JAM global QCD analysis of spin-dependent parton distributions

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Motivations for extracting spin PDFs (SPDFs)

- Spin sum rule:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g$$

- Large x behavior of SPDFs:

$$q^{\downarrow} \sim (1-x)^2 q^{\uparrow} \quad \rightarrow \quad \Delta q/q \rightarrow 1 \quad \text{as} \quad x \rightarrow 1$$

- Testing the Burkhardt-Cottingham sum rule:

$$\int_0^1 dx \; g_2(x) = 0$$

How to extract SPDFs?

- SPDFs are extracted from cross section asymmetries between different initial state polarizations.
- Inclusive DIS: $e + N \rightarrow e' + X$ (In this talk)
- Semi-Inclusive DIS: $e + N \rightarrow e' + h + X$
- Inclusive Jet production: $p + p \rightarrow J + X$
- Inclusive π^0 production: $p + p \rightarrow \pi^0 + X$
- W boson production: $p + p \rightarrow W \rightarrow l + X$

How to extract SPDFs

- In practice the polarized data covers a small range in Q^2 in comparison with unpolarized data.

- In order to maximally utilize the data, the JAM analysis set the cuts to $Q^2 \geq 1\text{GeV}^2$ and $W^2 \geq 3.5\text{GeV}^2$.

- For such cuts the extraction of g_1 and g_2 is potentially sensitive to:
  - Finite Q^2 corrections: TMC, HT
  - Nuclear corrections for nuclear targets (especially for high-x).

Polarized inclusive DIS

- The measured DIS asymmetries can be described in terms of g_1 and g_2 structure functions

$$A_{||} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Downarrow}} = D(A_1 + \eta A_2)$$

$$A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\Leftarrow}} = d(A_2 - \xi A_1)$$

$$A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \quad A_2 = \gamma \frac{(g_1 + g_2)}{F_1}$$

- In the JAM analysis g_1 and g_2 are parametrized as

$$g_1(x, Q^2) = g_1^{\text{T2+TMC}} + g_1^{\text{T3}} + g_1^{\text{T4}}$$
$$g_2(x, Q^2) = g_2^{\text{T2+TMC}} + g_2^{\text{T3}}$$

Polarized inclusive DIS

$$g_1(x, Q^2) = g_1^{\mathsf{T2+TMC}} + g_1^{\mathsf{T3}} + g_1^{\mathsf{T4}}$$
$$g_2(x, Q^2) = g_2^{\mathsf{T2+TMC}} + g_2^{\mathsf{T3}}$$

- Leading twist @ NLO without TMC

$$g_1^{\mathsf{T2}}(x) = \frac{1}{2} \sum_q e_q^2 \left[\Delta C_{qq} * \Delta q(x) + \Delta C_{qg} * \Delta g(x) \right]$$

- with TMC (Blumlein, Tkabladze NPB 553 427)

$$g_{1,2}^{\mathsf{T2+TMC}}(x) = \zeta_{1,2}^1 \; g_1^{\mathsf{T2}}(\xi) + \zeta_{1,2}^2 \int_\xi^1 \frac{dz}{z} g_1^{\mathsf{T2}}(z) \quad + \zeta_{1,2}^3 \int_\xi^1 \frac{dz}{z} g_1^{\mathsf{T2}}(z) \log(z/\xi)$$

- $\zeta_{1,2}^{1,2,3} = \zeta_{1,2}^{1,2,3}\left(x, \frac{M^2}{Q^2}\right)$ and $\xi = \xi\left(x, \frac{M^2}{Q^2}\right)$

Polarized inclusive DIS

$$g_1(x,Q^2) = g_1^{\text{T2+TMC}} + g_1^{\text{T3}} + g_1^{\text{T4}}$$
$$g_2(x,Q^2) = g_2^{\text{T2+TMC}} + g_2^{\text{T3}}$$

- Twist-3 part of g_2 (PRD 83,094023) with free parameters t_i

$$g_2^{\text{T3}}(x) = t_0\left[\log x + (1-x) + \frac{1}{2}(1-x)^2\right] + \sum_{i=1}^{4} t_i(1-x)^{i+2}$$

- Twist-4 part of g_1 (spline parametrization)

$$g_1^{\text{T4}}(x) = \frac{h(x)}{Q^2}$$

- Twist-3 part of g_1 can be obtained from g_2^{T3} (NPB,553)

$$g_1^{\text{T3}}(x) = \frac{4M^2x^2}{Q^2}\left[g_2^{\text{T3}}(x) - 2\int_x^1 \frac{dy}{y} g_2^{\text{T3}}(x)\right]$$

Nuclear corrections in polarized DIS

- To achieve flavour separation (at least between Δu and Δd) polarized data with deuterium and ^{3}He are used.

- Ignoring Fermi motion the "nuclear" structure functions are given by

$$g_i^A(x,Q^2) = P_{p/A}\, g_i^p(x,Q^2) + P_{n/A}\, g_i^n(x,Q^2)$$

- This is called *Effective Polarization Approximation* (EPA). (no dependence on x)

Nuclear corrections in polarized DIS

- At large x the nuclear smearing plays an important role (PRC 88,5)
- The nuclear corrections can be implemented as convolutions between the nuclear smearing functions (f_{ij}^N) and nucleon structure functions g_i^N

$$g_i^A(x, Q^2) = \sum_N \int \frac{dy}{y} f_{ij}^N(y, \gamma) g_j^N(x/y, Q^2)$$

SPDF parametrization

- For inclusive DIS we only need to parametrize $\Delta q^+ = \Delta q + \Delta \bar{q}$ for $q = u, d, s$ and Δg

- We use the following parametrization

$$\Delta q^+(x) = N x^a (1-x)^b (1+dx)$$

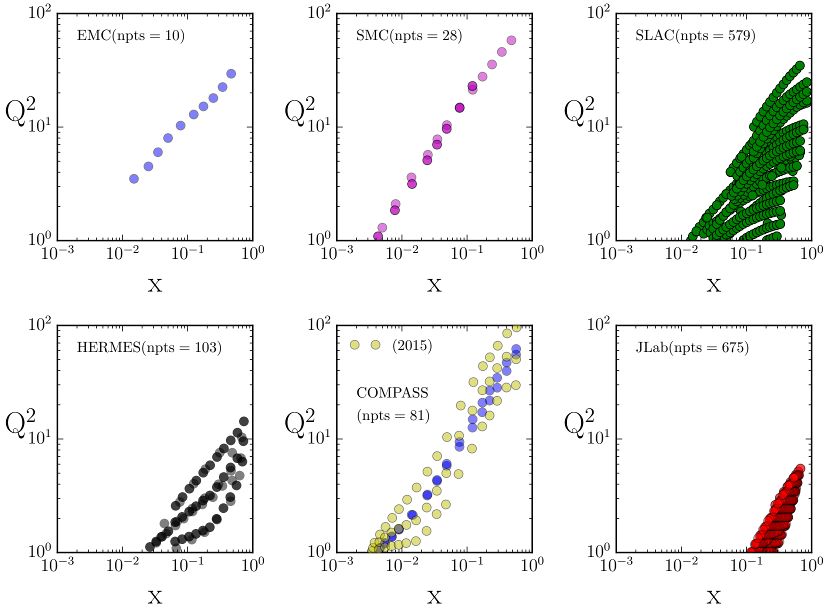
- In practice the inclusive DIS data is insensitive to Δs^+ and Δg. We fix their shape parameters using the results from DSSV collaboration except for the normalizations.

The fitting

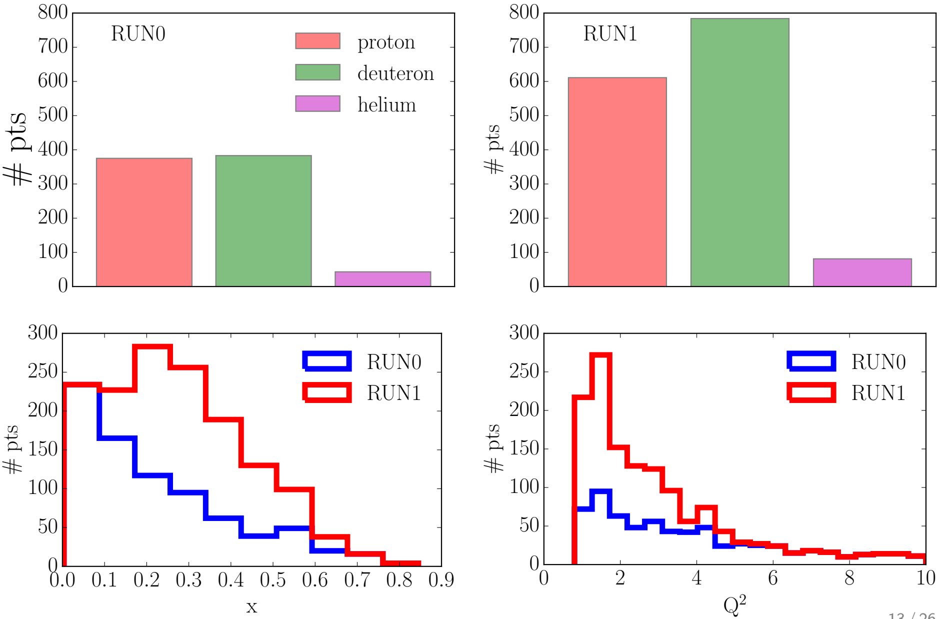
In this work two fits are considered

- RUN0: Global data of inclusive DIS without JLab data. We include the new COMPASS data (arXiv:1503.08935)
- RUN1: RUN0 + JLab data.
- In both runs we fit 26 parameters
  - 10 shape parameters for SPDFs
  - 4+4 parameters for proton T3 and T4
  - 4+4 parameters for neutron T3 and T4

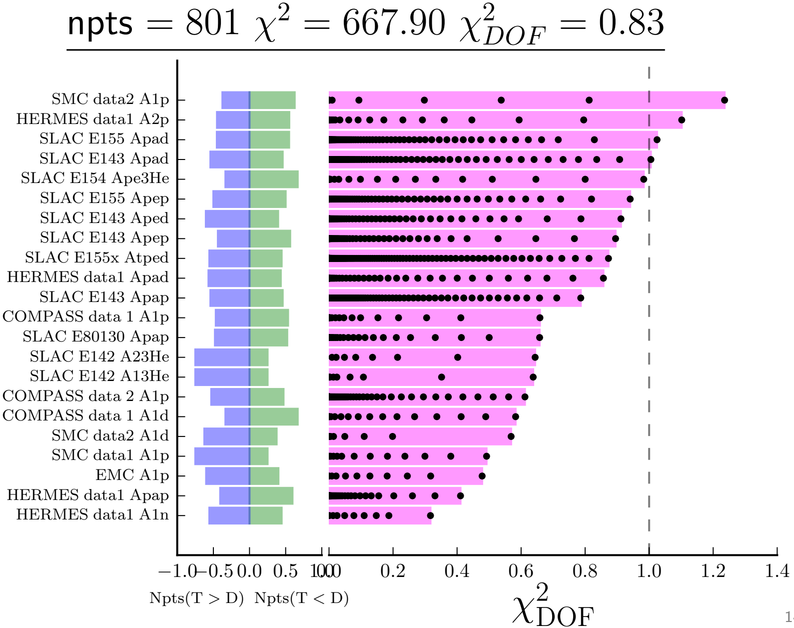
World Data kinematics ($Q^2 > 1GeV^2$, $W^2 > 3.5GeV^2$)



World Data kinematics

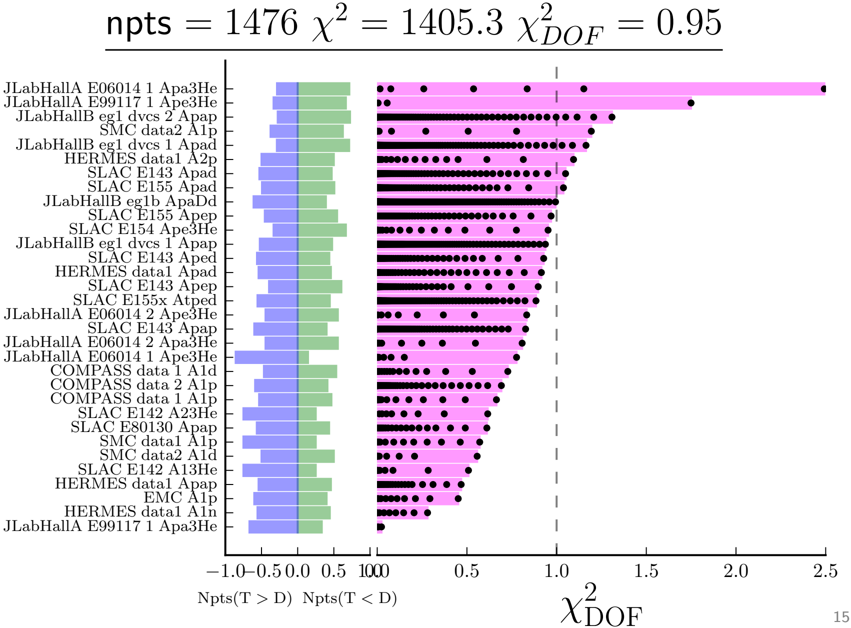


RUN0 fit



npts = 801 χ^2 = 667.90 χ^2_{DOF} = 0.83
SMC data2 A1p
HERMES data1 A2p
SLAC E155 Apad
SLAC E143 Apad
SLAC E154 Ape3He
SLAC E155 Apep
SLAC E143 Aped
SLAC E143 Apep
SLAC E155x Atped
HERMES data1 Apad
SLAC E143 Apap
COMPASS data 1 A1p
SLAC E80130 Apap
SLAC E142 A23He
SLAC E142 A13He
COMPASS data 2 A1p
COMPASS data 1 A1d
SMC data2 A1d
SMC data1 A1p
EMC A1p
HERMES data1 Apap
HERMES data1 A1n
−1.0 −0.5 0.0 0.5 1.0
Npts(T > D) Npts(T < D)
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
χ^2_{DOF}

RUN1 fit



npts = 1476 χ^2 = 1405.3 χ^2_{DOF} = 0.95
JLabHallA E06014 1 Apa3He
JLabHallA E99117 1 Ape3He
JLabHallB eg1 dvcs 2 Apap
SMC data2 A1p
JLabHallB eg1 dvcs 1 Apad
HERMES data1 A2p
SLAC E143 Apad
SLAC E155 Apad
JLabHallB eg1b ApaDd
SLAC E155 Apep
SLAC E154 Ape3He
JLabHallB eg1 dvcs 1 Apap
SLAC E143 Aped
HERMES data1 Apad
SLAC E143 Apep
SLAC E155x Atped
JLabHallA E06014 2 Ape3He
SLAC E143 Apap
JLabHallA E06014 2 Apa3He
JLabHallA E06014 1 Ape3He
COMPASS data 1 A1d
COMPASS data 2 A1p
COMPASS data 1 A1p
SLAC E142 A23He
SLAC E80130 Apap
SMC data1 A1p
SMC data2 A1d
SLAC E142 A13He
HERMES data1 Apap
EMC A1p
HERMES data1 A1n
JLabHallA E99117 1 Apa3He
−1.0 −0.5 0.0 0.5 1.0
Npts(T > D) Npts(T < D)
0.0 0.5 1.0 1.5 2.0 2.5
χ^2_{DOF}

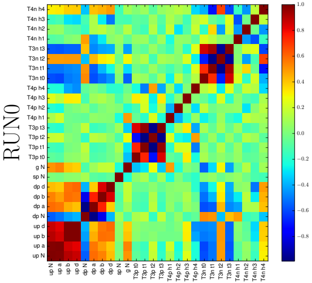
- We use the Hessian Method to propagate the parameter uncertainties.

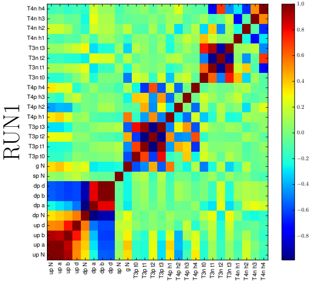
- The Hessian (H) and Covariance matrix (C):

$$\chi^2(\vec{p}) = \sum_i \frac{[\text{Data}_i - \text{Thy}_i(\vec{p})]^2}{\sigma^2_{\text{uncor},i} + \sigma^2_{\text{cor},i}}$$

$$H_{ij} = \frac{1}{2}\frac{\partial \chi^2(\vec{p})}{\partial p_i \partial p_j}\bigg|_{\vec{p}=\text{best}}, \quad C = H^{-1}$$

Correlation matrices for RUN0 and RUN1





- Correlation matrix ρ is defined as $\rho_{i,j} = C_{i,j}/\sqrt{C_{i,i}C_{j,j}}$
- The inclusion of the JLab data induces higher decorrelation between the HT (neutron) parameters and SPDF parameters.
- The uncertainties on the distributions are estimated by diagonalizing the covariance matrix.

The eigen-directions

- The shifts of the parameters from their best values can be parametrized with scale factors $\{t_i\}$ in the eigen-basis of the covariance matrix $\{\hat{e}_i\}$:

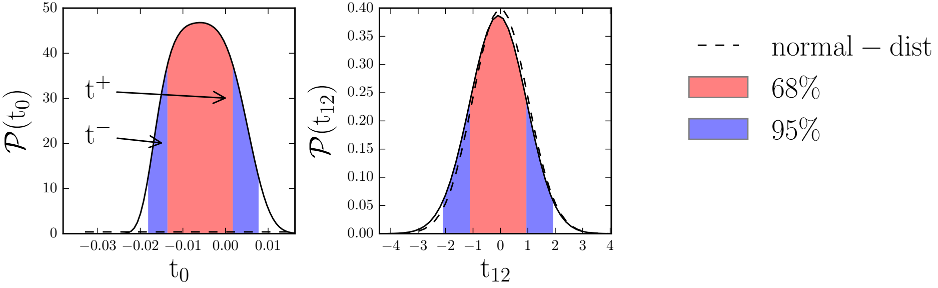
$$\Delta\vec{p} = \vec{p} - \vec{p}_0 = \sum_i t_i \hat{e}_i$$

- The Hessian method assumes that

$$\boxed{\mathcal{P}(\Delta\vec{p}) \approx \prod_i \mathcal{P}_i(t_i) \propto \prod_i \exp\left[-\frac{1}{2}\chi^2(t_i)\right]}$$

- The probability distribution function $\mathcal{P}$ for the shifts decorrelates in the eigen-space

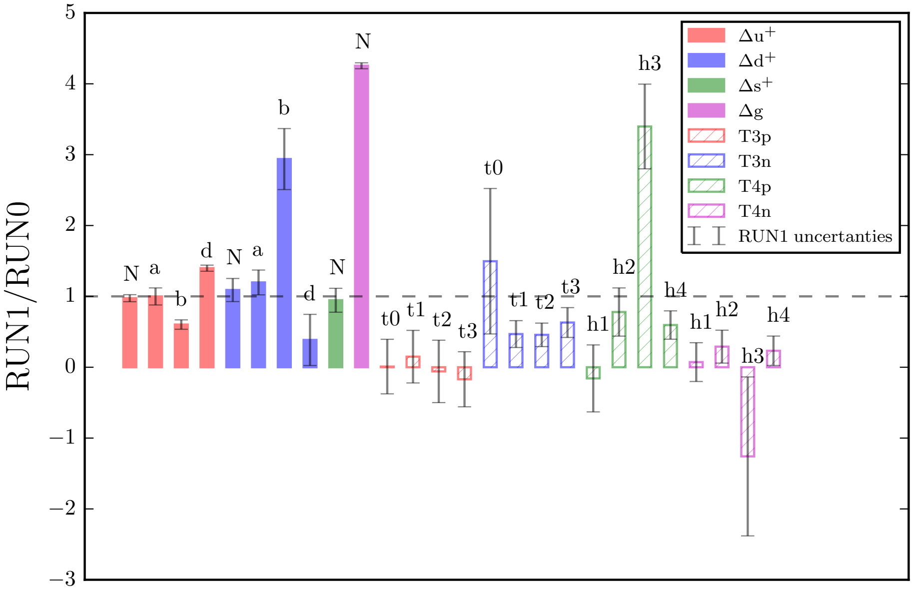
Distributions along eigen-directions



$$\delta\mathcal{O}^2 \simeq \frac{1}{2}\sum_i \left[\mathcal{O}(t_i^+) - \mathcal{O}(t_i^-)\right]^2$$

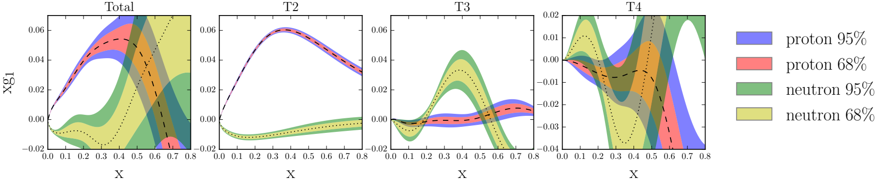
- $t_i^\pm$ are defined as the edges of the confidence region.

RUN1 parameters relative to RUN0

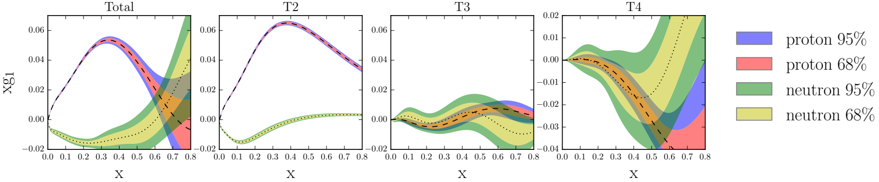


The extracted g_1 structure function at $Q^2 = 1\text{GeV}^2$

RUN0



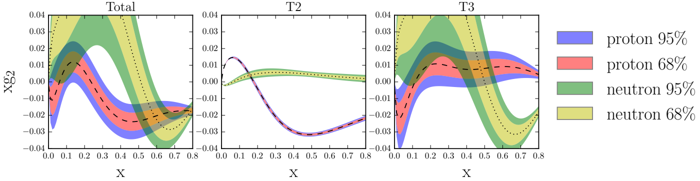
RUN1



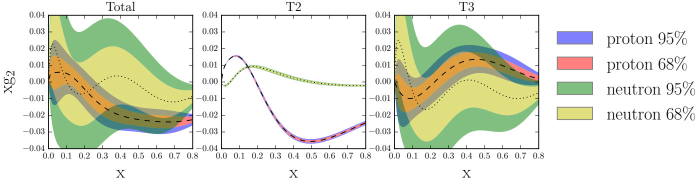
- Significant reduction of the uncertainties in g_1^{TOTAL}
- Most of the uncertainties are due to HT contributions.

The extracted g_2 structure function at $Q^2 = 1\text{GeV}^2$

RUN0



RUN1



- problem: large variations in the fitted HT → need to check goodness of fit.

Goodness of fit

- We use the likelihood-ratio test to compare the goodness of fit between fits with and without HT.
- The statistics t is defined as

$$\begin{aligned} t &= -2\ln\left(\frac{\mathcal{L}(\text{without HT})}{\mathcal{L}(\text{with HT})}\right) \\ &= \chi^2(\text{without HT}) - \chi^2(\text{with HT}) \end{aligned}$$

- t is assumed to be distributed as the Chi-squared distribution with

$$DOF = DOF(\text{without HT}) - DOF(\text{with HT})$$

Goodness of fit

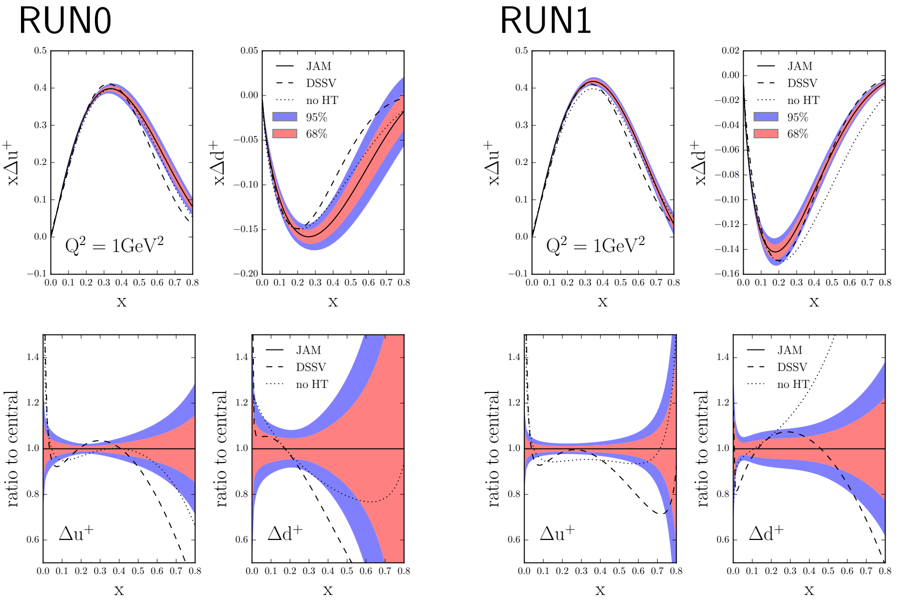
- For a given observed value of t we compute the p-value of its corresponding Chi-squared distribution.

- interpretation: The smaller the p-value, the larger the probability that the data can discriminate against the presence of HT.

	#pts	#params	χ^2	χ^2_{DOF}	p-value
RUN0	801	26	667.90	0.862	0.58
RUN0 w/o HT	801	10	682.11	0.862	
RUN1	1476	26	1405.3	0.969	8.4×10^{-12}
RUN1 w/o HT	1476	10	1492.41	1.018	

- HT is only relevant if we include Jlab data.

The extracted SPDFs at $Q^2 = 1\text{GeV}^2$



Conclusions

- An error propagation for global fits based on the Hessian method was discussed.
- The new JLab data conclusively favors the extraction of the polarized structure functions with HT contributions.

TODO:

- Inclusion of SIDIS and inclusive polarized Jets/pions.
- Extension of the global fit to include unpolarized PDFs fits.