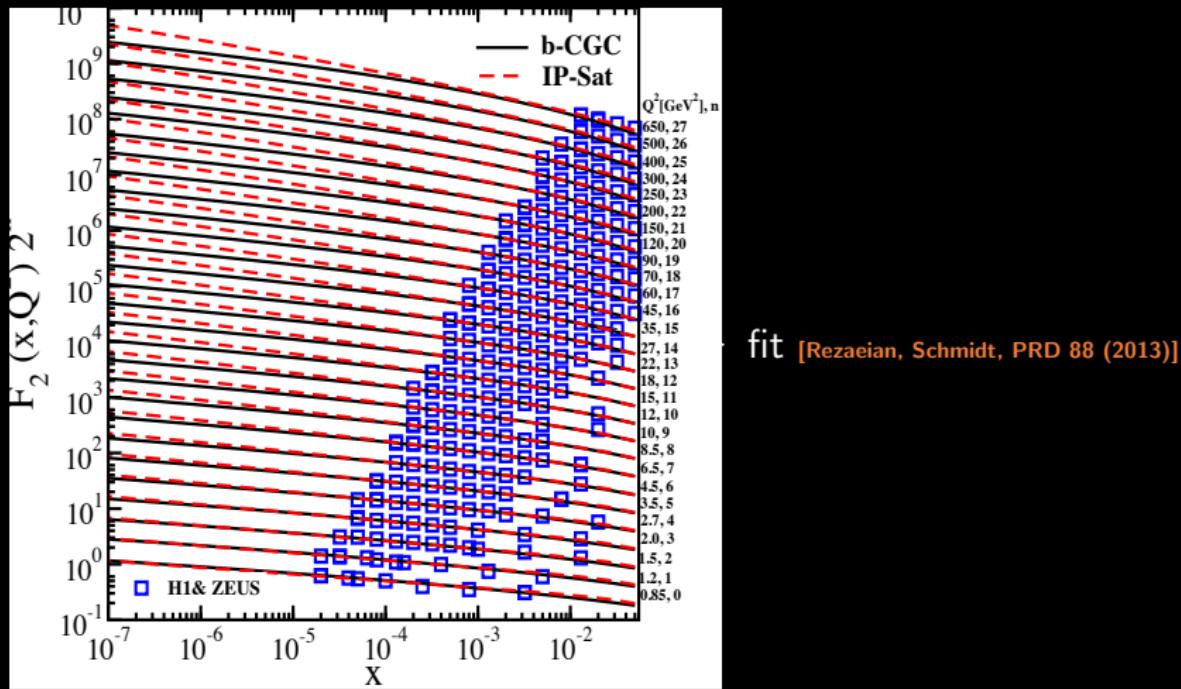
dipole amplitude \mathcal{N} : interaction of color dipole with target;

- (a) solution to BK/JIMWLK evolution equation with fitted input
- (b) model it \rightarrow (b)CGC-model [Iancu, Itakura, Munier; PLB 590 (2004)]

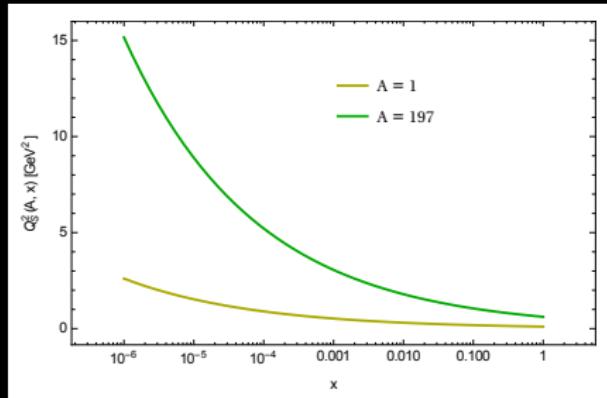
$$\mathcal{N}(x, r, b) = \begin{cases} N_0 \left(\frac{rQ_s}{2} \right)^{2\gamma_{eff}} & rQ_s \leq 2 \\ 1 - e^{-\mathcal{A} \ln^2(\mathcal{B}rQ_s)} & rQ_s > 2 \end{cases} \quad Q_s^2(x) = \left(\frac{x_0}{x} \right)^\lambda \text{GeV}^2$$

$$\gamma_{eff} = \gamma_s + \frac{1}{\kappa \lambda Y} \ln \frac{2}{rQ_s}$$

Recently fitted to combined HERA data



The idea ...

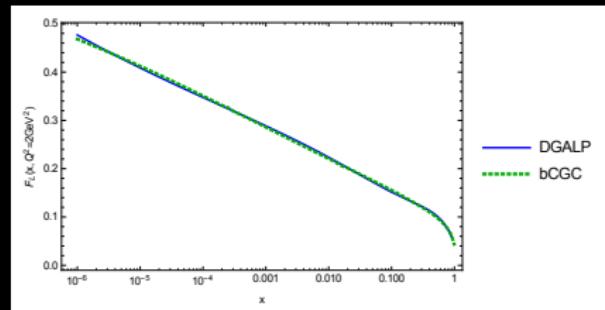


- ▶ simulate DIS on gold nucleus through $Q_s^2 \rightarrow Q_s^2 A^{1/3}$ → strong saturation effects ($Q_s/Q \sim 1$) at accessible x

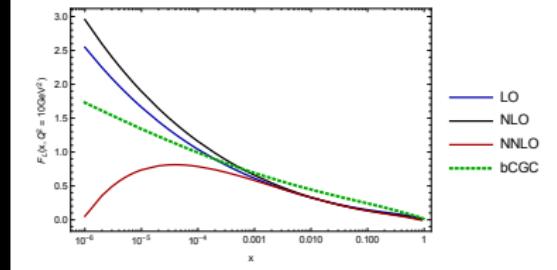
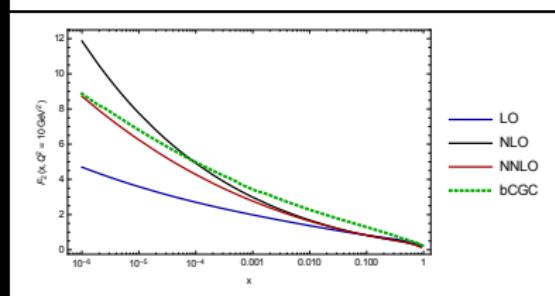
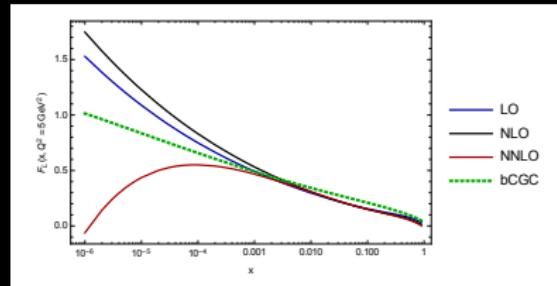
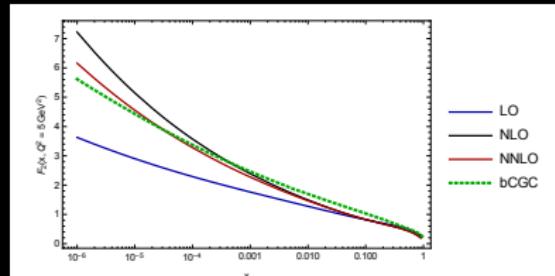
fit x -shape of bCGC at $Q^2 = 2\text{GeV}^2$ to
 $F_i = C(1-x)^b x^a(1+dx^{0.5}+ex+fx^{1.5})$

- ⊕ evolve this input with DGLAP
- ⊕ compare at higher values of Q^2
- deviations ≡ presence of saturation effects

[caveat: physical DGLAP evolution with massless heavy quarks]

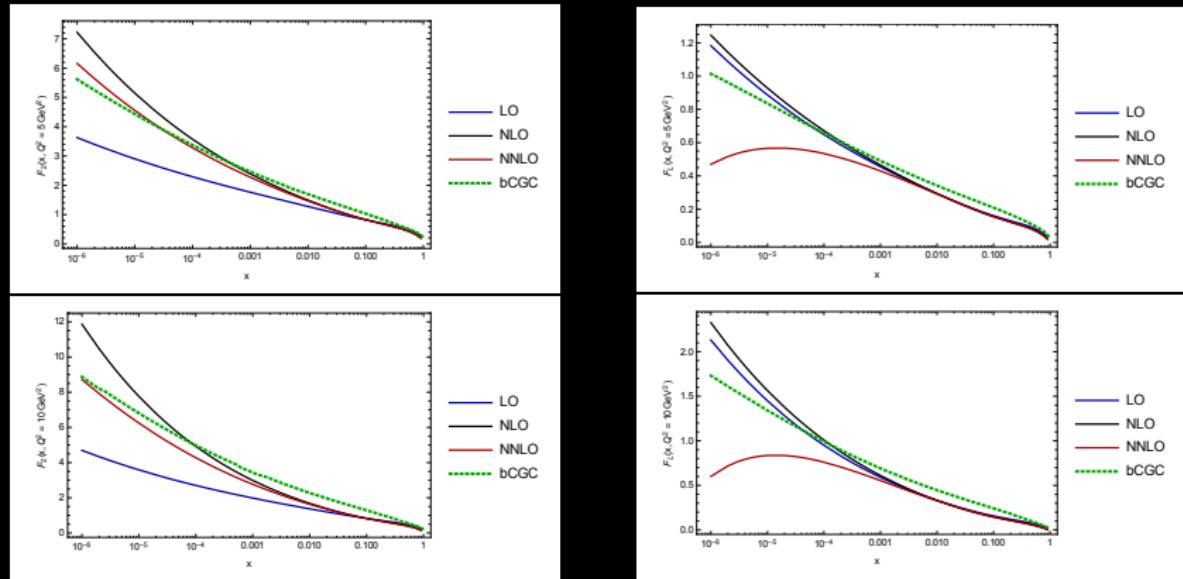


Results for physical evolution of (F_2, F_L)



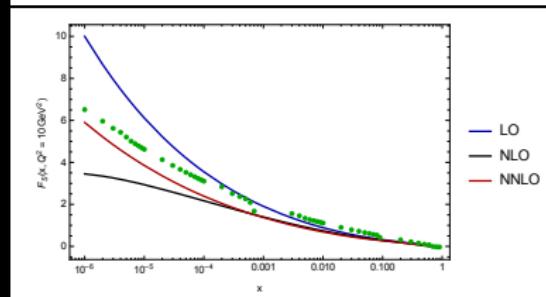
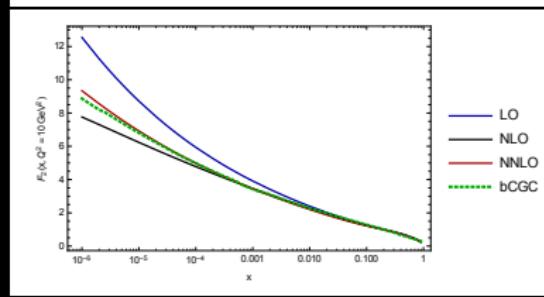
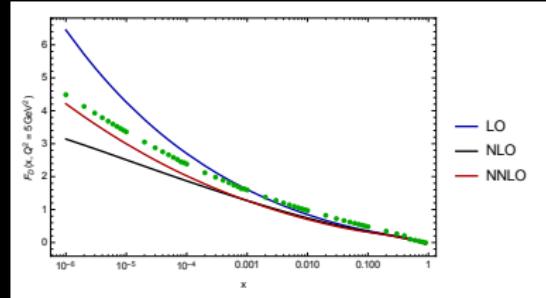
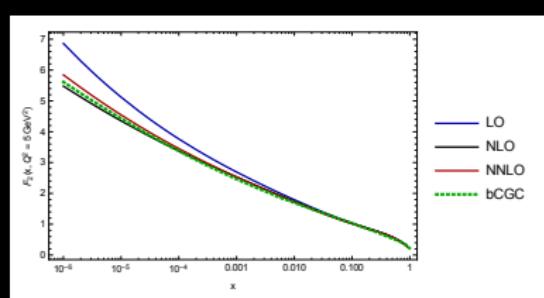
- NLO shows expected pattern: DLGAP overshoots saturation model
– NNLO unstable

Results for physical evolution of (F_2, F_L)



- NLO shows expected pattern: DLGAP overshoots saturation model
– NNLO unstable
- increase initial scale Q_0^2 $2\text{GeV}^2 \rightarrow 3\text{GeV}^2$ gives improvement, but not sufficient
- proper pheno requires resummation etc.

prelim. results for physical evolution of (F_2, F_S)



- slope more stable, even though difference NLO-NNLO still sizeable
- deviation from saturation model rather subtle

Summary & Conclusions

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Summary & Conclusions

- ▶ general: higher order \equiv higher precision only true if the perturbative series converges
- ▶ at moderate x and not too small initial Q^2 : physical evolution is very stable → increase in accuracy over pdf based analysis (absence of factorization scheme & scale ambiguities)
- ▶ instabilities at very small x → BFKL resummation; not really a surprise: $\ln 1/x \simeq 13.8$ for $x = 10^{-6}$ → requires $\alpha_s < 0.07$ for perturbative expansion to be valid; for pdfs:
 - White, Thorne; PRD 75 (2007)
 - Altarelli, Ball, Forte; NPB 742 (2006)
 - Ciafaloni, Colferai, Salam, Stasto; PLB 587 (2004)
- ▶ large correction to kernels associated with running coupling $\beta_{0,1}$ terms
→ another possibility of improvement
- ▶ not addressed: ambiguities due to different solutions of the evolution

Outlook

- ▶ method useful for searches for saturation effects/higher twist etc.
→ increase accuracy of DGLAP approach, but problems remain at small initial scales (should also affect standard pdf based DGLAP!)
- ▶ large enough initial input: useful for determination of running coupling from inclusive DIS; initial studies:
 - van Neerven, Vogt; NPB 568:263 (2000)]
 - Blümlein, Guffanti; AIP Conf.Proc. 792 (2005) 261

future colliders (EIC, LHeC): expect accurate F_L and/or $\frac{dF_2}{d \ln Q^2}$
HERA: scaling violations $\frac{dF_2}{d \ln Q^2}$ desireable