Holography for $\mathcal{N} = 2^*$ on S^4

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- Make progress by taking the planar limit for specific 4d $\mathcal{N} = 2$ theories. [Russo], [Russo-Zarembo], [Buchel-Russo-Zarembo]
- Evaluation of the partition function of planar $SU(N)$, $\mathcal{N} = 2^*$ SYM on S^4. An infinite number of quantum phase transitions as a function of $\lambda = g_{YM}^2 N$. [Russo-Zarembo]

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- $\mathcal{N}=2^*$ SYM is a theory of a $\mathcal{N}=2$ vector multiplet and a hyper multiplet in the adjoint. It is a massive deformation of $\mathcal{N}=4$ SYM. There is a unique supersymmetric Lagrangian on S^4. [Pestun], [Festuccia-Seiberg]

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- The result for $N, \lambda \gg 1$ is [Russo-Zarembo]

$$F_{S^4}^{\mathcal{N}=2^*} = -\log \mathcal{Z}_{S^4}^{\mathcal{N}=2^*} = -\frac{N^2}{2}(1+(mR)^2)\log \frac{\lambda(1+(mR)^2)e^{2\gamma+\frac{1}{2}}}{16\pi^2}\,,$$

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- Previous results on holography for $\mathcal{N} = 2^*$ on $\mathbb{R}^4$. [Pilch-Warner], [Buchel-Peet-Polchinski], [Evans-Johnson-Petrini] On S^4 the holographic construction is more involved.

Plan

- The $\mathcal{N} = 2^*$ SYM theory on S^4
- The gravity dual
- Holographic calculation of $F_{S^4}^{\mathcal{N}=2^*}$
- Comments on $\mathcal{N} = 1^*$ SYM theory on S^4
- Outlook

The $\mathcal{N} = 2^*$ SYM theory on S^4

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$$A_\mu \,, \qquad X_{1,2,3,4,5,6} \,, \qquad \lambda_{1,2,3,4} \,.$$

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Organize this into an $\mathcal{N} = 2$ vector multiplet

$$A_\mu \,, \qquad \psi_1 \equiv \lambda_4 \,, \qquad \psi_2 \equiv \lambda_3 \,, \qquad Z_3 \equiv \frac{1}{\sqrt{2}}\big(X_3 + iX_6\big) \,,$$

and one hypermultiplet

$$\chi_j = \lambda_j \,, \qquad Z_j = \frac{1}{\sqrt{2}}\big(X_j + iX_{j+3}\big), \qquad j = 1, 2 \,.$$

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The $\mathcal{N} = 2^*$ theory is obtained by giving a mass term for the hyper multiplet. In flat space this breaks the global symmetry to $SU(2)_V \times U(1)_H$.

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\begin{aligned}
\mathcal{L}^{S^4}_{\mathcal{N}=2^*} \;=\; \mathcal{L}^{S^4}_{\mathcal{N}=4} &+ \frac{2}{R^2}\,\mathrm{tr}\big(Z_1\widetilde{Z}_1 + Z_2\widetilde{Z}_2 + Z_3\widetilde{Z}_3\big) \\
&+ m^2\,\mathrm{tr}\left(Z_1\widetilde{Z}_1 + Z_2\widetilde{Z}_2\right) - \frac{m}{2}\,\mathrm{tr}\left(\chi_1\chi_1 + \chi_2\chi_2 + \tilde{\chi}_1\tilde{\chi}_1 + \tilde{\chi}_2\tilde{\chi}_2\right) \\
&- \sqrt{2}\,m\,\mathrm{tr}\left[(\widetilde{Z}_1 Z_2 - \widetilde{Z}_2 Z_1)Z_3 + (Z_1\widetilde{Z}_2 - Z_2\widetilde{Z}_1)\widetilde{Z}_3\right] \\
&+ \frac{im}{2R}\,\mathrm{tr}\left(Z_1^2 + Z_2^2 + \widetilde{Z}_1^2 + \widetilde{Z}_2^2\right) \;.
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\mathcal{O}_\psi &= \text{tr}\big(\chi_1\chi_1 + \chi_2\chi_2 + \tilde{\chi}_1\tilde{\chi}_1 + \tilde{\chi}_2\tilde{\chi}_2\big)\,, & \Delta_{\mathcal{O}_\psi} &= 3\,, \\
\mathcal{O}_\chi &= \text{tr}\big(Z_1^2 + Z_2^2 + \widetilde{Z}_1^2 + \widetilde{Z}_2^2\big)\,, & \Delta_{\mathcal{O}_\chi} &= 2\,.
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	$SU(2)_V$	$SU(2)_H$	$U(1)_R$
A_μ	0	0	0
Z_3	0	0	$+2$
$\psi_{1,2}$	$1/2$	0	$+1$
$\tilde{\psi}_{1,2}$	$1/2$	0	-1
$Z_{1,2}$	$1/2$	$1/2$	0
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In flat space the $\mathcal{N} = 2^*$ theory has $SU(2)_V \times U(1)_H$ global symmetry. The extra coupling $\mathcal{O}_\chi$ on S^4 breaks this to $U(1)_V \times U(1)_H$.

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There is a subtlety. The global symmetry is enhanced to to $U(1)_V \times U(1)_H \times U(1)_Y$ in the large N limit. The $U(1)_Y$ is the diagonal subgroup in $U(1)_R \times U(1)_{SL(2,\mathbb{R})}$.

This is proven in flat space. [Intriligator], [Intriligator-Skiba], [Pilch-Warner], [Buchel-Peet-Polchinski]

Expect the same symmetry to emerge on S^4.

Results from localization

The $SU(N)$ gauge symmetry is generally broken to $U(1)^{N-1}$ by a vev for Z_3.

$$Z_3 = \text{diag}(a_1, \ldots, a_N) \ .$$

After supersymmetric localization the path integral for the theory on S^4 reduces to a finite dimensional integral over the Coulomb moduli a_i. [Pestun]

$$\mathcal{Z} = \int \prod_{i=1}^{N} da_i \, \delta\left(\sum_{i=1}^{N} a_i\right) \prod_{i<j} (a_i - a_j)^2 \mathcal{Z}_{\text{1-loop}} |\mathcal{Z}_{\text{inst}}|^2 e^{-S_{\text{cl}}} \ ,$$

where

$$S_{\text{cl}} = \frac{8\pi^2 N}{\lambda} \sum_{i=1}^{N} a_i^2 \ , \qquad \mathcal{Z}_{\text{1-loop}} = \prod_{i=1}^{N} \frac{H^2(a_i - a_j)}{H(a_i - a_j + mR) H(a_i - a_j - mR)} \ ,$$

$$H(x) = \prod_{n=1}^{\infty} \left(1 + \frac{x^2}{n^2}\right)^n e^{-x^2/n} \ .$$

The function $\mathcal{Z}_{\text{inst}}$ is Nekrasov's partition function with parameters $\varepsilon_1 = \varepsilon_2 = 1/R$.

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This answer depends on the regularization scheme. The scheme independent quantity is

$$\boxed{\frac{d^3 F_{S^4}}{d(mR)^3} = -2N^2 \frac{mR((mR)^2 + 3)}{((mR)^2 + 1)^2}}$$

This is what one can aim to compute holographically.

Comments

For a CFT on S^4 of radius R with cutoff $\epsilon \to 0$

$$F_{S^4} = \alpha_4 \left(\frac{R}{\epsilon}\right)^4 + \alpha_2 \left(\frac{R}{\epsilon}\right)^2 + \alpha_0 - a_{\mathsf{anom}} \log\left(\frac{R}{\epsilon}\right) + \mathcal{O}(\epsilon/R) \ .$$

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For massive theories there is an extra scale, m, so $\alpha_2 = \alpha_2(m\epsilon)$ and $\alpha_0 = \alpha_0(m\epsilon)$. Expand this for small $m\epsilon$

$$\begin{aligned} \alpha_2 &= \tilde{\alpha}_2 + m^2\epsilon^2\beta_2 + \mathcal{O}(m^4\epsilon^4) \ , \\ \alpha_0 &= \tilde{\alpha}_0 + \mathcal{O}(m^2\epsilon^2) \ . \end{aligned}$$

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The nonuniversal contribution to the free energy is

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For nonsupersymmetric theories the 5th derivative of F_{S^4} w.r.t. mR is universal.

The gravity dual

Supergravity setup

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The result is that only 3 real, 2 Abelian gauge fields and the metric survive in the bosonic sector. The 3 scalars, $\{\phi, \psi, \chi\}$, are dual precisely to the 3 operators

$$\mathcal{O}_\phi\,, \qquad \mathcal{O}_\psi\,, \qquad \mathcal{O}_\chi\,.$$

It is convenient to use

$$\eta = e^{\phi/\sqrt{6}}\,, \qquad z = \frac{1}{\sqrt{2}}(\chi + i\psi)\,, \qquad \tilde{z} = \frac{1}{\sqrt{2}}(\chi - i\psi)\,.$$

In Euclidean signature the fields z and $\tilde{z}$ are independent.

For $\chi = 0$ there is $SU(2)_V \times U(1)_H \times U(1)_Y$ symmetry and we reproduce the Pilch-Warner truncation.

Supergravity setup

The Euclidean Lagrangian is

$$\mathcal{L} = \frac{1}{2\kappa^2}\left[-\mathcal{R} + \frac{12\partial_\mu\eta\partial^\mu\eta}{\eta^2} + \frac{4\,\partial_\mu z\partial^\mu\tilde{z}}{(1-z\tilde{z})^2} + \mathcal{V}\right],$$

$$\mathcal{V} \equiv -\frac{4}{L^2}\left(\frac{1}{\eta^4} + 2\eta^2\frac{1+z\tilde{z}}{1-z\tilde{z}} + \frac{\eta^8}{4}\frac{(z-\tilde{z})^2}{(1-z\tilde{z})^2}\right).$$

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To preserve the isometries of S^4 take the “domain-wall” Ansatz

$$ds^2 = L^2 e^{2A(r)} ds^2_{S^4} + dr^2\,,$$
$$\eta = \eta(r)\,, \qquad z = z(r)\,, \qquad \tilde{z} = \tilde{z}(r)\,.$$

The masses of the scalars are

$$m_\phi^2 L^2 = m_\chi^2 L^2 = -4\,, \qquad m_\psi^2 L^2 = -3\,.$$

The BPS equations

Plug the Ansatz in the supersymmetry variations of the 5d $\mathcal{N}=8$ theory and use the "conformal Killing spinors" on S^4

$$\hat{\nabla}_\mu \zeta = \frac{1}{2}\gamma_5 \gamma_\mu \zeta \,,$$

to derive the BPS equations

$$z' = \frac{3\eta'(z\tilde{z}-1)\left[2(z+\tilde{z})+\eta^6(z-\tilde{z})\right]}{2\eta\left[\eta^6\left(\tilde{z}^2-1\right)+\tilde{z}^2+1\right]}\,,$$

$$\tilde{z}' = \frac{3\eta'(z\tilde{z}-1)\left[2(z+\tilde{z})-\eta^6(z-\tilde{z})\right]}{2\eta\left[\eta^6\left(z^2-1\right)+z^2+1\right]}\,,$$

$$(\eta')^2 = \frac{\left[\eta^6\left(z^2-1\right)+z^2+1\right]\left[\eta^6\left(\tilde{z}^2-1\right)+\tilde{z}^2+1\right]}{9L^2\eta^2(z\tilde{z}-1)^2}\,,$$

$$e^{2A} = \frac{(z\tilde{z}-1)^2\left[\eta^6\left(z^2-1\right)+z^2+1\right]\left[\eta^6\left(\tilde{z}^2-1\right)+\tilde{z}^2+1\right]}{\eta^8\left(z^2-\tilde{z}^2\right)^2}\,.$$

UV expansion

The (constant curvature) metric on $\mathbb{H}^5$ is

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$$\begin{aligned} e^{2A} &= \frac{e^{2r}}{4} + \frac{1}{6}(\mu^2 - 3) + \dots \ , \\ \eta &= 1 + e^{-2r}\left[\frac{2\mu^2}{3} r + \frac{\mu(\mu + v)}{3}\right] + \dots \ , \\ \frac{1}{2}(z + \tilde{z}) &= e^{-2r}\Big[2\mu\, r + v\Big] + \dots \ , \\ \frac{1}{2}(z - \tilde{z}) &= \mu\, e^{-r} + e^{-3r}\left[\frac{4}{3}\mu\big(\mu^2 - 3\big)\, r + \frac{1}{3}\Big(2v(\mu^2 - 3) + \mu(4\mu^2 - 3)\Big)\right] + \dots \ . \end{aligned}$$

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Here μ and v are integration constants. Think of them as the "source" and "vev" for the operator $\mathcal{O}_\chi$. Compare to field theory to identify $\mu = imR$.

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$$e^{2A} = (r - r_*)^2 + \frac{7{\eta_0}^{12} + 20}{81{\eta_0}^4}(r - r_*)^4 + \dots\ ,$$

$$\eta = \eta_0 - \left(\frac{{\eta_0}^{12} - 1}{27{\eta_0}^3}\right)(r - r_*)^2\left[1 - \left(\frac{85 + 131{\eta_0}^{12}}{810{\eta_0}^4}\right)(r - r_*)^2 + \dots\right],$$

$$\frac{1}{2}(z + \tilde{z}) = \sqrt{\frac{{\eta_0}^6 - 1}{{\eta_0}^6 + 1}}\left[\frac{{\eta_0}^6}{{\eta_0}^6 + 2} - \frac{2{\eta_0}^8(4{\eta_0}^6 + 5)}{15({\eta_0}^6 + 2)^2}(r - r_*)^2 + \dots\right],$$

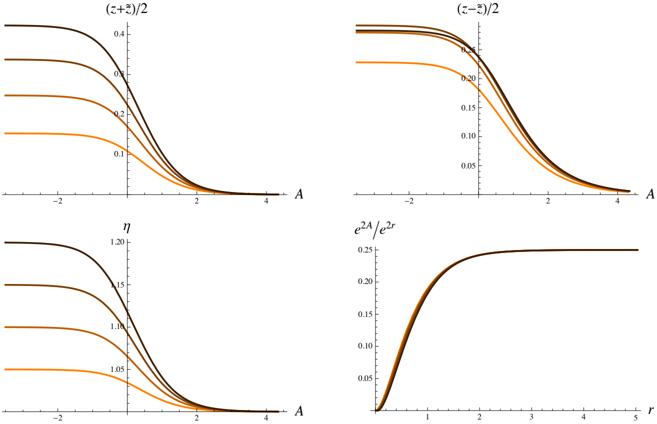
$$\frac{1}{2}(z - \tilde{z}) = \sqrt{\frac{{\eta_0}^6 - 1}{{\eta_0}^6 + 1}}\left[\frac{2}{{\eta_0}^6 + 2} + \frac{{\eta_0}^2(3{\eta_0}^{12} - 10{\eta_0}^6 - 20)}{15({\eta_0}^6 + 2)^2}(r - r_*)^2 + \dots\right].$$

Here, η_0 is the only free parameter since one can set $r_* = 0$ by the shift symmetry of the equations.

Numerical solutions

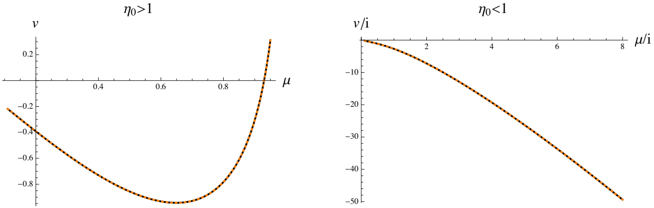
One can find numerical solutions by "shooting" from the IR to the UV. There is a one-parameter family parametrized by η_0, so

$$v = v(\eta_0)\,, \qquad \text{and} \qquad \mu = \mu(\eta_0)\,.$$



Numerical solutions

Qualitatively different behavior for $\eta_0 > 1$ and $\eta_0 < 1$.



From the numerical results one can extract the following dependence

$$\boxed{v(\mu) = -2\mu - \mu\,\log(1-\mu^2)}$$

Holographic calculation of $F_{S^4}^{\mathcal{N}=2^*}$

Calculating F from supergravity

- By the standard holographic dictionary the partition function of the field theory is mapped to the on-shell action of the supergravity at hand. [Maldacena], [GKP], [Witten]

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- There is a subtlety here. If we insist on using a supersymmetric regularization scheme there is a particular finite counterterm that has to be added. Only with it one can successfully compare $\frac{d^3F}{d\mu^3}$ with the field theory result.
- Without knowing this finite counterterm we can only hope to match $\frac{d^5F}{d\mu^5}$ with field theory.

Calculating F from supergravity

The 5d action + the boundary Gibbons-Hawking term is

$$S_{5\text{D}} = \int_M d^5x\,\sqrt{G}\Big\{-\frac{1}{4}\mathcal{R} + \frac{1}{2}(\partial\phi)^2 + \frac{K}{2}\Big((\partial\chi)^2 + (\partial\psi)^2\Big) + \mathcal{V}\Big\} - \frac{1}{2}\int_{\partial M}\sqrt{\gamma}\,\mathcal{K}\,,$$

with $K = \big(1 - \frac{1}{2}(\chi^2 + \psi^2)\big)^{-2}$.

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The (infinite) counterterm action obtained by holographic renormalization is

$$S_{\text{ct}} = \int_{\partial M_\epsilon} d^4x\,\sqrt{\gamma}\,\Big[\frac{3}{2} + \frac{1}{8}\mathcal{R}[\gamma] + \frac{1}{2}\psi^2 + \Big(1 + \frac{1}{\log\epsilon}\Big)(\phi^2+\chi^2) - \log\epsilon\,\Big\{\frac{1}{32}\Big[\mathcal{R}[\gamma]^{ij}\mathcal{R}[\gamma]_{ij} - \frac{1}{3}\mathcal{R}[\gamma]^2\Big] + \frac{1}{4}\psi\Box_\gamma\psi - \frac{1}{24}\mathcal{R}[\gamma]\,\psi^2 - \frac{1}{6}\psi^4\Big\}\Big]\,,$$

Interestingly S_{ct} is almost the same for many 5d models studied in the literature. [FGPW], [GPPZ], [Pilch-Warner],...

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Take $\chi = 0$, then 5d supergravity action action (on $\mathbb{R}^4$) can be written as

$$S = \int dr d^4x \Big\{ e^{4A} \Big[-3\left(A' - \frac{2}{3}\mathcal{W}\right)^2 + \frac{\sqrt{3}e^{-\frac{\phi}{\sqrt{6}}}}{\sqrt{2}}(\phi' + \partial_\phi \mathcal{W})^2 + \frac{2}{(2-\psi^2)^2}\left(\psi' + (1 - \frac{1}{2}\psi^2)^2 \partial_\psi \mathcal{W}\right)^2 \Big] - \frac{d}{dr}\left(e^{4A}\mathcal{W}\right) \Big\}$$

with

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The full counterterm action from the Bogomolnyi procedure is

$$S_{\mathcal{W}} = \int d^4x e^{4A} \mathcal{W} = \int d^4x \sqrt{\gamma} \left(\frac{3}{2} + \phi^2 + \frac{1}{2}\psi^2 + \sqrt{\frac{2}{3}} \phi \psi^2 + \frac{1}{4}\psi^4 + \ldots \right) .$$

The finite counterterm obtained by the Bogomolnyi trick is

$$\boxed{S_{\text{finite}} = \int d^4x \sqrt{\gamma} \, \frac{1}{4} \psi^4}$$

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Set $\mu = imR$ and compare this to field theory

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Comments on $\mathcal{N} = 1^*$

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- Perform a similar analysis for mass deformations of $\mathcal{N} = 4$ SYM with only $\mathcal{N} = 1$ supersymmetry.
- Supersymmetric localization does not seem to work for $\mathcal{N} = 1$ theories on S^4.
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- Supergravity may offer some insight about the dual field theory!

The Lagrangian for $\mathcal{N} = 1^*$ on S^4 of radius R is

$$\mathcal{L}^{S^4}_{\mathcal{N}=1^*} = \mathcal{L}^{S^4}_{\mathcal{N}=4} + \frac{(2 + {m_i}^2 R^2)}{R^2} \text{tr}\big(Z_i \widetilde{Z}_i\big) - \frac{m_i}{2} \text{tr}\left(\chi_i \chi_i + \tilde{\chi}_i \tilde{\chi}_i\right) - \frac{\sqrt{2} m_i}{2} \varepsilon^{ijk} \text{tr}\left(\widetilde{Z}_i Z_j Z_k + Z_i \widetilde{Z}_j \widetilde{Z}_k\right) + \frac{i m_i}{2R} \text{tr}\left(Z_i^2 + \widetilde{Z}_i^2\right) .$$

In supergravity this will mean at least 8 real scalars. Possible in principle but too cumbersome...

$\mathcal{N} = 1^*$

There are special cases which allow for an explicit analysis.

- Take $m_1 = m_2 = m_3$. In flat space this is the GPPZ flow. It has $SO(3)$ global symmetry. On S^4 we need 4 supergravity scalars.
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For both examples we have explicit supergravity truncations and BPS solutions on S^4.

The hard part is to extract the third derivative of the free energy from the numerical solutions.

No results from localization to guide us.

A glimpse of the field theory answer from conformal perturbation theory in the planar limit and in the small m expansion.

Summary

- We found a 5d supegravity dual of $\mathcal{N}=2^*$ SYM on S^4.
- After careful holographic renormalization we computed the universal part of the free energy of this theory.
- The result is in exact agreement with the supersymmetric localization calculation in field theory.
- This is a precision test of holography in a non-conformal Euclidean setting.
- The results extend immediately to $\mathcal{N}=2^*$ mass deformations of quiver gauge theories obtained by $\mathbb{Z}_k$ orbifolds of $\mathcal{N}=4$ SYM. [Azeyanagi-Hanada-Honda-Matsuo-Shiba]
- Extension of these results to $\mathcal{N}=1^*$ SYM.

Outlook

- Uplift of the $\mathcal{N} = 2^*$ solution to IIB supergravity. Will allow for a holographic calculation of the Wilson line vev. In addition one can study probe D3-branes.
- Holography for $\mathcal{N} = 2^*$ on other 4-manifolds.
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- Extensions to other dimensions.
- Can we see some of the large N phase transitions argued to exist by Russo-Zarembo in IIB string theory?
- Revisit supersymmetric localization for $\mathcal{N} = 1$ theories on S^4. Can one find the exact partition function (modulo ambiguities)? [Gerchkovitz-Gomis-Komargodski]
- Massive deformations of class $\mathcal{S}$ SCFTs from holography?
- Broader lessons for holography from localization?

THANK YOU!