The perturbative Pomeron with NLO accuracy: Jet-Gap-Jet Observables

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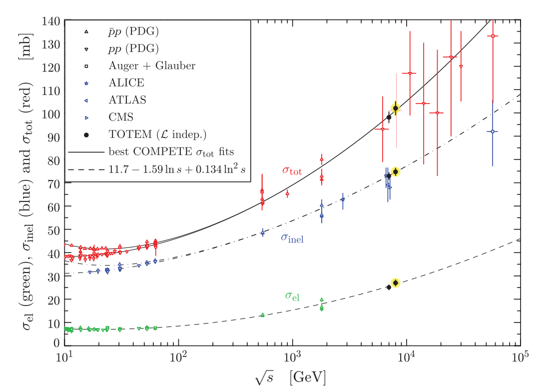
based on common work with

J. D. Madrigal Martínez, B. Murdaca, A. Sabio Vera

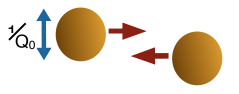
[Phys.Lett.B 735 (2014) 168, Nucl.Phys.B 887 & 889 (2014)]

What is a Pomeron?

proton-proton scattering @ LHC



[TOTEM Collaboration, PRL 111, 012001 (2013)]

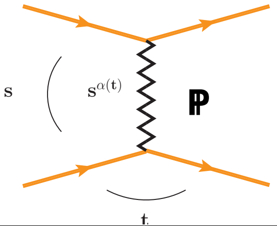


scale $Q_0 \sim \Lambda_{QCD}$

non-perturbative

$$\sigma_{tot} \sim s^{\alpha(0)-1} = s^{0.1}$$

[Donnachie, Landshoff, PLB 727 (2013) 500]



soft Pomeron exchange

perturbative description: BFKL Pomeron

microscopic description in terms of quarks & gluons
→ process with hard scale $Q^2 \gg Q_0^2 \to \alpha_S(Q^2) \ll 1$

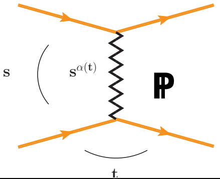
requires:
expansion of perturbative amplitudes in 1/s

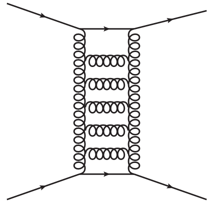
+ resummation of enhanced terms ($\alpha_S(Q^2) \ln s)^n \sim 1$
to all orders in α_S

⇛ BFKL equation

LL: [Fadin, Kuraev, Lipatov, PLB 60 (1975) 50],
[Balitsky, Lipatov, SJNP (1978 822)]

NLL: [Fadin, Lipatov; PLB 429 (1998) 127];
[Ciafaloni, Camici; PLB 430 (1998) 349]



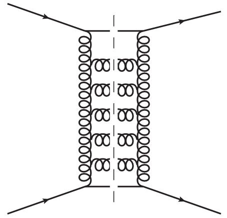


BFKL at cross-section & amplitude level

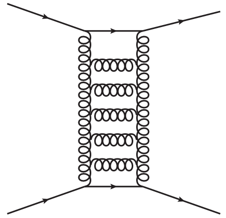
- total cross-section related to elastic scattering amplitude

$$\sigma_{\text{tot}} = \frac{1}{s}\Im\text{m}\mathcal{A}(s, t = 0)$$

- BFKL Pomeron describes also high enery limit of $\mathcal{A}(s, t)$ with vacuum number exchange for *finite* t.

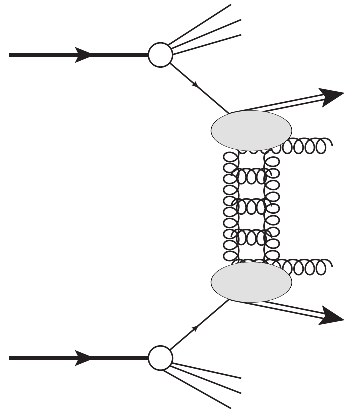


'cut' Pomeron: high multiplicity events (X-sec level)



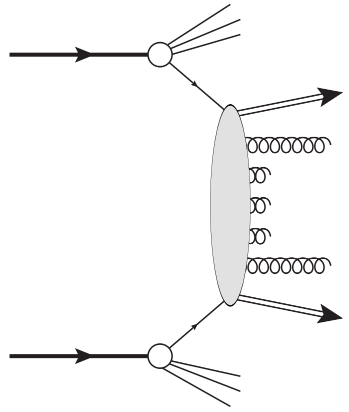
'uncut' Pomeron: diffractive/elastic scattering (amplitude level)

dijets with rapidity gap as a probe of the BFKL Pomeron



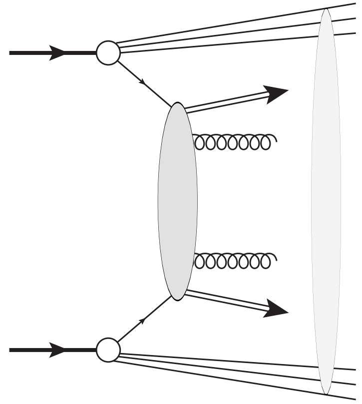
- observable: two jets in the final state separated by a large rapidity
- gap = no hadronic activity in the detectors
- exchange of color charge demands radiation → would destroy gap
- no emission of hadrons → color singlet/vacuum quantum numbers

dijets with rapidity gap - a challenge



- gap is never really empty: experimental resolution E_{gap}
- color charge exchange (=one gluon @ leading order): Sudakov suppressed
- color singlet exchange (= two gluons @ leading order) α_S^2 suppressed

dijets with rapidity gap - a challenge



perturbative treatment: **[Forshaw, Seymour, Siodmok, JHEP 1211 (2012)]**

✓ p_T of jet provides hard scale $\alpha_s(p_T^2) \ll 1 \to$ QCD perturbation theory

BUT: exclusive final state

breakdown of collinear factorization theorems $\to$ uncanceled Coulomb singularities

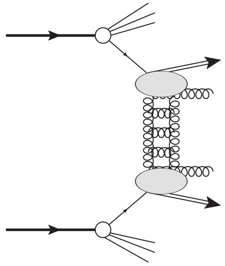
- physics: soft rescattering destroys gap $\to$ rapidity gap survival probability factor

Description using high energy factorization

- probe BFKL formalism
- pQCD: LO = NNLO of inclusive dijets
- high energy expansion in $e^{-\Delta y_{\text{gap}}} \ll 1$: only currently available analytic (= non MC, no model) treatment of such processes
  $\rightarrow$ reduce model dependence + constrain MC generators

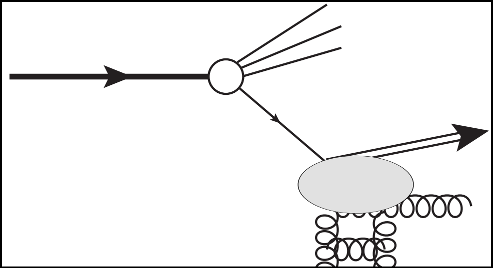
elements of perturbative high energy factorization

description requires two elements



- universal **Green's function**
  - the actual "Pomeron"
  - resums $(\alpha_S \log s)^n$, $(\alpha_S \text{ rapidity})^n$ etc.
  - know up to NLL
- process dependent **impact factors**
  - determines scales (coupling, reggeization scale, coll. factorization, …)
  - NLO corrections known only for a few observables → new: jet+gap+jet

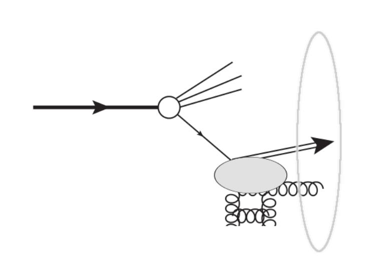
The coupling of the color singlet to the proton-jet system



- High energy factorized amplitude: Coulomb singularities $\Leftrightarrow$ Green's function, t-channel gluons

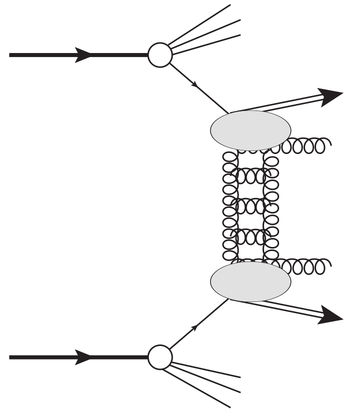
- coupling 2 gluons to proton: finite
  - ★ reason: inklusive Pomeron-hadron cross-section
  - ★ explicitly confirmed at NLO

no elastic scattering, but diffractive dissociation



- observable exclusive in the gap, but inclusive in the forward direction → diffractive system with mass M_X^2 associated with jet
- present already at LO through collinear initial state radiation, NLO explicitly from matrix element

inclusive version: double diffractive dissociation



two observables, one matrix element

- forward-backward jet + rapidity gap
- inclusive version: large t -tail of double-diffractive dissociation
- LO: both are the same!
  NLO: distinct through jet definition

Why NLO corrections ….?

- NLO: realistic jet > 1 parton
- NLO corrections: reduce scale uncertainties due to
  1. renormalization scale μ_R
  2. collinear factorization scale μ_F
  3. reggeization scale s_0

BFKL: NLO corrections generally found to be large → need to calculate them

how to calculate (NLO corrections) to impact factors

inclusive cross-section up to NLL:

- high energy limit of scattering amplitude = single (reggeized) gluon exchange
- factorization into products of single Reggeon exchange & impact factors

color singlet exchange = 2 (reggeized) gluons

- factorization as **convolution** in transverse momentum space

need a tool:

→Lipatov's "gauge invariant high energy effective action"
[Lipatov, Nucl. Phys. B452 (1995)]

High energy effective action [Lipatov, NPB452 (1995)]

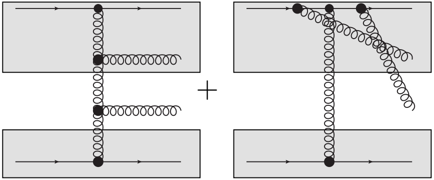
effective action proposed by L. N. Lipatov:

divide final state particles into clusters of particles "local in rapidity"

for each cluster

- integrate out specific details of fast $+/-$ fields
- dynamics in local cluster: QCD Lagrangian $+$ universal eikonal factor

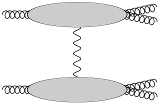
(up to power suppressed corrections)

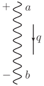


→ effective field theory for **each cluster** of particles local in rapidity

elements of high energy factorized amplitudes

decompose scattering amplitude into gauge invariant subsectors;





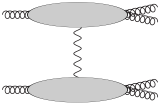
$$= \quad \delta^{ab}\frac{i/2}{\boldsymbol{q}^2}$$

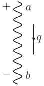
propagator at Born level

connected by the new effective degree of freedom

elements of high energy factorized amplitudes

decompose scattering amplitude into gauge invariant subsectors;





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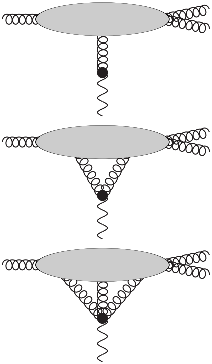


coupling through eikonal operator

$$W_\pm[v] = -\partial_+ \frac{1}{g} U(x)$$

$$U(x) =$$

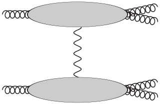
$$\mathrm{P}\exp -\frac{1}{2} \int_{-\infty}^{x_\pm} dx'^\pm v_\pm$$

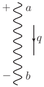
projected on anti- symmetric color octet $\mathbf{8_A}$

automatic at tree-level; projection for loops [MH; NPB 859, 129 (2012)]

elements of high energy factorized amplitudes

decompose scattering amplitude into gauge invariant subsectors;





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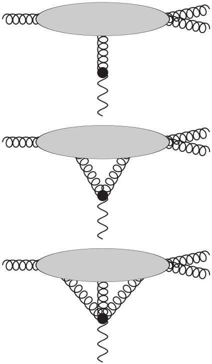

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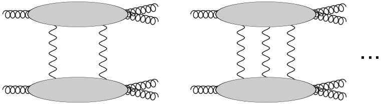
projected on anti- symmetric color octet $\mathbf{8_A}$

automatic at tree-level;

projection for loops [MH; NPB 859, 129 (2012)]

other color channels:

multiple reggeized gluon exchange



Explicit NLO results from Lipatov's effective action

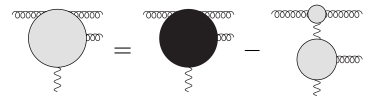
impact factors for inclusive high energy dijets (Mueller-Navelet)

- Quark initiated forward jet at NLO

[MH, Sabio Vera, PRD 85 (2012) 056006]

- Gluon initiated forward jet at NLO

[Chachamis, MH, Madrigal, Sabio Vera, PRD 87 (2013) 076009]



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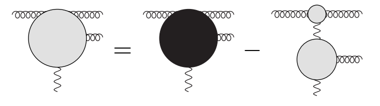
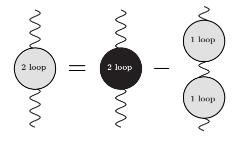


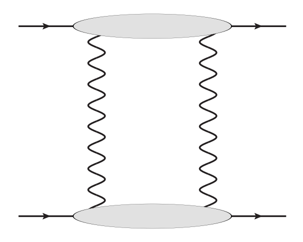
gluon Regge trajectory up to 2-loop

- Quark contribution [Chachamis, MH, Madrigal, Sabio Vera, NPB 861 (2012) 133]
- Gluon contribution [Chachamis, MH, Madrigal, Sabio Veram NPB 876 (2013) 453]



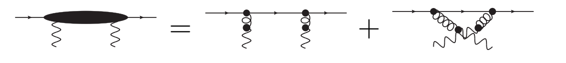
result in full agreement with literature

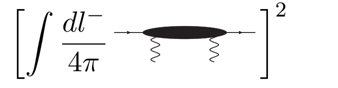
the partonic Mueller-Tang cross-section at Born level



high energy effective action: impact factors from matrix element of 2 reggeized gluon field & partons

- impact factor determined from 1-loop parton-parton amplitude with color singlet exchange
- high energy limit: 2 reggeized gluon exchange





integrate matrix elements over light-cone component of t-channel loop momentum

The LO partonic cross-section

$$h_{MT}^{(0)} = C_f^2 \cdot \frac{\alpha_s^2(\mu^2)}{\mu^{4\epsilon}\Gamma^2(1-\epsilon)(N_c^2-1)}$$

LO impact factor: a constant

cross-section

$$d\sigma_{ab} = h_{aMT}^{(0)} h_{bMT}^{(0)} \left[\int \frac{d^{2+2\epsilon}\boldsymbol{l}_1}{\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2}\right]\left[\int \frac{d^{2+2\epsilon}\boldsymbol{l}_2}{\boldsymbol{l}_2^2(\boldsymbol{k}-\boldsymbol{l}_2)^2}\right] d[\boldsymbol{k}] \qquad d[\boldsymbol{k}] \equiv d^{2+2\epsilon}\boldsymbol{k}$$

in agreement with **[Mueller, Tang; PLB 284 (1992) 123]**

same for gluon with $C_f^2 \leftrightarrow C_a^2$ in $h_{MT}^{(0)}$

transverse integrals divergent

$\rightarrow$ LO: Coulomb singularity only canceled for sum of all color channels

$\rightarrow$ Note: Regge factors $s^{\alpha(t)}$ can improve convergence

$\Rightarrow$ asymptotic finitness for resummed amplitude

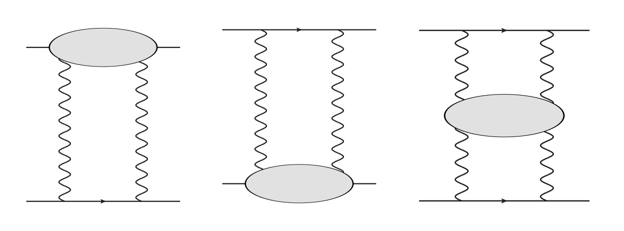
Including the BFKL Green's function

$$\frac{\mathrm{d}\hat{\sigma}_{\mathrm{ij}}}{\mathrm{d}^2\mathrm{k}} = h^{(0)}_{i,\mathrm{a}} h^{(0)}_{j,\mathrm{b}} \int \frac{\mathrm{d}^2\boldsymbol{l}_1 \mathrm{d}^2\boldsymbol{l}'_1}{\pi} \frac{\mathrm{d}^2\boldsymbol{l}_2 \mathrm{d}^2\boldsymbol{l}'_2}{\pi} G\left(\boldsymbol{l}_1, \boldsymbol{l}'_1, \boldsymbol{k}, \frac{s}{s_0}\right) G\left(\boldsymbol{l}_2, \boldsymbol{l}'_2, \boldsymbol{k}, \frac{s}{s_0}\right),$$

BFKL resummation

- replace two (reggeized) gluons by BFKL Green's function G
- contains complete dependence on center of mass energy^2/gap size & universal

types of NLO corrections in the high energy limit



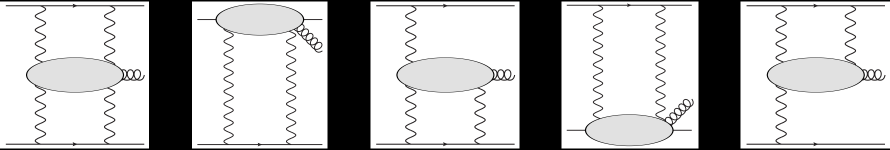
virtual corrections known (NOT from the h.e. effective action approach)

- central corrections: non-forward BFKL kernel (up to NLL) [Fadin, Fiore, PRD 60 (1999), PLB 440 (1998) 359]

  iteration → Green's function

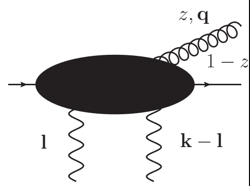
- partonic impact factor at 1-loop [Fadin, Fiore, Kotsky Pap[a, PRD 61 (2000) 094005 & 094006]

types of NLO corrections in the high energy limit



- ★ central production in $\mathbb{1} \otimes \mathbb{1}$ color channel: zero by color algebra
- ★ central production in $\mathbb{1} \otimes \mathbb{8}$ and $\mathbb{8} \otimes \mathbb{1}$ color channel: not present by definition of observable, but important consistency check for our result
- ✓ quasi-elastic production in fragmentation region: real corrections to Mueller-Tang impact factor

differential impact factor at Xsec-level for initial partons j=q,g



$\boldsymbol{l}_{1,2}$: 'Pomeron' transverse loop momenta in amplitude and its complex conjugate resp.

$$h^{(1)}_{r,ij}\mathrm{d}\Gamma^{(2)} = \frac{\mathrm{h}^{(0)}(1+\epsilon)}{\mu^{2\epsilon}\Gamma(1-\epsilon)}\frac{\alpha_{\mathrm{s},\epsilon}}{2\pi}\mathrm{P_{ij}}(\mathrm{z},\epsilon)$$

$$\left[A^{(1)}_{ij}\frac{\boldsymbol{\Delta}}{\boldsymbol{\Delta}^2} - A^{(2)}_{ij}\frac{\boldsymbol{q}}{\boldsymbol{q}^2} - A^{(3)}_{ij}\frac{\boldsymbol{p}}{\boldsymbol{p}^2} - \frac{1}{2}A^{(4)}_{ij}\left(\frac{\boldsymbol{q}-\boldsymbol{l}_1}{(\boldsymbol{q}-\boldsymbol{l}_1)^2} + \frac{\boldsymbol{l}_1-\boldsymbol{p}}{(\boldsymbol{l}_1-\boldsymbol{p})^2}\right)\right]\cdot\left[\{\boldsymbol{l}_1\leftrightarrow\boldsymbol{l}_2\}\right]\mathrm{d}\Gamma^{(2)},$$

$$ij = gq, gg, qg$$

$$P_{gq}(z,\epsilon) = C_f\frac{1+(1-z)^2+\epsilon z^2}{z} \quad P_{gg}(z,\epsilon) = 2C_a\frac{(1-z(1-z))^2}{z(1-z)} \quad P_{qg}(z,\epsilon) = \frac{1}{2}\left(1-\frac{2z(1-z)}{1+\epsilon}\right)$$

$$A^{(k)}_{gq} = \frac{1}{1+\epsilon}\left(C_f, C_f, C_a, C_a\right) \quad A^{(k)}_{gg} = \frac{1}{2!}\left(C_a, C_a, C_a, C_a\right) \quad A^{(k)}_{gq} = \left(C_a, C_f, C_f, 2(C_f - C_a)\right)$$

$$\mathrm{d}\Gamma^{(2)} = \mathrm{dzd}^{2+2\epsilon}\mathrm{q}/\pi^{1+\epsilon} \quad \Delta = \mathrm{q}-\mathrm{zk}$$

how to obtain a jet

jet: select a sub-set of partons in the forward region
formal: jet function S_J & jet phase space $\mathrm{dJ} = \mathrm{dy}\ \mathrm{d}^2\boldsymbol{p}_T$

$$\frac{d\hat{\sigma}_J}{dJ_1 dJ_2 d^2\boldsymbol{k}} = d\hat{\sigma} \otimes S_{J_1} S_{J_2}$$

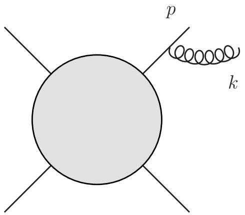
LO: identify final state parton with jet

$$S_J^{(2)}(\boldsymbol{p}, x) = x\delta\left(x - \frac{|\boldsymbol{k}_J| e^{y_J}}{\sqrt{s}}\right)\delta^{2+2\epsilon}(\boldsymbol{p} - \boldsymbol{k})$$

$$\begin{aligned} S_J^{(3,\mathrm{cone})}(\vec{k}_2, \vec{k}_1, x\beta_1; x) = {} & S_J^{(2)}(\vec{k}_2; x(1-\beta_1))\Theta\left(\left[\Delta y^2 + \Delta\phi^2\right] - \left[\frac{|\vec{k}_1| + |\vec{k}_2|}{\max(|\vec{k}_1|, |\vec{k}_2|)} R_{\mathrm{cone}}\right]^2\right) \\ & + S_J^{(2)}(\vec{k}_1; x\beta_1)\Theta\left(\left[\Delta y^2 + \Delta\phi^2\right] - \left[\frac{|\vec{k}_1| + |\vec{k}_2|}{\max(|\vec{k}_1|, |\vec{k}_2|)} R_{\mathrm{cone}}\right]^2\right) \\ & + S_J^{(2)}(\vec{k}_1 + \vec{k}_2; x)\Theta\left(\left[\frac{|\vec{k}_1| + |\vec{k}_2|}{\max(|\vec{k}_1|, |\vec{k}_2|)} R_{\mathrm{cone}}\right]^2 - \left[\Delta y^2 + \Delta\phi^2\right]\right), \end{aligned}$$

NLO: jet algorithm to select partons participating in the jet

Collinear & soft singularities



additional emission of massless particle → 2 singular phase space configurations

$$\frac{1}{(p+k)^2} = \frac{1}{E_P E_k (1 - \cos\theta_{pk})}$$

a) collinear $\theta_{pk} = 0$

b) soft $E_k = 0$

soft singularities — Bloch-Nordsieck theorem [Bloch, Nordsieck; Phys. Rev. 52 (1937) 54]; [Yennie, Frautschi, Suura, Ann. Phys. 13 (1961) 379] : cancel against singularities of virtual corrections

final state collinear emission — Kinoshita- Lee-Nauenberg theorem [Kinoshita; J. Math. Phys. 3 (1962) 650], [Lee, Nauenberg; Phys. Rev. 133 (1964) B 1549] : cancel for inclusive observables

initial state collinear emission — counter-term of collinear factorization

infrared finiteness

✓ inclusive impact factor (diffractive dissociation of the proton at large t):
can calculate explicit analytic expression of the NLO impact factor for the limit $M_{X,max}^2 \to \infty$

✓ infra-red finiteness verified

jet impact factor: jet algorithm does not allow for explicit analytic calculations

But: infra-red safety requirements on jet algorithm

→ extract real NLO singularities (phase space slicing) & verify their cancelation

Our result - a compact summary

fully resummed result at hadronic level

$$\frac{\mathrm{d}\sigma_{\mathrm{H_1H_2}}}{\mathrm{dJ_1dJ_2d^2k}} = \frac{1}{\pi^2}\int \mathrm{dl_1dl_1'dl_2dl_2'}\frac{\mathrm{dV(l_1,l_2,k,p_{J,1},y_1,s_0)}}{\mathrm{dJ_1}} \times G\left(\boldsymbol{l}_1,\boldsymbol{l}_1',\boldsymbol{k},\frac{\hat{s}}{s_0}\right) G\left(\boldsymbol{l}_2,\boldsymbol{l}_2',\boldsymbol{k},\frac{\hat{s}}{s_0}\right)\frac{\mathrm{dV(l_1',l_2',k,p_{J,2},y_2,s_0)}}{\mathrm{dJ_2}}$$

new element: jet vertices V

Our result - the NLO jet vertex

$$\frac{\mathrm{dV}}{\mathrm{dJ}} = \sum_{j=\{q,\bar{q},g\}} \int_{x_0}^{1} \mathrm{dx}\, \mathrm{f}_{\mathrm{j/H}}(\mathrm{x}, \mu_\mathrm{F}^2) \left(\frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j}^{(0)}}{\mathrm{dJ}} + \frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j}^{(1)}}{\mathrm{dJ}} \right), \qquad \mathrm{x}_0 = \frac{-\mathrm{t}}{\mathrm{M}^2_{\mathrm{x,max}} - \mathrm{t}}$$

$$\frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j}^{(0)}}{\mathrm{dJ}} = \frac{\alpha_s^2 C_j^2}{N_c^2 - 1} S_J^{(2)}(\boldsymbol{k}, x), \qquad C_{q,\bar{q}} = C_f, \quad C_g = C_a$$

$$\frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j}^{(1)}}{\mathrm{dJ}} = \left(\frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j,\,v}^{(1)}}{\mathrm{dJ}} + \frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j,\,r}^{(1)}}{\mathrm{dJ}} + \frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j,\,UV\,ct.}^{(1)}}{\mathrm{dJ}} + \frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j,\,col.\,ct.}^{(1)}}{\mathrm{dJ}} \right),$$

$$\frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j,v}^{(1)}}{\mathrm{dJ}} = h_{v,j}\; S_J^{(2)}(\boldsymbol{k}, x),$$

$$\frac{\mathrm{d}\hat{\mathrm{V}}_\mathrm{j,r}^{(1)}}{\mathrm{dJ}} = \int d\Gamma^{(2)} \sum_i h_{r,ij}^{(1)}\; S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, zx, x) \;.$$

- virtual corrections extracted from **[Fadin, Fiore, Kotsky, Papa, PRD 61 (2000) 094005 & 094006]**
- ultraviolet & collinear counter-terms: standard $\overline{\mathrm{MS}}$ expressions
- combined impact factor infrared & ultraviolet finite

result defined in terms of the following functions

$$\alpha_s = \alpha_s(\mu^2), \qquad \phi_i = \arccos \frac{\boldsymbol{l}_i \cdot (\boldsymbol{k} - \boldsymbol{l}_i)}{|\boldsymbol{l}_i||\boldsymbol{k} - \boldsymbol{l}_i|},$$

$$P_0(z) = C_a\left[\frac{2(1-z)}{z} + z(1-z)\right], \quad P_1(z) = C_a\left[\frac{2z}{[1-z]_+} + z(1-z)\right], \quad P_{qq}^{(0)}(z) = C_f\left(\frac{1+z^2}{1-z}\right)_+,$$

$$P_{qg}^{(0)}(z) = \frac{z^2 + (1-z)^2}{2} \quad P_{gq}^{(0)}(z) = C_f \frac{1+(1-z)^2}{z}, \quad P_{gg}^{(0)}(z) = P_0(z) + P_1(z) + \frac{\beta_0}{2}\delta(1-z),$$

$$J_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_i, z) = \frac{1}{4}\left[2\frac{\boldsymbol{k}^2}{\boldsymbol{p}^2}\left(\frac{(1-z)^2}{\boldsymbol{\Delta}^2} - \frac{1}{\boldsymbol{q}^2}\right) - \left(\frac{(\boldsymbol{l}_i - z\boldsymbol{k})^2}{\boldsymbol{\Delta}^2(\boldsymbol{q}-\boldsymbol{l}_i)^2} - \frac{\boldsymbol{l}_i^2}{\boldsymbol{q}^2(\boldsymbol{q}-\boldsymbol{l}_i)^2}\right)\right.$$
$$\left. - \left(\frac{(\boldsymbol{l}_i - (1-z)\boldsymbol{k})^2}{\boldsymbol{\Delta}^2(\boldsymbol{p}-\boldsymbol{l}_i)^2} - \frac{(\boldsymbol{l}_i - \boldsymbol{k})^2}{\boldsymbol{q}^2(\boldsymbol{p}-\boldsymbol{l}_i)^2}\right)\right],$$

$$J_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_1, \boldsymbol{l}_2) = \frac{1}{4}\left[\frac{\boldsymbol{l}_1^2}{\boldsymbol{p}^2(\boldsymbol{p}-\boldsymbol{l}_1)^2} + \frac{(\boldsymbol{k}-\boldsymbol{l}_1)^2}{\boldsymbol{p}^2(\boldsymbol{q}-\boldsymbol{l}_1)^2} + \frac{\boldsymbol{l}_2^2}{\boldsymbol{p}^2(\boldsymbol{p}-\boldsymbol{l}_2)^2} + \frac{(\boldsymbol{k}-\boldsymbol{l}_2)^2}{\boldsymbol{p}^2(\boldsymbol{q}-\boldsymbol{l}_2)^2}\right.$$
$$\left. - \frac{1}{2}\left(\frac{(\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{q}-\boldsymbol{l}_1)^2(\boldsymbol{q}-\boldsymbol{l}_2)^2} + \frac{(\boldsymbol{k}-\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{p}-\boldsymbol{l}_1)^2(\boldsymbol{q}-\boldsymbol{l}_2)^2} + \frac{(\boldsymbol{k}-\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{q}-\boldsymbol{l}_1)^2(\boldsymbol{p}-\boldsymbol{l}_2)^2} + \frac{(\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{p}-\boldsymbol{l}_1)^2(\boldsymbol{p}-\boldsymbol{l}_2)^2}\right)\right],$$

and phase space slicing parameter $\lambda \to 0$

partonic result for initial quark

$$\frac{d\hat{V}_q^{(1)}(x,k,l_1,l_2;x_J,k_J;M_{X,\mathsf{max}},s_0)}{dJ} = v^{(0)}\frac{\alpha_s}{2\pi}(Q_1+Q_2+Q_3)$$

$$Q_1 = S_J^{(2)}(\boldsymbol{k},x)C_f^2\Bigg[-\frac{\beta_0}{4}\Bigg\{\Bigg[\ln\left(\frac{\boldsymbol{l}_1^2}{\mu^2}\right)+\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\mu^2}\right)+\{1\leftrightarrow 2\}\Bigg]-\frac{20}{3}\Bigg\}-4C_f+\frac{C_a}{2}$$
$$\Bigg(\Bigg\{\frac{3}{2\boldsymbol{k}^2}\Bigg[\boldsymbol{l}_1^2\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\boldsymbol{l}_1^2}\right)+(\boldsymbol{l}_1-\boldsymbol{k})^2\cdot\ln\left(\frac{\boldsymbol{l}_1^2}{(\boldsymbol{l}_1-\boldsymbol{k})^2}\right)-4|\boldsymbol{l}_1||\boldsymbol{l}_1-\boldsymbol{k}|\phi_1\sin\phi_1\Bigg]-\frac{3}{2}\Bigg[\ln\left(\frac{\boldsymbol{l}_1^2}{\boldsymbol{k}^2}\right)+$$
$$\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\boldsymbol{k}^2}\right)\Bigg]-\ln\left(\frac{\boldsymbol{l}_1^2}{\boldsymbol{k}^2}\right)\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{s_0}\right)-\ln\left(\frac{(\boldsymbol{l}_1-\boldsymbol{k})^2}{\boldsymbol{k}^2}\right)\cdot\ln\left(\frac{\boldsymbol{l}_1^2}{s_0}\right)-2\phi_1^2+\{1\leftrightarrow 2\}\Bigg\}+2\pi^2+\frac{14}{3}\Bigg)\Bigg]$$

$$Q_2 = \int_{z_0}^{1}dz\; S_J^{(2)}(k,zx)\Bigg[\ln\frac{\lambda^2}{\mu_F^2}\left(C_f^2P_{qq}^{(0)}(z)+C_a^2P_{gq}^{(0)}(z)\right)+C_f(1-z)\left(C_f^2-\frac{2C_a^2}{z}\right)+2C_f(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+\Bigg]$$

$$Q_3 = \int_0^1 dz\int\frac{d^2q}{\pi}\Bigg[\Theta\left(\hat{M}_{X,\max}^2-\frac{(p-zk)^2}{z(1-z)}\right)S_J^{(3)}(p,q,(1-z)x,x)C_f^2P_{qq}^{(0)}(z)\Theta\left(\frac{|q|}{1-z}-\lambda^2\right)\frac{k^2}{q^2(p-zk)^2}$$
$$+\Theta\left(\hat{M}_{X,\max}^2-\frac{\boldsymbol{\Delta}^2}{z(1-z)}\right)S_J^{(3)}(\boldsymbol{p},\boldsymbol{q},zx,x)P_{gq}^{(0)}(z)\big\{C_fC_a[J_1(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1)+J_1(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_2)]$$
$$+C_a^2J_2(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2)\Theta(\boldsymbol{p}^2-\lambda^2)\big\}\Bigg].$$

partonic result for initial gluon - part I

$$\frac{d\hat{V}^{(1)}(x, k, l_1, l_2; x_J, k_J; M_{X,\,\max}, s_0)}{dJ} = v^{(0)} \frac{\alpha_s}{2\pi}\big(G_1 + G_2 + G_3\big)$$

$$G_1 = C_a^2\, S_J^{(2)}(\boldsymbol{k}, x)\Bigg[C_a\left(\pi^2 - \frac{5}{6}\right) - \beta_0\left(\ln\frac{\lambda^2}{\mu^2} - \frac{4}{3}\right) + \left(\frac{\beta_0}{4} + \frac{11C_a}{12} + \frac{n_f}{6C_a^2}\right)\left(\ln\frac{\boldsymbol{k}^8}{\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1^2)\boldsymbol{l}_2^2(\boldsymbol{k}-\boldsymbol{l}_2)^2}\right) +$$

$$\frac{1}{2}\Bigg\{C_a\left(\ln^2\frac{\boldsymbol{l}_1^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2} + \ln\frac{\boldsymbol{k}^2}{\boldsymbol{l}_1^2}\ln\frac{\boldsymbol{l}_1^2}{s_0} + \ln\frac{\boldsymbol{k}^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}\ln\frac{(\boldsymbol{k}-\boldsymbol{l}_1)^2}{s_0}\right) - \left(\frac{n_f}{3C_a^2} + \frac{11C_a}{6}\right)\frac{\boldsymbol{l}_1^2 - (\boldsymbol{k}-\boldsymbol{l}_1)^2}{\boldsymbol{k}^2}\ln\frac{\boldsymbol{l}_1^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}$$

$$-\,2\left(\frac{n_f}{C_a^2} + 4C_a\right)\frac{(\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2)^{\frac{1}{2}}}{\boldsymbol{k}^2}\phi_1\sin\phi_1 + \frac{1}{3}\left(C_a + \frac{n_f}{C_a^2}\right)\Bigg[16\frac{(\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2)^{\frac{3}{2}}}{(\boldsymbol{k}^2)^3}\phi_1\sin^3\phi_1$$

$$-\,4\frac{\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2}{(\boldsymbol{k}^2)^2}\left(2 - \frac{\boldsymbol{l}_1^2 - (\boldsymbol{k}-\boldsymbol{l}_1)^2}{\boldsymbol{k}^2}\ln\frac{\boldsymbol{l}_1^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}\right)\sin^2\phi_1 + \frac{(\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2)^{\frac{1}{2}}}{(\boldsymbol{k}^2)^2}\cos\phi_1$$

$$\left(4\boldsymbol{k}^2 - 12(\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2)^{\frac{1}{2}}\phi_1\sin\phi_1 - (\boldsymbol{l}_1^2 - (\boldsymbol{k}-\boldsymbol{l}_1)^2)\ln\frac{\boldsymbol{l}_1^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}\right)\Bigg] - 2C_a\phi_1^2 + \{\boldsymbol{l}_1 \leftrightarrow \boldsymbol{l}_2, \phi_1 \leftrightarrow \phi_2\}\Bigg\}\Bigg]$$

$$G_2 = \int_{z_0}^{1} dz\, S_J^{(2)}(k, zx)\Bigg\{2n_f P_{qg}^{(0)}(z)\left(C_f^2\ln\frac{\lambda^2}{\mu_F^2} + C_a^2\ln(1-z)\right)$$

$$+\, C_a^2 P_{gg}^{(0)}(z)\ln\frac{\lambda^2}{\mu_F^2} + C_f^2 n_f + 2C_a^3 z\left((1-z)\ln(1-z) + 2\left[\frac{\ln(1-z)}{1-z}\right]_+\right)$$

partonic result for the initial gluon - part II

$$G_3 = \int_0^1 \mathrm{dz} \int \frac{\mathrm{d^2q}}{\pi} \Big\{ \mathrm{n_f P_{qg}^{(0)}(z)} \Big[\mathrm{C_a^2} \Theta \left(\mathrm{\hat{M}^2_{X,max}} - \frac{\mathrm{zp^2}}{\mathrm{(1-z)}} \right) \mathrm{S_J^{(3)}(k - zq, zq, zx, x)}$$

$$\left[\frac{\Theta(\boldsymbol{p}^2 - \lambda^2)\boldsymbol{k}^2}{(\boldsymbol{p}^2 + \boldsymbol{q}^2)\boldsymbol{p}^2} + \frac{\boldsymbol{k}^2}{(\boldsymbol{p}^2 + \boldsymbol{q}^2)\boldsymbol{q}^2} \right] - \Theta \left(\hat{M}^2_{X,\max} - \frac{\boldsymbol{\Delta}^2}{z(1-z)} \right) S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, zx, x) \Big(C_a^2 \frac{\boldsymbol{k}^2}{(\boldsymbol{p}^2 + \boldsymbol{q}^2)\boldsymbol{q}^2}$$

$$- 2C_f^2 \frac{\boldsymbol{k}^2 \Theta(\boldsymbol{q}^2 - \lambda^2)}{(\boldsymbol{p}^2 + \boldsymbol{q}^2)\boldsymbol{q}^2} \Big) \Big] + P_1(z) \Theta \left(\hat{M}^2_{X,\max} - \frac{(\boldsymbol{p} - z\boldsymbol{k})^2}{z(1-z)} \right) S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, (1-z)x, x) \frac{(1-z)^2 \boldsymbol{k}^2}{(1-z)^2(\boldsymbol{p} - z\boldsymbol{k})^2 + \boldsymbol{q}^2}$$

$$\left[\Theta \left(\frac{|\boldsymbol{q}|}{1-z} - \lambda \right) \frac{1}{\boldsymbol{q}^2} + \Theta \left(\frac{|\boldsymbol{p} - z\boldsymbol{k}|}{1-z} - \lambda \right) \frac{1}{(\boldsymbol{p} - z\boldsymbol{k})^2} + \Theta \left(\hat{M}^2_{X,\max} - \frac{\boldsymbol{\Delta}^2}{z(1-z)} \right) S_J^{(3)}(\boldsymbol{p}, \boldsymbol{q}, zx, x) \right.$$

$$\left[\frac{n_f}{C_a^2} P_{qg}^{(0)} \Big(J_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_1, \boldsymbol{l}_2) - \frac{\boldsymbol{k}^2}{\boldsymbol{p}^2(\boldsymbol{q}^2 + \boldsymbol{p}^2)} \Big) - n_f P_{qg}^{(0)} \Big(J_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_1, z) \right.$$

$$+ J_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_2, z) \Big) + P_0(z) \Big(J_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_1) + J_1(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_2) + J_2(\boldsymbol{q}, \boldsymbol{k}, \boldsymbol{l}_1, \boldsymbol{l}_2) \Theta(\boldsymbol{p}^2 - \lambda^2) \Big) \Big] \Big\}.$$

That's all nice, but we want numbers ....

How to implement this?

bottleneck:
the BFKL Green's function ...

2 possibilities ...

outlook – getting numbers

1:

- "traditional approach": solution of non-forward BFKL in terms of conformal eigenfunctions → requires projection of result on eigenfunction with so-called "Mueller-Tang prescription"
- requires calculations of non-trivial integrals both for impact factors & Green's function, see *e.g.* [Ross; PLB 668 (2008) 233] → maybe only numerically?

outlook – getting numbers

1:

- "traditional approach": solution of non-forward BFKL in terms of conformal eigenfunctions → requires projection of result on eigenfunction with so-called "Mueller-Tang prescription"
- requires calculations of non-trivial integrals both for impact factors & Green's function, see *e.g.* [Ross; PLB 668 (2008) 233] → maybe only numerically?

2:

Monte-Carlo solution of NLL non-forward BFKL Greens function directly in kT space

- promises higher flexibility & closer to experiment (exclusive distributions!)
- so far successfully explored for LL+running coupling and N=4 maximal supersymmetric Yang-Mills theory [Caporale, Chachamis, Madrigal, Murdaca, Sabio Vera, PLB 724 (2013) 127], [Chachamis, Sabio Vera, Salas, PRD 87 (2013) 016007],[Chachamis, Sabio Vera, PLB 717 (2013 458), PLB 709 (2012) 301]
- fully exclusive description & implementation of NLO jet impact factors is to be achieved

Summary

- microscopic description of the perturbative Pomeron in QCD: high energy factorization & BFKL evolution
- suitable observable for its study in diffraction: jet + gap + jet
- calculated jet vertex for coupling of hard Pomeron to a forward jet at next-to-leading order accuracy $\rightarrow$ all elements needed for a NLO description now available
- major benefits:
    - realistic jet
    - reduce scale & scheme uncertainties