DIS 2015
XXIII International Workshop on
Deep-Inelastic Scattering and
Related Subjects

Dallas, Texas
April 27 – May 1, 2015

WG2 Highlights: Small-x, Diffraction and Vector Mesons

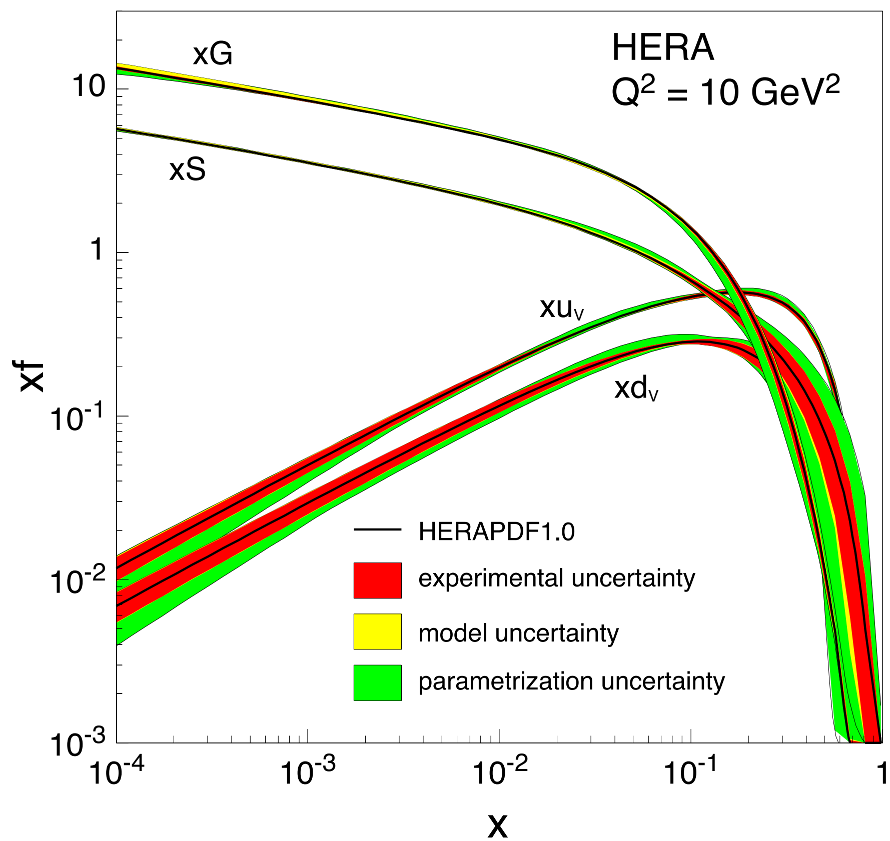
Robert Ciesielski, Yuri Kovchegov,
Eugenio Scapparone

16 experimental and 22 theory talks

Small-x Physics and Saturation

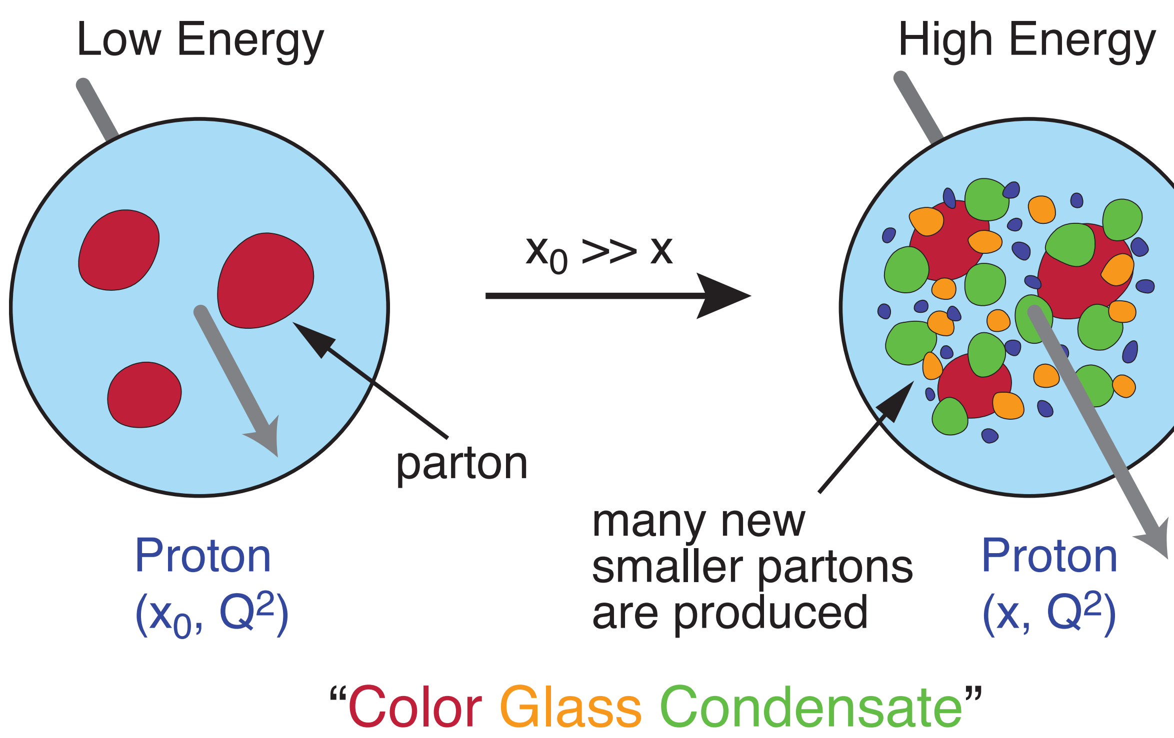
Gluons at Small-x

- There is a large number of small-x gluons (and quarks) in a proton:



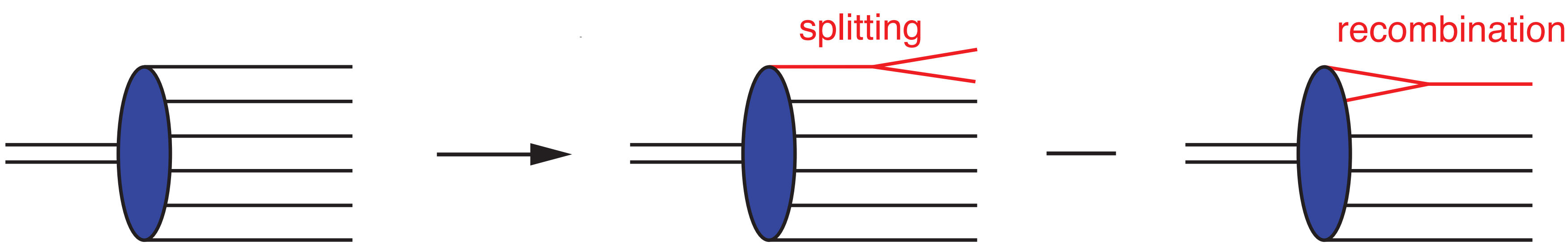
High Density of Gluons

- High number of gluons populates the transverse extent of the proton or nucleus, leading to a very dense saturated wave function known as the Color Glass Condensate (CGC):



Nonlinear Equation

At very high energy gluon recombination becomes important. As energy (rapidity) increases, gluons not only split into more gluons, but also recombine. Recombination reduces the number of gluons in the wave function. Here Y~ln s~ln 1/x is rapidity, s is cms energy.



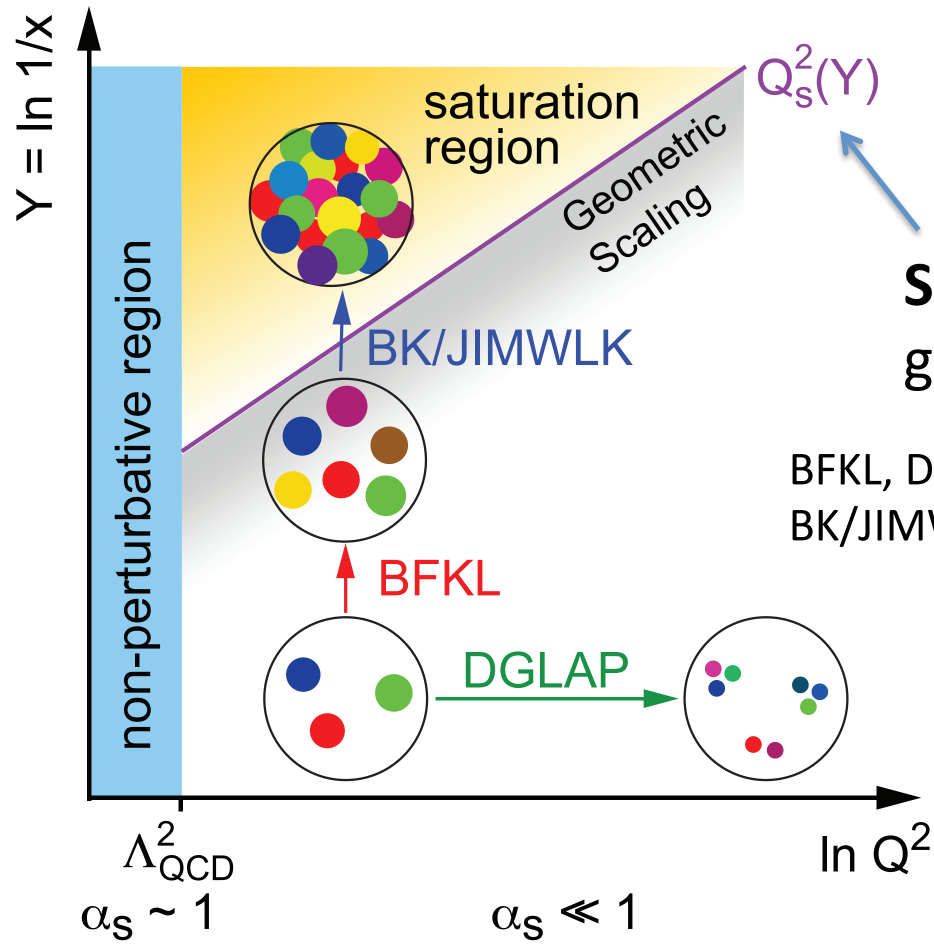
$$\frac{\partial}{\partial Y} N(x, k_T^2) = \alpha_s \, K_{BFKL} \otimes N(x, k_T^2) - \alpha_s \, [N(x, k_T^2)]^2$$

Number of gluon pairs ~ N^2

I. Balitsky ’96, Yu. K. ’99; JIMWLK ‘98-’01

Map of High Energy QCD

energy



Saturation Scale
grows with energy

BFKL, DGLAP – linear equations
BK/JIMWLK – nonlinear

size of gluons

DIS at Small-x

Geometric Scaling (GS)

- One of the predictions of the JIMWLK/BK evolution equations is geometric scaling:

  DIS cross section should be a function of one parameter:

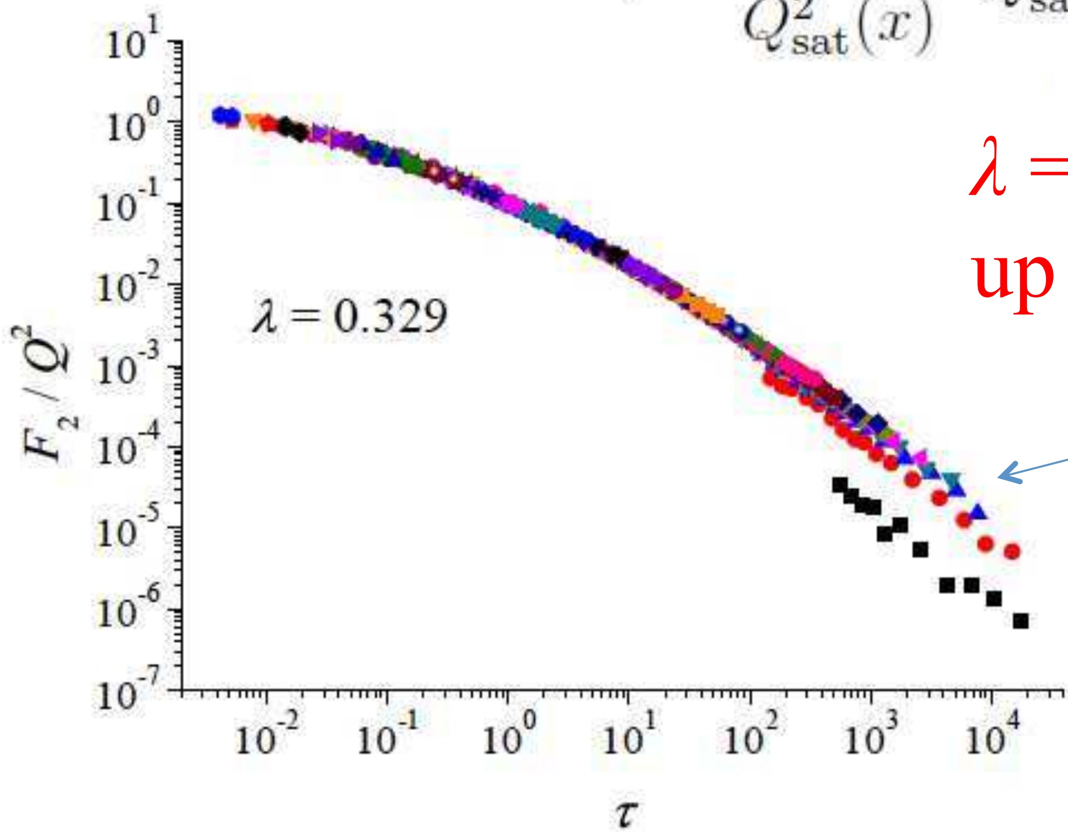
$$\sigma_{DIS}\,(x, Q^2) = \sigma_{DIS}\,(Q^2 / Q_S^2(x))$$

(Levin, Tuchin ' 99; Iancu, Itakura, McLerran ' 02)

Saturation scale: energy and x dependence

talk by Michal PRASZALOWICZ

$$\tau = \frac{Q^2}{Q^2_{\rm sat}(x)} \quad Q^2_{\rm sat}(x) = Q_0^2 \left(\frac{x}{x_0}\right)^{-\lambda}$$



$\lambda = 0.329 \pm 0.005$

up to $x = 0.08$ (!)

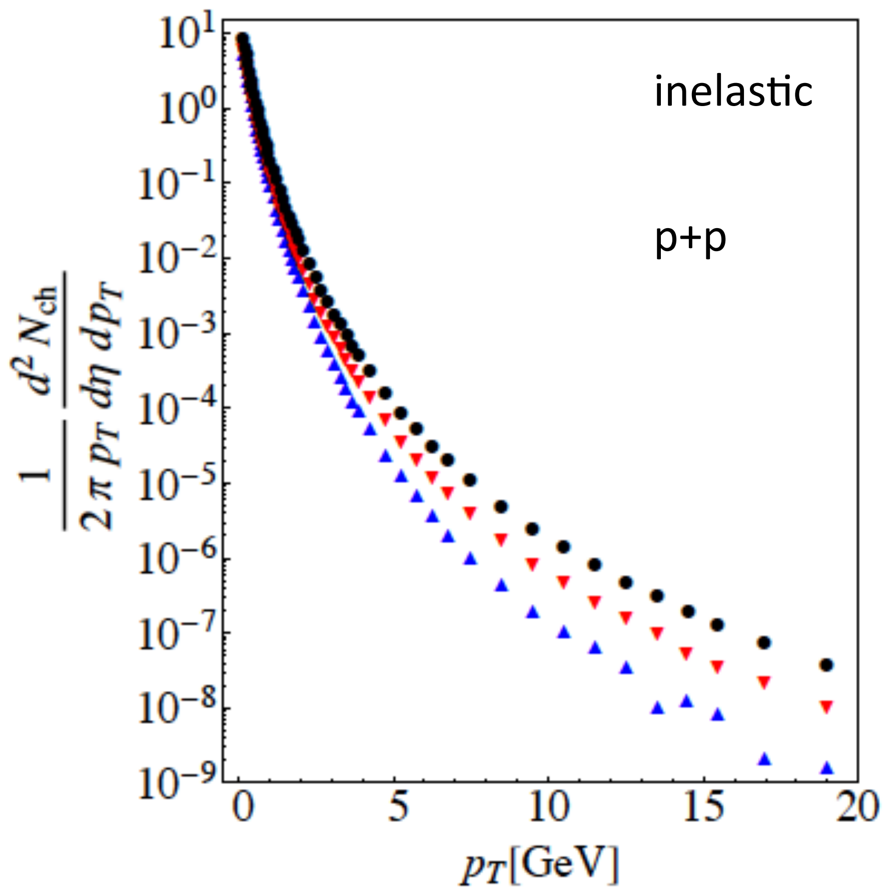
large x

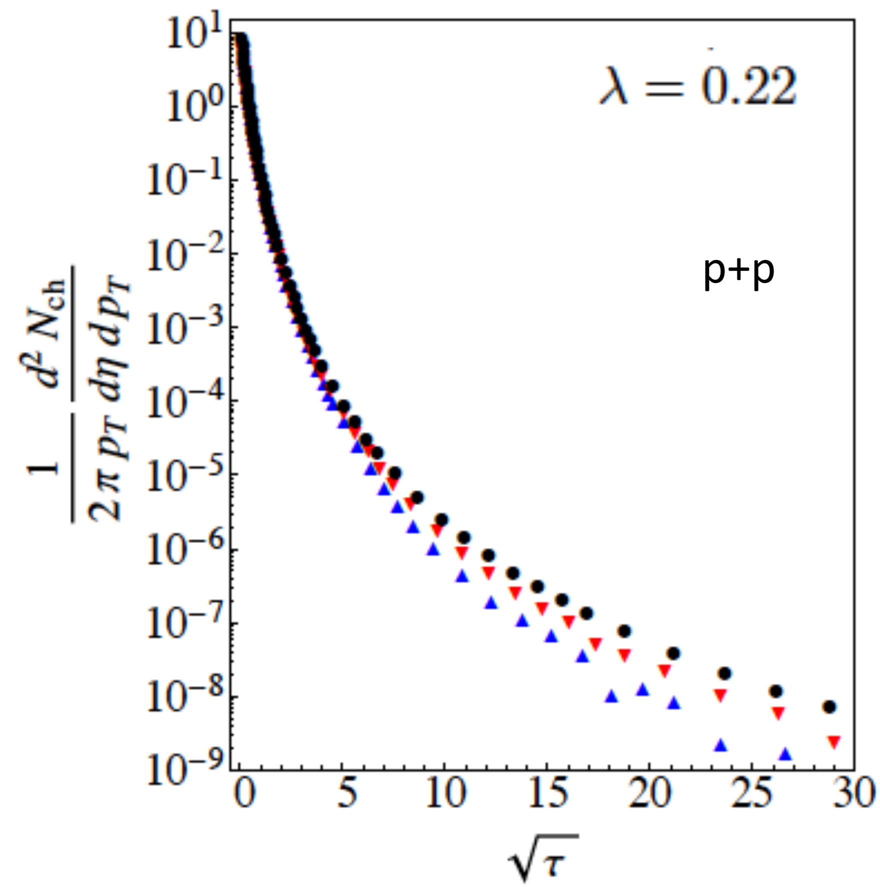
more "sophisticated" scaling variables do not work well talk by Michal PRASZALOWICZ

Determination of lambda

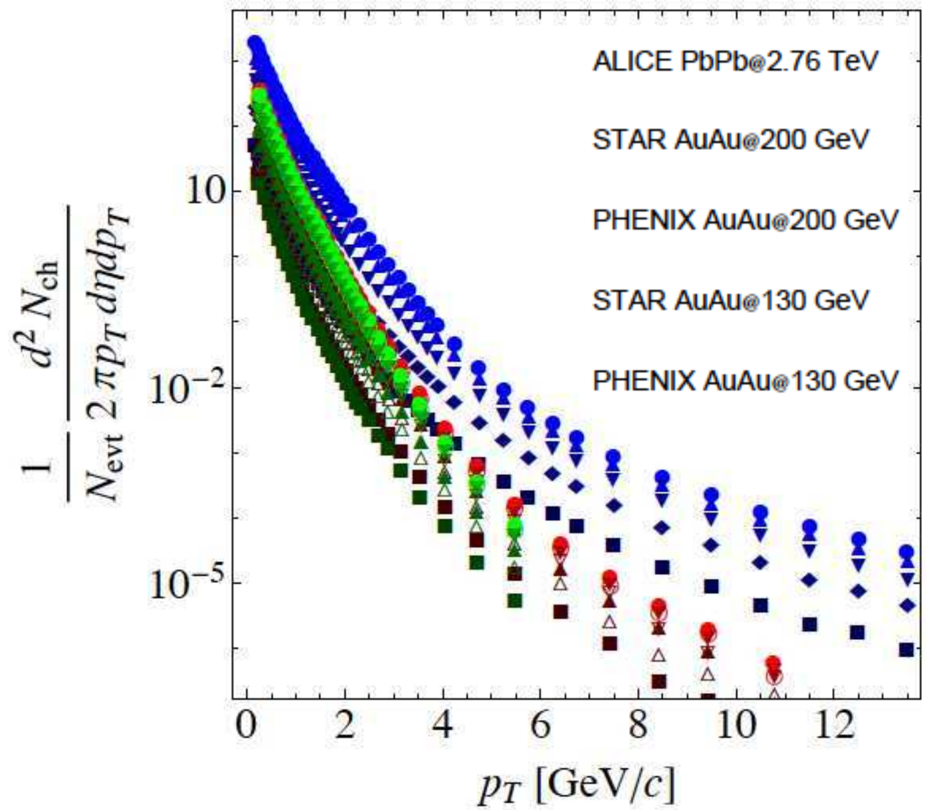
$$\frac{dN_{\rm ch}}{dyd^2p_{\rm T}} = S_\perp \mathcal{F}(\tau) \qquad \tau = \frac{p_{\rm T}^2}{Q_{\rm sat}^2(p_{\rm T}/\sqrt{s})} = \frac{p_{\rm T}^2}{1\,{\rm GeV}^2}\left(\frac{p_{\rm T}}{\sqrt{s}\times 10^{-3}}\right)^\lambda$$

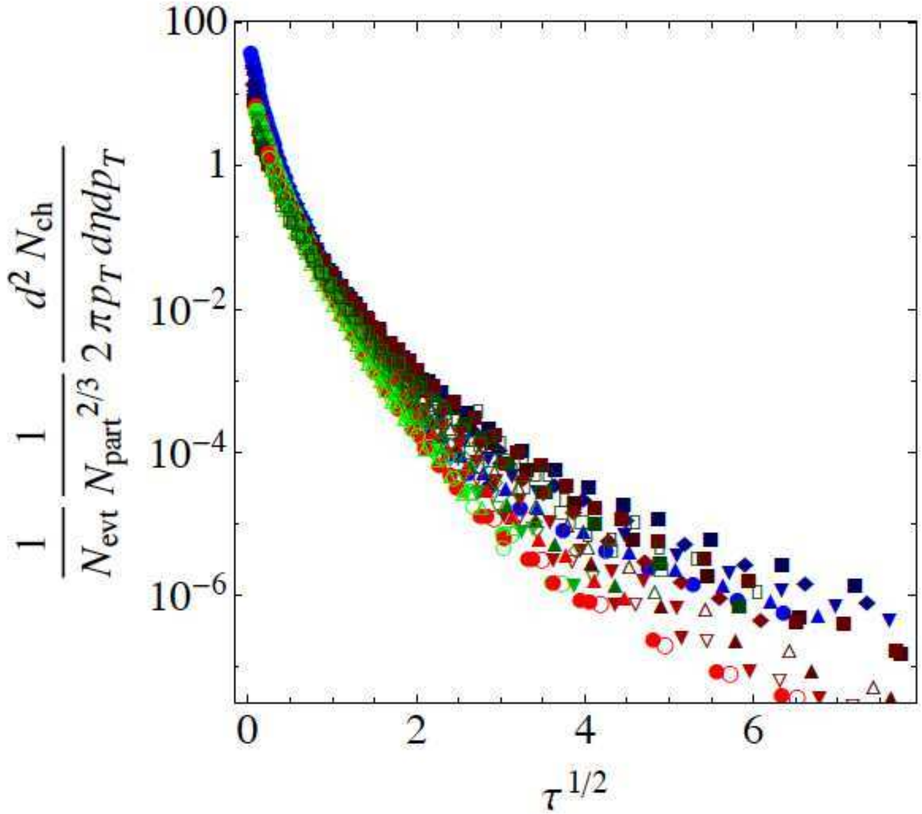
ALICE 1307.1093 [nucl-ex], Eur.Phys.J C73 (2013) 2662





Geometric Scaling in Heavy Ions





talk by Michal PRASZALOWICZ

$$\tau = \frac{p_{\rm T}^2}{Q_{\rm sat}^2(p_{\rm T}/\sqrt{s})} = \frac{p_{\rm T}^2}{1\,{\rm GeV}^2}\left(\frac{p_{\rm T}}{\sqrt{s}\times 10^{-3}}\right)^{\lambda}$$

Small-x Evolution: NLO Corrections

Nonlinear Evolution

$$\frac{\partial}{\partial Y} N(x, k_T^2) = \alpha_s \, K_{BFKL} \otimes N(x, k_T^2) - \alpha_s \, [N(x, k_T^2)]^2$$

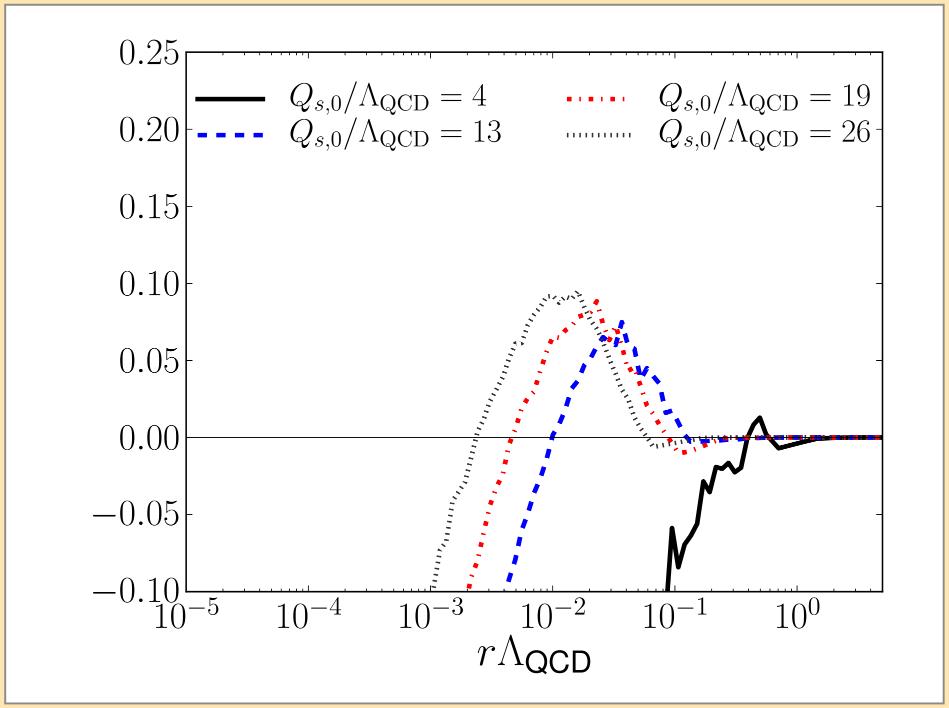
- NLO corrections to BFKL evolution were found in 1998 (Fadin & Lipatov, Catani & Ciafaloni).
- NLO corrections to BK evolution were found in 2007 (Balitsky & Chirilli).
- NLO JIMWLK were found recently and were presented here (Balitsky & Chirilli; Kovner, Lublinsky and Mulian; Grabovsky, all 2013)

talks by M. Lublinsky, A. Grabovsky, G. Chirilli

Can we solve the NLO Nonlinear Evolution Equation?

Tuomas Lappi: for very small dipoles the dipole amplitude decreases with energy, such that numerical solution becomes negative unless one decreases numerical steps in rapidity.

The problem seems to be associated with the certain double-log terms in the NLO evolution kernel.

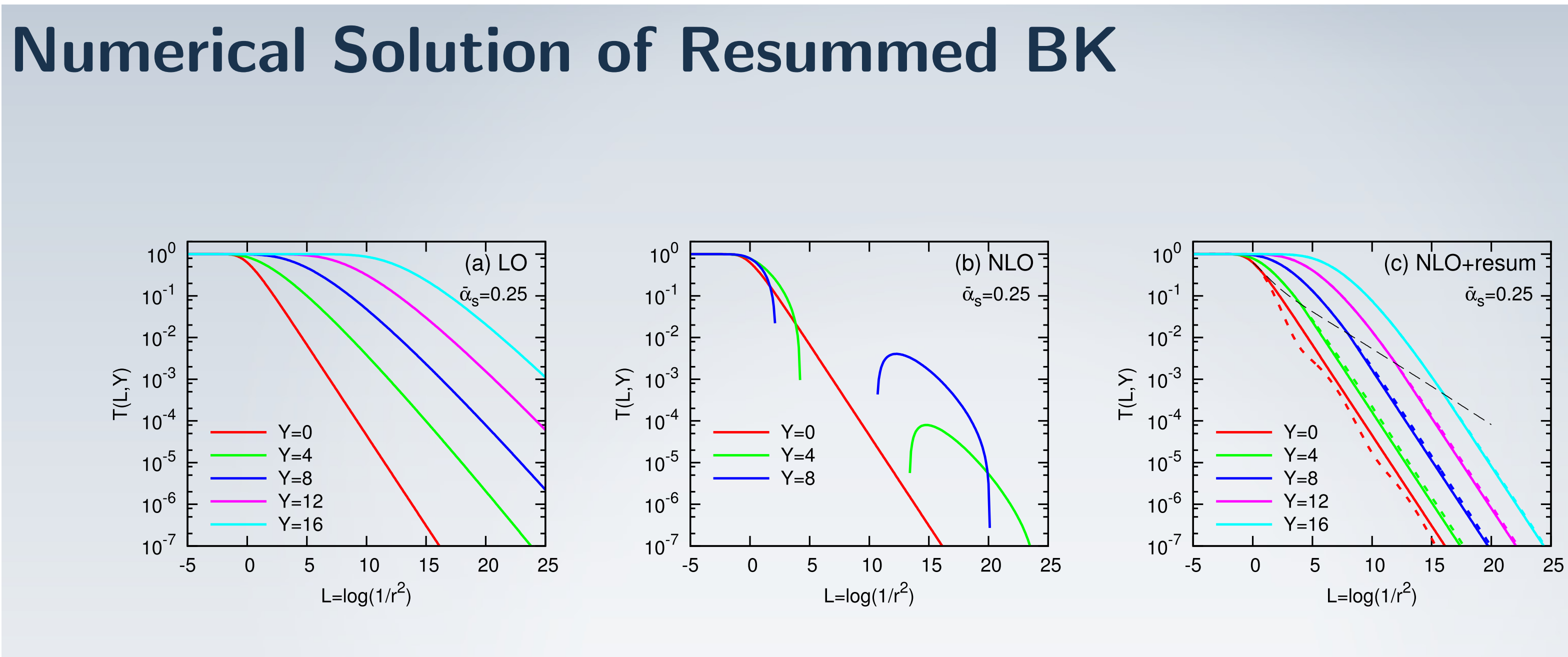


$\partial_y N/N$

Is there a cure?

Jose Madrigal: sum double transverse logarithms in the kernel to all orders:

$$\frac{\partial \tilde{T}_{\boldsymbol{xy}}(Y)}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi}\int \mathrm{d}^2\boldsymbol{z}\,\mathcal{M}_{\boldsymbol{xyz}}\,\mathcal{K}_{\mathrm{DLA}}\left(\sqrt{L_{\boldsymbol{xz}r}L_{\boldsymbol{yz}r}}\right)$$
$$\times\left[\tilde{T}_{\boldsymbol{xz}}(Y)+\tilde{T}_{\boldsymbol{zy}}(Y)-\tilde{T}_{\boldsymbol{xy}}(Y)-\tilde{T}_{\boldsymbol{xz}}(Y)\tilde{T}_{\boldsymbol{zy}}(Y)\right]$$



Particle Production

NLO bites.

Initial Hybrid fits of RHIC data were reasonable.

And then we got greedy (or honest).

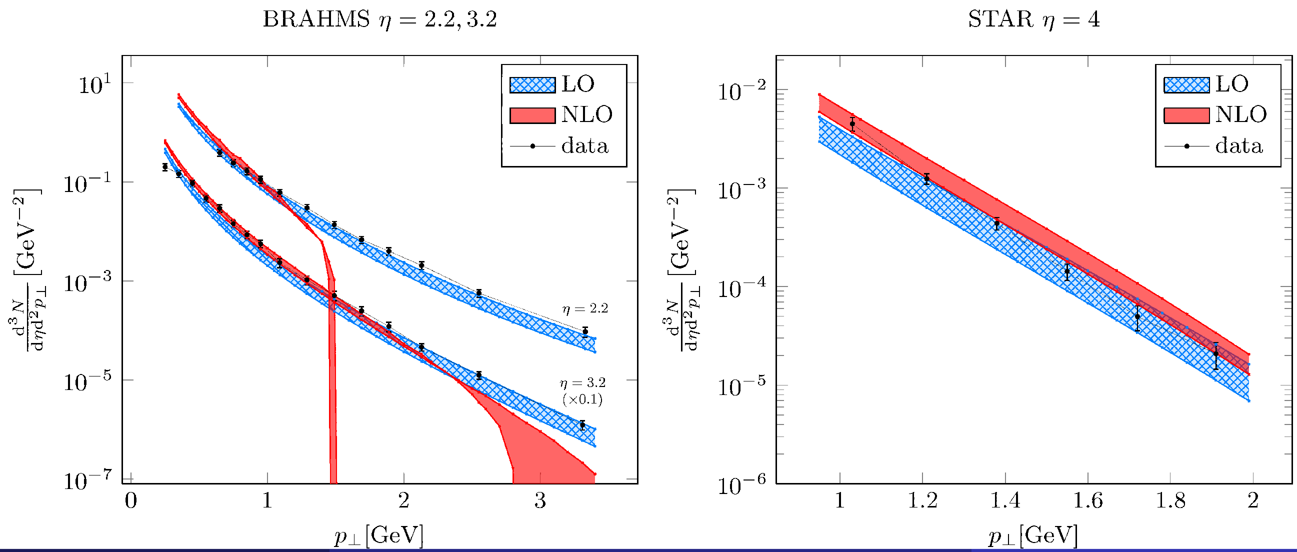
NLO corrections:

T.Altinoluk and A. K. - 2011 Elastic and Inelastic contributions (part of NLO); G.A. Chirilli, B.W. Xiao, F. Yuan - 2012 Full NLO calculation...

Numerical resuts:

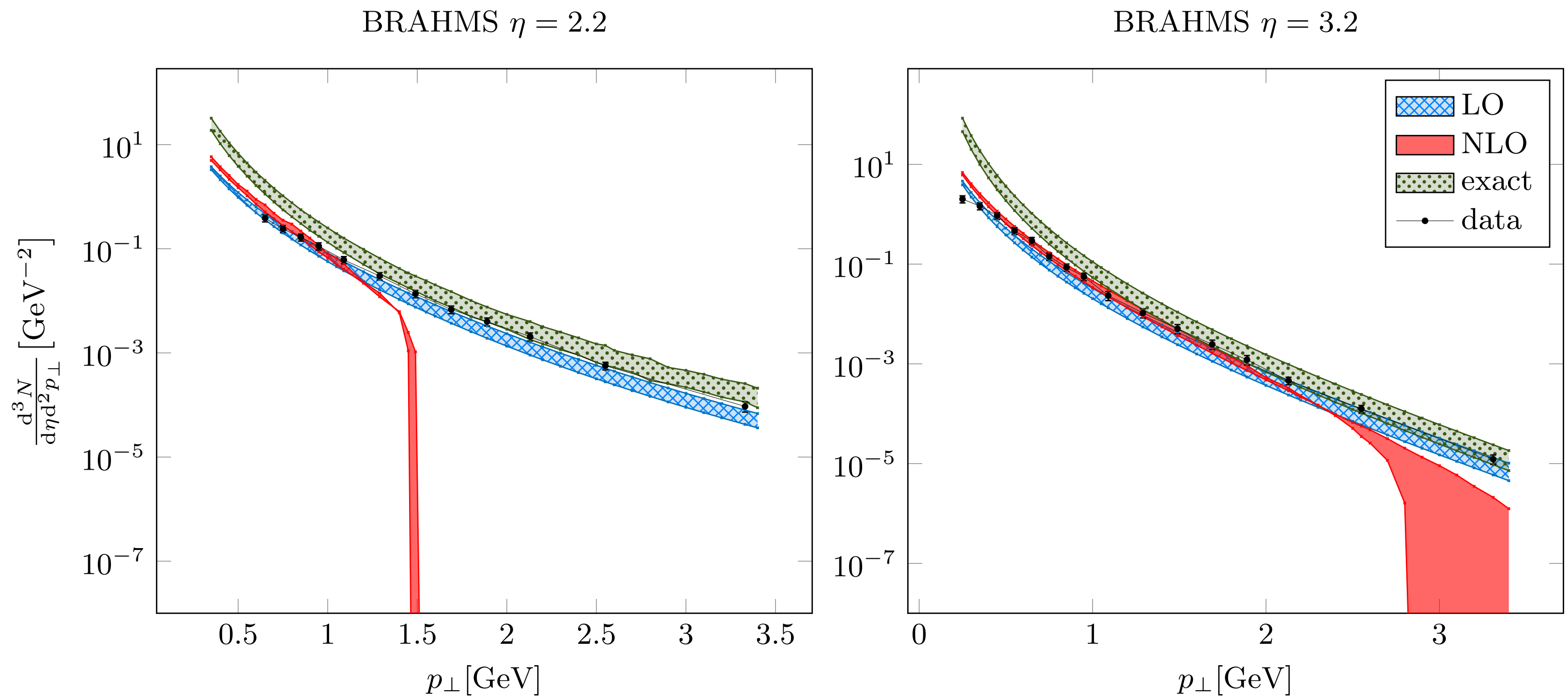
J. Jalilian Marian and A. Rezaeian- 2011; A.M.Stasto, B.W.Xiao, D. Zaslavsky, - 2013 Numerical analysis...

Trouble: effect of NLO corrections very large, and disturbingly negative



How do we fix negative cross sections?

Anna Stasto: match small-x result onto DGLAP at large p_T.



Matching between small x and collinear approximation with exact kinematics.

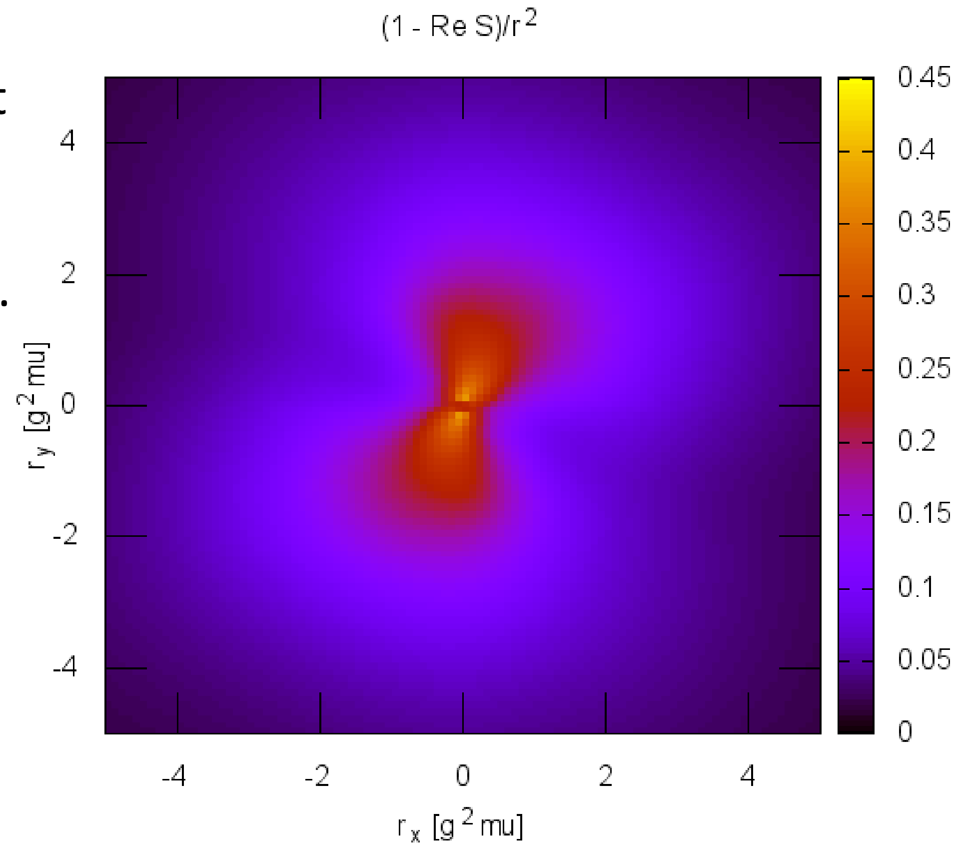
The small x gives good prediction up to $p_T \sim Q_s$

Collinear gives good description at large transverse momenta and overpredicts data for low transverse momenta.

Heavy Ions Azimuthal Harmonics

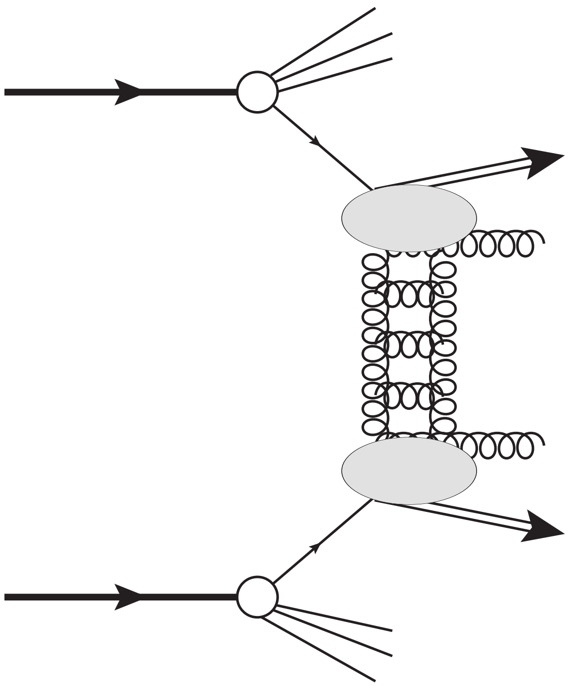
Adrian Dumitru: event-by-event get all types of asymmetric particle distributions. However, saturation approach only give even harmonics.

Could some clever NLO corrections lead to odd harmonics?



Doug Wertepny: let us calculate saturation corrections in the projectile. Needed to understand particle production in A+A.

Jet-Gap-Jet Production



A test of NLO BFKL evolution

talk by Martin Hentschinski

fully resummed result at hadronic level

$$\frac{d\sigma_{H_1H_2}}{dJ_1 dJ_2 d^2k} = \frac{1}{\pi^2}\int dl_1 dl_1' dl_2 dl_2' \frac{dV(l_1, l_2, k, p_{J,1}, y_1, s_0)}{dJ_1} \times G\left(\boldsymbol{l}_1, \boldsymbol{l}_1', \boldsymbol{k}, \frac{\hat{s}}{s_0}\right) G\left(\boldsymbol{l}_2, \boldsymbol{l}_2', \boldsymbol{k}, \frac{\hat{s}}{s_0}\right) \frac{dV(l_1', l_2', k, p_{J,2}, y_2, s_0)}{dJ_2}$$

A word of caution about scattering on nuclei

talk by Kirill Tuchin

- Coulomb corrections in high energy pA collisions:

  - In photon production up to 7% (Au), 3% (Cu) at k_T<1 GeV. Below 1% at higher k_T.

  - Dilepton production receives Coulomb corrections at two stages: (i) virtual photon emission and (ii) photon splitting into a dilepton pair. Photon splitting in Coulomb field contributes up to 10% at small M and semi-peripheral b.

- DIS: Coulomb corrections a important in DIS off heavy nuclei in a wide range of x and Q^2. 10% in semi-inclusive and 25% in diffractive cross section at low x and $Q^2 < Q_s^2$.

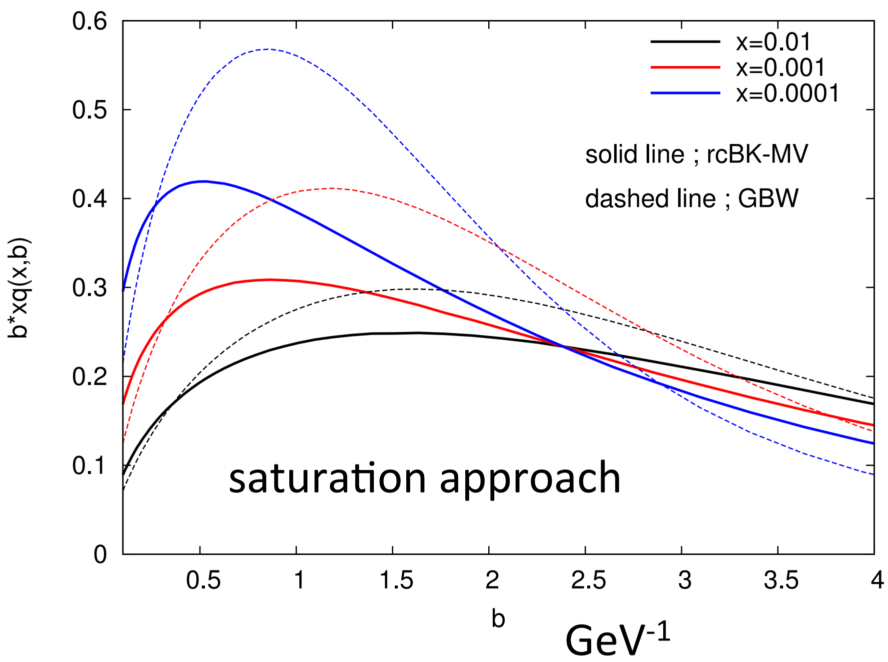
- Coulomb corrections violate the geometric scaling.

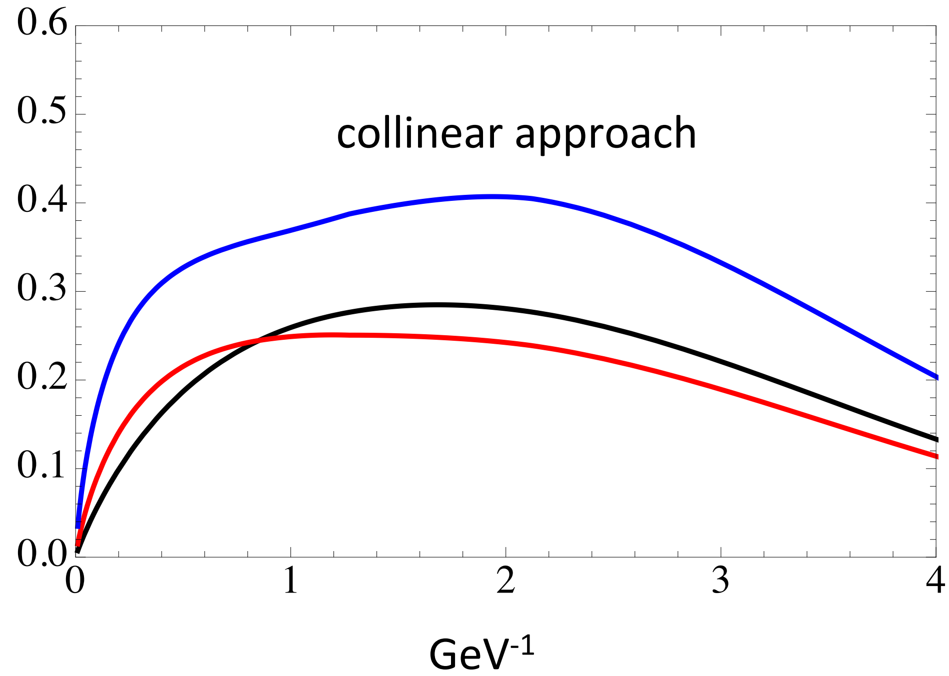
- Beware of the Coulomb corrections at the EIC!

TMDs and small-x

What can we learn about TMDs from small-x physics?

Feng Yuan: TMDs can be calculated explicitly at small-x, providing a cross check of the TMDs extracted using the "standard" procedure.





TMD – Transverse Momentum Distribution f(x, k_T, Q^2)

Andrei Tarasov: An evolution equation for TMDs: reduces to DGLAP equation and Sudakov form factor in the appropriate limits. Also gives small-x BK/ JIMWLK evolution of gluon TMDs.

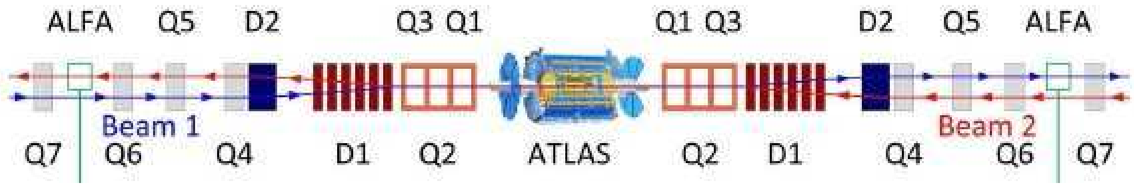
Conclusions

- NLO corrections have been calculated for BK/JIMWLK evolution.
- NLO corrections for particle production have been and are being calculated for a variety of processes.
- LO small-x calculations are beginning to provide important feedback for gluon and quark TMDs.
- Saturation physics continues to provide tantalizing phenomenological hints of its existence, such as geometric scaling.

Experimental part

Elastic/Total pp cross section from ATLAS/ALFA

L. Adamczyk talk

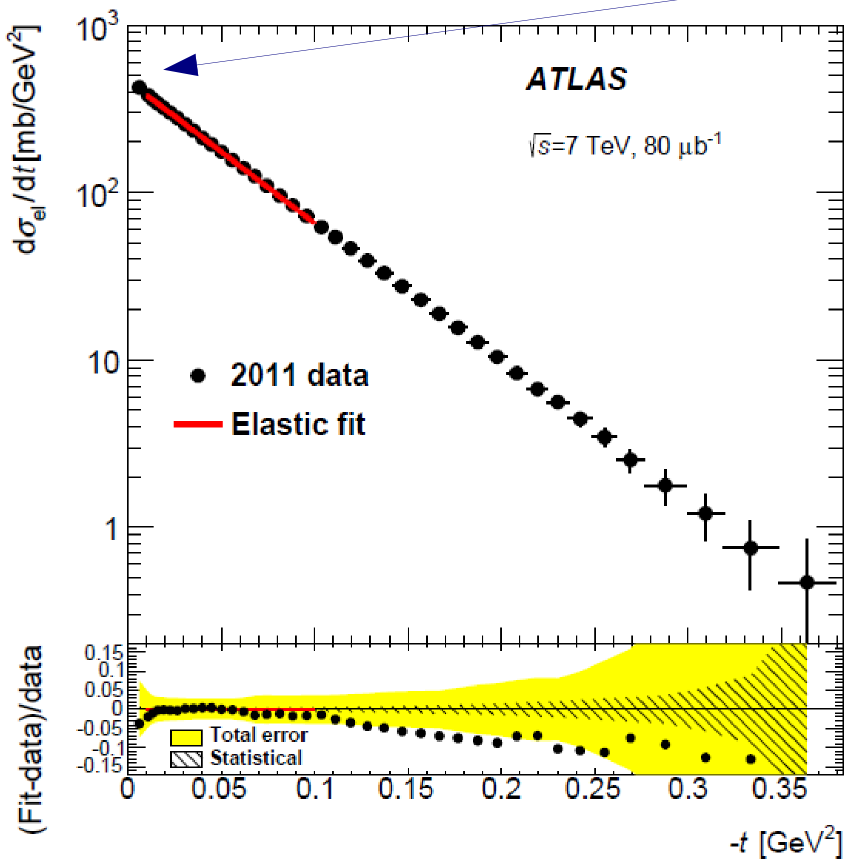


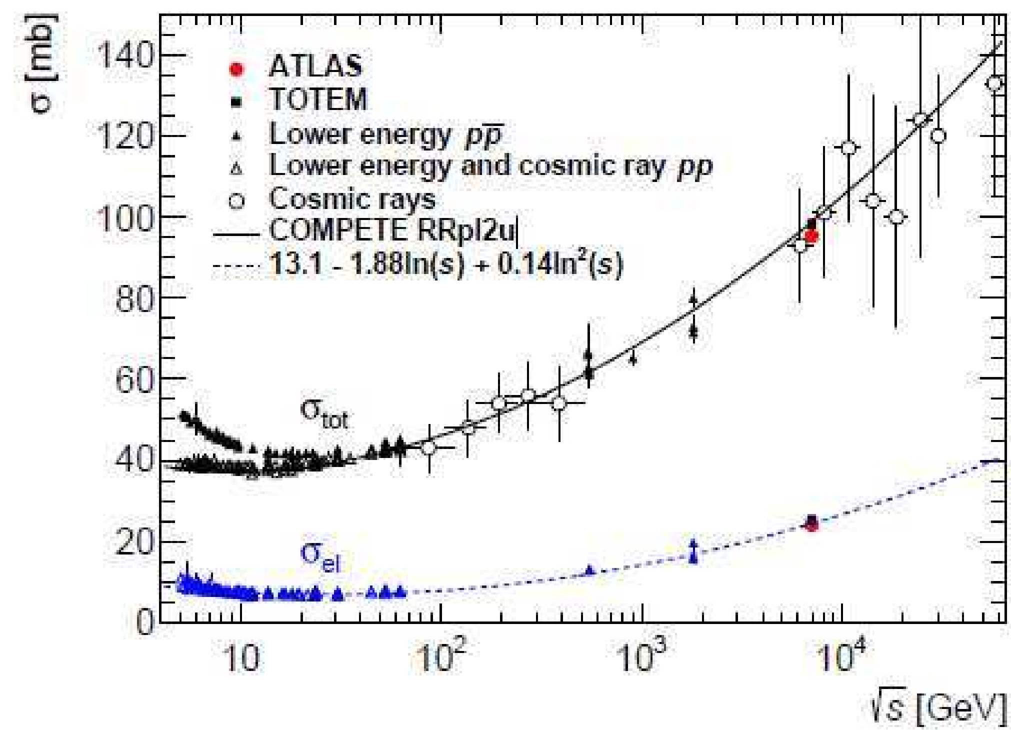
$$\sigma_{\mathrm{tot}}^2 = \frac{1}{L}\frac{16\pi}{1+\rho^2}\frac{\mathrm{d}N_{\mathrm{el}}}{\mathrm{d}t}\Big|_{t\to 0}$$

ALFA - tracking detectors with scintillating fibers at z= ±240 m

β*=90m optics, 700 kevts

$$\rho = \frac{\mathrm{Re}(f_{\mathrm{el}})}{\mathrm{Im}(f_{\mathrm{el}})}\Big|_{t\to 0} = 0.14$$





Exponential fit for 0.01 < |t| < 0.1 GeV^2

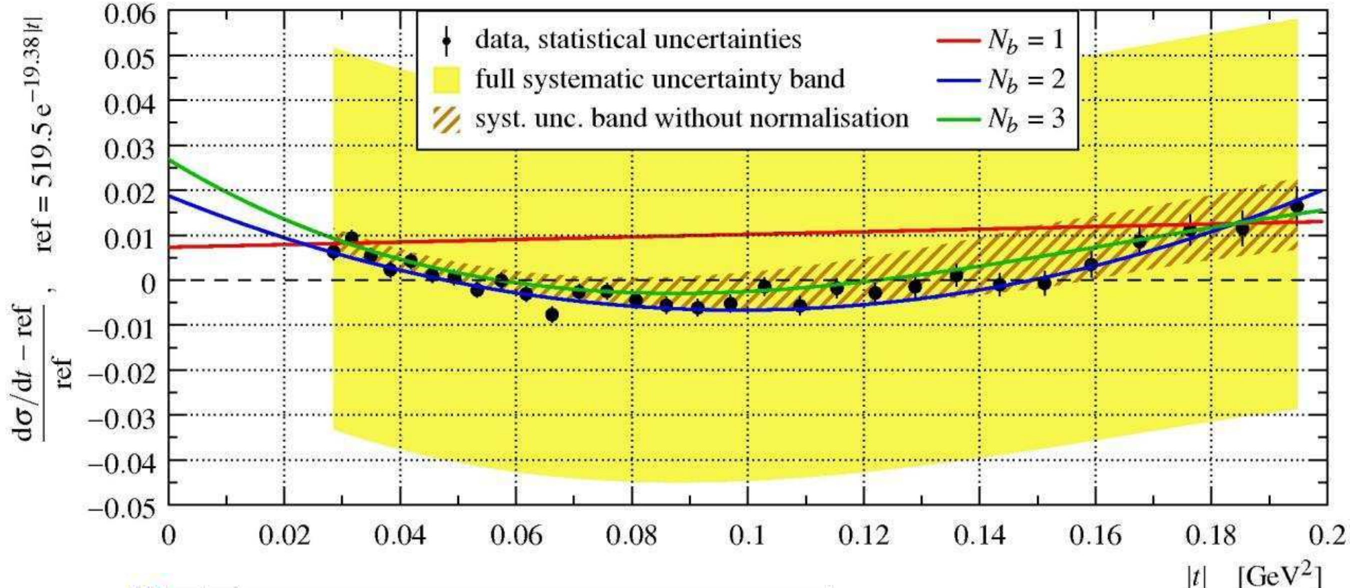
ATLAS: $\sigma_{\mathrm{tot}} = 95.4 \pm 1.4$ mb $\quad B = 19.7 \pm 0.3$ GeV^{-2}

TOTEM: $\sigma_{\mathrm{tot}} = 98.6 \pm 2.2$ mb $\quad B = 19.9 \pm 0.3$ GeV^{-2}

Elastic at low and very low |t| - TOTEM

Mario Deile talk

- High statistics dataset (β^*=90m, 2012), 7 Mevt
  0.027 GeV2< |t| < 0.2 GeV2 (Coulomb effects negligible)



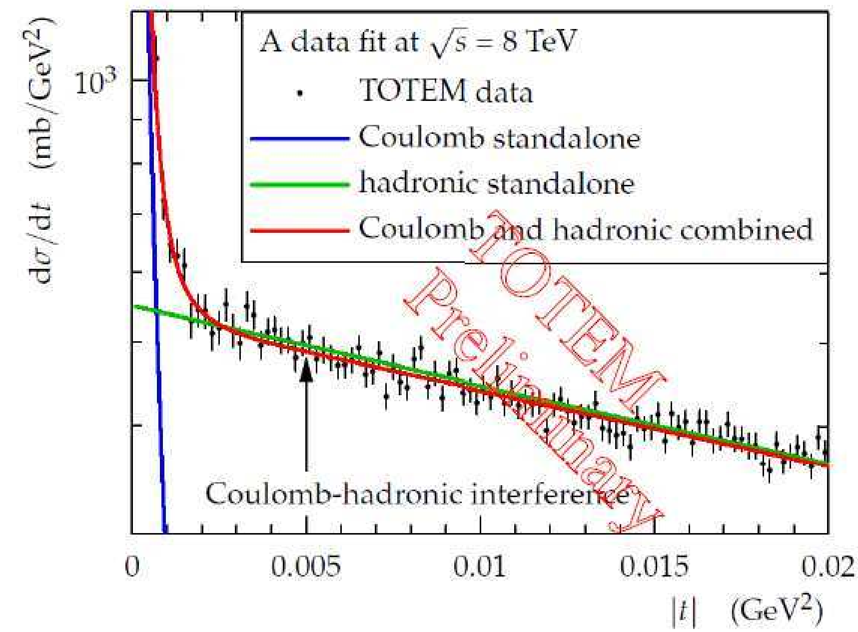
Relative deviation from exponential
+ fit with: $d\sigma/dt = A\, e^{-B(t)\,|t|}$ and

$B(t) = b_0$

$B(t) = b_0 + b_1 t$

$B(t) = b_0 + b_1 t + b_2 t^2$

Pure exponential dependence excluded at 7.2 σ significance.



- Very-low |t| dataset (β^*=1000m, 2012)
  $|t| > 6 \cdot 10^{-4}$ GeV^2

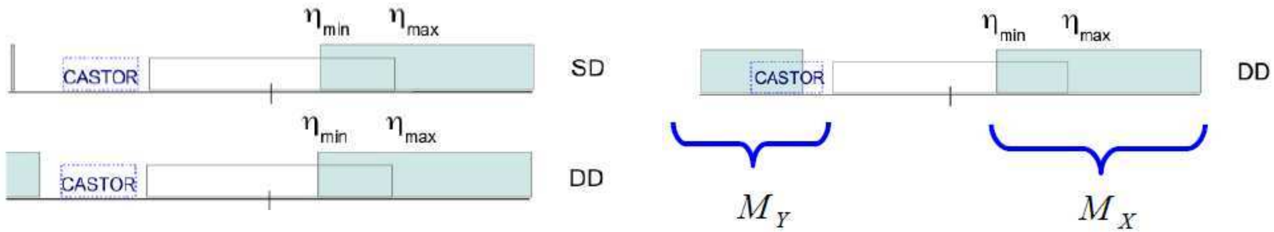
Constrain models of Coulomb-nuclear interference (nuclear phase Ψ, B(t))

$$F^{C+H} = F^C + F^H e^{i\alpha\Psi}$$

SD and DD cross sections - CMS

G. Brona talk

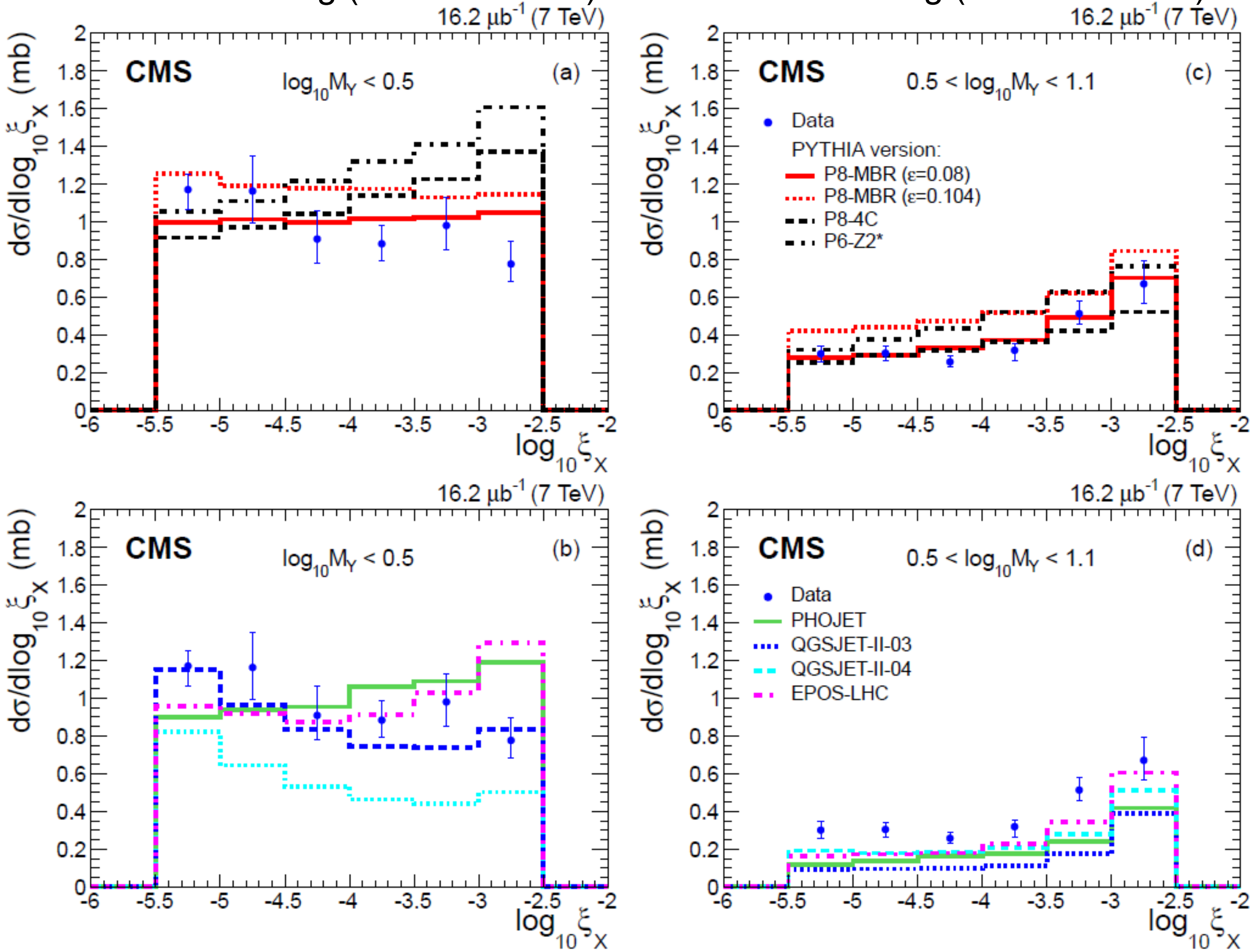
- Forward rapididy gap + CASTOR (-6.6<η< -5.2)



$$\xi_X = \frac{M_X^2}{s}$$

for 12 < M_X < 394 GeV

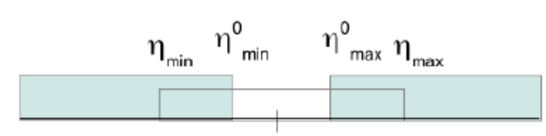
no CASTOR-tag (SD dominated) CASTOR-tag (DD dominated)



Test of diffraction (and hadronization) models

PYTHIA8-MBR describes all aspects of the data

- Also central gap (DD dominated, not shown)



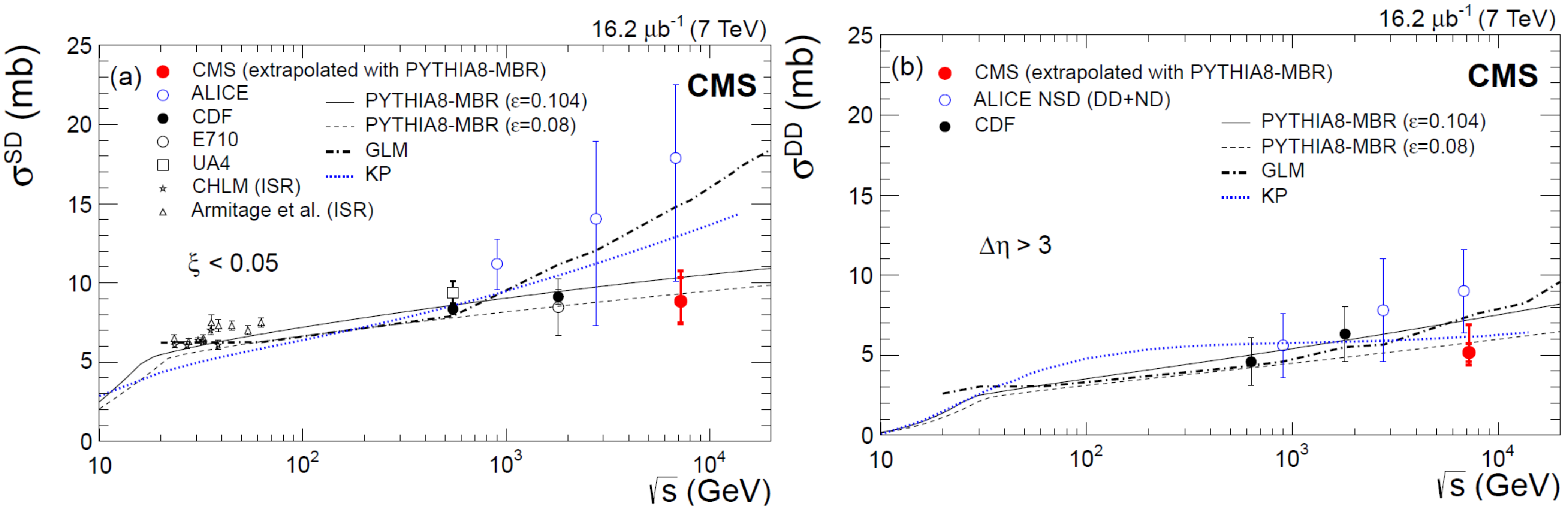
SD and DD cross sections - CMS

G. Brona talk

Background subtraction (with small uncertainties) and extrapolation to the not observed region with PYTHIA8-MBR (K. Goulianos talk)

$$\sigma^{SD} = 8.84 \pm 0.08(stat)^{+1.49}_{-1.38}(syst)^{+1.17}_{-0.37}(extr)\,mb \qquad \xi_{X(Y)} < 0.05$$

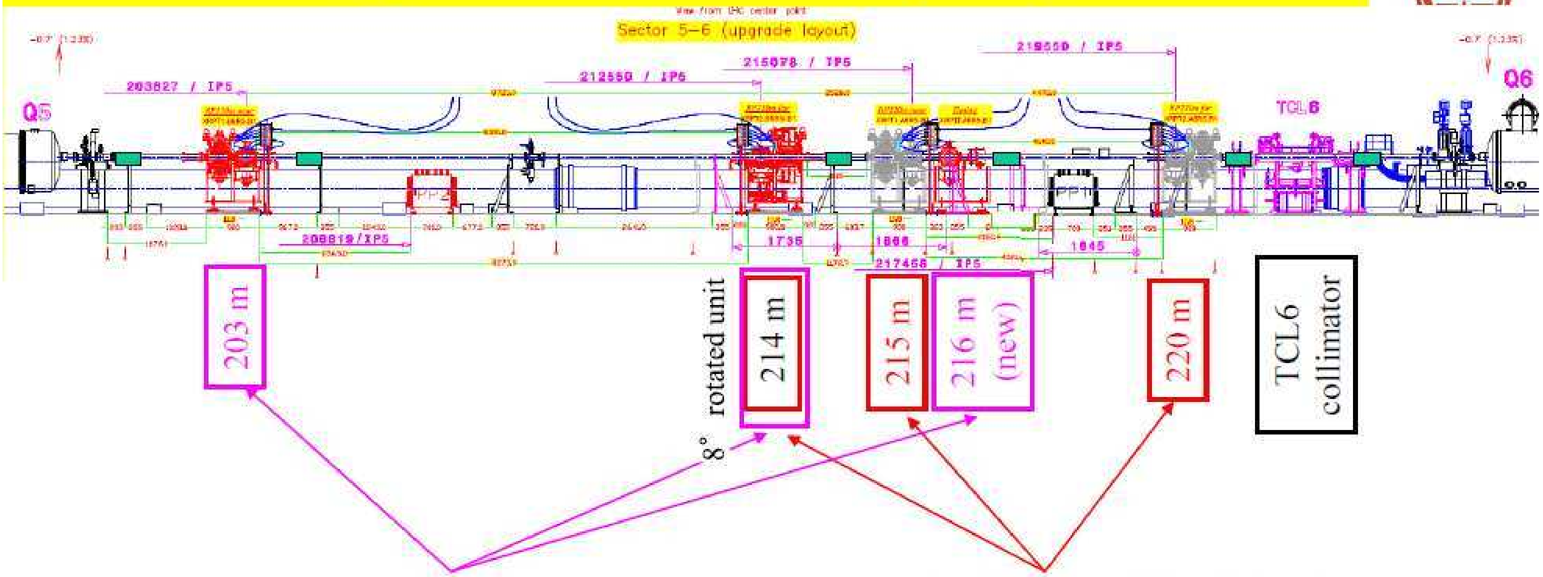
$$\sigma^{DD} = 5.17 \pm 0.08(stat)^{+0.55}_{-0.57}(syst)^{+1.62}_{-0.51}(extr)\,mb \qquad \Delta\eta > 3$$



SD, DD cross section weakly rising with energy

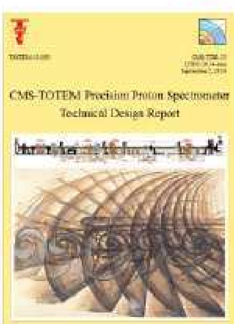
CMS+TOTEM, CT-PPS future plans

Two Upgrade Technical Design Reports



Operation at low β* (< 1 m), high luminosity, standard runs

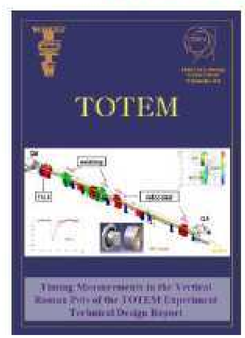
Operation at high β* (19 m, 90 m, > 1 km), Low - medium luminosity, special runs



CMS-TOTEM Precision Proton Spectrometer (CT-PPS)

M. Albrow talk

High statistics CEP:
DPE exclusive dijets,
photon-photon WW and
BSM EWK couplings.
2016-2017



Timing Measurements in the Vertical Roman Pots of the TOTEM Experiment

M. Deile talk

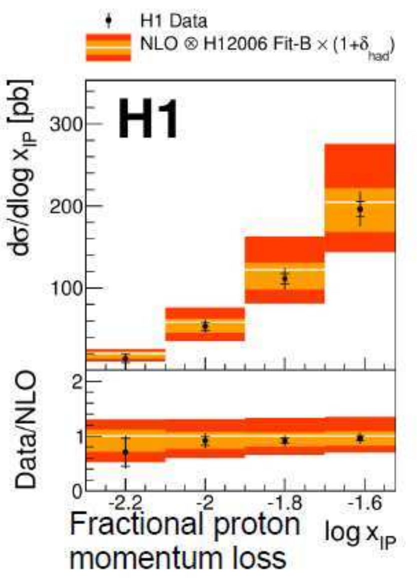
Diffractive processes with TOTEM+CMS,
e.g.: SD J/Psi, Y, W, Z, dijet
DPE dijets, hadron spectroscopy (gluballs)
2015-2016

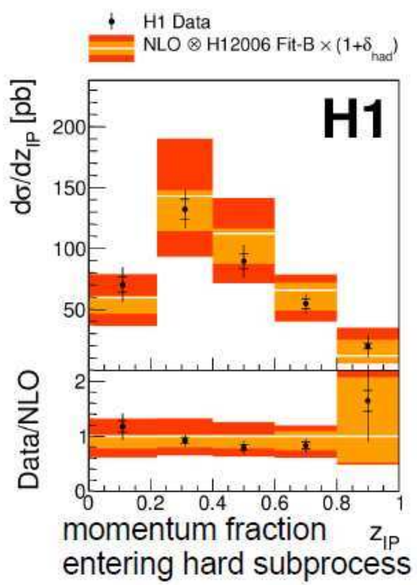
Similar physics program for ATLAS-ALFA and AFP (ATLAS Forward Physics) project

L. Adamczyk talk

Diffractive dijet production at HERA (H1)

S. Schmidt talks

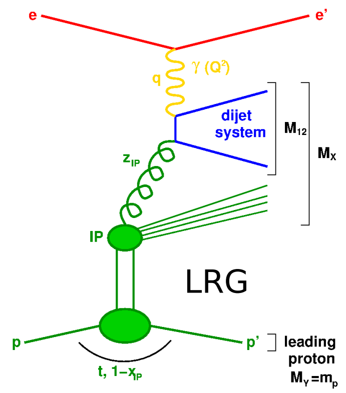




- Dijet in DIS with Large Rapidity Gap method

Precision of 3% on integrated cross section (10% systematic uncertainty)

NLO QCD describes the data well
Potential to further constrain diffractive PDFs

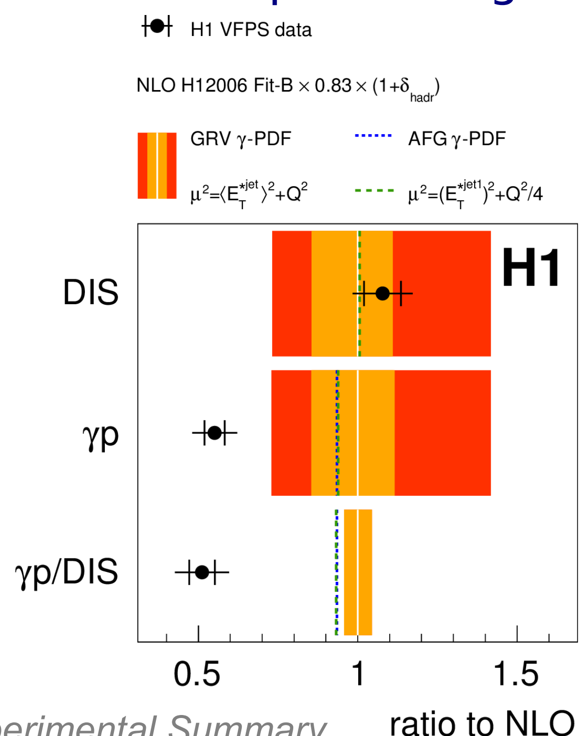


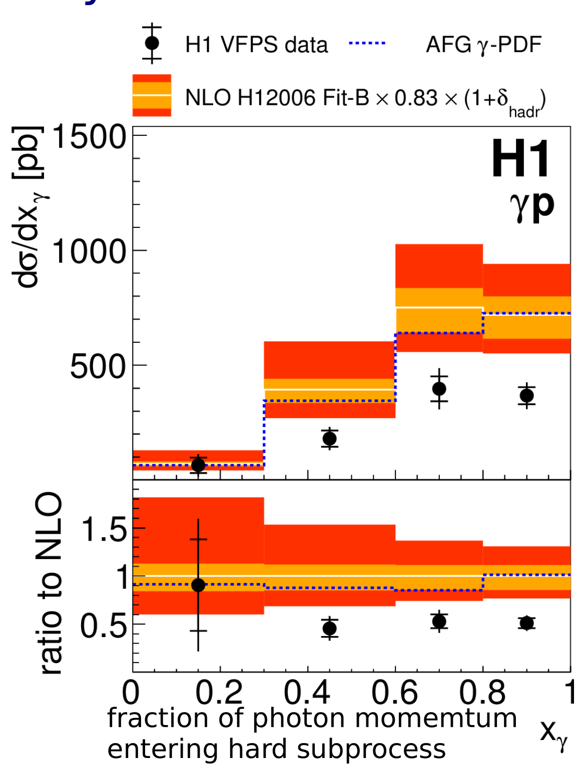
- Dijet in Photoproduction (Q^2=0) and DIS with proton tag in Very Forward Proton Spectrometer at z=220m

NLO predictions based on HERA DPDFs fail to describe diffractive jet production in hadron colliders (data/NLO suppression ~ 0.2).

Is suppression observed at HERA in photoproduction?

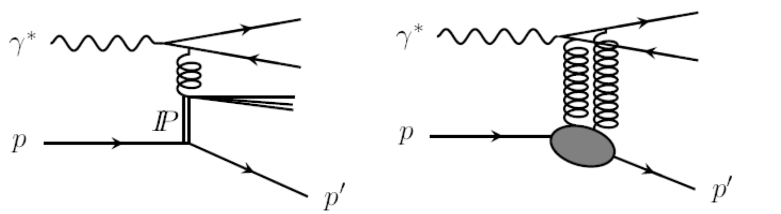
Photoproduction supressed by ~0.5.
Not related to proton dissociation (p-tag).
No clear dependence on any variable.





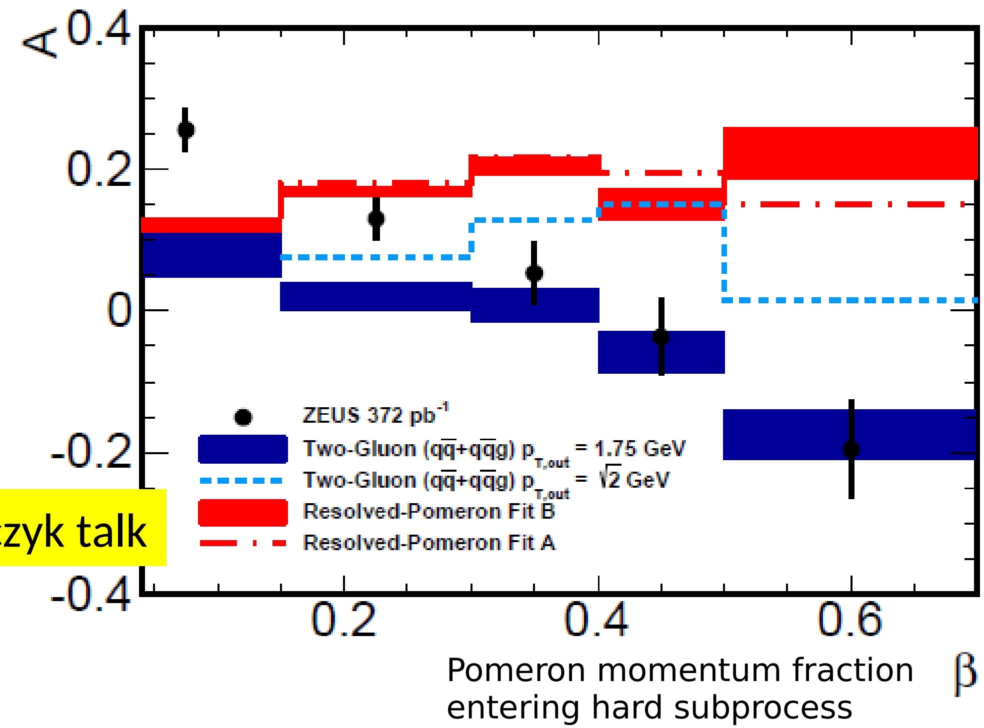
Exclusive dijet production (DIS) and photon-jet photoproduction at HERA (ZEUS)

φ - angle between lepton plane and jet plane
φ distribution $\propto 1 + A \cos 2\varphi$
A: >0 for Resolved-Pomeron model (Ingelman-Schlein)
<0 for two-gluon exchange model (Bartels-Jung)

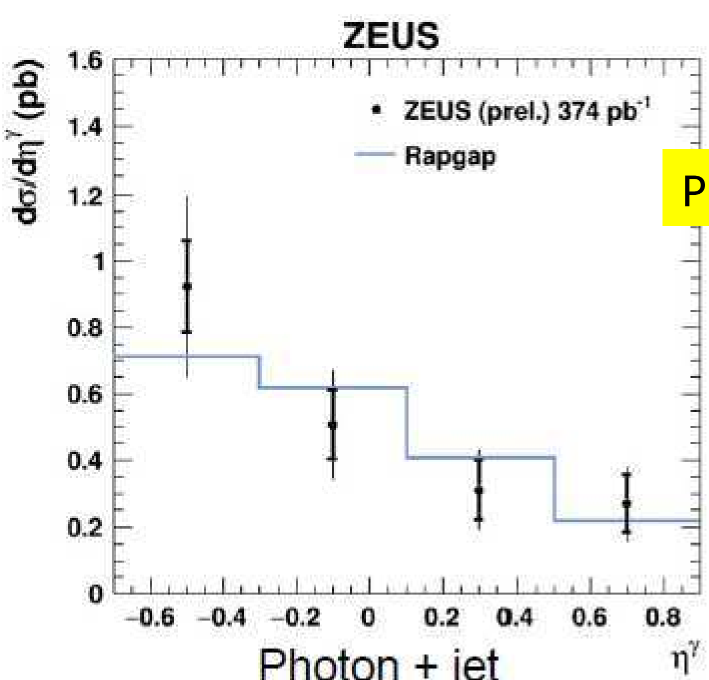


Resolved-Pomeron vs Two-gluon Exchange

Two-gluon exchange model favored



L. Adamczyk talk



P. Bussey talk

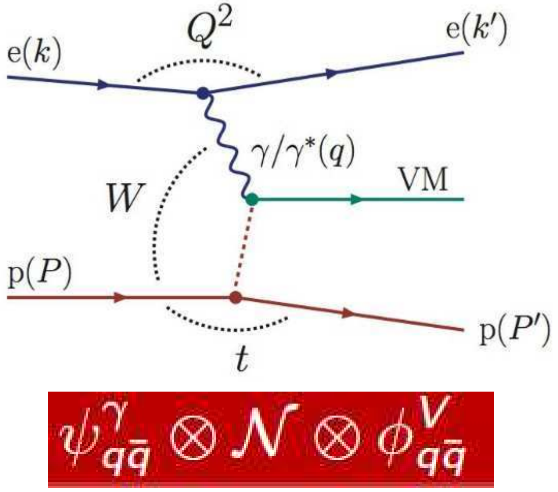
Isolated prompt photons from hard partonic interaction
Sensitivity to quark component of diffractive PDFs

~250 events with $E_{\gamma/jet}$>4/5 GeV (central)
Dominated by direct photoproduction

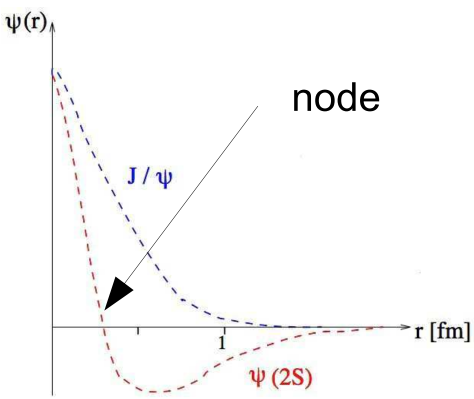
Good description by RAPGAP MC

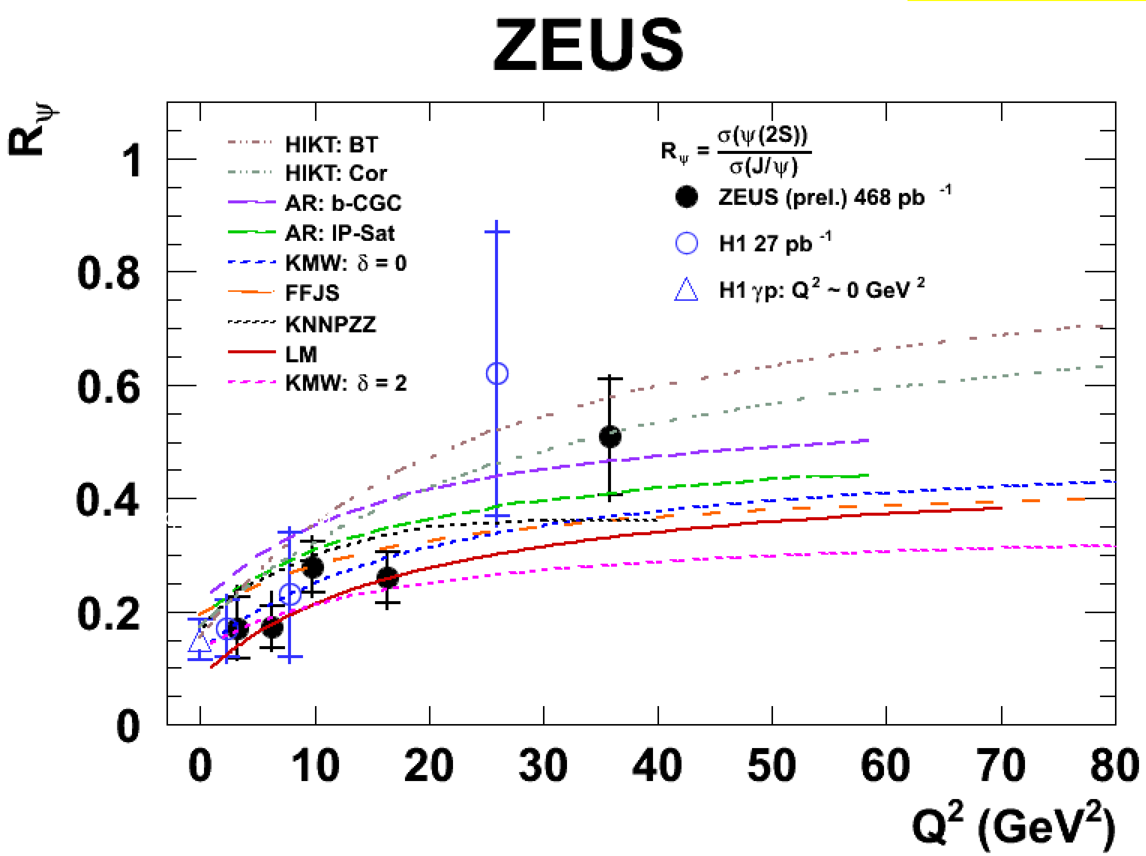
Vector Mesons.
ψ(2S)/J/ψ ratio in DIS ep (ZEUS)

N. Kovalchuk talk



Ratio R of ψ(2S) to J/ψ cross sections suppressed due to difference in ψ(2S) and J/ψ wave functions:

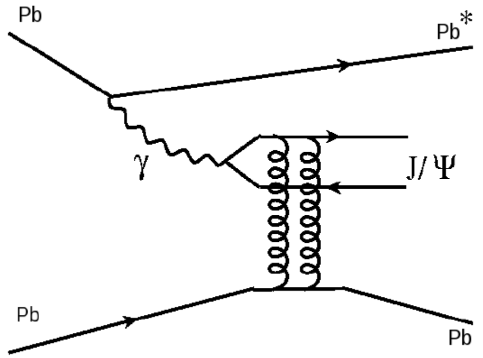




R rises with Q2. Independent on W and |t|.

Results can help constrain model predictions.

Vector Meson production in Pb-Pb collsions

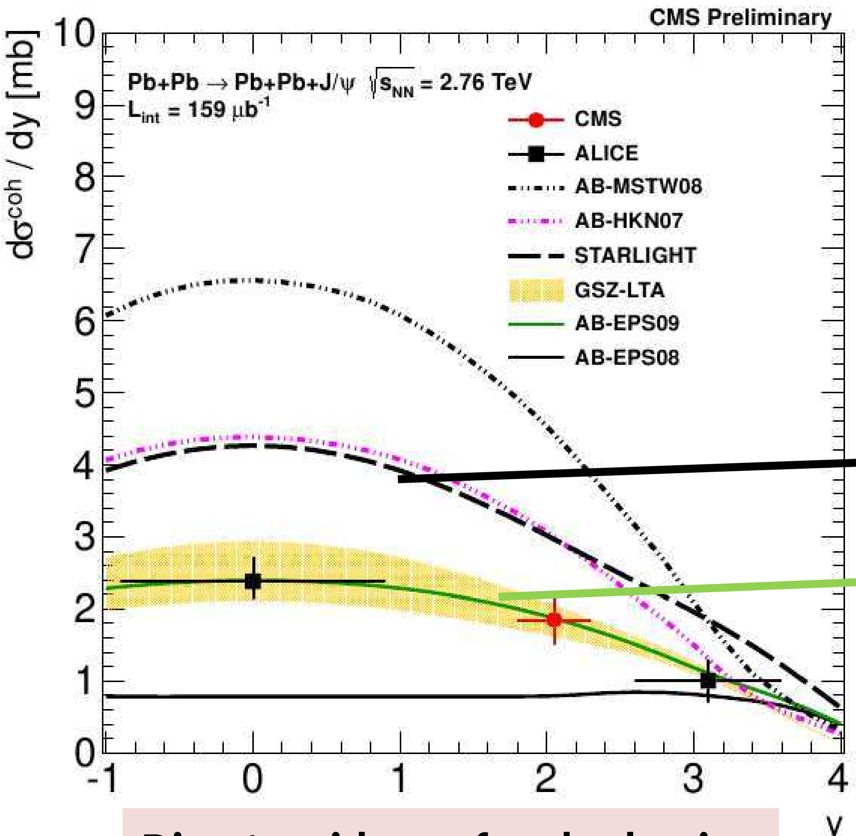


Is the nucleus gluon field equivalent to to those of A nucleons ?
→ Hunting for shadowing

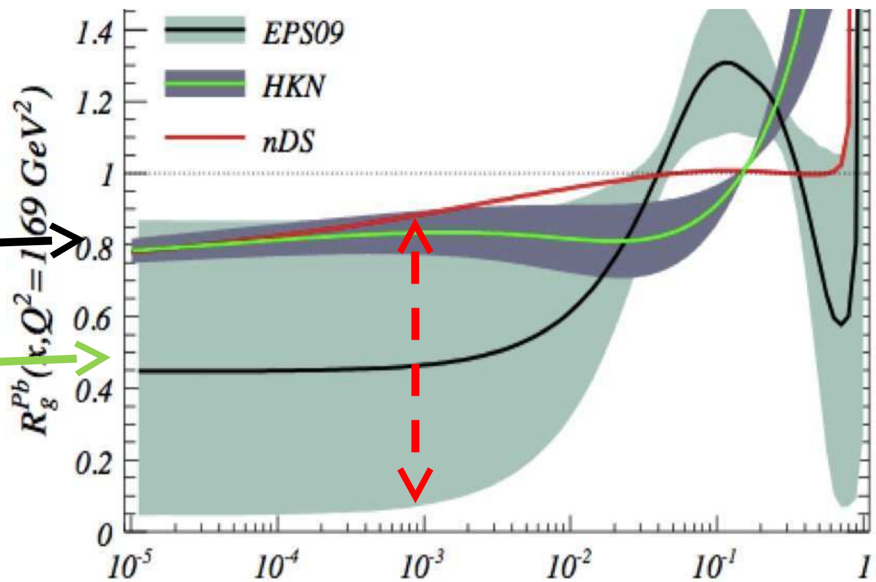
$$\left.\frac{d\sigma_{\gamma A\to J/\Psi A}}{dt}\right|_{t=0} = \xi_{J/\Psi}\left(\frac{16\pi^3\alpha_s^2\Gamma_{l^+l^-}}{3\alpha M_{J/\Psi}^5}\right)\left[xG_A(x,\mu^2)\right]^2$$

LO

M. Murray, J. Adams talks



100% uncertainty has to be updated

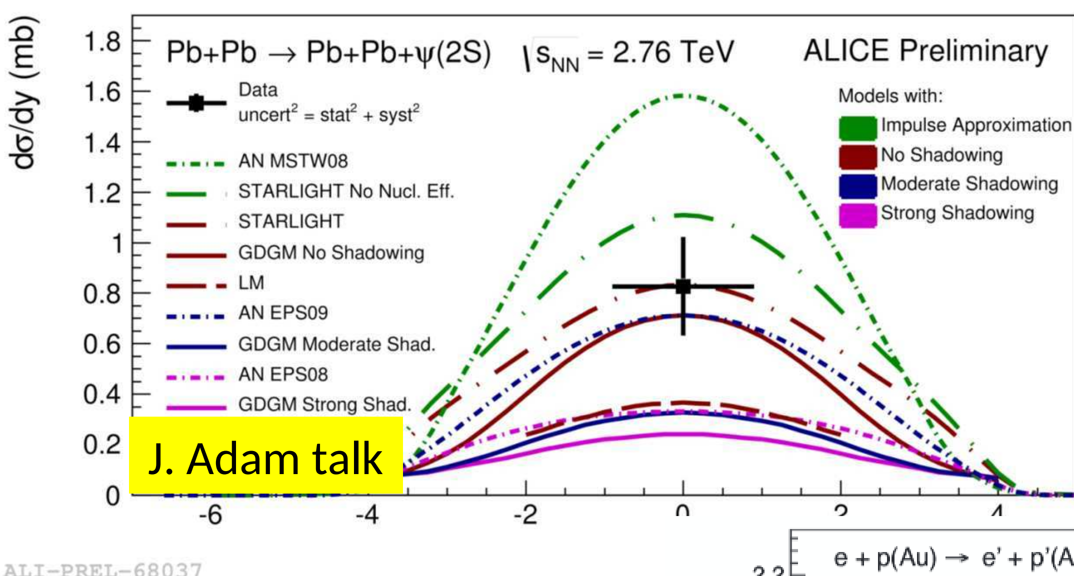


Direct evidence for shadowing

Vector Meson production in Pb-Pb collsions

Impressive theoretical effort in this field (J. Nystrand, B. Gay Ducati and T. Lappi talks)

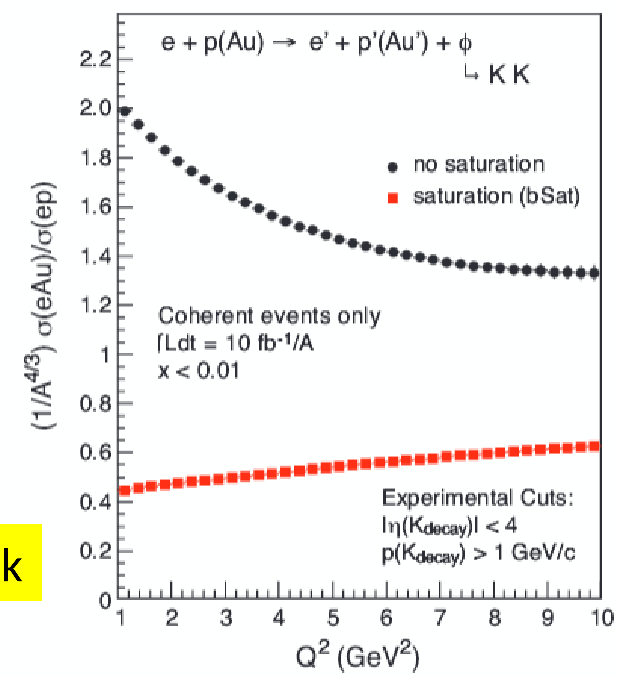
Other VM at LHC in Pb-Pb



J. Adam talk

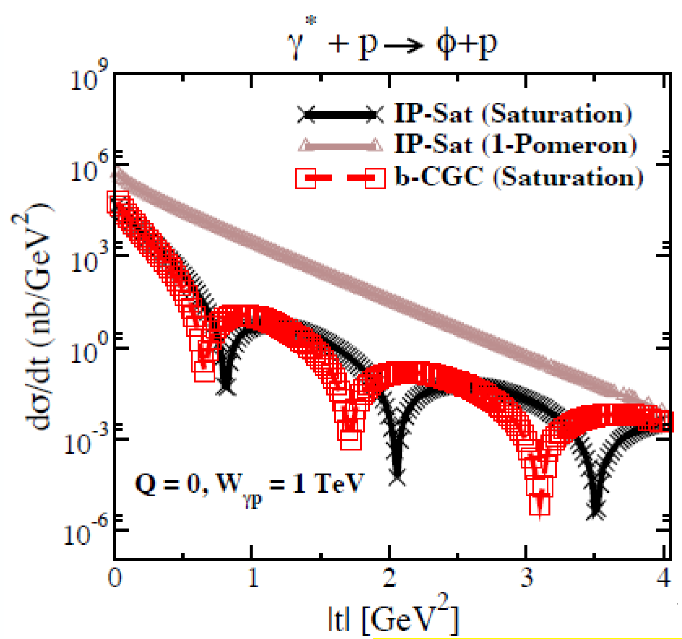
Very strong shadowing and no-nuclear-effect disfavored. But more statistics and more theoretical effort required (e.g. uncertainty of Ψ(2s) wave function).

Vector meson are a key tool to study saturation at EIC: Φ meson well suited for this job



R. Ciesielski, Y. Kovchegov, E. Scappar

T. Ullrich talk

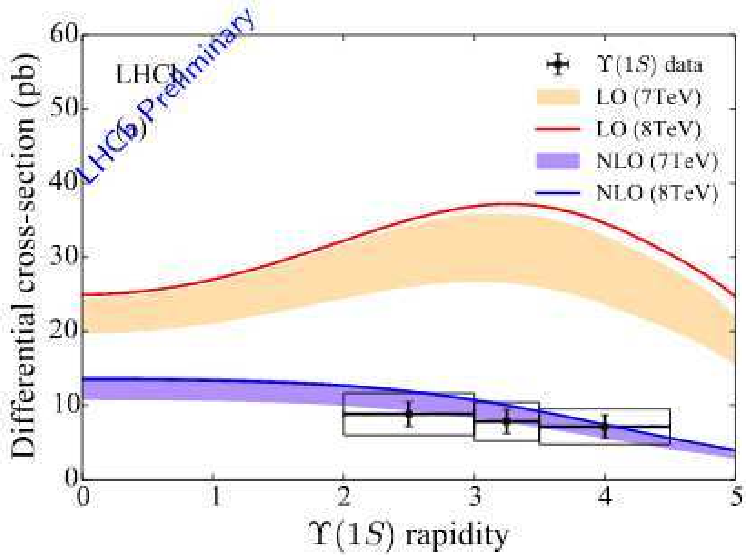


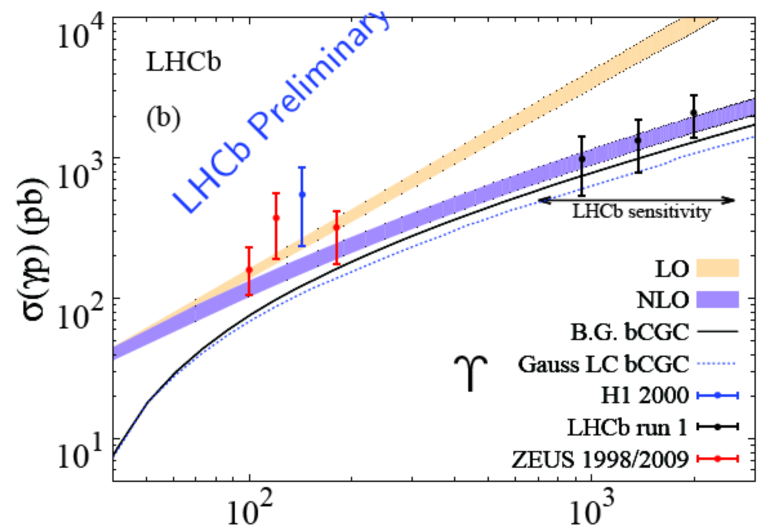
A. Rezaeian talk

Υ in pp (LHCb), $\pi^+\pi^-$ in DPE (CDF)

R. Wallace talk

ϒ production in pp collisions by LHCb

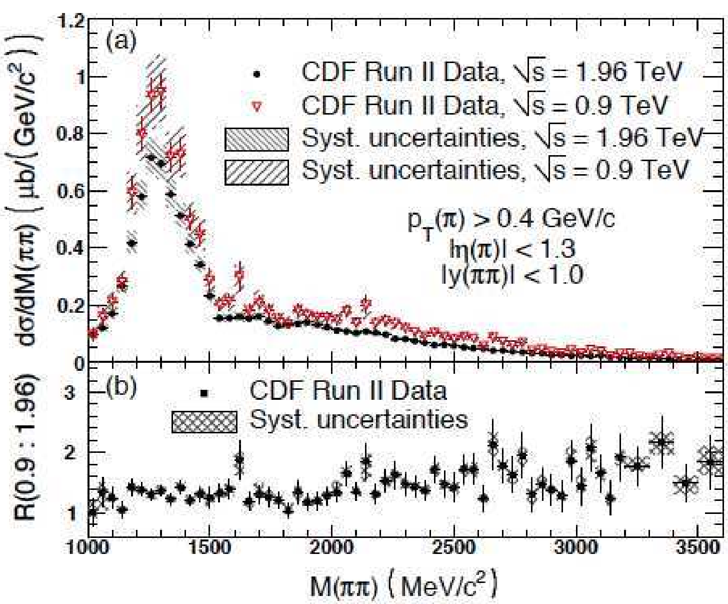


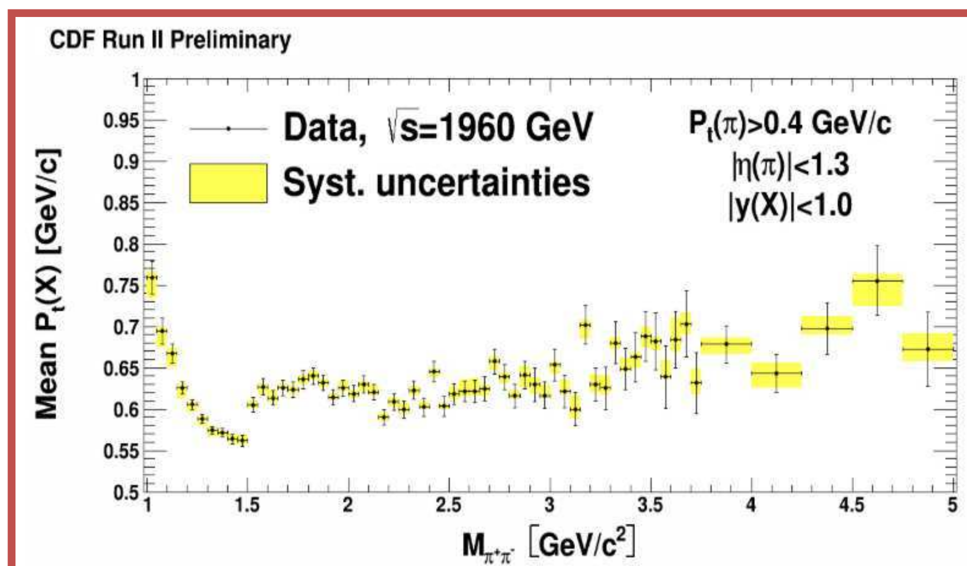


NLO strongly preferred

Final data from Tevatron (CDF) on di-pion production in DPE → glueball are still elusive

M. Albrow talk



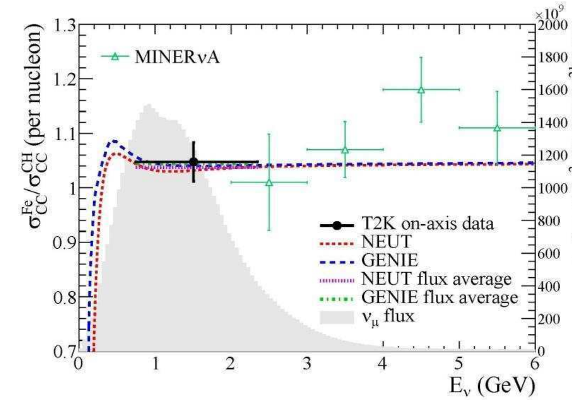


Neutrino-Nucleon Ineractions

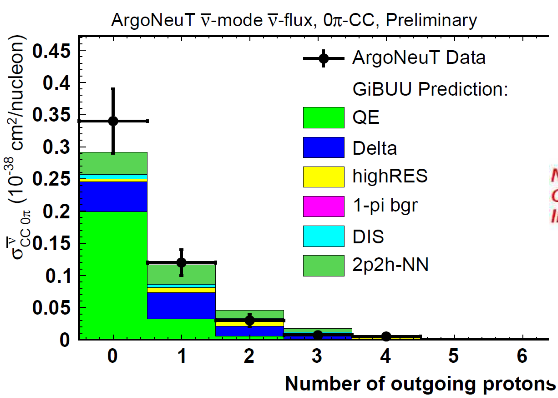
Neutrino-nucleus cross sections: EW physics + nuclear effects:
- short-range nucleon-nucleon correlations
- meson exchange currents
- finale state interactions (multi-nucleon nockout)

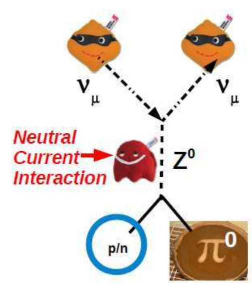
E. Reinherz-Aronis talk

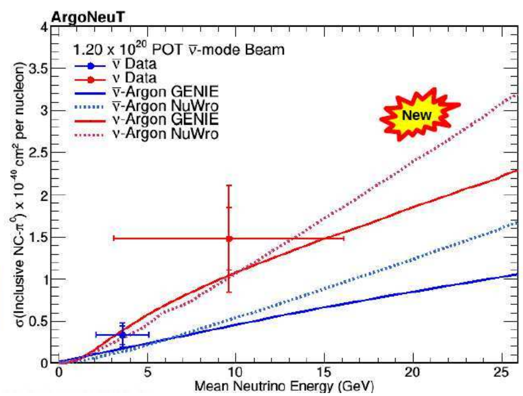
CC cross section ratio of Fe to scintillator from T2K $\frac{\sigma_{CC}^{Fe}}{\sigma_{CC}^{CH}}$



J. Assadi talk



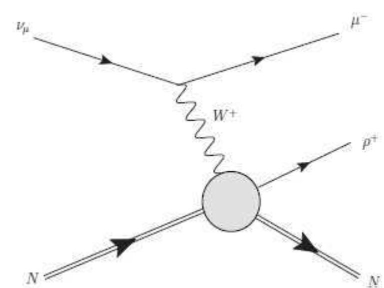


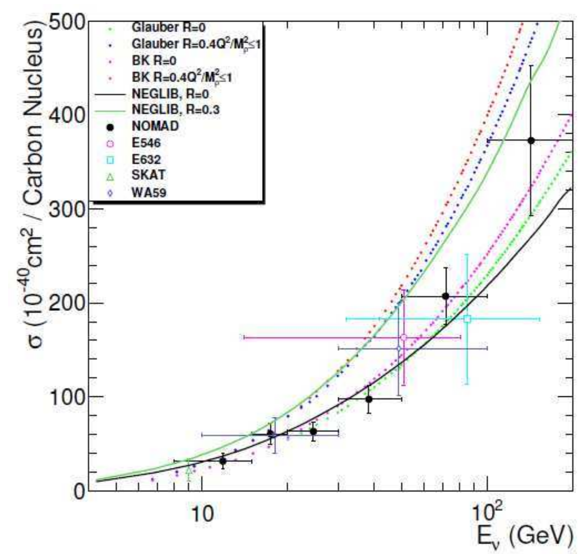


Charge current zero-π and Neutral Current π^0 final states from ArgoNeuT

X. Tian talk

Coherent ρ^+ and ρ^0 production from NOMAD





Valuable constrains for theoretical models.

Summary

- **Measurements of total, elastic, diffractive cross section at LHC**
  - **Good agreement between TOTEM and ATLAS/ALFA results of total and elastic cross sections**
  - **Elastic process studied down to very low |t| ($6x10^{-4}$ GeV)**
  - **SD, DD cross sections rising slowly with energy**
- **Interesting future diffractive program with tagged protons with TOTEM+CMS, CT-PPS, AFS, at low and high lumi**
- **Hard diffractive dijet and jet-photon cross sections from HERA:**
  - **constrain diffractive PDF**
  - **test factorization and production mechanism**
  - **test quark compontent of diffractive PDFs and hard emissions**
- **Vector meson production in pp and PbPb at LHC**
  - **test pQCD, nuclear shadowing, saturation**
- **Neutrino-nucleus cross section measurements**
  - **test nuclear effects in nuclei**

Thank you for your attention!
Thank you to all the WG2 speakers!

Backup Slides

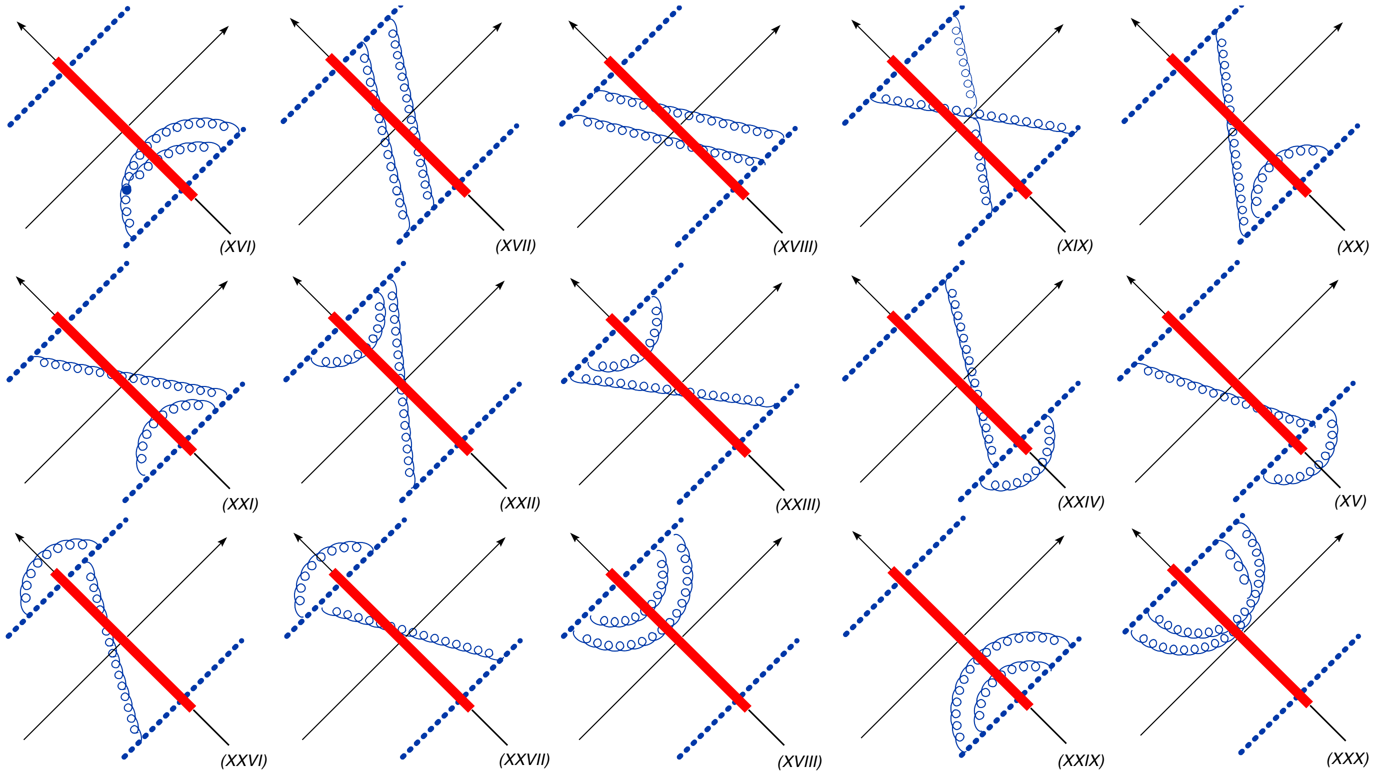
NLO BK

$$
\begin{aligned}
\frac{d}{d\eta}\mathrm{Tr}\{U_x U_y^\dagger\} \;=\; & \frac{\alpha_s}{2\pi^2}\int d^2z\,\Big([\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_y^\dagger\} - N_c \mathrm{Tr}\{U_x U_y^\dagger\}] \\
& \times \Big\{\frac{(x-y)^2}{X^2Y^2}\Big[1 + \frac{\alpha_s N_c}{4\pi}\big(\frac{11}{3}\ln(x-y)^2\mu^2 + \frac{67}{9} - \frac{\pi^2}{3}\big)\Big] \\
& - \frac{11}{3}\frac{\alpha_s N_c}{4\pi}\frac{X^2-Y^2}{X^2Y^2}\ln\frac{X^2}{Y^2} \;-\; \frac{\alpha_s N_c}{2\pi}\frac{(x-y)^2}{X^2Y^2}\ln\frac{X^2}{(x-y)^2}\ln\frac{Y^2}{(x-y)^2}\Big\} \\
& + \frac{\alpha_s}{4\pi^2}\int d^2z'\,\Big\{[\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_{z'}^\dagger\}\{U_{z'} U_y^\dagger\} - \mathrm{Tr}\{U_x U_z^\dagger U_{z'} U_y^\dagger U_z U_{z'}^\dagger\} \\
& - (z' \to z)]\frac{1}{(z-z')^4}\Big[-2 + \frac{X'^2Y^2 + Y'^2X^2 - 4(x-y)^2(z-z')^2}{2(X'^2Y^2 - Y'^2X^2)}\ln\frac{X'^2Y^2}{Y'^2X^2}\Big] \\
& + [\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_{z'}^\dagger\}\{U_{z'} U_y^\dagger\} - \mathrm{Tr}\{U_x U_{z'}^\dagger U_z U_y^\dagger U_{z'} U_z^\dagger\} - (z' \to z)] \\
& \times \Big[\frac{(x-y)^4}{X^2Y'^2(X^2Y'^2 - X'^2Y^2)} + \frac{(x-y)^2}{(z-z')^2X^2Y'^2}\Big]\ln\frac{X^2Y'^2}{X'^2Y^2}\Big\}\Big)
\end{aligned}
$$

Balitsky, Chirilli ‘07

NLO Evolution

Diagrams with 2 gluons interaction



talk by Giovanni Chirilli

JIMWLK Hamiltonian @ NLO

$$H^{NLO\ JIMWLK} = \int_{x,y,z} K_{JSJ}(x,y;z)\left[J_L^a(x)J_L^a(y) + J_R^a(x)J_R^a(y) - 2J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z')\left[f^{abc}f^{def}J_L^a(x)S_A^{be}(z)S_A^{cf}(z')J_R^d(y) - N_c J_L^a(x)S_A^{ab}(z)J_R^b(y)\right]$$

$$+ \int_{x,y,z,z'} K_{q\bar{q}}(x,y;z,z')\left[2\,J_L^a(x)\,tr[S^\dagger(z)\,t^a\,S(z')t^b]\,J_R^b(y) \;-\; J_L^a(x)\,S_A^{ab}(z)\,J_R^b(y)\right]$$

$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z')f^{acb}\Big[J_L^d(x)\,J_L^e(y)\,S_A^{dc}(z)\,S_A^{eb}(z')\,J_R^a(w) - J_L^a(w)\,S_A^{cd}(z)\,S_A^{be}(z')\,J_R^d(x)\,J_R^e(y)\Big]$$

$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\,f^{bde}\left[J_L^d(x)\,J_L^e(y)\,S_A^{ba}(z)\,J_R^a(w) - J_L^a(w)\,S_A^{ab}(z)\,J_R^d(x)\,J_R^e(y)\right]$$

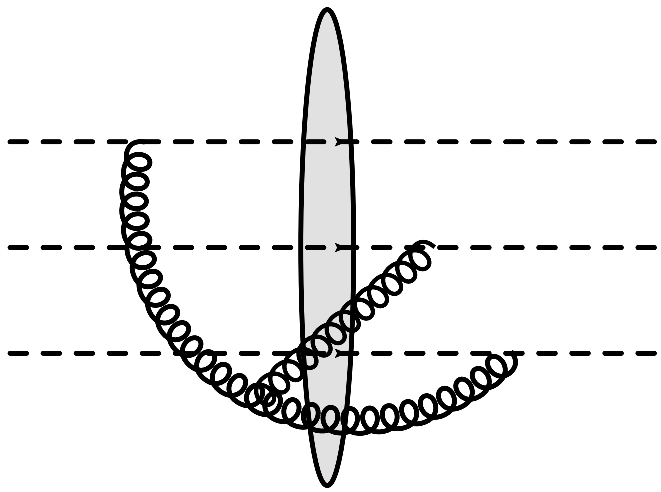
$$+ \int_{w,x,y} K_{JJJ}(w;x,y)f^{deb}\left[J_L^d(x)\,J_L^e(y)\,J_L^b(w) \;-\; J_R^d(x)\,J_R^e(y)\,J_R^b(w)\right].$$

NLO JIMWLK has been constructed by Balitsky & Chirilli; Kovner, Lublinsky and Mulian; Grabovsky (all 2013).

talk by Michael Lublinsky

NLO corrections

NLO evolution of 2 Wilson lines with open indices from Balitsky and Chirilli 2013

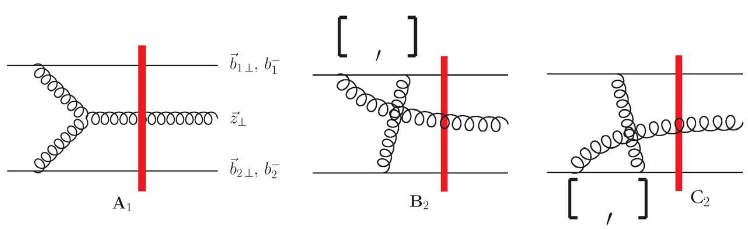


$$
\begin{aligned}
\frac{\partial B_{123}}{\partial \eta} = \frac{\alpha_s(\mu^2)}{8\pi^2} \int d\vec{r}_0 \Bigg[& (B_{100}B_{320} + B_{200}B_{310} - B_{300}B_{210} - 6B_{123}) \\
& \times \left\{ \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} - \frac{3\alpha_s}{4\pi}\beta \left[\ln\left(\frac{\vec{r}_{01}^{\,2}}{\vec{r}_{02}^{\,2}}\right)\left(\frac{1}{\vec{r}_{02}^{\,2}} - \frac{1}{\vec{r}_{01}^{\,2}}\right) - \frac{\vec{r}_{12}^{\,2}}{\vec{r}_{01}^{\,2}\vec{r}_{02}^{\,2}} \ln\left(\frac{\vec{r}_{12}^{\,2}}{\tilde{\mu}^2}\right) \right] \right\} \\
& - \frac{\alpha_s}{\pi} \ln\frac{\vec{r}_{20}^{\,2}}{\vec{r}_{21}^{\,2}} \ln\frac{\vec{r}_{10}^{\,2}}{\vec{r}_{21}^{\,2}} \left\{ \frac{1}{2}\left[\frac{\vec{r}_{13}^{\,2}}{\vec{r}_{10}^{\,2}\vec{r}_{30}^{\,2}} - \frac{\vec{r}_{32}^{2}}{\vec{r}_{30}^{2}\vec{r}_{20}^{\,2}} \right] (B_{100}B_{320} - B_{200}B_{310}) \right. \\
& \left. - \frac{\vec{r}_{12}^{2}}{\vec{r}_{10}^{2}\vec{r}_{20}^{\,2}} \left(9B_{123} - \frac{1}{2}\left[2\left(B_{100}B_{320} + B_{200}B_{130}\right) - B_{300}B_{120} \right] \right) \right\} + (1 \leftrightarrow 3) + (2 \leftrightarrow 3) \Bigg]
\end{aligned}
$$

Saturation Effects in Projectile

Doug Wertepny

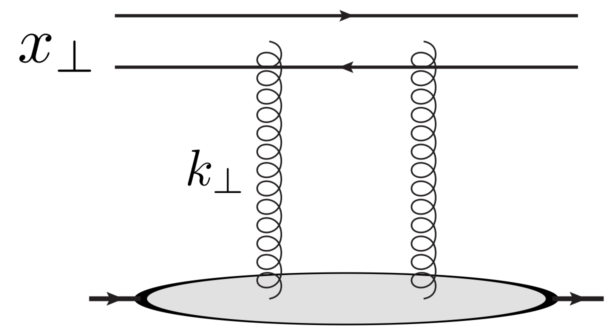
$$
\begin{aligned}
&\sum_i A_i + \sum_i B_i + \sum_i C_i \\
&= -\frac{g^3}{4\,\pi^4} \int d^2x_1\, d^2x_2\, \delta[(\vec{z}_\perp - \vec{x}_{1\perp}) \times (\vec{z}_\perp - \vec{x}_{2\perp})] \left[\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{x}_{1\perp})}{|\vec{x}_{2\perp} - \vec{x}_{1\perp}|^2} \, \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{1\perp} - \vec{b}_{1\perp})}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_{1\perp}}{|\vec{z}_\perp - \vec{x}_{1\perp}|^2} \cdot \frac{\vec{x}_{2\perp} - \vec{b}_{2\perp}}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \right. \\
&\quad \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{x}_{2\perp} - \vec{b}_{2\perp})}{|\vec{x}_{2\perp} - \vec{b}_{2\perp}|^2} \, \frac{\vec{x}_{1\perp} - \vec{b}_{1\perp}}{|\vec{x}_{1\perp} - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{z}_\perp - \vec{x}_{2\perp}}{|\vec{z}_\perp - \vec{x}_{2\perp}|^2} \right] f^{abc} \left[U^{bd}_{\vec{x}_{1\perp}} - U^{bd}_{\vec{b}_{1\perp}} \right] \left[U^{ce}_{\vec{x}_{2\perp}} - U^{ce}_{\vec{b}_{2\perp}} \right] \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \\
&\quad + \frac{i\, g^3}{4\,\pi^3} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \int d^2x \left[U^{bd}_{\vec{b}_{1\perp}} \left(U^{ce}_{\vec{x}_\perp} - U^{ce}_{\vec{b}_{2\perp}} \right) \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right. \right. \\
&\quad \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \\
&\quad - \left(U^{bd}_{\vec{x}_\perp} - U^{bd}_{\vec{b}_{1\perp}} \right) U^{ce}_{\vec{b}_{2\perp}} \left(\frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \, \frac{\vec{z}_\perp - \vec{x}_\perp}{|\vec{z}_\perp - \vec{x}_\perp|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \right. \\
&\quad \left. \left. - \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \right) \right] \\
&\quad - \frac{i\, g^3}{4\,\pi^2} f^{abc} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \left[(U^{bd}_{\vec{z}_\perp} - U^{bd}_{\vec{b}_{1\perp}})\, U^{ce}_{\vec{b}_{2\perp}} \, \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{1\perp})}{|\vec{z}_\perp - \vec{b}_{1\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{2\perp}|\, \Lambda} - U^{bd}_{\vec{b}_{1\perp}} \, (U^{ce}_{\vec{z}_\perp} - U^{ce}_{\vec{b}_{2\perp}}) \, \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{b}_{2\perp})}{|\vec{z}_\perp - \vec{b}_{2\perp}|^2} \ln \frac{1}{|\vec{z}_\perp - \vec{b}_{1\perp}|\, \Lambda} \right] \\
&\quad - \frac{i\, g^3}{4\,\pi^3} \int d^2x \, \left[U^{ab}_{x_\perp} - U^{ab}_{z_\perp} \right] f^{bde} \left(V_{\vec{b}_{1\perp}} t^d \right)_1 \left(V_{\vec{b}_{2\perp}} t^e \right)_2 \frac{\vec{\epsilon}_\perp^{\,\lambda *} \cdot (\vec{z}_\perp - \vec{x}_\perp)}{|\vec{z}_\perp - \vec{x}_\perp|^2} \, \frac{\vec{x}_\perp - \vec{b}_{1\perp}}{|\vec{x}_\perp - \vec{b}_{1\perp}|^2} \cdot \frac{\vec{x}_\perp - \vec{b}_{2\perp}}{|\vec{x}_\perp - \vec{b}_{2\perp}|^2} \, \mathrm{Sign}(b_2^- - b_1^-)
\end{aligned}
$$



$$\vec{x}_\perp \times \vec{y}_\perp = x_1\, y_2 - x_2\, y_1$$

$$\Lambda = \text{IR cutoff}$$

Dipole Amplitude as a Probe of Spatial Gluon Distribution



- Dipole amplitude is related to gluon distribution.
- It is related to the Wigner distribution for low-x gluons:

$$N(\vec{x}_\perp, \vec{b}_\perp, Y = \ln 1/x_{Bj}) \Leftarrow (\text{Fourier transform}) \Rightarrow W(\vec{k}_\perp, \vec{b}_\perp, x_{Bj})$$

- Just like for the Wigner distribution, one can extract the gluon transverse momentum distribution (TMD) out of it:

$$\int d^2 b_\perp \, N(\vec{x}_\perp, \vec{b}_\perp, Y = \ln 1/x_{Bj}) \Leftarrow (\text{Fourier transform}) \Rightarrow f(\vec{k}_\perp, x_{Bj})$$

- Dipole amplitude gives us information about the spatial distribution of small-x gluons.

Geometric Scaling in DIS

Geometric scaling has been observed in DIS data by Stasto, Golec-Biernat, Kwiecinski in ’00.

Here they plot the total DIS cross section, which is a function of 2 variables - Q^2 and x, as a function of just <u>one</u> variable:

$$\tau = \frac{Q^2}{Q_s^2}$$